Power Iteration Summary

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This document discusses the Power Iteration algorithm, as it applies to finding k_∞, the reactor eigenvalue for an infinite homogeneous problem.

Consider a group structure with G energy groups, and fully known group constants. With Σ_{tg} as the group g macroscopic total cross section, we define the total cross section matrix $\mathbf{T}$:

$$\mathbf{T} = \begin{bmatrix} \Sigma_{t1} & & \\ & \ddots & \\ & & \Sigma_{tG} \end{bmatrix}. \tag{1}$$

Next, we define the scattering matrix $\mathbf{S}$, with $\Sigma_{sgg'}$ as the group g' to g scattering cross section:

$$\mathbf{S} = \begin{bmatrix} \Sigma_{s11} & \dots & \Sigma_{s1G} \\ \vdots & \ddots & \vdots \\ \Sigma_{sG1} & \dots & \Sigma_{sGG} \end{bmatrix}. \tag{2}$$

Let $\vec{\boldsymbol{\chi}}^{j_f}$ and $\overrightarrow{\boldsymbol{\nu\Sigma}}_f^{j_f}$ denote the group fission yield spectrum and group production cross section vectors for fissionable isotope j_f, respectively. Let the operator "$\otimes$" denote the outer (Kronecker) product of two vectors, as follows:

$$\vec{\mathbf{a}} \otimes \vec{\mathbf{b}} = \vec{\mathbf{a}}\,\vec{\mathbf{b}}^{\mathrm{T}}. \tag{3}$$

The fission matrix $\mathbf{F}$ is then defined as:

$$\mathbf{F} = \sum_{\text{all } j^f} \vec{\boldsymbol{\chi}}^{j_f} \otimes \overrightarrow{\boldsymbol{\nu\Sigma}}_f^{j_f}, \tag{4}$$

which with one fissionable isotope j_f, or with a isotope-independent group fission yield spectrum $\vec{\boldsymbol{\chi}}$ reduces to:

$$\mathbf{F} = \vec{\boldsymbol{\chi}} \otimes \overrightarrow{\boldsymbol{\nu\Sigma}}_f = \begin{bmatrix} \chi_1\nu\Sigma_{f1} & \dots & \chi_1\nu\Sigma_{fG} \\ \vdots & \ddots & \vdots \\ \chi_G\nu\Sigma_{f1} & \dots & \chi_G\nu\Sigma_{fG} \end{bmatrix}. \tag{5}$$

The infinite homogeneous reactor's eigenvalue problem is the following:

$$\mathbf{T}\vec{\boldsymbol{\phi}} = \mathbf{S}\vec{\boldsymbol{\phi}} + \frac{1}{k_\infty}\mathbf{F}\vec{\boldsymbol{\phi}}. \tag{6}$$

Equation (6) is a general eigenvalue problem. The fission matrix $\mathbf{F}$ has rank J_f (the number of fissionable isotopes in Eq. (4)), and so may only be nonsingular if $J_f \geq G$; this is usually not the case, so we can assume $\mathbf{F}$ to be singular. This is not relevant for power iteration however, but may make a difference for some eigenvalue search methods.

The power iteration algorithm can be summarized as Algorithm 1:

Algorithm 1 Power Iteration Algorithm

1: Set initial guesses k_∞^0 and $\vec{\boldsymbol{\phi}}^0$ ▷ Can be arbitrary
2: Set iteration limit i_{max} and fractional eigenvalue tolerance ϵ_k
3: Set fractional group flux tolerance ϵ_ϕ ▷ Often skipped
4: $i \leftarrow 0$ ▷ Iteration counter
5: $\vec{\mathbf{S}}^i \leftarrow \mathbf{F}\vec{\boldsymbol{\phi}}^i$
6: **repeat**
7:     Solve $\left(\mathbf{T} - \mathbf{S}\right)\vec{\boldsymbol{\phi}}^{i+1} = \frac{1}{k_\infty^i}\vec{\mathbf{S}}^i$ for $\vec{\boldsymbol{\phi}}^{i+1}$ ▷ Using a regular linear solver
8:     $\vec{\mathbf{S}}^{i+1} \leftarrow \mathbf{F}\vec{\boldsymbol{\phi}}^{i+1}$
9:     $k_\infty^{i+1} \leftarrow \dfrac{\sum_{\text{all } g} S_g^{i+1}}{\frac{1}{k_\infty^i}\sum_{\text{all } g} S_g^i}$
10:     $i \leftarrow i + 1$
11: **until** $\left(\left|\dfrac{k_\infty^i - k_\infty^{i-1}}{k_\infty^i}\right| < \epsilon_k \text{ and } \max_{\text{all } g}\left|\dfrac{\phi_g^i - \phi_g^{i-1}}{\phi_g^i}\right| < \epsilon_\phi\right)$ or $i > i_{max}$ ▷ ϵ_ϕ often skipped
12: **if** $i \leq i_{max}$ **then**
13:     Power iterations converged, algorithm successful
14:     Normalize $\vec{\boldsymbol{\phi}}^i$ as required
15: **else**
16:     Iteration limit exceeded, algorithm failed
17: **end if**

Note, that the linear problem on line 7 in Algorithm 1 could easily be extended to a heterogeneous problem with a specified fission source. In such cases, the fission source ratio on line 9 is taken as the ratio of integrals over the entire reactor core. Such problems are referred to as the "fixed source problems," and are the basis of most eigenvalue search codes. If an iterative procedure is used to solve the fixed source problem (which is frequently the case with linear problems), it is referred to as the "inner iterations," while the power iterations on lines 6–11 are referred to as the "outer iterations."