Jacobian-Free Newton- Krylov (JFNK) Methods for Nonlinear Neutronics/Thermal-Hydrualic Equations

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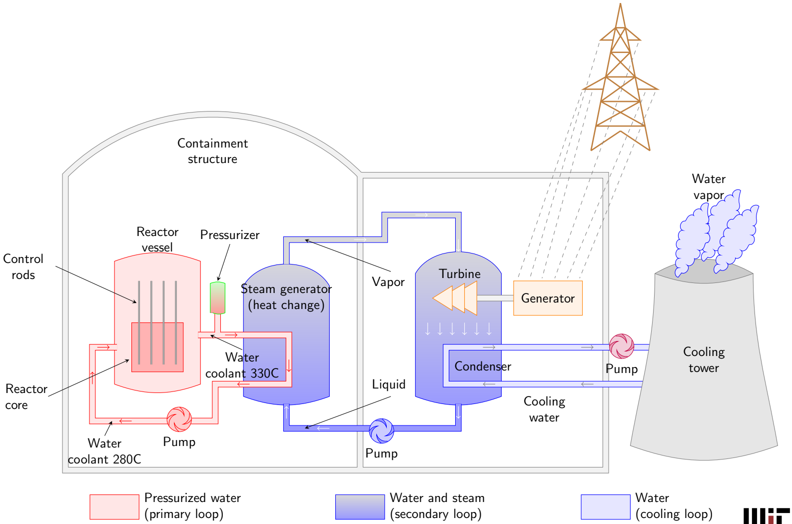
Outline

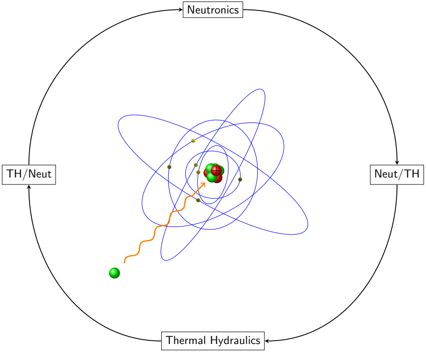
Motivation

- Eventually will be part of thesis work
- JFNK method not currently used in nuclear reactor analysis
- Incorporates a lot of ideas from 2.29 class
- Coupled physics is fun!

Nuclear Reactor Plant

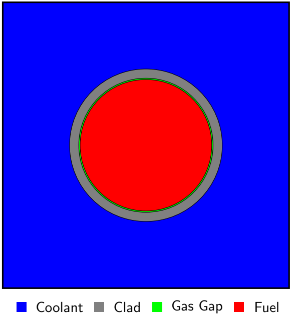


Nuclear Feedback

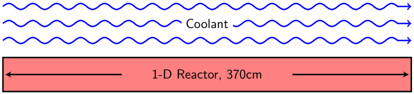


- Fuel Temperature Feedback - $T_f \uparrow$, U-238 Capture $\uparrow$, Fission Rate $\downarrow$, Power $\downarrow$
- Coolant Density Feedback - $\rho \downarrow$, $E_n \uparrow$, Fission Rate $\downarrow$, Power $\downarrow$

1-D Slab Reactor Geometry



Top-view
Fuel Rod Unit-Cell



Side-view (vertical)
1-D Model of Reactor

Neutronics

- Basic Neutron Conservation:

$$Change + Leakage + Interactions = Scattering + Fission$$

- Neutron Transport Equation:

$$\underbrace{\frac{1}{v}\frac{\partial \varphi}{\partial t}}_{\text{time}-\text{dependent}} + \underbrace{\boldsymbol{\Omega} \cdot \nabla \varphi\left(\mathbf{r}, E, \boldsymbol{\Omega}, t\right)}_{\text{neutron leakage}} + \underbrace{\Sigma_t\left(\mathbf{r}, E, t\right)\varphi\left(\mathbf{r}, E, \boldsymbol{\Omega}, t\right)}_{\text{interation of neutrons with medium}} = \underbrace{Q\left(\mathbf{r}, E, \boldsymbol{\Omega}, t\right)}_{\text{neutron source}}$$

- Neutron Diffusion Equation (1-D Energy Integrated)

$$\frac{1}{v}\frac{\partial \phi}{\partial t} - D\left(x,t\right)\frac{\partial^2 \phi}{\partial^2 x} + \Sigma_a\left(x,t\right)\phi\left(x,t\right) = \frac{1-\beta}{k_{eff}}\nu\Sigma_f\left(x,t\right)\phi\left(x,t\right) + \lambda_d c\left(x,t\right)$$

$$\frac{\partial c}{\partial t} = \frac{\beta}{k_{eff}}\nu\Sigma_f\left(x,t\right)\phi\left(x,t\right) - \lambda_d c\left(x,t\right)$$

Discretization of Neutronics Equations

Assumptions:

1. One-dimensional finite volume spatial discretization
2. Central difference scheme for diffusion term
3. No-incoming current of neutrons at boundaries
4. Implicit Euler time discretization

- Discretized neutronics equation for interior cell

$$\frac{1}{v}\frac{d\bar{\phi}_i}{dt}+-\frac{2}{\Delta x^2}\frac{D_i D_{i-1}}{D_i+D_{i-1}}\bar{\phi}_{i-1}+\left(\frac{2}{\Delta x^2}\frac{D_{i+1}D_i}{D_{i+1}+D_i}+\frac{2}{\Delta x^2}\frac{D_i D_{i-1}}{D_i+D_{i-1}}+\Sigma_{a,i}\right)\bar{\phi}_i-\frac{2}{\Delta x^2}\frac{D_{i+1}D_i}{D_{i+1}+D_i}\bar{\phi}_{i+1}=\frac{1-\beta}{k_{eff}}\nu\Sigma_{f,i}\bar{\phi}_i+\lambda_d\bar{c}_i$$

- Matrix-form of neutronics equations

$$\bar{\mathbf{\Phi}}^{n+1}-\bar{\mathbf{\Phi}}^{n}+v\Delta t\left(\mathbb{M}\bar{\mathbf{\Phi}}^{n+1}-(1-\beta)\,\lambda\mathbb{F}\bar{\mathbf{\Phi}}^{n+1}-\lambda_d\bar{\mathbf{c}}^{n+1}\right)=0$$

- Matrix-form of precursors

$$\bar{\mathbf{c}}^{n+1}-\bar{\mathbf{c}}^{n}+\Delta t\left(\lambda_d\bar{\mathbf{c}}^{n+1}-\beta\lambda\mathbb{F}\bar{\mathbf{\Phi}}^{n+1}\right)=0$$

Thermal Hydraulics

- Energy Equation - single phase fluid and inviscid fluid

$$\frac{\partial\left(\rho h\right)}{\partial t}+\nabla\cdot\left(\rho h\mathbf{u}\right)=-\nabla\cdot\mathbf{q}''+q'''$$

- Assuming fissions are a volumetic heat source in 1-D

$$\rho A\frac{\partial h}{\partial t}+\dot{m}\frac{\partial h}{\partial x}=q'$$

- For an incompressible fluid, $dh=c_p dT$

$$\rho A c_p\frac{\partial T}{\partial t}+\dot{m}c_p\frac{\partial T}{\partial x}=q'$$

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Discretization of Energy Equation

1. One-dimensional finite volume spatial discretization
2. Upwind difference scheme for diffusion term
3. Specify inlet conditions and mass flow rate
4. Implicit Euler time discretization

- spatial discretization

$$\frac{\rho A \Delta x}{w}\frac{d\bar{T}_i}{dt} + \bar{T}_i - \bar{T}_{i-1} = \frac{1}{2wc_p}\left(Q_{i-1} + Q_i\right)$$

- Matrix-form with time discretization

$$\bar{\mathbf{T}}^{n+1} - \bar{\mathbf{T}}^{n} + \frac{w\Delta t}{\mathcal{P}^{n+1} A\Delta x}\left(\mathbb{S}\bar{\mathbf{T}}^{n+1} - \mathbb{R}\mathbf{Q}^{n+1}\right) = 0$$

Note: $\mathcal{P}$ is a vector of cell-averaged coolant densities

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Physics Coupling

Neutronics - Thermal Hydraulics

- Neutrons cause fissions
- Large portion of fission energy deposited in coolant
- This is represented by

$$\mathbf{Q} = \tilde{c}\mathbb{E}\bar{\mathbf{\Phi}}\Delta x$$

where $\mathbb{E} = \mathrm{diag}\left\{\kappa\Sigma_f\right\}$ characterizes energy per fission and $\tilde{c}$ is flux-power normalization constant

Thermal-Hydraulics to Neutroncs

- Diffusion theory parameters depend on coolant density
- This dependence is determined with a transport theory code
- D, Σ_a, $\nu\Sigma_f$, $\kappa\Sigma_f$ are all affect by coolant density variations
- Data is fitted with a linear regression of the form:

$$\mathbf{\Sigma} = \mathbf{\Sigma}^{ref} + \frac{\partial \Sigma}{\partial \rho}\left(\mathcal{P} - \rho^{ref}\right)$$

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The Steady-State Eigenvalue Problem

- The steady state equations must be solved first
- Reducing the neutronics equation to steady-state:

$$\mathbb{M}\bar{\mathbf{\Phi}} = \lambda \mathbb{F}\bar{\mathbf{\Phi}}, \qquad \lambda = \frac{1}{k_{eff}}$$

- Eigenvalue, λ, and eigenvector, $\mathbf{\Phi}$, must be determined
- Flux-power normalization constant determined from reactor power

$$Q_R = \tilde{c} \int_0^L dx \kappa \Sigma_f (x) \phi (x) = \tilde{c} \sum_i \kappa \Sigma_{f,i} \bar{\phi}_i \Delta x = \tilde{c} \kappa \mathbf{\Sigma}_f^{\mathrm{T}} \bar{\mathbf{\Phi}} \Delta x$$

- λ and $\tilde{c}$ are input as constants for time-dependent calculations

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Newton's Method

Krylov Subspace Methods

Generalized Minimal RESidual Method

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Inexact Newton's Method

Jacobian-Free Approximation

Steady State

Conclusions