NUCLEAR REACTOR KINETICS

Lecture 2

Delay Neutrons

Point Kinetics Equations and Numerical Solution

Massachusetts
Institute of
Technology

Course Outline

22.213 (22.S904) Calendar					
Lecture #	**Date**	**Topic**	**LECTURER**	**Read**	**Assignment Handed Out**
1	5-Sep	Course Overview and First Day Exam	Smith		
2	10-Sep	Review of Delayed Neutrons and Point Kinetics Equations	Smith		PSET # 1: Point Kinetics
3	12-Sep	Review Steady-State Finite-Difference Diffusion Methods (1D, 2D)	Smith		
4	17-Sep	Generalized PKEs from Spatial Finite-Difference Diffusion	Smith		PSET # 2: 2-D Steady-State Diffusion
5	19-Sep	Basic Transient Finite-Difference with Direct Solutions	Smith		
6	24-Sep	Higher-order Time Integration and Runga-Kutta	Smith		PSET # 3: 2-D Fully-Implicit Diffusion
7	26-Sep	Time Stepping for Automatic Error Control	Smith		
8	1-Oct	PKE with Feedback: Operator Splitting/Exact Integration	Smith		PSET # 4: PKE from 2-D Diffusion
9	3-Oct	Quasi-Static Time-Integration and Synthesis Methods	Smith		
	8-Oct	Columbus Holiday (8th and 9th)			
10	10-Oct	2D F-I Iterative Numerical Methods: PJ, GS, SOR	Smith		PSET # 5: PKE Time Step Control
11	15-Oct	Iterative Numerical Methods: CG, GMRES,???	Smith		
12	17-Oct	Coarse Mesh Rebalance & Nonlinear Diffusion Acceleration	Smith		PSET # 6: PKE with Nonlinear Feedback
13	22-Oct	Nodal Methods: Kinetic Distortion and Frequency Transformation	Smith		
	24-Oct	Midterm Exam			
14	29-Oct	Midterm Detailed Exam Solution/2D LRA SS Comparisons	Smith		PSET # 7: CMR and NDA acceleration
15	31-Oct	Multigrid Acceleration Methods	Smith		
16	5-Nov	JFNK for Non-linear Systems	Smith		2-D LRA Rod Ejection Contest
17	7-Nov	Transient Sn	Smith		
	12-Nov	Veterns Day Holiday			
	14-Nov	Special Project Work Period	ANS Meeting		
18	19-Nov	Transient MOC	Smith		
19	21-Nov	Parallel Solver Technologies (PetSc)	Herman/Roberts		
20	26-Nov	So You Want To Be A Professor? Student Lectures	?????		
21	28-Nov	So You Want To Be A Professor? Student Lectures	?????		
22	3-Dec	So You Want To Be A Professor? Student Lectures	?????		
23	5-Dec	So You Want To Be A Professor? Student Lectures	?????		
24	10-Dec	So You Want To Be A Professor? Student Lectures	?????		
25	12-Dec	Last Day of Class General Wrapup, Cats and Dogs, Critique	Smith		
	17-21 Dec	Finals Week - No Exam for 22.S904 (22.213)			

Lecture Objectives

- Understand delayed neutron precursor data (beta)
- Understand neutron emission spectra (chi)
- Understand delayed neutron physics
- Understand delayed neutron models
- Derive Point Kinetics Equations (PKEs) from transport (or diffusion) equation
- Solve PKEs (e.g. MATLAB's cheating way today)
- Derive and use prompt jump approximation
- Derive and use in-hour equation
- Derive and use inverse point kinetics
- Understand some reactor reactivity measurement techniques

Steady-State Neutron Diffusion Equation

9/10

- In steady-state neutron diffusion (or transport) equation, we do not treat delayed neutrons directly.

$$-\nabla \cdot D(\vec{r},E,t)\nabla\,\phi(\vec{r},E,t)+\Sigma_t(\vec{r},E,t)\,\phi(\vec{r},E,t)=$$
$$+\int_0^\infty \Sigma_s(\vec{r},E'\rightarrow E,t)\,\phi(\vec{r},E',t)\,dE'$$
$$+\chi(E)\int_0^\infty \nu\Sigma_f(\vec{r},E',t)\,\phi(\vec{r},E',t)\,dE'$$

* The fission emission spectrum must be properly weighted with prompt and delayed contributions from each fissioning species

  - Chicken-and-egg problem in reality: need fission rates to get chi, and chi to get fission rates:

$$-\nabla \cdot D(\vec{r},E,t)\nabla\,\phi(\vec{r},E,t)+\Sigma_t(\vec{r},E,t)\,\phi(\vec{r},E,t)=$$
$$+\int_0^\infty \Sigma_s(\vec{r},E'\rightarrow E,t)\,\phi(\vec{r},E',t)\,dE'$$
$$+\sum_j \chi_T^j(E)\int_0^\infty \left[N^j\nu\sigma_f^j(\vec{r},E',t)\right]\phi(\vec{r},E',t)\,dE'$$

Time-Dependent Neutron Diffusion Equation

- In transient diffusion equations, the delayed neutrons must be treated explicitly:

$$\frac{\partial}{\partial t}\left[\frac{1}{v}\phi(\vec{r},E,t)\right] = \nabla\cdot D(\vec{r},E,t)\nabla\,\phi(\vec{r},E,t) - \Sigma_t(\vec{r},E,t)\,\phi(\vec{r},E,t)$$

$$+\int_0^\infty \Sigma_s(\vec{r},E'\rightarrow E,t)\,\phi(\vec{r},E',t)\,dE'$$

prompt

$$+\sum_j \chi_p^j(E)(1-\beta^j)\int_0^\infty \left[N^j\nu\sigma_f^j(\vec{r},E',t)\right]\phi(\vec{r},E',t)\,dE'$$

delayed

$$+\sum_i \chi_d^i(E)\lambda_i C_i(\vec{r},t)$$

- $\nu * \beta$ absolute yield
- 10^{-4} sec is small time
- 6-8 delayed precursor

and

$$\frac{\partial}{\partial t}C_i(\vec{r},t) = \sum_j \beta_i^j\int_0^\infty \left[N^j\nu\sigma_f^j(\vec{r},E',t)\right]\phi(\vec{r},E',t)\,dE' - \lambda_i C_i(\vec{r},t)$$

where

$$\beta^j = \sum_i \beta_i^j$$

6-Group Yields and Decay Constants Depend on Fissioning Species

understand how data is processed

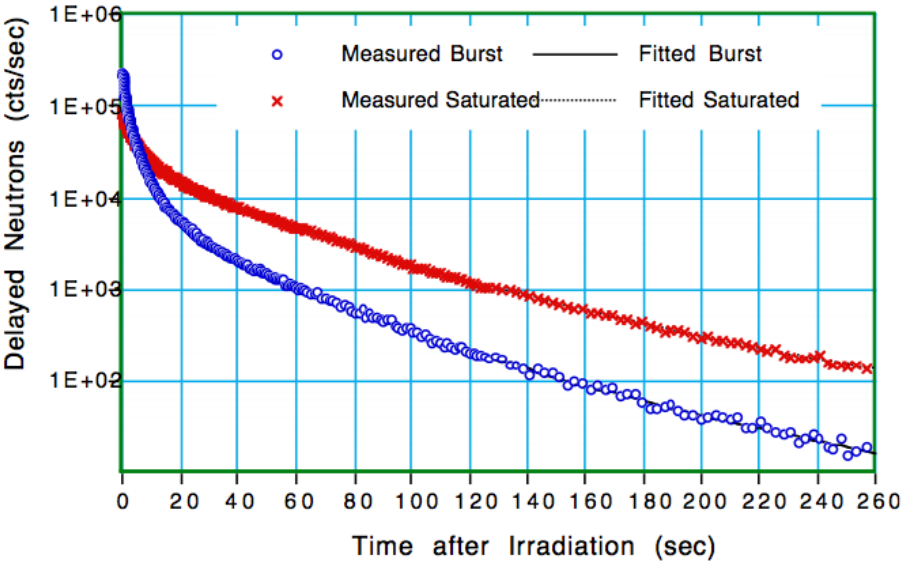


TABLE II. Thermal-fission delayed-neutron data.[a–e]

divide by ν for β

Group index i	Half-life, T_i	Relative abundance, a_i/a	Absolute group yield (%)
	U^{235} (99.9% 235; $n/F=0.0158\pm0.0005$)		
1	55.72 ±1.28	0.033±0.003	0.052±0.005
2	22.72 ±0.71	0.219±0.009	0.346±0.018
3	6.22 ±0.23	0.196±0.022	0.310±0.036
4	2.30 ±0.09	0.395±0.011	0.624±0.026
5	0.610±0.083	0.115±0.009	0.182±0.015
6	0.230±0.025	0.042±0.008	0.066±0.008
	Pu^{239} (99.8% 239; $n/F=0.0061\pm0.0003$)		
1	54.28 ±2.34	0.035±0.009	0.021±0.006
2	23.04 ±1.67	0.298±0.035	0.182±0.023
3	5.60 ±0.40	0.211±0.048	0.129±0.030
4	2.13 ±0.24	0.326±0.033	0.199±0.022
5	0.618±0.213	0.086±0.029	0.052±0.018
6	0.257±0.045	0.044±0.016	0.027±0.010
	U^{233} (100% 233; $n/F=0.0066\pm0.0003$)		
1	55.00 ±0.54	0.086±0.003	0.057±0.003
2	20.57 ±0.38	0.299±0.004	0.197±0.009
3	5.00 ±0.21	0.252±0.040	0.166±0.027
4	2.13 ±0.20	0.278±0.020	0.184±0.016
5	0.615±0.242	0.051±0.024	0.034±0.016
6	0.277±0.047	0.034±0.014	0.022±0.009

Delayed Yields (betas) Depend on Fissioning Species and Neutron Energy

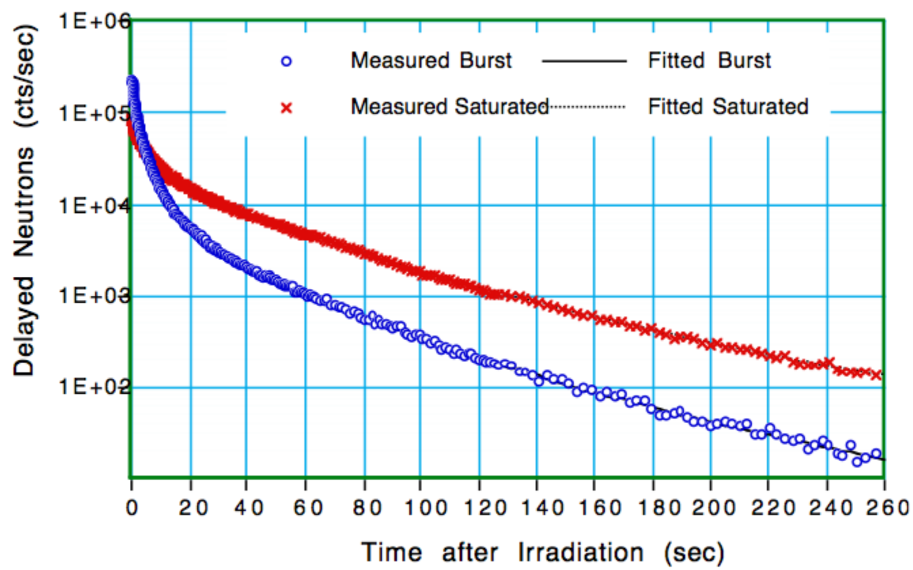


TABLE III. Absolute total yields of delayed neutrons.

Fissile nuclide	Absolute yield (delayed neutrons/fission for pure isotope) Fast fission	Thermal fission
Pu^{239}	0.0063±0.0003	0.0061±0.0003
U^{233}	0.0070±0.0004	0.0066±0.0003
Pu^{240}	0.0088±0.0006	...
U^{235}	0.0165±0.0005	0.0158±0.0005
U^{238}	0.0412±0.0017	...
Th^{232}	0.0496±0.0020	...

big

Delayed Yields Measurements/Evaluations are Expensive and Difficult

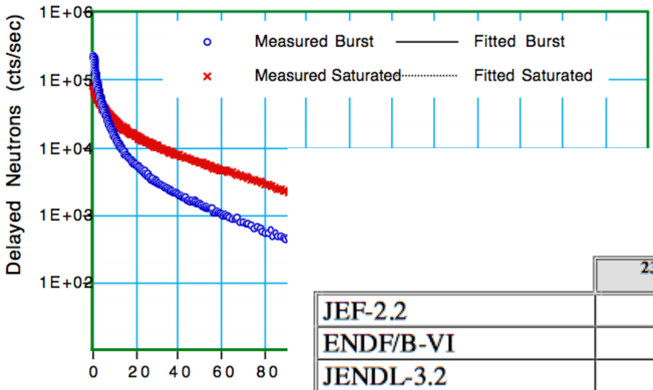


Table 8. Thermal and fast reactor spectrum-averaged values

	^{235}U thermal	^{235}U fast	^{238}U fast	^{239}Pu thermal	^{239}Pu fast
JEF-2.2	0.01654	0.01658	0.0468*	0.00647	0.00646
ENDF/B-VI	0.01670	0.01667	0.0429	0.00645	0.00644
JENDL-3.2	0.0160	0.0161	0.0471*	0.00622	0.00627
Tuttle (1975) [66]	0.01654±2.5%	0.01714±1.3%	0.0451±1.4%	0.00624±3.8%	0.00664±2.0%
Tuttle (1979) [63]	0.01621±3.1%	0.01673±2.1%	0.0439±2.3%	0.00628±6.0%	0.00630±2.5%
Blachot (1990) [1]	0.0166±3.0%	0.0166±3.0%	0.045±4.5%	0.00654±4.0%	0.00654±4.0%
D'Angelo (1990) [86]		0.0165±2.0%	0.0457±3.8%		0.0066±2.9%
Kaneko (1988) [93]	0.01650±1.2%				
Piksaikin (1997) [61]		0.0168±5%**			
Parish (1997) [4]	0.0159±2.5%	0.0167±4.8%			
Borzakov (1997) [65]				0.00686±5%	
Sakurai and Okajima (2002) [130]	0.01586	0.0160±1.8%	0.0456±3.6%	0.00638±3.6%	0.00642±3.6%
Fort, *et al*. (2002) [131]***	0.01621±1.3%	0.01658±1.6%	0.0469±2.4%	0.00651±1.7%	0.00656±2.6%
Values proposed in Appendix 2	0.0162	0.0163	0.0465	0.00650	0.00651
% difference from Tuttle (1979)	0%	-2.6%	+5.6%	+3.4%	+3.2%

* The delayed neutron data for ^{238}U in JEF-2.2 were adopted from JENDL-3.2. The value of 0.0468 is an average for the cores studied by Fort, *et al*. [131]. The value used by Okajima, *et al*. [130], starting from the same energy-dependent data, is 0.0471, this being the appropriate value for the FCA XIX series of fast spectrum cores.

** The value quoted for Piksaikin, *et al*. (1997) is the value measured at 1.165 MeV (0.01709) reduced by 1.9% on the assumption of a rate of increase of 2% per MeV below this energy (Tuttle's estimate of the variation).

*** The values given here are those derived in Appendix 2 and are not precisely the same as the spectrum-averaged values given by Fort, *et al*. in their paper. The uncertainties given here are relative and do not take into account all sources of uncertainty.

Why not use direct measured fission product yields and measured fission product neutron emission rates?

Can compute from MC by sampling fission prod.

TABLE III. Total delayed neutron yields ($\bar{\nu}_d$ per 100 fissions).

Fissionable nuclide[a]	Experimental[b]	Calculated I[c]	Calculated II[d]
^{232}Th *F*	5.31±0.23	5.38	5.54
^{233}U *T*	0.67±0.03	0.77	0.80
^{235}U *T*	1.62±0.05	1.63	1.72
^{235}U *F*	1.67±0.04	1.95	2.03
^{235}U *H*	0.93±0.03	1.02	1.06
^{238}U *F*	4.39±0.10	3.24	3.46
^{238}U *H*	2.73±0.08	2.52	2.64
^{239}Pu *T*	0.63±0.04	0.70	0.74
^{239}Pu *F*	0.63±0.02	0.61	0.65
^{241}Pu *T*	1.52±0.11	1.27	1.35

[a] *T* denotes thermal neutron, *F* denotes fast neutron (fission spectrum), *H* denotes high energy neutron (≃14 MeV).

[b] R. J. Tuttle, Nucl. Sci. Eng. 56, 37 (1975).

[c] P_n values from Ref. 15 and the present work.

[d] P_n values from Refs. 15 and 6.

Current state of data is insufficient to use directly for production analysis

Why not keep track of all know delayed precursors?

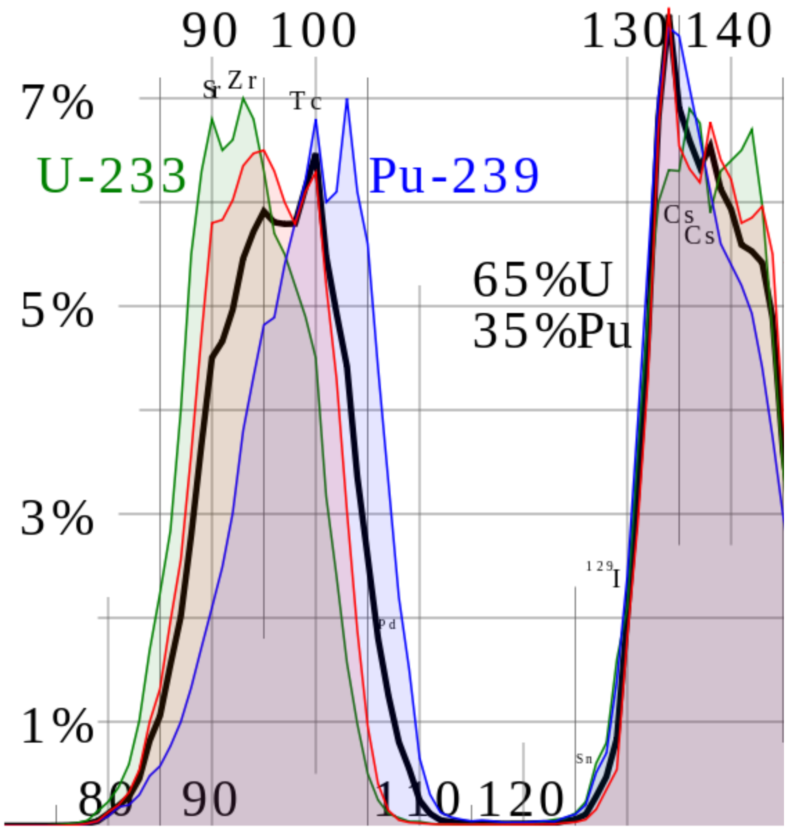
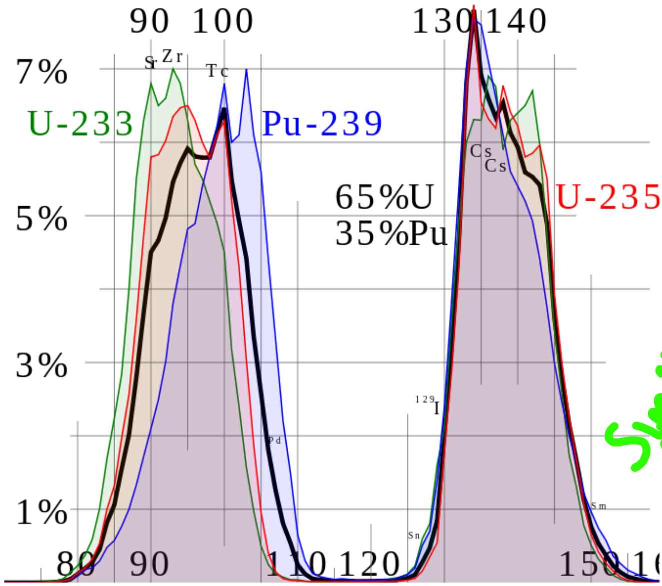


Table 2.6 Seven-group Parameters for Thermal Fission of ^{235}U [37]

Group number	Precursor	$T_{1/2,i}$ sec	a_i (per 100 fissions)	Average $T_{1/2,j}$, sec	Group yield (per 100 fissions)
1	Br-87	55.6	0.054	55.6	0.054
2	I-137	24.5	0.233	24.5	0.233
3	Te-136	17.7	0.015	16.4	0.134
	Br-88	16.23	0.119		
4	I-138	6.27	0.087	4.99	0.309
	Rb-93	5.91	0.046		
	Br-89	4.37	0.176		
5	Rb-94	2.73	0.162	2.07	0.664
	Te-137	2.50	0.014		
	I-139	2.29	0.059		
	Y-98m	2.05	0.105		
	As-85	2.02	0.103		
	Br-90	1.91	0.131		
	Cs-143	1.79	0.024		
	Sb-135	1.70	0.025		
	Y- 99	1.48	0.041		
	Kr-93	1.29	0.010		
6	Cs-144	0.99	0.014	0.494	0.159
	As-86	0.95	0.013		
	Y-100	0.73	0.005		
	I-140	0.61	0.015		
	Cs-145	0.586	0.013		
	Y-98	0.549	0.010		
	Br-91	0.542	0.034		
	As-87	0.490	0.007		
	I-141	0.470	0.008		
	Se-89	0.410	0.007		
	Y-101	0.38	0.007		
	Rb -95	0.379	0.060		
7	Br-92	0.31	0.008	0.218	0.045
	Y-102	0.30	0.008		
	Kr-94	0.22	0.006		
	Rb-96	0.199	0.020		
	Rb-97	0.169	0.009		

Modern Trend: Use 8-group Fixed Decay Constants For All Species



Similar decay constants

			Half life			Delayed neutron fraction
92-U-235	thermal	1	55.6	0.012467	0.0328 ± 0.0042	0.0218 ± 0.0029
		2	24.5	0.028292	0.1539 ± 0.0068	0.1023 ± 0.0056
		3	16.3	0.042524	0.091 ± 0.009	0.0605 ± 0.0063
		4	5.21	0.133042	0.197 ± 0.023	0.131 ± 0.016
		5	2.37	0.292467	0.3308 ± 0.0066	0.2200 ± 0.0083
		6	1.04	0.666488	0.0902 ± 0.0045	0.0600 ± 0.0036
		7	0.424	1.634781	0.0812 ± 0.0016	0.0540 ± 0.0021
		8	0.195	3.554600	0.0229 ± 0.0095	0.0152 ± 0.0064
		Total	9.020	0.076849	1.000 ± 0.029	0.665 ± 0.021
92-U-235	fast	1	55.6	0.012467	0.0084 ± 0.0013	0.0139 ± 0.0023
		2	24.5	0.028292	0.1040 ± 0.0022	0.1716 ± 0.0097
		3	16.3	0.042524	0.0375 ± 0.0008	0.0619 ± 0.0035
		4	5.21	0.133042	0.137 ± 0.020	0.226 ± 0.036
		5	2.37	0.292467	0.294 ± 0.012	0.485 ± 0.033
		6	1.04	0.666488	0.1980 ± 0.0023	0.327 ± 0.018
		7	0.424	1.634781	0.128 ± 0.013	0.211 ± 0.025
		8	0.195	3.554600	0.0931 ± 0.0034	0.1536 ± 0.0098
		Total	5.315	0.130409	1.000 ± 0.027	1.650 ± 0.086
94-Pu-239	thermal	1	55.6	0.012467	0.032 ± 0.012	0.0072 ± 0.0028
		2	24.5	0.028292	0.237 ± 0.034	0.0533 ± 0.0081
		3	16.3	0.042524	0.0826 ± 0.0016	0.01859 ± 0.00098
		4	5.21	0.133042	0.182 ± 0.052	0.041 ± 0.012
		5	2.37	0.292467	0.294 ± 0.029	0.0662 ± 0.0073
		6	1.04	0.666488	0.0816 ± 0.0016	0.01836 ± 0.00097
		7	0.424	1.634781	0.072 ± 0.031	0.0162 ± 0.0071
		8	0.195	3.554600	0.0185 ± 0.0004	0.00416 ± 0.00023
		Total	10.698	0.064794	1.000 ± 0.077	0.225 ± 0.011

Table I. Half-Lives for 8-Group Model

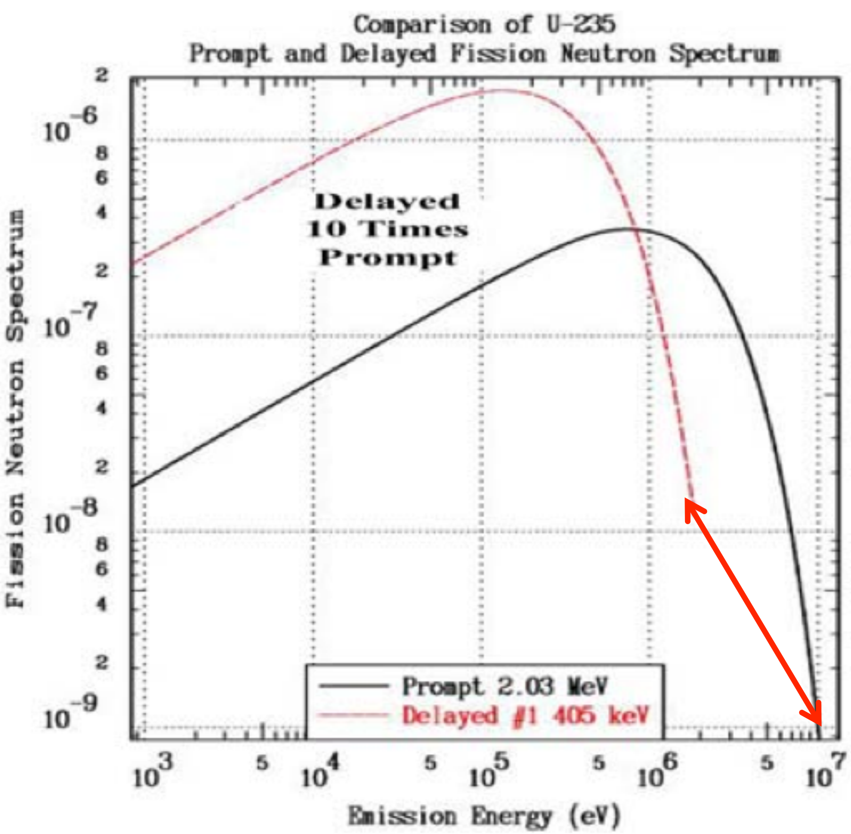
Group	Precursor	Half-life (s)
1	Br-87	55.6
2	I-137	24.5
3	Br-88	16.3
4	Br-89	4.35
5	Br-90	1.91
6	Y-98	0.548
7	Rb-95	0.378
8	Rb-96	0.203

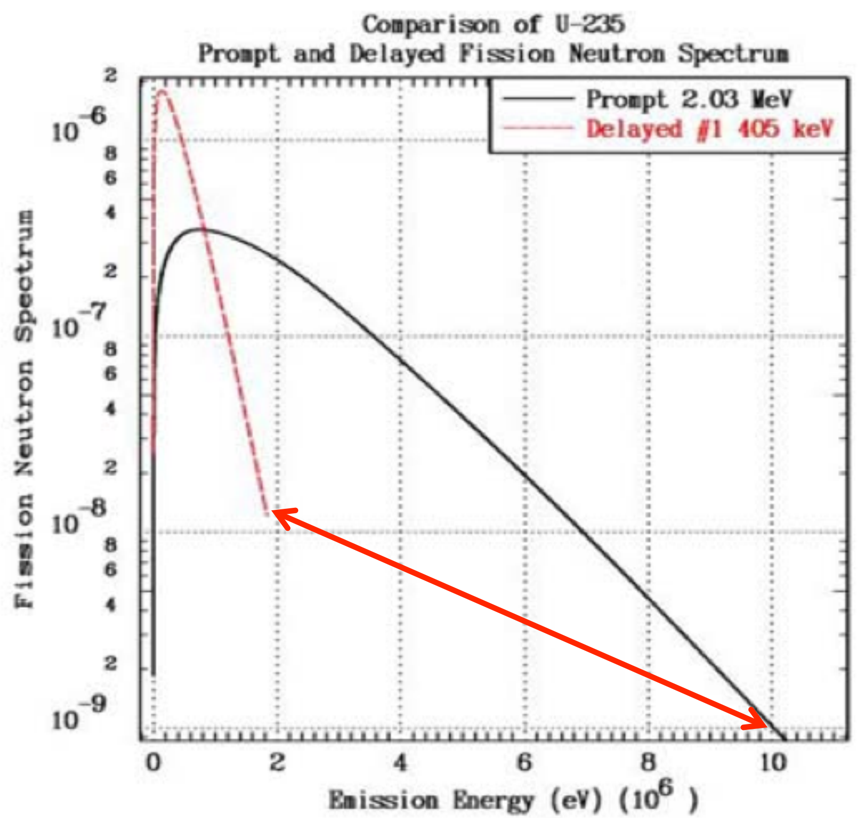
Note: delay neutron fraction = absolute yield/nu-bar

Delayed Neutron Spectra

- Average prompt neutron emission energy is about 2.0 MeV
- Average delayed neutron emission energy is about 0.4 MeV

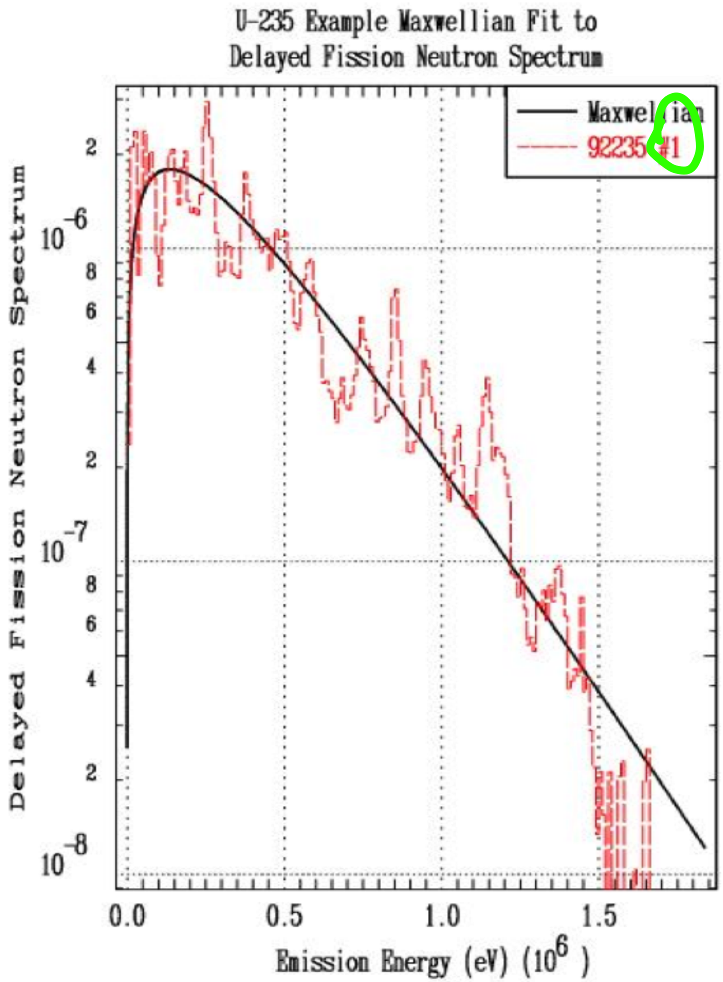
2 competing reactions effects.

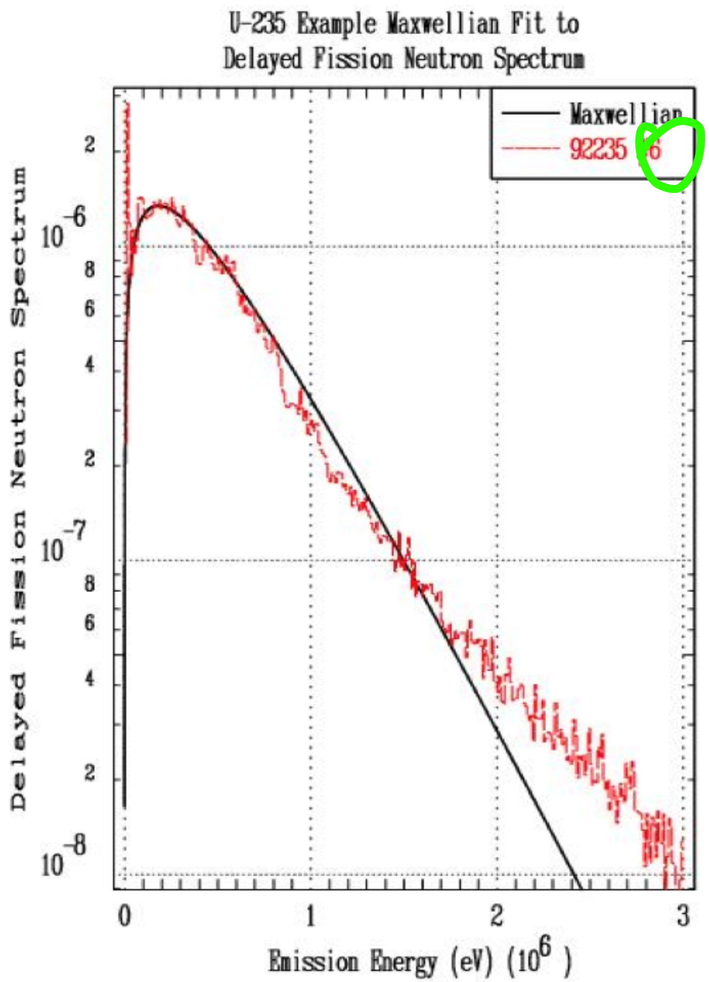




Delayed Neutron Spectra

- Delayed neutron spectra have similar shapes for all delayed groups
- Smooth shape is an approximation: real data is often discrete in nature





Delayed Neutron Spectra by Fissioning Species

- Delayed spectra only vary slightly for different fissioning nuclides
- Delayed spectra depend significantly on delayed neutron group

most codes don't separate chi

Table 9. Mean energies of the eight-group spectra (in keV) for the major actinide isotopes

Data taken from LA-UR-99-4000

Group	1	2	3	4	5	6	7	8	Sum
Half-life (secs)	55.6	24.5	16.3	5.21	2.37	1.04	0.424	0.195	
^{235}U thermal	211	612	269	441	516	512	616	619	494
^{235}U fast	211	609	265	453	542	534	603	572	501
^{238}U fast	211	613	289	433	539	515	671	569	535
^{239}Pu thermal	211	617	289	418	475	473	555	549	481
^{239}Pu fast	211	615	284	421	484	477	586	523	488

Point Kinetics Equations

Assume that the flux can be separated into a space/energy term and a time-dependent term: $\phi(\vec{r},E,t)=S(\vec{r},E)T(t)$,

$$\frac{\partial}{\partial t}\left[\frac{1}{v}S(\vec{r},E)T(t)\right]=\left[\begin{array}{l}\nabla\cdot D(\vec{r},E,t)\nabla S(\vec{r},E)-\Sigma_t(\vec{r},E,t)\,S(\vec{r},E)+\int_0^\infty\Sigma_s(\vec{r},E'\to E,t)\,S(\vec{r},E')\,dE'\\ +\sum_j\left\{\chi_p^j(E)(1-\beta^j)+\sum_i\chi_d^i(E)\beta_i^j\right\}\int_0^\infty\nu\Sigma_f^j(\vec{r},E',t)\,S(\vec{r},E')\,dE'\end{array}\right]T(t)$$
$$-\left[\sum_j\chi_p^j(E)\beta^j\int_0^\infty\nu\Sigma_f^j(\vec{r},E',t)\,S(\vec{r},E')\,dE'\right]T(t)$$
$$+\sum_i\chi_d^i(E)\lambda_i C_i(\vec{r},t)$$
$$+Q(\vec{r},E,t)$$

and

$$\frac{\partial}{\partial t}C_i(\vec{r},t)=\left[\sum_j\beta_i^j\int_0^\infty\nu\Sigma_p^j(\vec{r},E',t)\,S(\vec{r},E')\,dE'\right]T(t)-\lambda_i C_i(\vec{r},t)$$

If one integrates over all space and energy and normalize $\int dE\int d\vec{r}\,\frac{1}{v}S(\vec{r},E)\equiv 1.0$,

$$\frac{d}{dt}\left[T(t)\right]=\int dE\int d\vec{r}\left[\begin{array}{l}\nabla\cdot D(\vec{r},E,t)\nabla S(\vec{r},E)-\Sigma_t(\vec{r},E,t)\,S(\vec{r},E)+\int_0^\infty\Sigma_s(\vec{r},E'\to E,t)\,S(\vec{r},E')\,dE'\\ +\sum_j\left\{\chi_p^j(E)(1-\beta^j)+\sum_i\chi_d^i(E)\beta_i^j\right\}\int_0^\infty\nu\Sigma_f^j(\vec{r},E',t)\,S(\vec{r},E')\,dE'\end{array}\right]T(t)$$
$$-\int dE\int d\vec{r}\left[\sum_j\chi_p^j(E)\beta^j\int_0^\infty\nu\Sigma_f^j(\vec{r},E',t)\,S(\vec{r},E')\,dE'\right]T(t)$$
$$+\int dE\int d\vec{r}\left[\sum_i\chi_d^i(E)\lambda_i C_i(\vec{r},t)\right]$$
$$+\int dE\int d\vec{r}\left[Q(\vec{r},E,t)\right]$$

and

$$\frac{d}{dt}\left[\int dE\int d\vec{r}\,C_i(\vec{r},t)\right]=\int dE\int d\vec{r}\left[\sum_j\beta_i^j\int_0^\infty\nu\Sigma_p^j(\vec{r},E',t)\,S(\vec{r},E')\,dE'\right]T(t)-\lambda_i\int dE\int d\vec{r}\,C_i(\vec{r},t)$$

→ Go through this a little tricky.

Point Kinetics Equations

Defining:

$$\rho(t) \equiv \frac{\int dE \int d\vec{r} \left[\nabla \cdot D(\vec{r},E,t) \nabla S(\vec{r},E) - \Sigma_t(\vec{r},E,t)\, S(\vec{r},E) + \int_0^\infty \Sigma_s(\vec{r},E' \to E,t)\, S(\vec{r},E')\, dE' + \sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}$$

reactivity $\cong \dfrac{k_{eff} - 1.0}{k_{eff}}$

$$\beta_i(t) \equiv \frac{\int dE \int d\vec{r} \left[\sum_j \chi_d^i(E)\beta_i^j \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}$$

beta $\cong \beta_i$

$$\Lambda(t) \equiv \frac{\int dE \int d\vec{r}\, \frac{1}{v} S(\vec{r},E)}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}$$

promt neutron lifetime $\cong \dfrac{1}{v\, \upsilon\Sigma_f} \cong \dfrac{1}{v\, \Sigma_a}$

$C_i(t) \equiv \int dE \int d\vec{r}\; C_i(\vec{r},t)$ i-th precursor

$Q(t) \equiv \int dE \int d\vec{r}\; Q(\vec{r},t)$ total eternal source

*W*e obtain the "point" kinetics equations

$$\frac{d}{dt}T(t) = \frac{\rho(t) - \sum_i \beta_i(t)}{\Lambda(t)} T(t) + \sum_i \lambda_i C_i(t) + Q(t)$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i(t)}{\Lambda(t)} T(t) - \lambda_i C_i(t)$$

$$\phi(\vec{r},E,t) = S(\vec{r},E)T(t),$$

Remember this approximation is not always valid!

MATLAB Example of Solving PKEs

- Assume no external source and constant betas/lambda, to get classic PKEs:

$$\frac{d}{dt}T(t) = \frac{\rho(t) - \sum_i \beta_i}{\Lambda} T(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda} T(t) - \lambda_i C_i(t)$$

- If we start from a steady state solution, at power level T_0, we know that:

$$\frac{d}{dt}C_i(0) = 0 \quad \Rightarrow C_i(0) = \frac{\beta_i}{\Lambda\, \lambda_i} T_0$$

$$\frac{d}{dt}T(0) = 0 \quad \Rightarrow \quad \frac{\rho(0)}{\Lambda} T_0 = 0 \quad \Rightarrow \quad \rho(0) = 0,$$

- We will assume that the reactor is operating at low enough flux that there is no feedback (coefficients are then independent of solution).

- Hence, if reactivity as a function of time is known, one can solve for reactor power as a function of time.

Matrix Equation for 1st Order ODE

$$\frac{d}{dt}T(t) = \frac{\rho(t) - \sum_i \beta_i}{\Lambda} T(t) + \sum_i \lambda_i C_i(t)$$

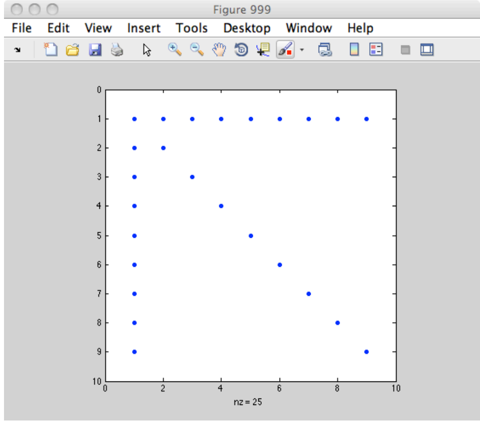
$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda} T(t) - \lambda_i C_i(t)$$

$$\frac{d}{dt}\left[N(t)\right] = \quad [N(t)]$$

$$Let \quad N = \begin{bmatrix} T \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} \quad \Rightarrow \frac{d}{dt}\left[N(t)\right] = \begin{bmatrix} \frac{\rho(t)-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & & & & & \\ \frac{\beta_2}{\Lambda} & & -\lambda_2 & & & & \\ \frac{\beta_3}{\Lambda} & & & -\lambda_3 & & & \\ \frac{\beta_4}{\Lambda} & & & & -\lambda_4 & & \\ \frac{\beta_5}{\Lambda} & & & & & -\lambda_5 & \\ \frac{\beta_6}{\Lambda} & & & & & & -\lambda_6 \end{bmatrix} [N(t)]$$

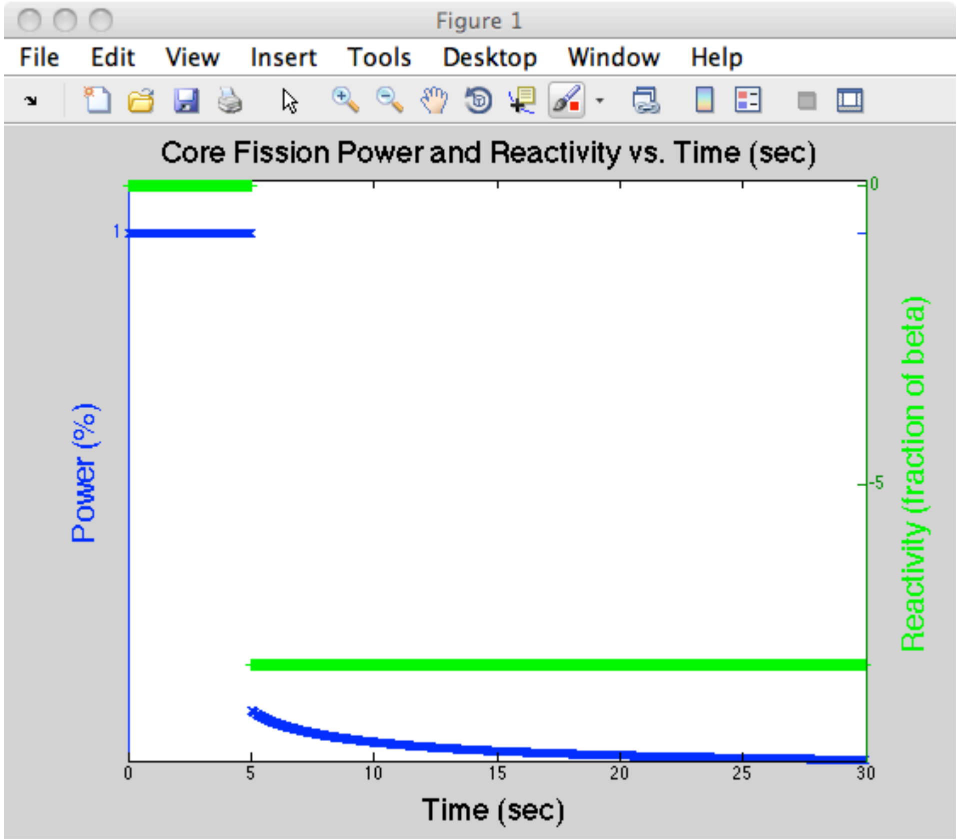
Simple MATLAB Code

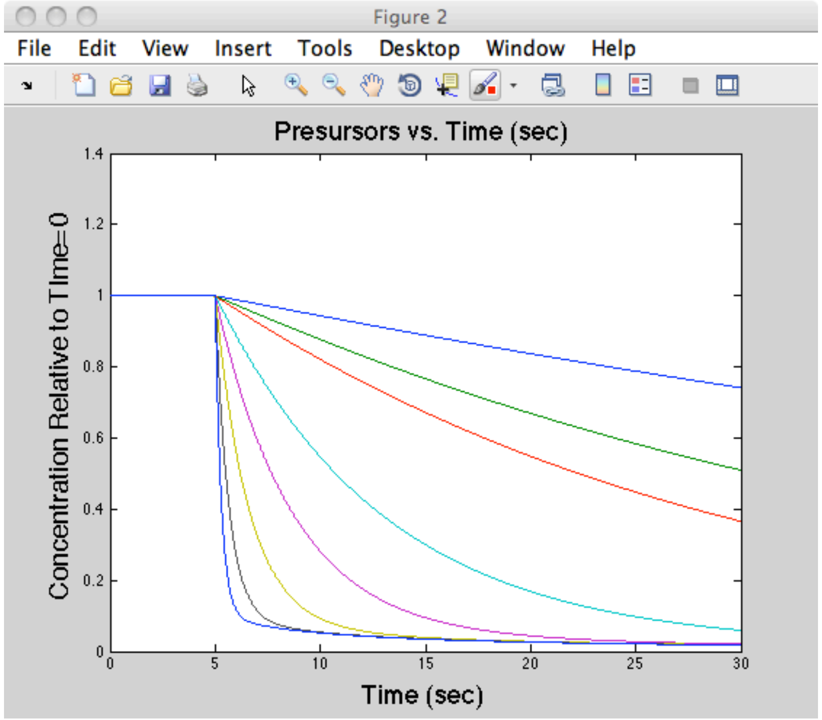
```
function Point_kinetics_one
%
% model point kinetics with ramp perturbations
%
betai  = [.000218 .001023 .000605 .00131 .00220 .00060 .000540 .000152];   % no units, 8-group data
halfli = [55.6     24.5     16.3     5.21    2.37   1.04   0.424   0.195 ];   % half-life in seconds
decayi = log(2)./halfli;                                                     % sec-1
sigf=.05;                                    % cm-1, approximate cross sections for pin cell

powden=100.0;                                % w-cm-3, pin-cell power density
flux0=powden/sigf/(1.602e-13*200);           % (w/cc)/(cm-1)/(w/MeV*200MeV/fission)
nu    = 2.45;                                % no units
vel   = 2200.*100.*(.100/.0253)^.5;          % cm/sec
pnl   = 1./(sigf*nu*vel);                    % sec
betat= sum(betai);                           % no units

nztime = [0   5.0   5.01    30];             % time in seconds
rho    = [00  00   -8   -8];                 % reactivity in units of beta
deltat =.01;                                 % time-step in seconds

nz=length(nztime); nzt=nztime; nzt=nzt; endtime=nzt(nz);
tend=endtime/deltat; tt([1:tend])=0.0; rhott(1:tend)=0.0;

ND=length(betai)+1; N([1:ND],[1:tend])=0.0; N(1,1)=flux0;                  % initialize flux
for i=2:ND; N(i,1)=betai(i-1)/(pnl*decayi(i-1))*N(1,1); end;                % initialize precursors

A0([1:ND],[1:ND])=0;                                                         % construct production matrix
for i=2:ND; A0(i,1)=betai(i-1)/pnl; A0(i,i)=-decayi(i-1); A0(1,i)=decayi(i-1); end
A0(1,1)=(-betat)/pnl; figure (999); spy(A0);
time=0; nzone=1;

for t=2:1:endtime/deltat
  time=time+deltat;
  if time<nzt(nzone+1); else nzone=nzone+1; if nzone>nz-1;nzone=nz-1; end; end;
  frac=(time-nztime(nzone))/(nztime(nzone+1)-nztime(nzone));
  rhon=rho(nzone)+frac*(rho(nzone+1)-rho(nzone));
  tt(t)=time; rhott(t)=rhon;
  A=A0;  A(1,1)=(rhon*betat-betat)/pnl;                                      % fill rho term in production
  EXPM=expm(A*deltat); N(:,t)=EXPM*N(:,t-1);                                 % update flux/concentrations
end
```



Example 1: Instantaneous Reactor Scram (ρ = - 8.0*beta)

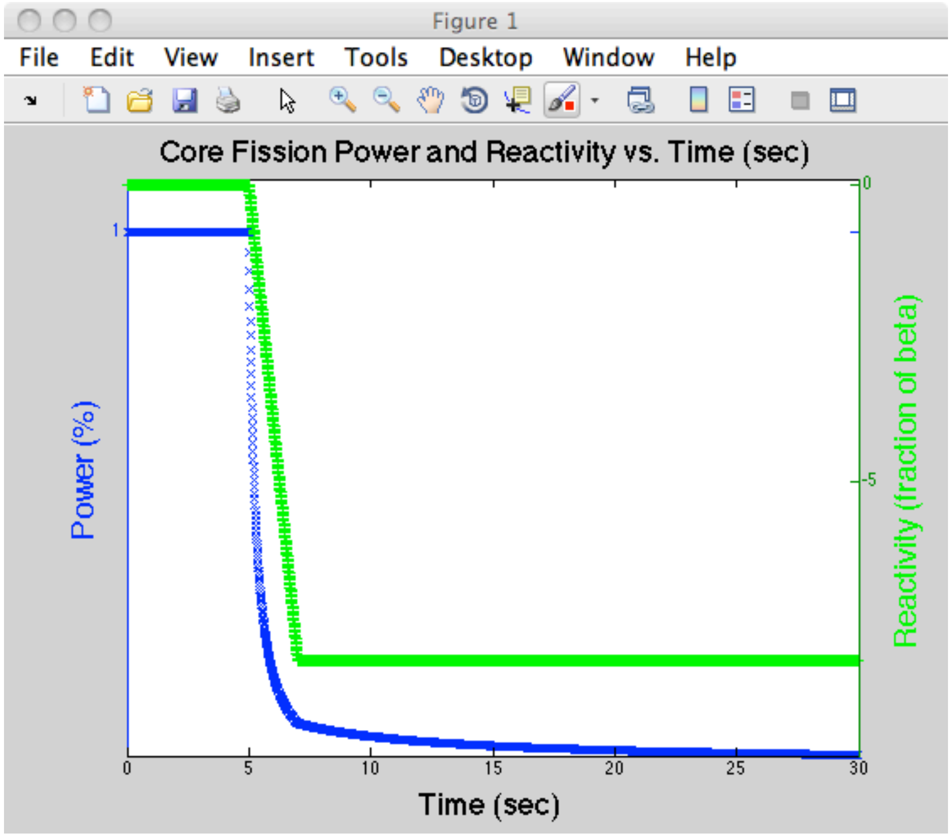
- SCRAM from Critical:
- Power goes almost instantly to ~10%
- Power ultimately decays with time constant of longest lived precursor (55 sec)

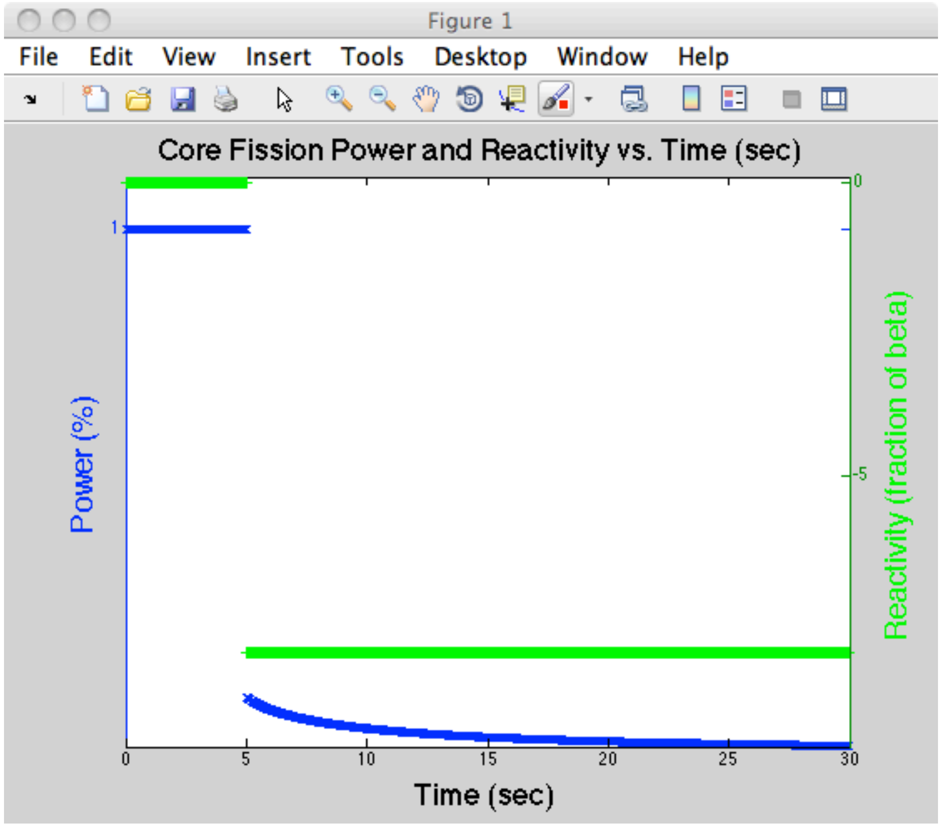




Example 2: Two Second Rod drop (ρ = - 8.0*beta)

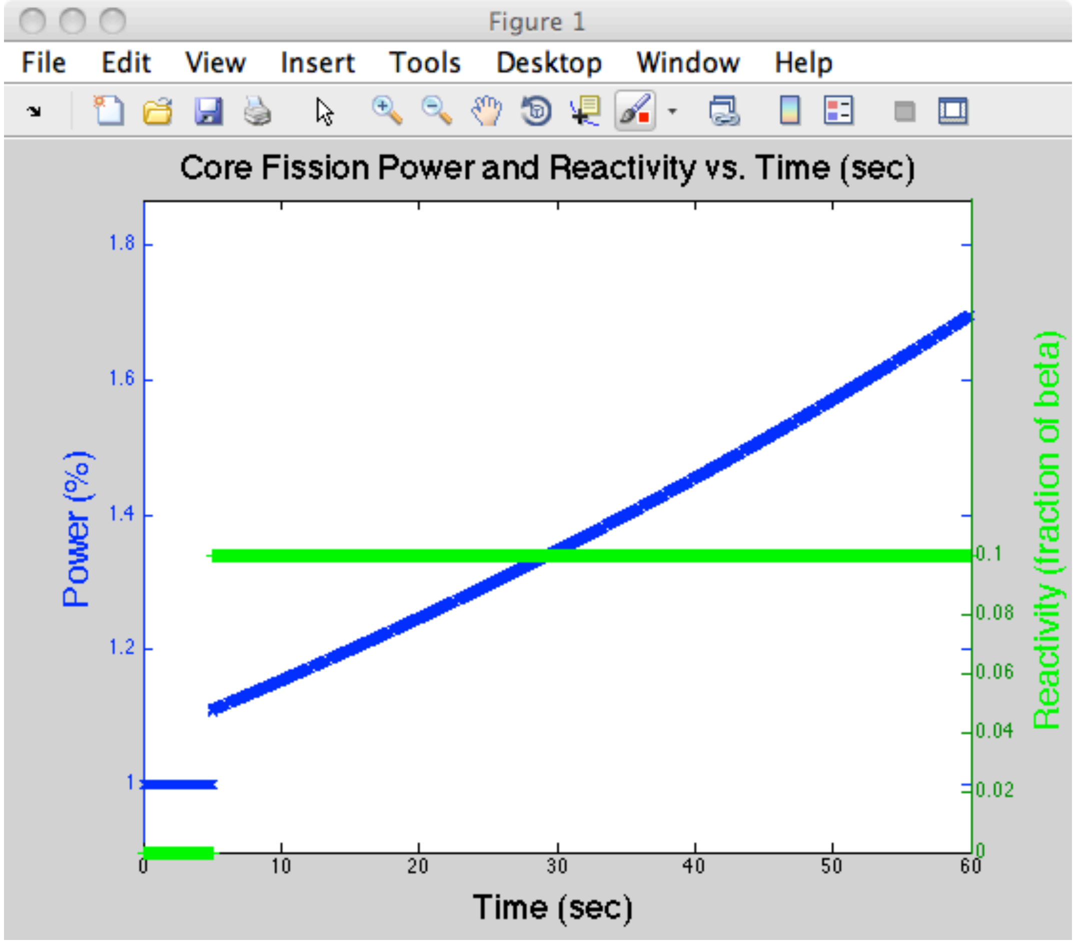
- SCRAM from Critical:
- Power goes to ~10% when rod fully inserted
- Power ultimately decays with time constant of longest lived precursor (55 sec)





Example 3: Rod Withdrawal at Power (ρ = +.1*beta)

- One delayed Neutron precursor group
- Rod withdrawn instantly
- Power increases with time constant of longest precursor group (55 sec)



Prompt Jump Approximation

$$\frac{d}{dt}T(t) \cong 0 = \frac{\rho-\beta}{\Lambda}T(t) + \lambda C(t)$$

$$\Rightarrow T(0^+) = \frac{\Lambda\lambda}{\beta-\rho}\left[C(0^+)\right]$$

$$\Rightarrow T(0^+) = \frac{\Lambda\lambda}{\beta-\rho}\left[\frac{\beta}{\Lambda\lambda}T_0\right]$$

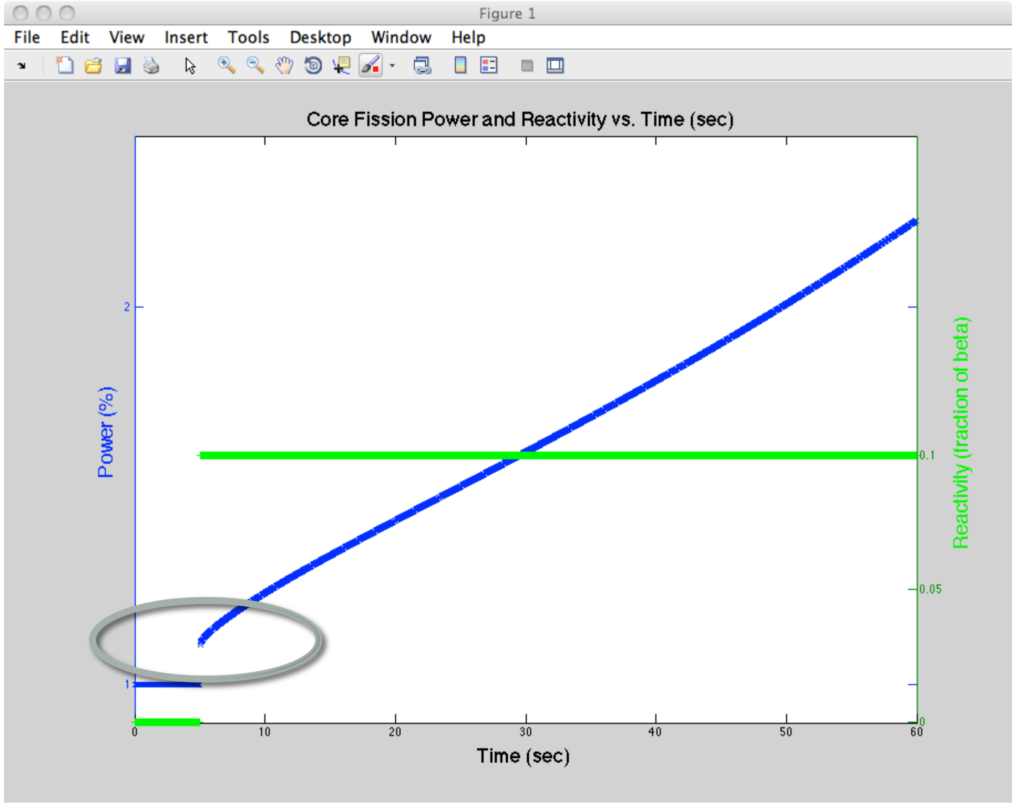
$$\Rightarrow \frac{T(0^+)}{T_0} = \frac{\beta}{\beta-\rho}$$

here

$$\frac{T(0^+)}{T_0} = \frac{\beta}{\beta-.1\beta} = 1/.9 = 1.111$$

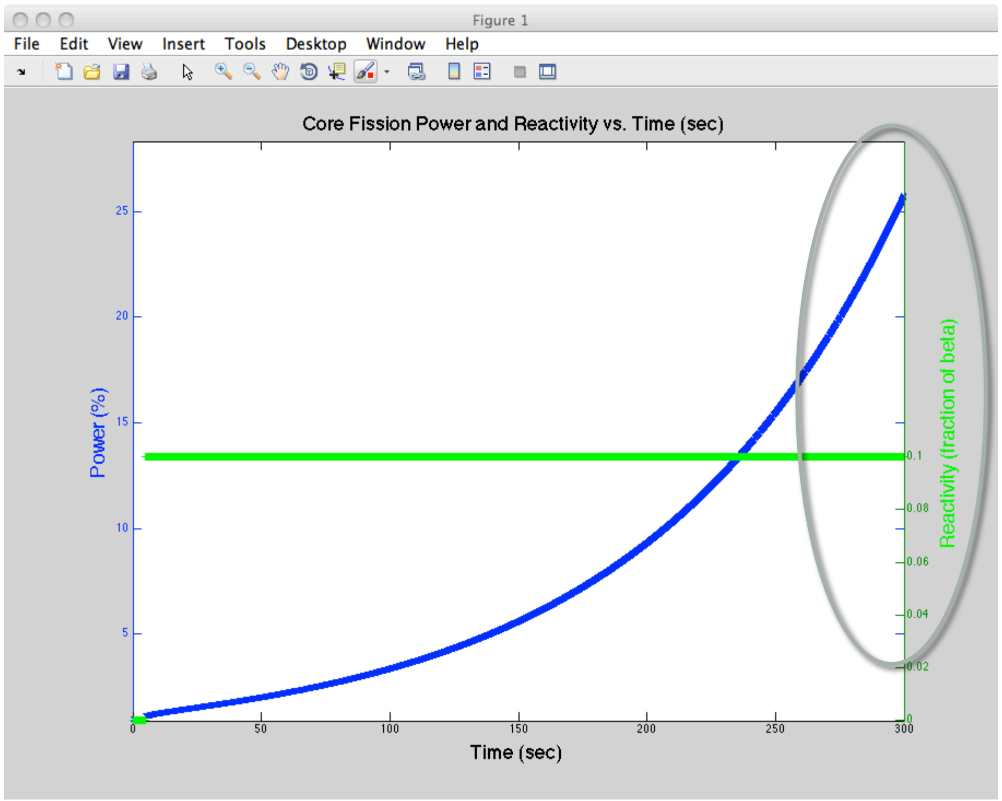
Example 4: Rod Withdrawal at Power (8 vs. 1 delayed groups)

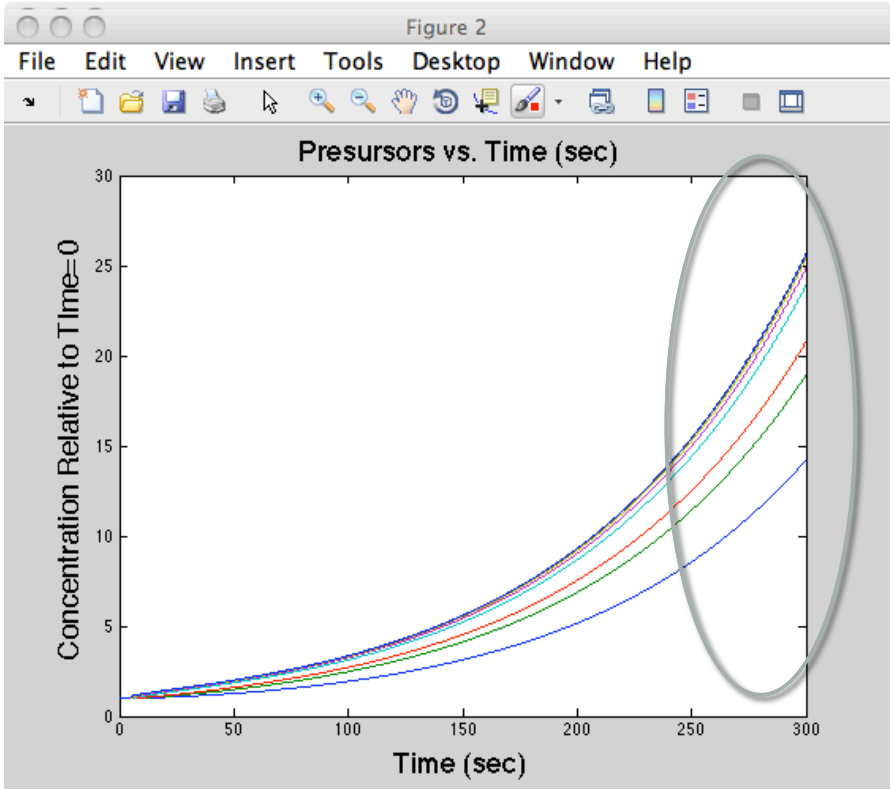
- 8 Delayed Neutron Groups
- Rod withdrawn instantly
- Initial part of transient is slightly more complicated than 1-group case
- Single delayed group is very sensitive to the chosen time-constant



Example 4b: Rod Withdrawal at Power

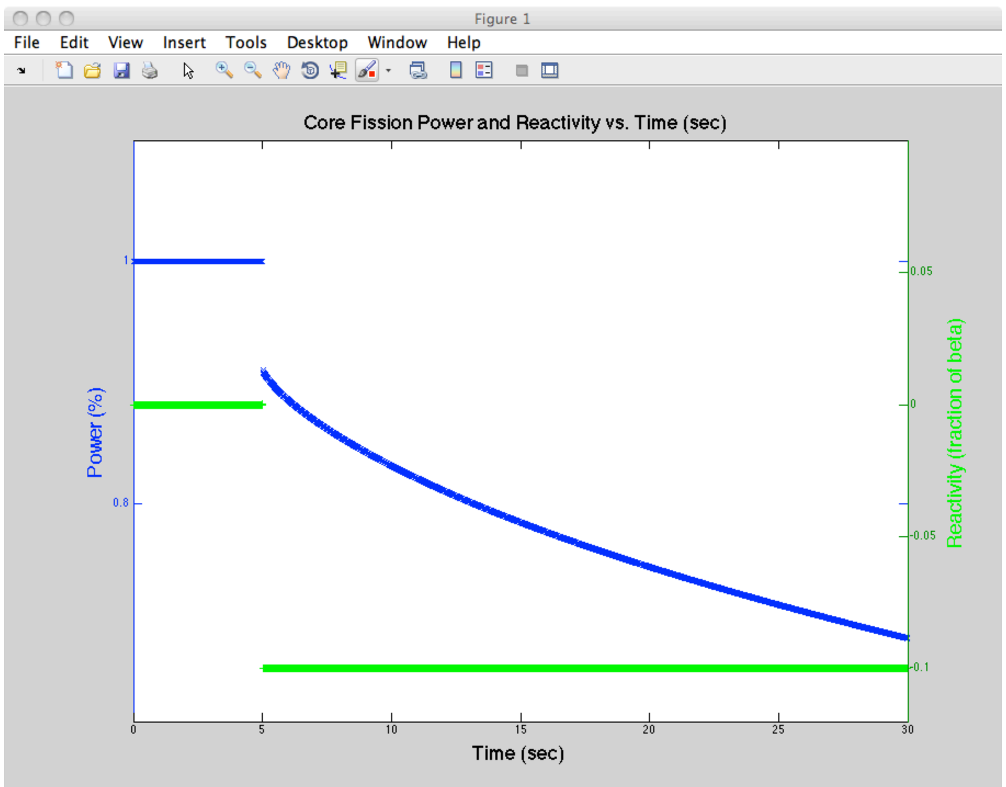
- 8 Delayed Neutron Groups
- Rod withdrawn instantly
- Eventually becomes an asymptotic exponential rise in power
- e.g., develops secular equilibrium between power and precursor rate



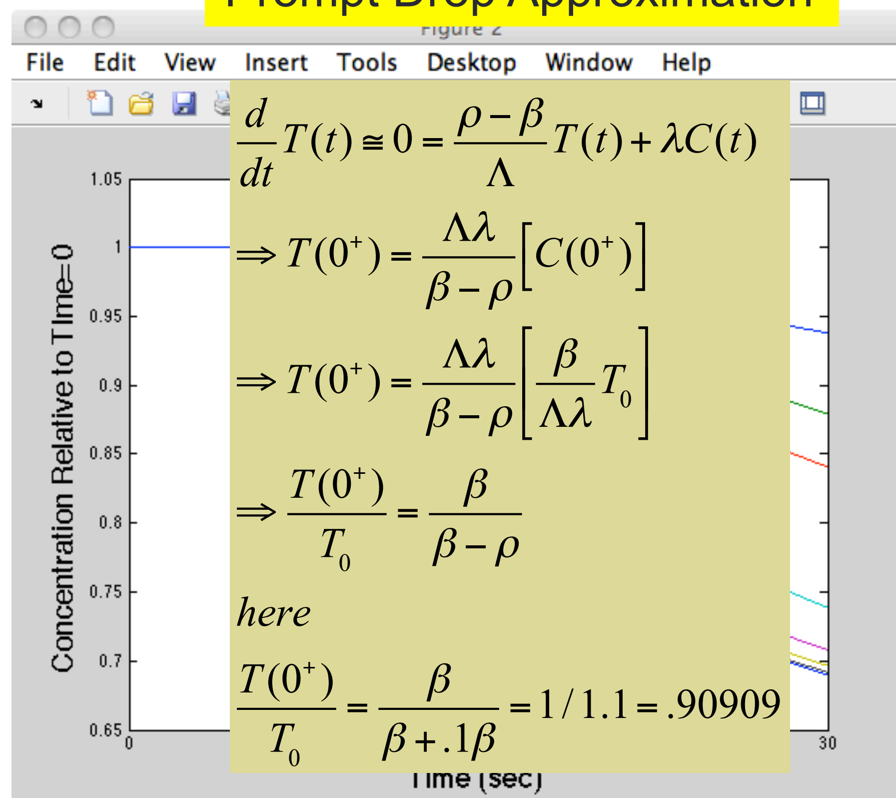


Example 5: Regulating Rod Insertion at Power

- 8 Delayed Neutron Groups
- Rod inserted instantly
- -0.1 beta



Prompt Drop Approximation



$$\frac{d}{dt}T(t) \cong 0 = \frac{\rho-\beta}{\Lambda}T(t) + \lambda C(t)$$

$$\Rightarrow T(0^+) = \frac{\Lambda\lambda}{\beta-\rho}\left[C(0^+)\right]$$

$$\Rightarrow T(0^+) = \frac{\Lambda\lambda}{\beta-\rho}\left[\frac{\beta}{\Lambda\lambda}T_0\right]$$

$$\Rightarrow \frac{T(0^+)}{T_0} = \frac{\beta}{\beta-\rho}$$

here

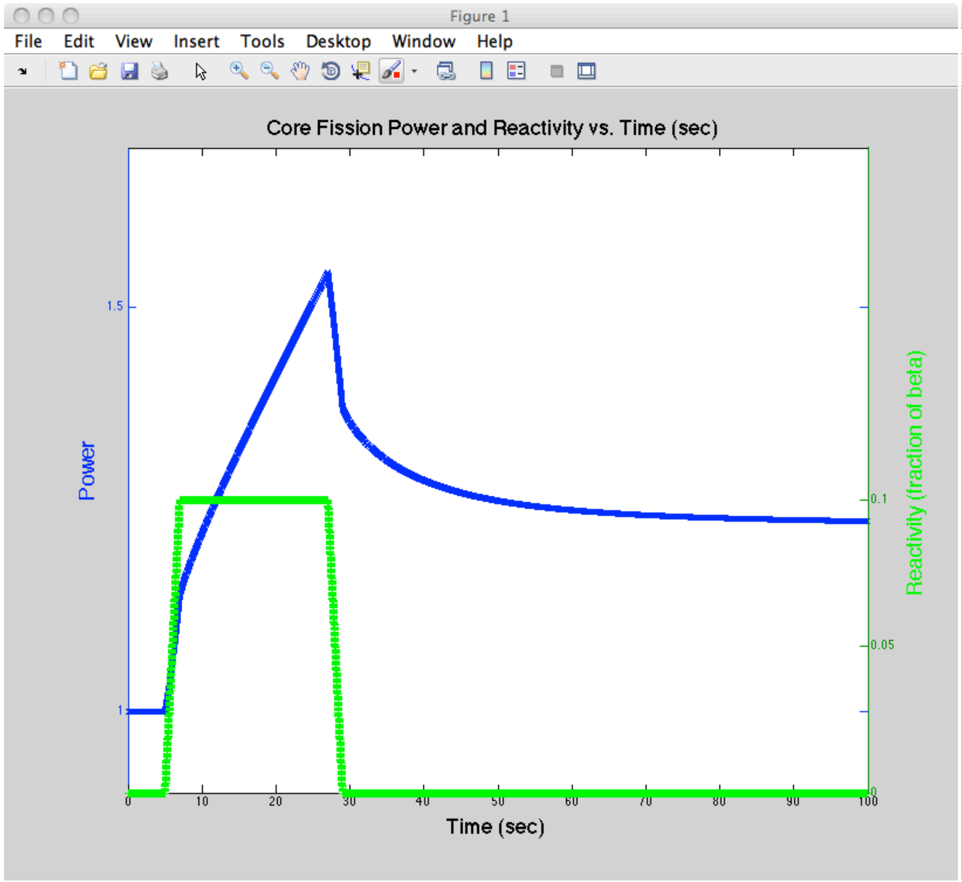
$$\frac{T(0^+)}{T_0} = \frac{\beta}{\beta+.1\beta} = 1/1.1 = .90909$$

Know how to derive this!

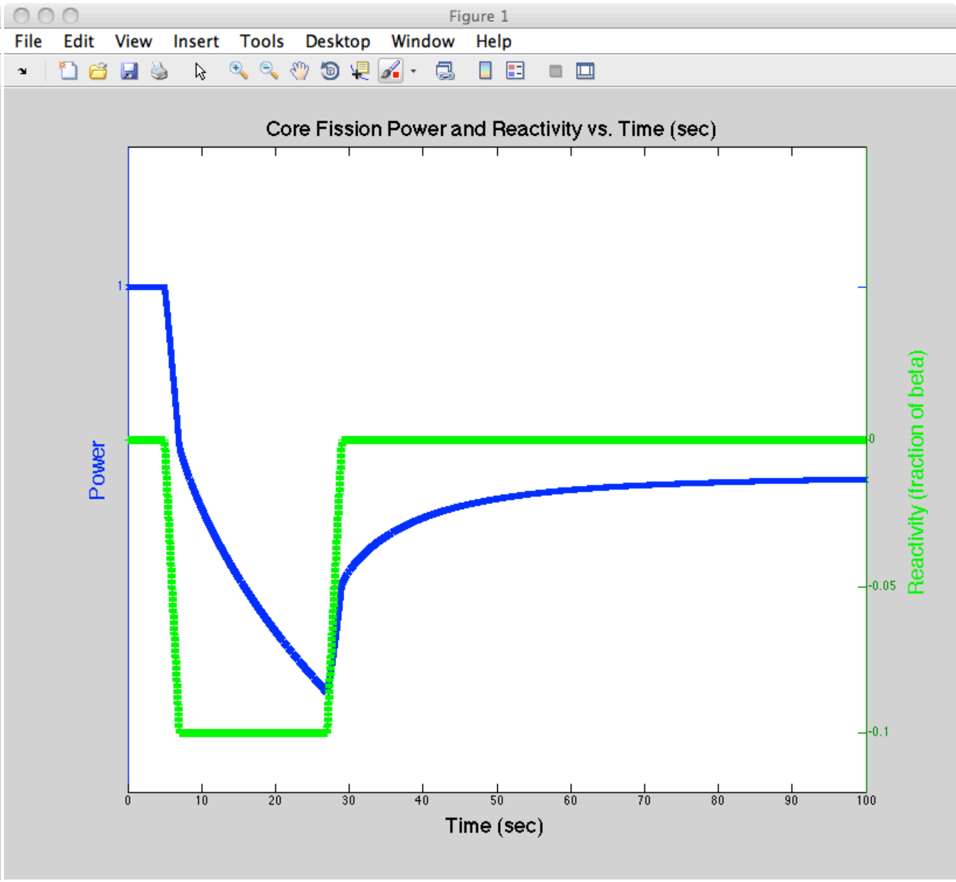
Example 6: Regulating Rod Withdrawal/Insertion

- Rod moved in 2 sec, hold for 20 sec, re-positioned in 2 sec
- Reactor does not return to original state - why not??

Withdrawal



Insertion

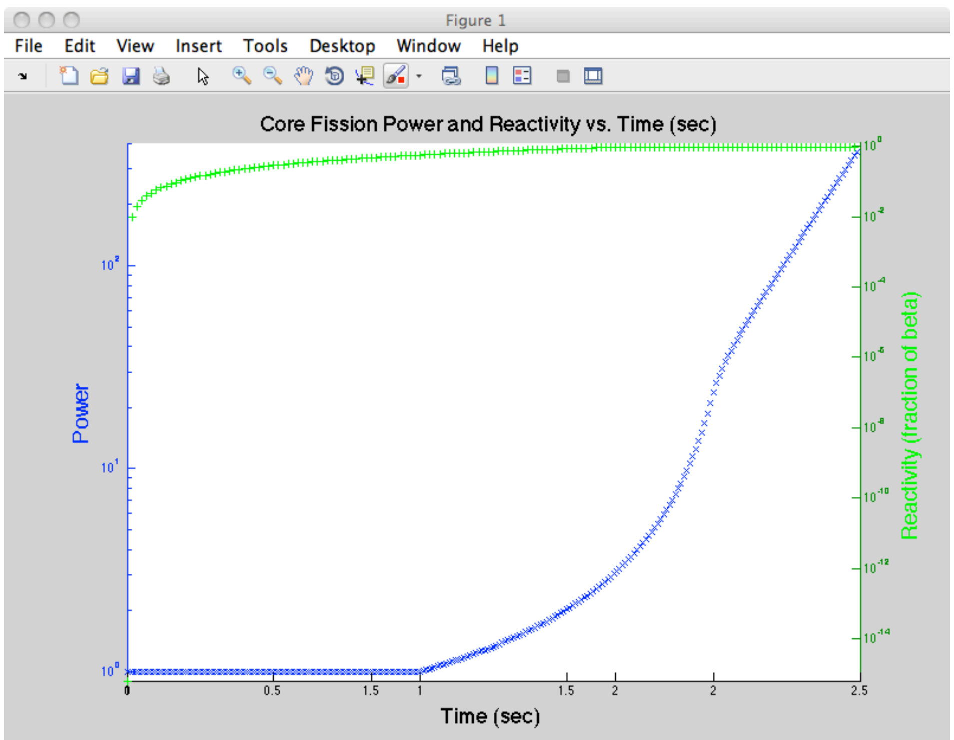


Example 6: Rod Ejection (Failed Housing)

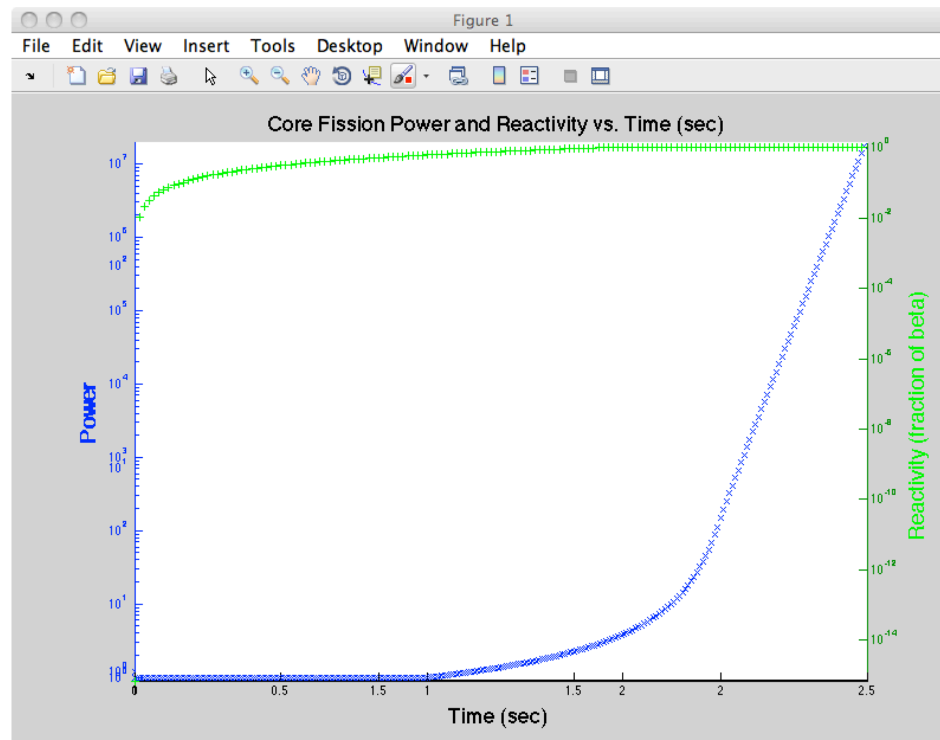
- Rod moved in 1.0 sec
- Power ascension is very sensitive as reactivity approaches 1 beta
- When reactivity surpasses beta, power does not wait for delayed neutrons

or 1.5$ worth

0.95 beta



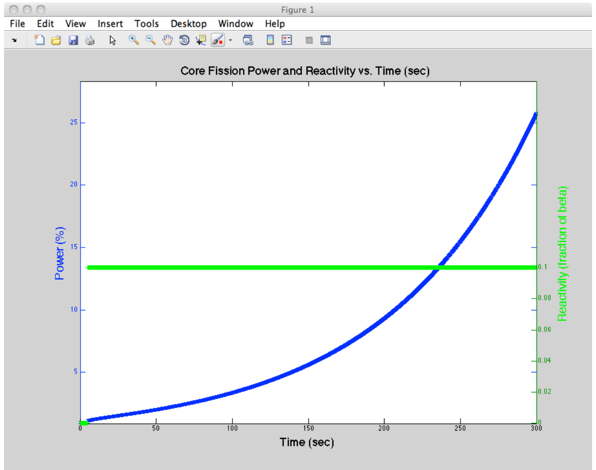
1.05 beta



Inhour Equation

$$\frac{d}{dt}T(t) = \frac{\rho(t) - \sum_i \beta_i}{\Lambda} T(t) + \sum_i \lambda_i C_i(t)$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda} T(t) - \lambda_i C_i(t)$$



$$T(t) \approx C_i(t) \approx e^{\omega t} \Rightarrow \frac{d}{dt}T(t) \approx \frac{d}{dt}T(t) \approx \omega\, T(t)$$

$$\omega T(t) = \frac{\rho - \beta}{\Lambda} T(t) + \sum_i \lambda_i C_i(t)$$

$$\omega C_i(t) = \frac{\beta_i}{\Lambda} T(t) - \lambda_i C_i(t)$$

$$(\omega + \lambda_i) C_i(t) = \frac{\beta_i}{\Lambda} T(t) \Rightarrow C_i(t) = \frac{\beta_i}{\Lambda(\omega + \lambda_i)} T(t)$$

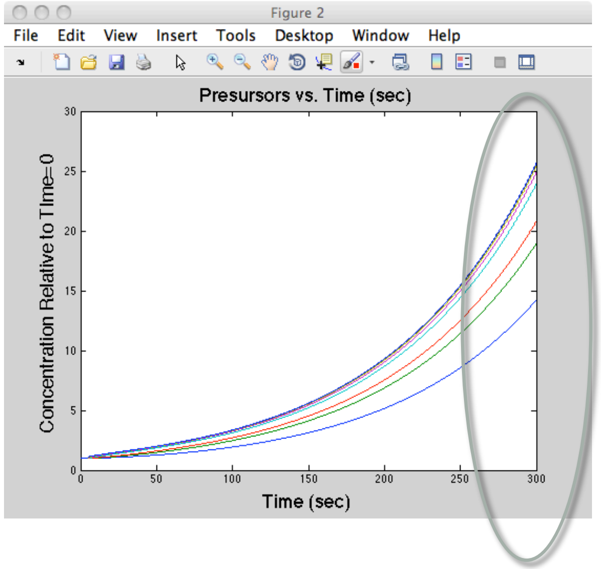
$$\omega \cancel{T(t)} = \frac{\rho - \beta}{\Lambda} \cancel{T(t)} + \sum_i \frac{\lambda_i \beta_i}{\Lambda(\omega + \lambda_i)} \cancel{T(t)}$$

$$\omega \Lambda = \rho - \beta + \sum_i \frac{\lambda \beta_i}{(\omega + \lambda_i)}$$

$$\rho = \omega \Lambda + \sum_i \beta_i - \sum_i \frac{\lambda \beta_i}{(\omega + \lambda_i)}$$

$$\rho = \omega \Lambda + \sum_i (\beta_i - \frac{\lambda \beta_i}{(\omega + \lambda_i)})$$

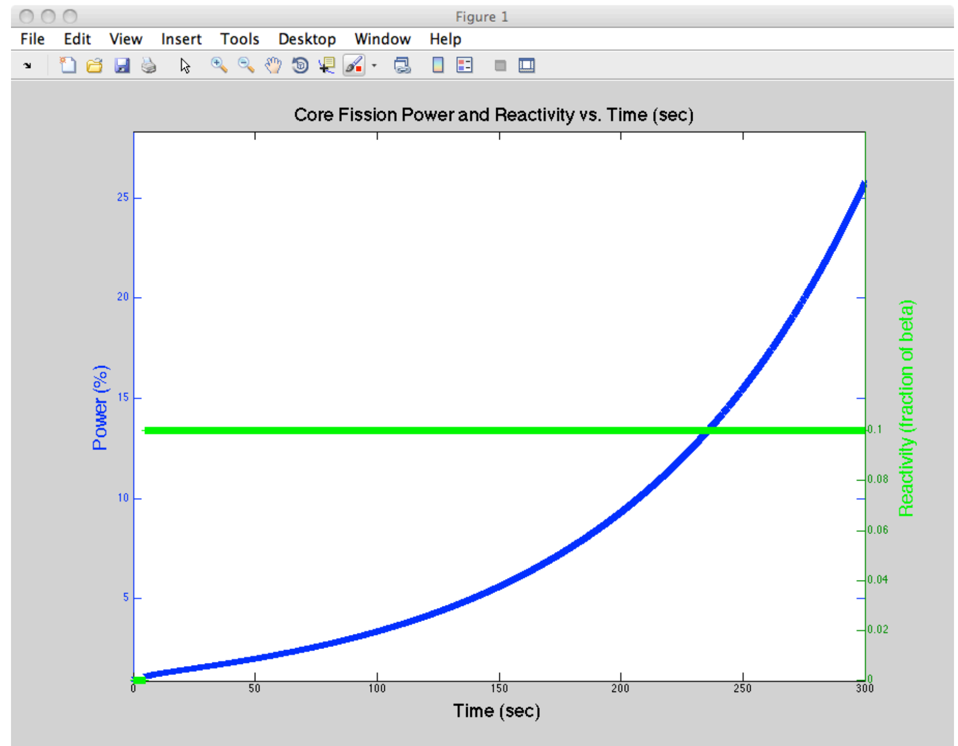
$$\boxed{\rho = \omega \Lambda + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)}}$$



Know how to derive this!

In-hour Equation for Reactor Measurements

$$\rho = \cancelto{0}{\omega\Lambda} + \sum_i \frac{\beta_i \omega}{(\omega + \lambda_i)}$$



1. Withdraw reactor control rod
2. Measure reactor period
3. Use inhour equation to compute reactivity

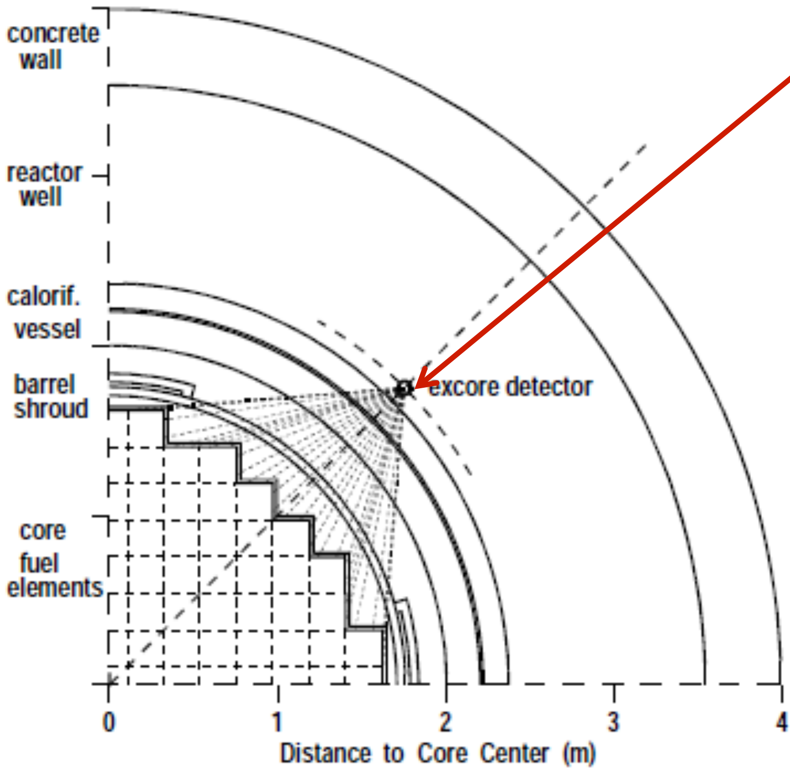
This is how control rods are “calibrated”.

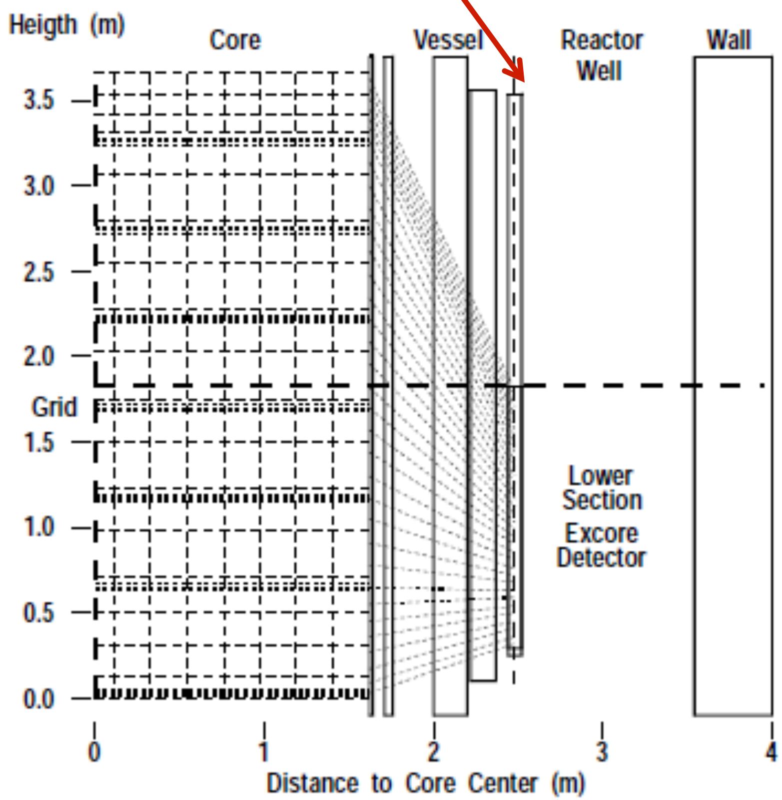
This is how small sample worths are measured.
Very useful for reactor operations.

Make sure reactor period is constant!!!
Note direct sensitivity to “input” beta (and the constant shape function assumption)

PWR Ex-Core Detectors

- Monitor core flux (power) level
- Monitor core axial power shape

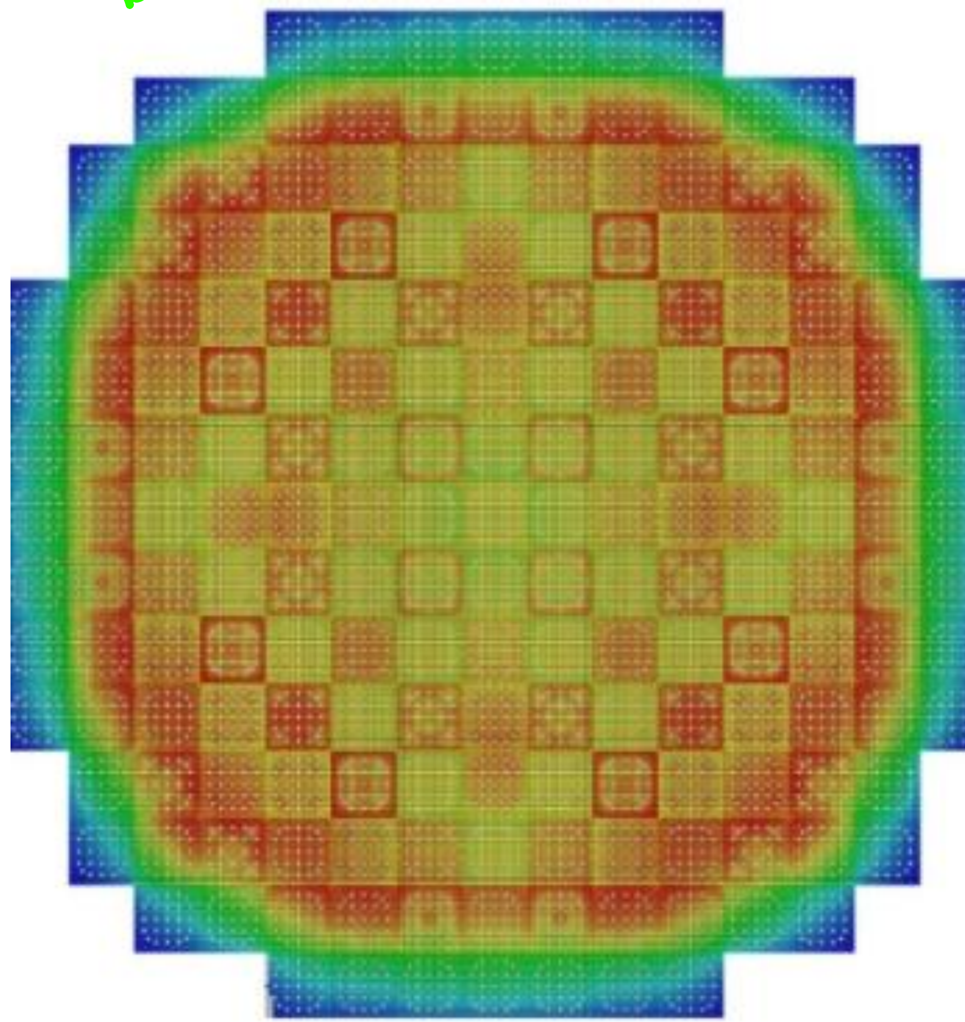




Don't Forget Rod Bank Worth may be affected by changes in beta-effective

can change by a lot depending on what bank is pushed

- Rods preferential in in center of core
- High burnup fuel on outside of core
- Pu's low beta contribution increases as rods are inserted.
- Beta-effective can change by 3%
- All "measured" rod worths use six or eight group effective betas put into the process computer.
- We usually post-correct the measurements.



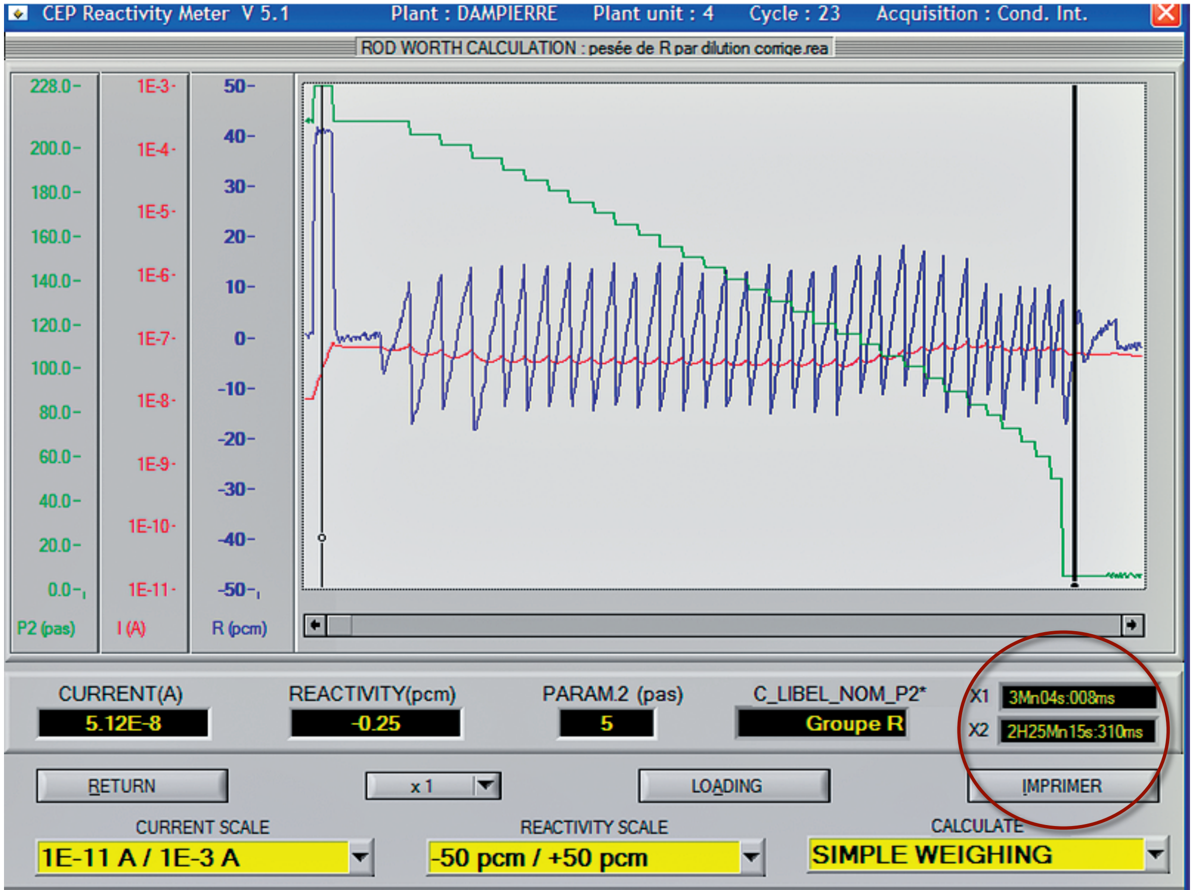
Confusing Reactivity Units

- Delta-k (actual units of PKEs) (e.g. 0.01)
- % delta-k (e.g. 1%)
- pcm = delta-k*10^5 (e.g., 1000 pcm)
- Dollars = delta-k/beta-effective ($1.5)
- Cents = Dollars*100 (e.g., 150 cents)
- milli-beta = Dollars*1000 (e.g., 1500 milli-beta)

Don't' get confused when you solve the point kinetics equations!

HZP Rod Worth Measurement: Boron Dilution

1. Make reactor stable
2. Start boron dilution
3. Move control rod in small number of steps and then wait
4. Use point kinetics to convert ex-core detector signal into reactivity
5. Infer differential and integral rod worths



Time > 1 hour/rod

General Inverse Kinetics

- Example: assume we want to know how to operate our reactor to produce a desired reactor power shape vs. time from an initial steady-state condition.
- Thus, T(t) is given, so we can solve the precursor equations

$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda}T(t) - \lambda_i C_i(t)$$

$$\frac{d}{dt}C_i(t) + \lambda_i C_i(t) = \frac{\beta_i}{\Lambda}T(t)$$

$$\frac{d}{dt}[C(t)] + Diag[\lambda_i][C(t)] = Diag[\beta_i]\frac{T(t)}{\Lambda}$$

$$\frac{d}{dt}[C(t)] + [A][C(t)] = [B]T(t) \equiv [Y(t)]$$

multiplying the o.d.e. by the IF

$$[e^{At}]\frac{d}{dt}[C(t)] + [e^{At}][A][C(t)] = [e^{At}][Y(t)]$$

recognizing that

$$\frac{d}{dt}\left[[e^{At}][C(t)]\right] = [e^{At}]\frac{d}{dt}[C(t)] + [A][e^{At}][C(t)]$$

we obtain

$$\frac{d}{dt}\left[[e^{At}][C(t)]\right] = [e^{At}][Y(t)]$$

Put Power(t) in, get C(t) out

Numerical Solution of Inverse Kinetics Equations

integrating both sides with respect to t:

$$[e^{At}][C(t)] = [A]^{-1}[e^{At}][Y(t)] + [c]$$

using the particular solution that $C_o = C(t=0)$

$$[C_0] = [A]^{-1}[Y] + [c] \Rightarrow [c] = [C_0] - [A]^{-1}[Y]$$

so

$$[e^{At}][C(t)] = [A]^{-1}\{[e^{At}][Y(t)] - [Y]\} + [C_0]$$

and finally,

$$[C(t)] = [e^{-At}][C_0] + [e^{-At}][A]^{-1}\{[e^{At}][Y] - [Y]\}$$

$$where \quad Y(t) \equiv diag[\beta_i]\frac{T(t)}{\Lambda}$$

- We can solve for time-varying concentrations of precursors for any desired reactor power shape in time by applying this equation successively for discrete steps.

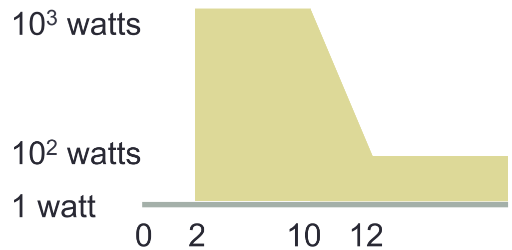
- From the PKEs, we can solve for reactivity in terms of precursor concentration/reactor power vs. time by making finite-difference approximation for derivative term:

$$\frac{d}{dt}T(t) = \frac{\rho(t) - \beta}{\Lambda}T(t) + \sum_i \lambda_i C_i(t)$$

$$\Rightarrow \rho(t) = \frac{\Lambda}{T(t)}\frac{d}{dt}T(t) + \beta - \frac{\Lambda}{T(t)}\sum_i \lambda_i C_i(t)$$

$$\Rightarrow \rho_n = \frac{\Lambda}{T_n}\frac{\left(T_n - T_{n-1}\right)}{\left(t_n - t_{n-1}\right)} + \beta - \frac{\Lambda}{T_n}\sum_i \lambda_i C_{i,n}$$

Example of Inverse Kinetics Application

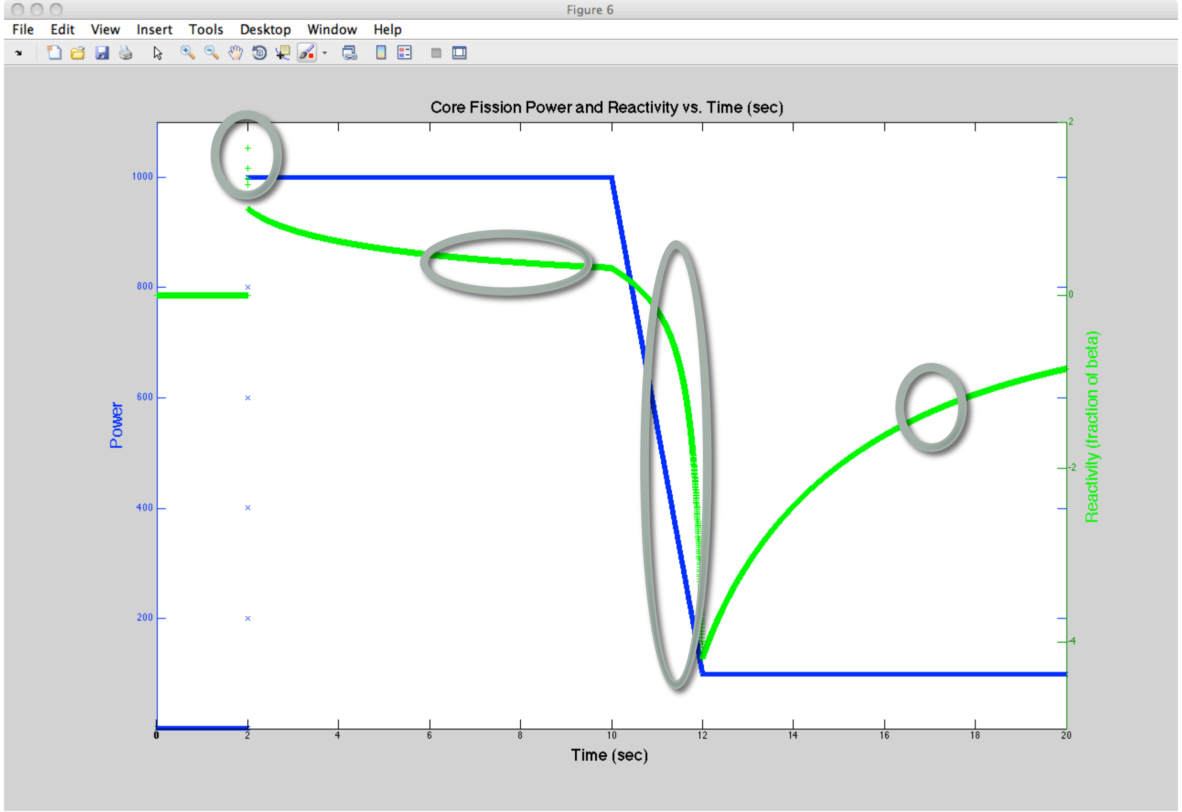


$$A = diag[\lambda_i]$$

$$Y(t) \equiv diag[\beta_i]\frac{T(t)}{\Lambda}$$

$$[C(t)] = [e^{-At}][C_0] + [e^{-At}][A]^{-1}\{[e^{At}][Y] - [Y]\}$$

$$\Rightarrow \rho(t) = \frac{\Lambda}{T(t)}\frac{d}{dt}T(t) + \beta - \frac{\Lambda}{T(t)}\sum_i \lambda_i C_i(t)$$



Inverse Kinetics Equation For SCRAM Reactivity

S. TAMURA

$$\Rightarrow \rho(t)=\frac{\Lambda}{T(t)}\frac{d}{dt}T(t)+\beta-\frac{\Lambda}{T(t)}\sum_i \lambda_i C_i(t)$$

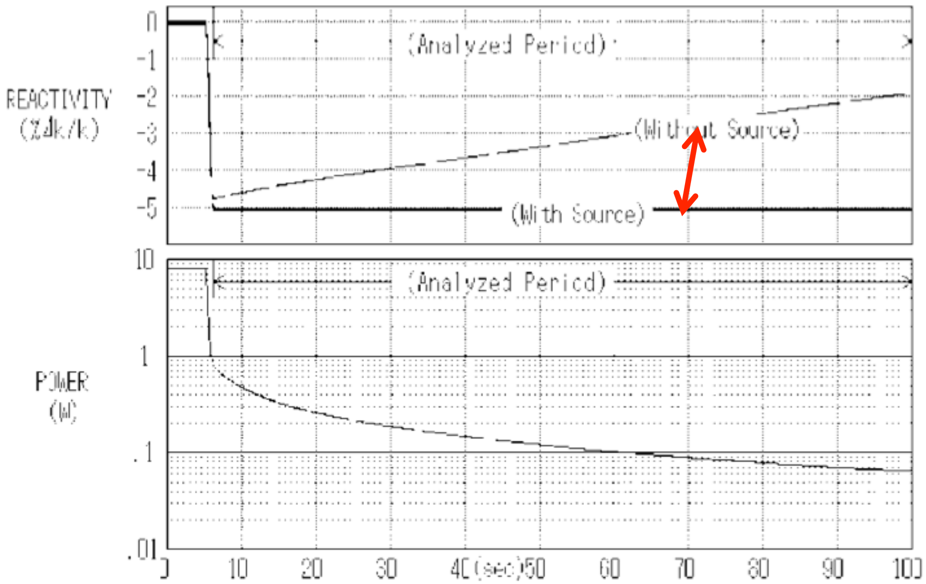


Fig. 1 Rod drop experiment (no fluctuation)

With External Neutron Sources

$$\Rightarrow \rho(t)=\frac{\Lambda}{T(t)}\frac{d}{dt}T(t)+\beta-\frac{\Lambda}{T(t)}\sum_i \lambda_i C_i(t)-\frac{\Lambda}{T(t)}S$$

Prompt Jump Approximation

$$\Rightarrow \frac{T(0^+)}{T_0}=\frac{\beta}{\beta-\rho}$$

here

$$.111=\frac{\beta}{\beta+\rho}\Rightarrow \rho=8\beta\cong .05\ \Delta k$$

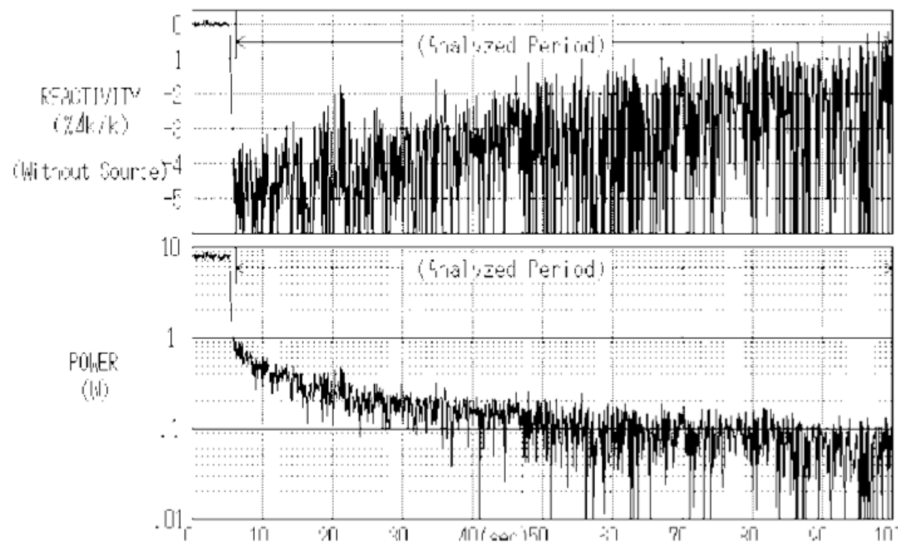


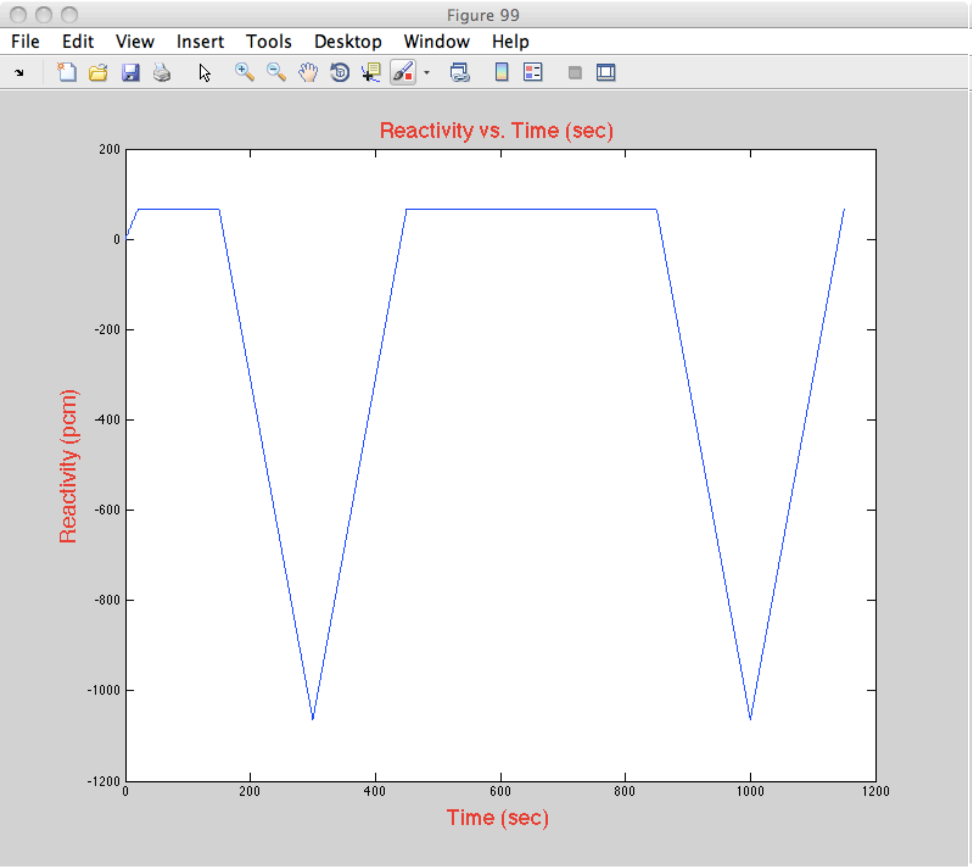
Fig. 2 Rod drop experiment (with fluctuation)

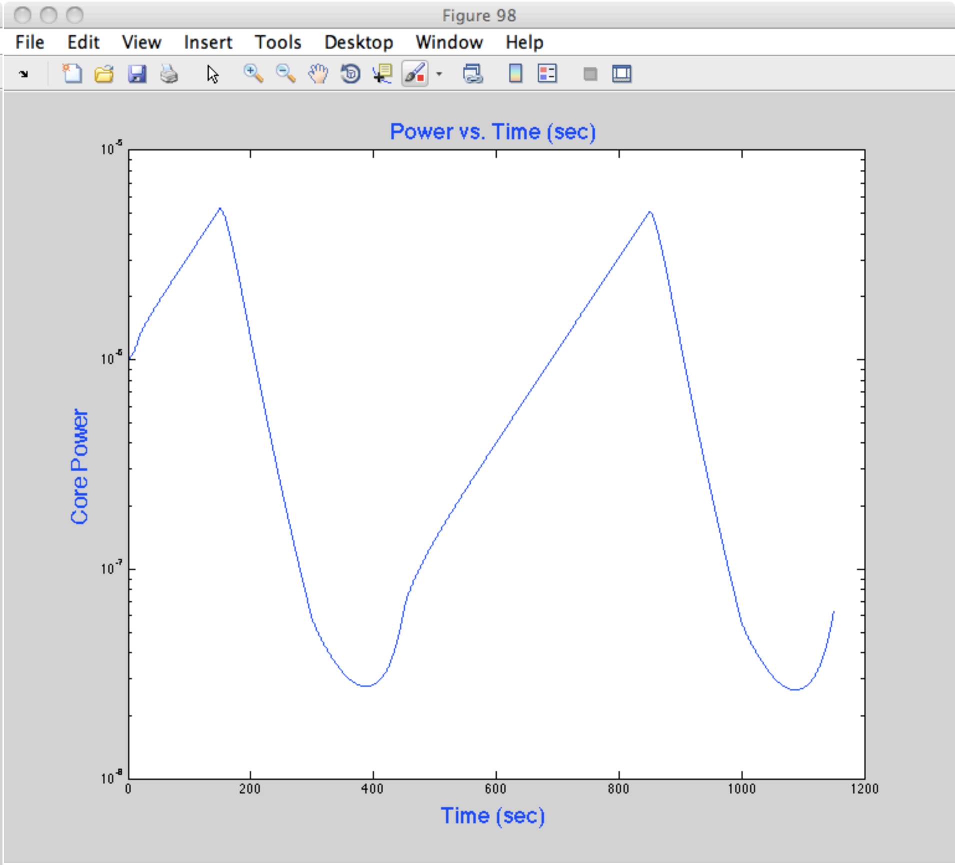
Inverse Kinetics Application: Dynamic Rod Worth Measurement

$$Y(t) \equiv diag[\beta_i]\frac{T(t)}{\Lambda}$$

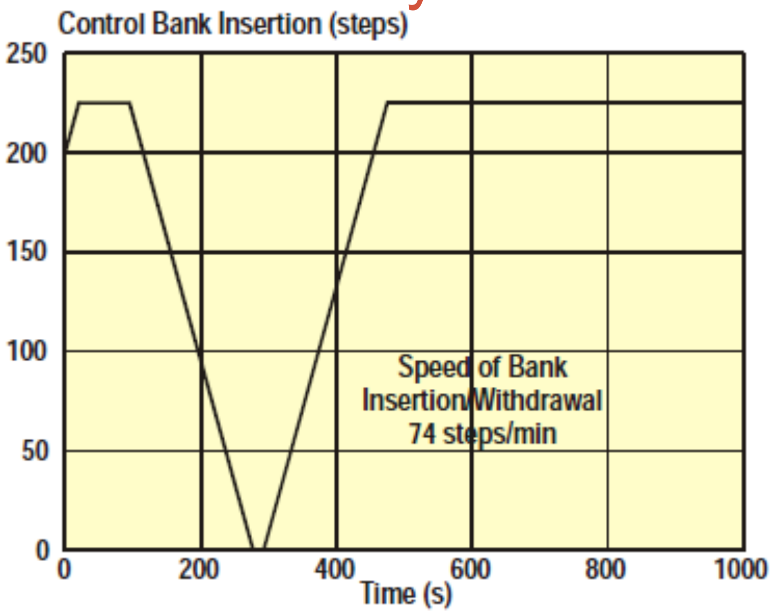
$$[C(t)] = [e^{-At}][C_0] + [e^{-At}][A]^{-1}\{[e^{At}][Y] - [Y]\}$$

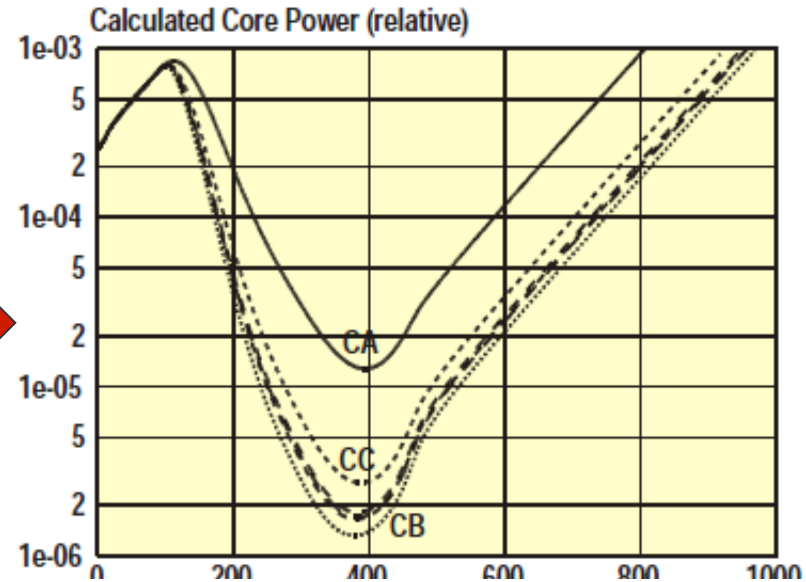
$$\Rightarrow \rho(t) = \frac{\Lambda}{T(t)}\frac{d}{dt}T(t) + \beta - \frac{\Lambda}{T(t)}\sum_i \lambda_i C_i(t)$$

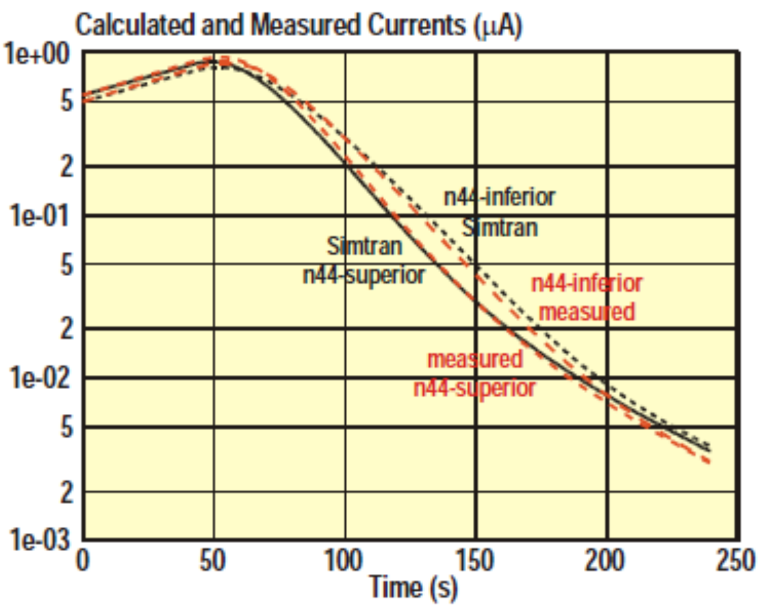


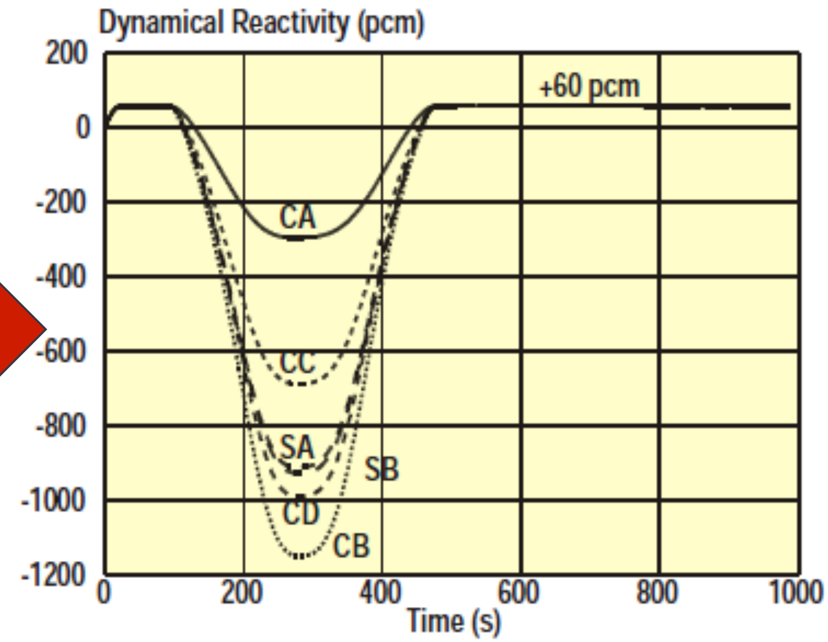


Dynamic Rod Worth: 15 Minutes/Rod





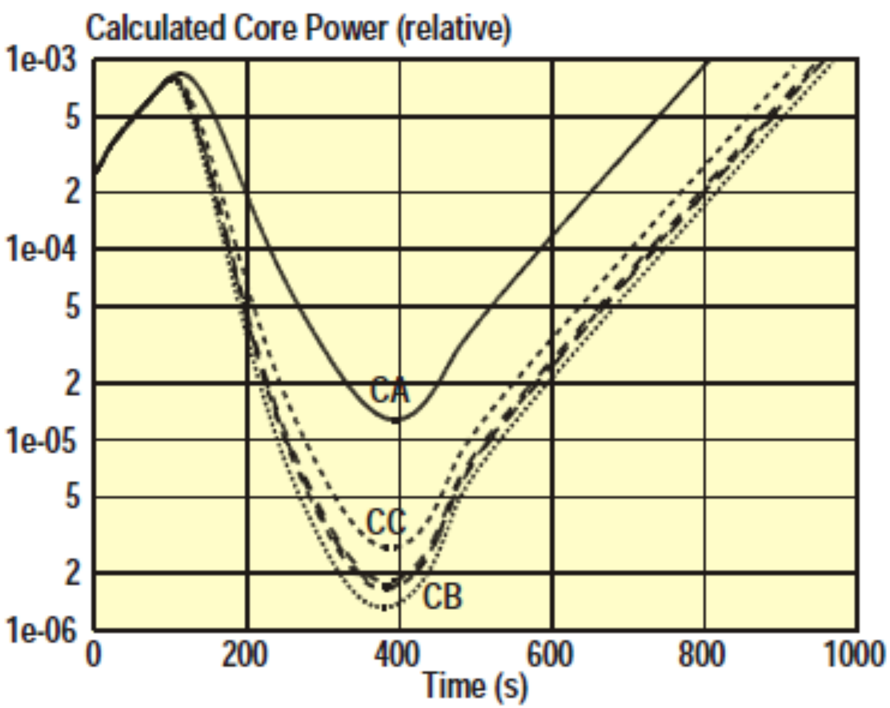




Be Careful Of Reactor Power Level!!!!

- Approximate ranges

	5 x 10^9 watts
Power range	10^9 watts
(Sensible heat)	10^8 watts
	10^7 watts
	10^6 watts
Intermediate range	10^5 watts
	10^4 watts
	10^3 watts
Critical zero power	10^2 watts
	10^1 watts
Source range	1 watt



PSET # 1
POINT KINETICS

Pset 1

Massachusetts
Institute of
Technology

PSet 1

PART A: Write a computational tool for solving PKEs

Following the MATLAB example used in class to solve the PKEs, write your own solver (in any language you choose) and incorporate the following features:

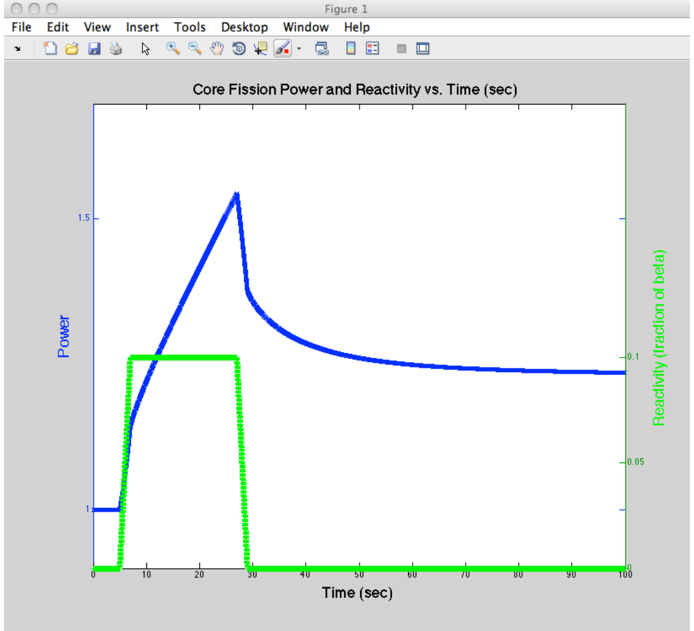
1. 8-group delayed neutron data for U^{235} fission
2. neutron velocity corresponding to 0.1 eV, sigma-fission cross section of 0.05 cm-1, and a nu-bar value of 2.45 for modeling prompt neutron lifetime
3. Initialize for steady-state operation at the start of each transient to a normalized power of 1.0
4. Incorporate ramp inputs of reactivity (as fractions of beta) vs. time so your code simulates reactor response to input reactivity changes
5. Produce plots of reactivity and power versus time

PSet 1

PART B: Testing your PKE Tool

Produce the following results to demonstrate that your tool functions correctly:

1. The value of total delayed neutron fraction (beta)
2. The value of prompt neutron lifetime (pnl) in seconds
3. Produce plot of relative power and reactivity vs. time for 100 seconds following a 2 second ramp insertion of +.1 beta, a hold for 20 seconds, and a 2 second ramp back to zero reactivity. (see Ex 6)
4. Make sure time-step size is small enough that results are converged.

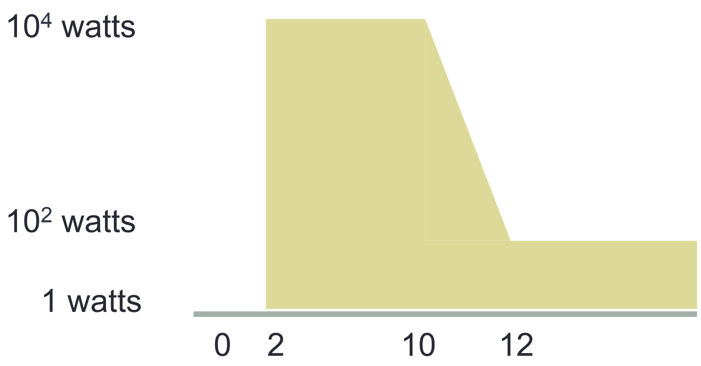


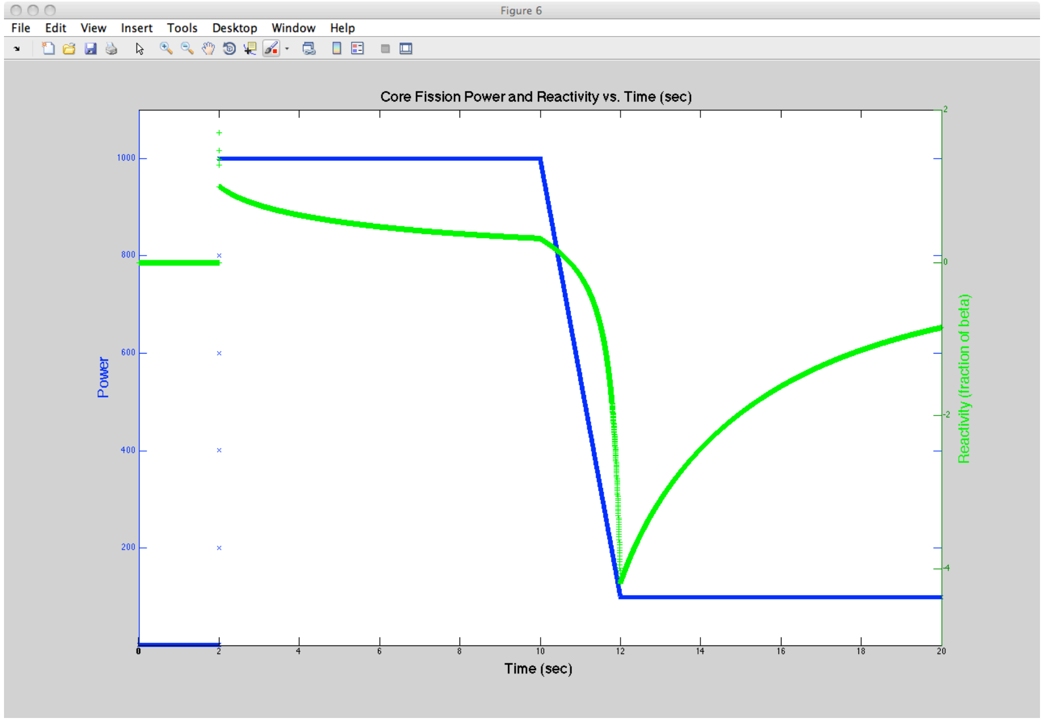
PSet 1

PART C: Write Inverse Kinetics Tool

- Solve IKE for reactivity vs. time that will match desired power shape

- Run PKE with your reactivity shape and prove it reproduces desired power shape





Assignment for Next Class

- Review basic physics: e.g., Neutron Physics by Paul Reuss for PKEs
- Make sure you are can use MATLAB or some low-level language:
  - Logical tests
  - Array-based computations
  - Simple loops
  - Informative plots
- Get on with PSet 1
- Start looking at solution techniques s for 1-D and 2D finite-difference diffusion equations.