NUCLEAR REACTOR KINETICS

Lecture 4

Point Kinetics Derived from Static Spatial Diffusion Solutions of the 2-group Diffusion Equations

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Course Outline

22.213 (22.S904) Calendar					
Lecture #	Date	Topic	LECTURER	Read	Assignment Handed Out
1	5-Sep	Course Overview and First Day Exam	Smith		
2	10-Sep	Review of Delayed Neutrons and Point Kinetics Equations	Smith		PSET # 1: Point Kinetics
3	12-Sep	Review Steady-State Finite-Difference Diffusion Methods (1D, 2D)	Smith		
4	17-Sep	Generalized PKEs from Spatial Finite-Difference Diffusion	Smith		PSET # 2: 2-D Steady-State Diffusion
5	19-Sep	Basic Transient Finite-Difference with Direct Solutions	Smith		
6	24-Sep	Higher-order Time Integration and Runga-Kutta	Smith		PSET # 3: 2-D Fully-Implicit Diffusion
7	26-Sep	Time Stepping for Automatic Error Control	Smith		
8	1-Oct	PKE with Feedback: Operator Splitting/Exact Integration	Smith		PSET # 4: PKE from 2-D Diffusion
9	3-Oct	Quasi-Static Time-Integration and Synthesis Methods	Smith		
	8-Oct	Columbus Holiday (8th and 9th)			
10	10-Oct	2D F-I Iterative Numerical Methods: PJ, GS, SOR	Smith		PSET # 5: PKE Time Step Control
11	15-Oct	Iterative Numerical Methods: CG, GMRES,???	Smith		
12	17-Oct	Coarse Mesh Rebalance & Nonlinear Diffusion Acceleration	Smith		PSET # 6: PKE with Nonlinear Feedback
13	22-Oct	Nodal Methods: Kinetic Distortion and Frequency Transformation	Smith		
	24-Oct	Midterm Exam			
14	29-Oct	Midterm Detailed Exam Solution/2D LRA SS Comparisons	Smith		PSET # 7: CMR and NDA acceleration
15	31-Oct	Multigrid Acceleration Methods	Smith		
16	5-Nov	JFNK for Non-linear Systems	Smith		2-D LRA Rod Ejection Contest
17	7-Nov	Transient Sn	Smith		
	12-Nov	Veterns Day Holiday			
	14-Nov	Special Project Work Period	ANS Meeting		
18	19-Nov	Transient MOC	Smith		
19	21-Nov	Parallel Solver Technologies (PetSc)	Herman/Roberts		
20	26-Nov	So You Want To Be A Professor? Student Lectures	?????		
21	28-Nov	So You Want To Be A Professor? Student Lectures	?????		
22	3-Dec	So You Want To Be A Professor? Student Lectures	?????		
23	5-Dec	So You Want To Be A Professor? Student Lectures	?????		
24	10-Dec	So You Want To Be A Professor? Student Lectures	?????		
25	12-Dec	Last Day of Class General Wrapup, Cats and Dogs, Critique	Smith		
	17-21 Dec	Finals Week - No Exam for 22.S904 (22.213)			

Today's Lecture: Goals

- Discuss final questions about PSet 1 (due at midnight)
- Explain PSet 2 assignment to solve 1-D, 2-group finite-difference problems
- Reiterate some points about 1-D, 2-group finite-difference equations
- Reiterate some important points about numerical solutions
- Derive more general Point Kinetics Equations
- Understand adjoint fluxes for critical reactor multiplication factor
- Understand some implications of using adjoint-weighted PKEs
- Understand strategy for upcoming lectures/assignments

PSET # 1
POINT KINETICS

Due: Sept 17, 2012

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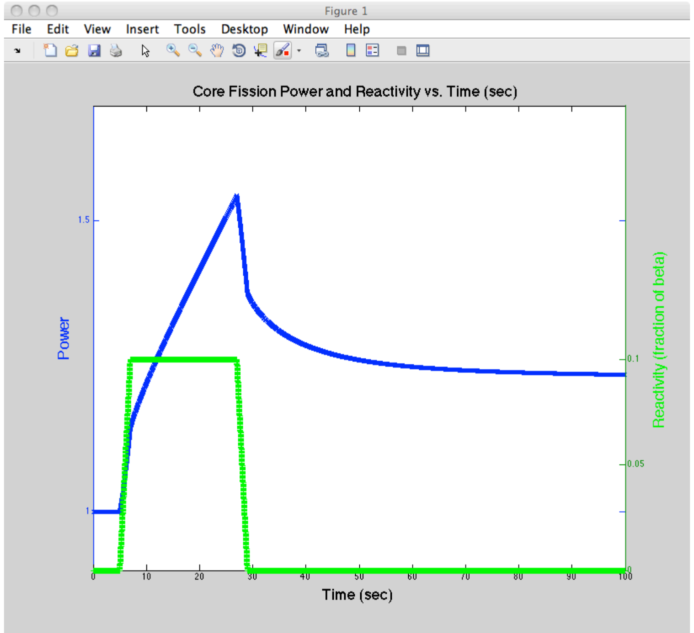
PSet 1

PART B: Testing your PKE Tool

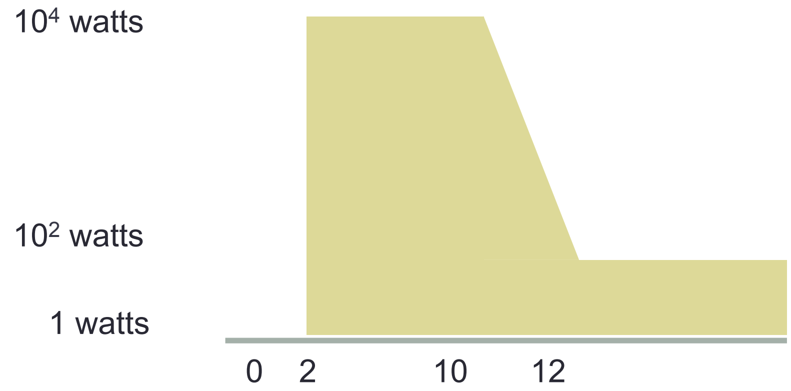
Produce the following results to demonstrate that your tool functions correctly:

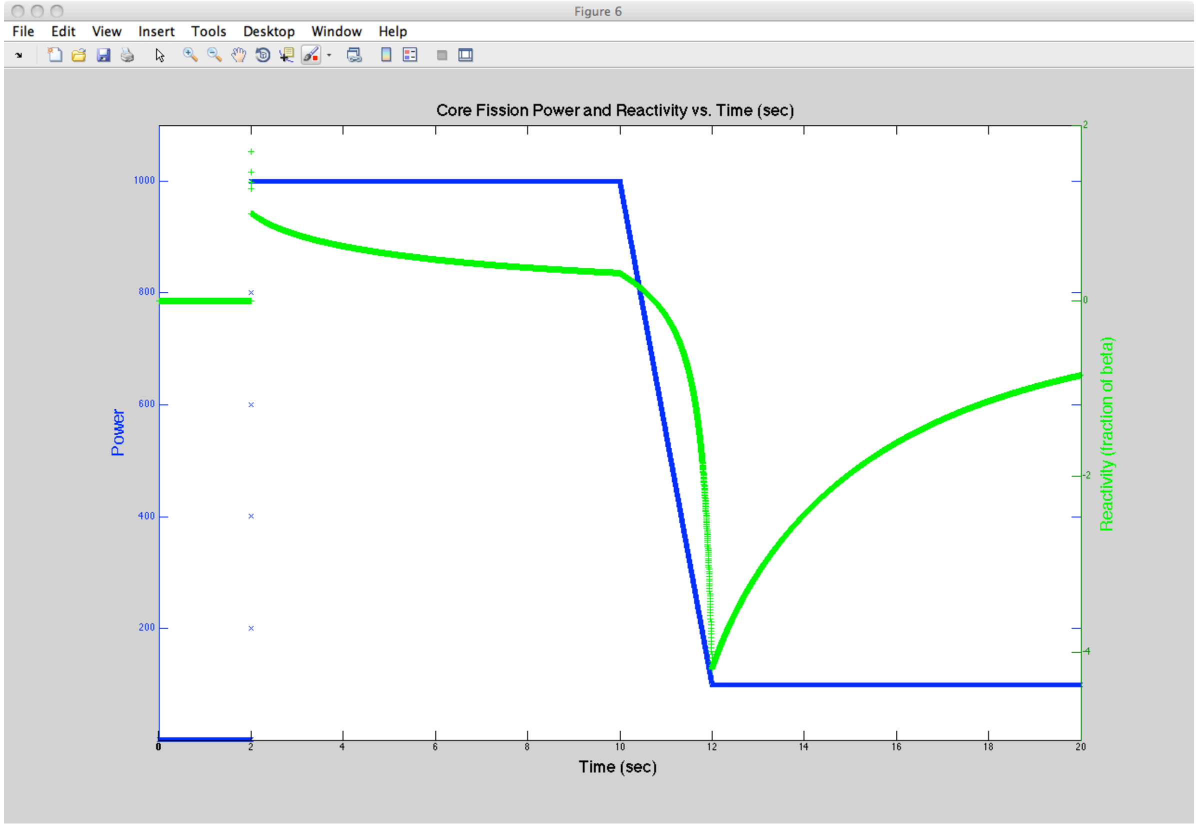
1. The value of total delayed neutron fraction (beta)
2. The value of prompt neutron lifetime (pnl) in seconds
3. Produce plot of relative power and reactivity vs. time for 100 seconds following a 2 second ramp insertion of +.1 beta, a hold for 20 seconds, and a 2 second ramp back to zero reactivity. (see Ex 6)
4. Make sure time-step size is small enough that results are converged.



PSet 1

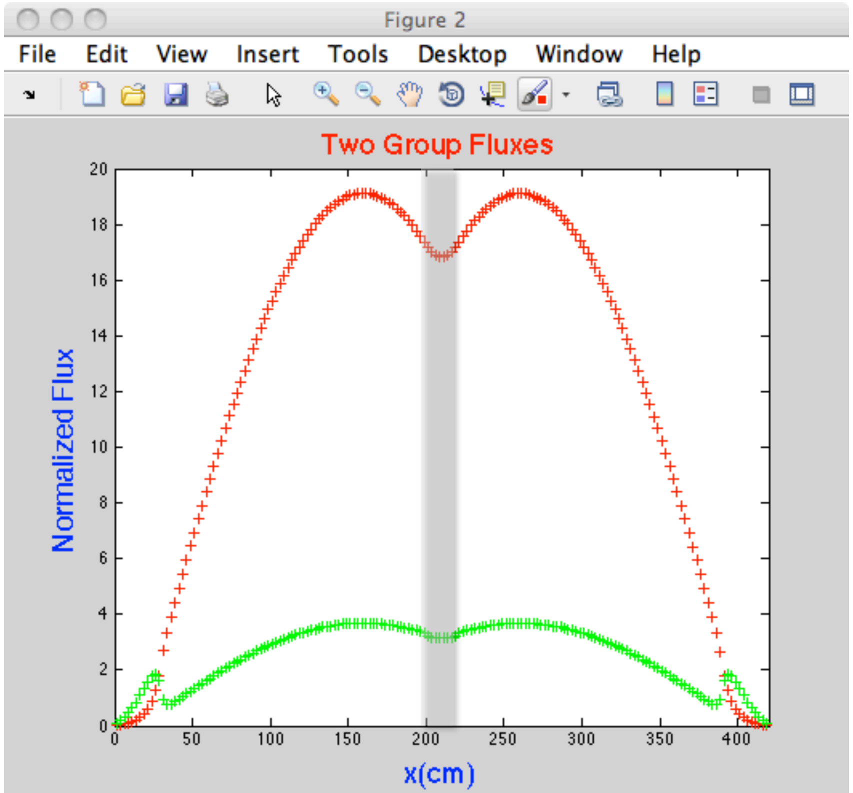
PART C: Write Inverse Kinetics Tool

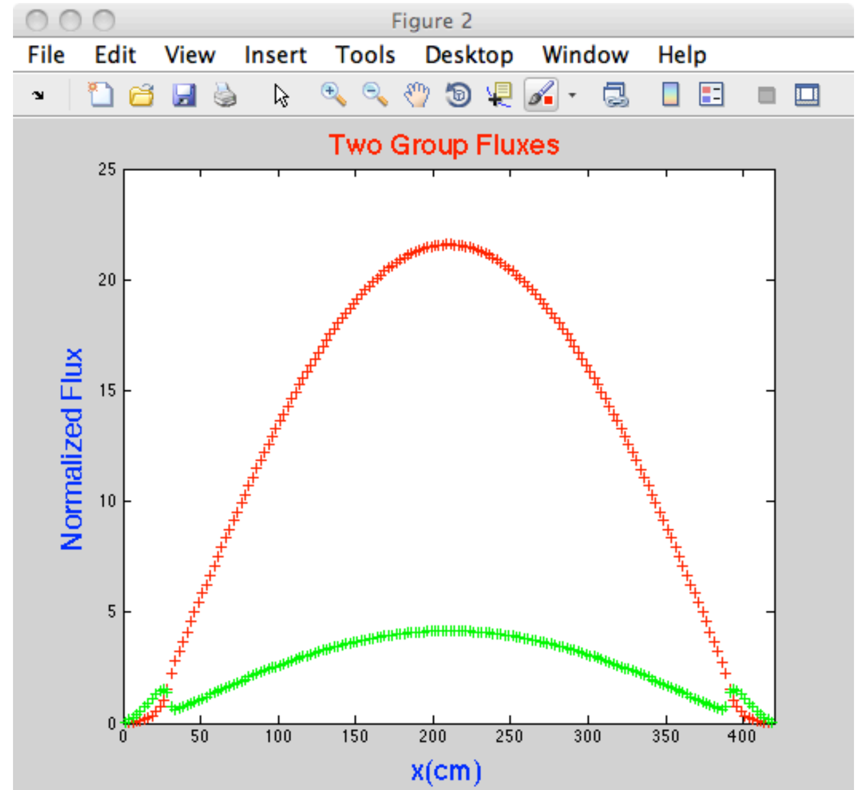




Pset 2: Starting Point for Transient Calculations: Steady-State Solutions

Transient Control Rod Withdrawal in a 1-D Reflected Core: Beginning/Ending Shapes





Simulations in 1-D:

Static Solutions

Dynamic Solutions

Point-Kinetics

Generalized Point-Kinetics

Improved Quasi-Static Methods

PSET # 2
SOLVING 1-D, 2-GROUP FINITE-DIFFERENCE EQUATIONS

Due: Sept 24, 2012

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Pset 2

Solve the Rodded and Unrodded, 1-D, 2-Group Diffusion Problems for the Following Materials, Geometry, and b.c.s

D1, D2, Sigma-a1, Sigma-a2, Sig1→2, Nu*sigma-f1, Nu*sigma-f2

```
%
% define two group cross sections for all potential materials
%
  nmat=5;
  xs(1,1)=1.300; xs(2,1)=0.500; xs(3,1)=0.0098; xs(4,1)=0.114; xs(5,1)=0.022; xs(6,1)=0.006; xs(7,1)=0.1950;  % 3% enriched fuel
  xs(1,2)=1.300; xs(2,2)=0.500; xs(3,2)=0.0105; xs(4,2)=0.134; xs(5,2)=0.022; xs(6,2)=0.008; xs(7,2)=0.2380;  % 4% enriched fuel
  xs(1,3)=1.500; xs(2,3)=0.500; xs(3,3)=0.0002; xs(4,3)=0.010; xs(5,3)=0.032; xs(6,3)=0.000; xs(7,3)=0.0000;  % water
  xs(1,4)=1.300; xs(2,4)=0.500; xs(3,4)=999.99; xs(4,4)=999.9; xs(5,4)=0.020; xs(6,4)=0.000; xs(7,4)=0.0000;  % black absorber
  xs(1,5)=1.300; xs(2,5)=0.500; xs(3,5)=0.0098; xs(4,5)=0.118; xs(5,5)=0.022; xs(6,5)=0.006; xs(7,5)=0.1950;  % 3% enriched + rod
%
% define problem geometry and assign materials:
%
%   #zones=NZONE w(nzone),n(nzone), mat(nzone)
%
%  bc | slab 1| slab 2| slab 3| ..... slab(NZONE) | bc
%

  NZONE=5;                                              % numer of material zones
  w(1)= 30; w(2)=170; w(3)=20;  w(4)=170; w(5)=30;     % width    per zone
  n(1)= 3;  n(2)=17;  n(3)=2;   n(4)=17;  n(5)=3;      % mesh     per zone
  m(1)= 3;  m(2)=1 ;  m(3)=5;   m(4)=1;   m(5)=3;      % material per zone
  n=n*4;                                                % mesh refinement factor

  bc=2;          % 0=zero flux, 1=zero incoming, 2=reflective bc
```

1. b.c. | Reflector | Fuel-1 | Rod | Fuel-1| Reflector | b.c.
2. Cross sections, zone widths, and cross section given above
3. Reflective b.c. on outer surfaces (you will see why later)

Pset 2

Write your own diffusion solver (in any language you choose) using power iteration with P-J and G-S iterative flux inversion

PART A: Difference Equations

- Derive the expression for the first-order finite-difference net current at a nodal interface for the case of variable mesh spacing/material properties.

PART B: Spatial Convergence

- Plot iteratively-converged eigenvalue and L_2 norm of nodal power error (using 10 cm nodes) vs. mesh spacing until the L_2 norm of error is converged to $< 1.e-6$ for the rodded and unrodded cores.

PART C: Dominance Ratios

- Plot the asymptotic dominance ratio vs. mesh spacing for the rodded and unrodded cores.

Pset 2

PART D: Iterative Convergence of P-J

- Plot the number of fission source iterations needed to achieve L_2 norm of changes of nodal powers for successive fission source iterations < 1.e-6 vs. flux iteration point-wise L_2 norm for flux convergence criteria of 1.e-1, 1.e-2, 1.e-3, 1.e-4, and 1.e-5 for the rodded and unrodded cores.

PART E: Iterative Convergence of G-S

- Plot the number of fission source iterations needed to achieve L_2 norm of changes of nodal powers for successive fission source iterations < 1.e-6 vs. flux iteration point-wise L_2 norm for flux convergence criteria of 1.e-1, 1.e-2, 1.e-3, 1.e-4, and 1.e-5 for the rodded and unrodded cores.

PART F: Real vs. Adjoint Fluxes

1. What are the spatially and iteratively converged real and adjoint eigenvalues for the rodded and unrodded problems?

2. What is the static rod worth in pcm?

3. Plot the spatially and iteratively converged real and adjoint fluxes for the rodded and unrodded problems.

2-Group LWR Neutron Diffusion Equation

Group structure for LWR analysis:

- Group 1: E ≥ 0.625 eV → fast group
- Group 2: E < 0.625 eV → thermal group
- χ_1 = 1.0 and χ_2 = 0.0 which implies that fission source in thermal group = 0

Scattering source:

- Define effective down-scatter, so up-scattering is zero → $\Sigma_{2\to 1}(\vec{r}) = 0$

Final 2-group diffusion equations:

$$-\nabla \cdot D_1(\vec{r})\nabla\phi_1(\vec{r}) + [\Sigma_{a1}(\vec{r}) + \Sigma_{s12}(\vec{r})]\phi_1(\vec{r}) = \nu\Sigma_{f1}(\vec{r})\phi_1(\vec{r}) + \nu\Sigma_{f2}(\vec{r})\phi_2(\vec{r})$$

$$-\nabla \cdot D_2(\vec{r})\nabla\phi_2(\vec{r}) + \Sigma_{a2}(\vec{r})\phi_2(\vec{r}) = \Sigma_{s12}(\vec{r})\phi_1(\vec{r})$$

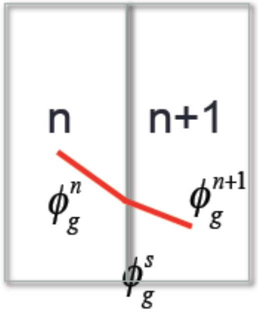
Be able To Write This In Your Sleep!!!

2-Group Finite-Difference Diffusion Equations: Uniform Mesh

Consider two neighboring cells having the same width, and make a linear flux approximation between the cell centers and cell edges:

$$J_n^+ = -D_g^n \frac{d}{dx}\phi_g^n(x)\Big|_{n+} = -D_g^n \frac{\phi_g^s - \phi_g^n}{\Delta/2}$$

$$J_n^- = -D_g^{n+1} \frac{d}{dx}\phi_g^{n+1}(x)\Big|_{n+} = -D_g^{n+1} \frac{\phi_g^{n+1} - \phi_g^s}{\Delta/2}$$



and by continuity of net current one solves for interface flux

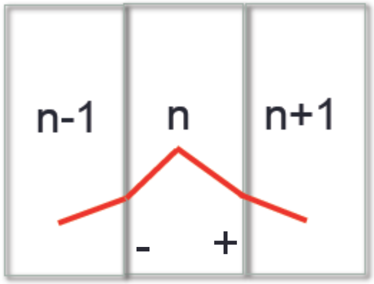
$$-D_g^n \frac{\phi_g^s - \phi_g^n}{\Delta/2} = -D_g^{n+1} \frac{\phi_g^{n+1} - \phi_g^s}{\Delta/2} \Rightarrow \phi_g^s = \frac{D_g^n \phi_g^n + D_g^{n+1}\phi_g^{n+1}}{(D_g^n + D_g^{n+1})}$$

one gets an approximation for the net current in terms of mesh-averaged fluxes:

$$J_n^+ = -\frac{2D_g^n}{\Delta}\left[\frac{D_g^n \phi_g^n + D_g^{n+1}\phi_g^{n+1}}{(D_g^n + D_g^{n+1})} - \phi_g^n\right] = \frac{2D_g^n D_g^{n+1}}{\Delta(D_g^n + D_g^{n+1})}\phi_g^{n+1} - \frac{2D_g^n D_g^{n+1}}{\Delta(D_g^n + D_g^{n+1})}\phi_g^n$$

2-Group Finite-Difference Diffusion Mesh Balance Equations

Consider two neighboring cells having the <u>same width</u>, and make a linear flux approximation between the cell centers and cell edges:



$$J_n^+ = -\hat{D}_g^{n,n+1}\left[\phi_g^{n+1} - \phi_g^n\right] \qquad J_n^- = -\hat{D}_g^{n-1,n}\left[\phi_g^n - \phi_g^{n-1}\right] \quad \textit{where} \quad \hat{D}_g^{n,n+1} \equiv \frac{2D_g^n D_g^{n+1}}{\Delta(D_g^n + D_g^{n+1})}$$

and one obtains two group balance equations:

$$\hat{D}_1^{n-1.n}\left[\phi_1^n - \phi_1^{n-1}\right] - \hat{D}_1^{n,n+1}\left[\phi_1^{n+1} - \phi_1^n\right] + \Sigma_{r1}^n \Delta\phi_1^n = \Sigma_{f1}^n \Delta\phi_1^n + \nu\Sigma_{f2}^n \Delta\phi_2^n$$

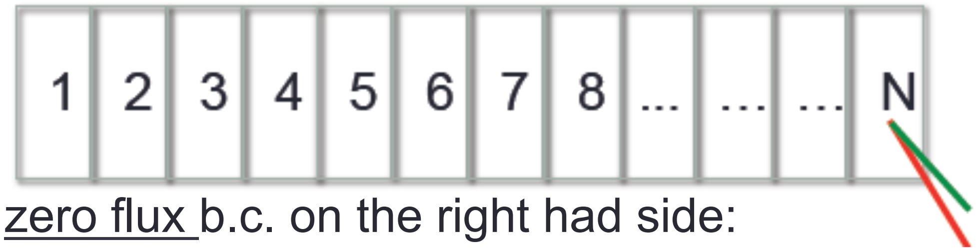
$$\hat{D}_2^{n-1.n}\left[\phi_2^n - \phi_2^{n-1}\right] - \hat{D}_2^{n,n+1}\left[\phi_2^{n+1} - \phi_2^n\right] + \Sigma_{a2}^n \Delta\phi_1^n = \Sigma_{s12}^n \Delta\phi_1^n$$

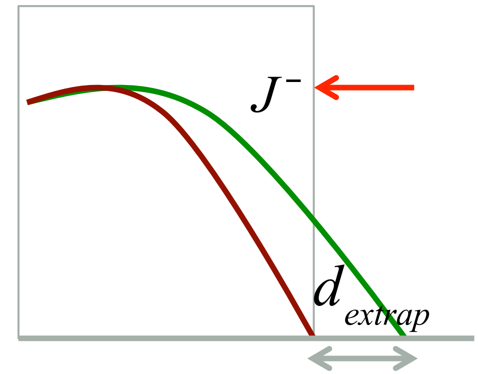
and simplifying one obtains the linear finite-difference diffusion equations:

$$-\hat{D}_1^{n-1.n}\phi_1^{n-1} + \left[\Sigma_{r1}^n\Delta + \hat{D}_1^{n-1.n} + \hat{D}_1^{n,n+1}\right]\phi_1^n - \hat{D}_1^{n,n+1}\phi_1^{n+1} = \Sigma_{f1}^n \Delta\phi_1^n + \nu\Sigma_{f2}^n \Delta\phi_2^n$$

$$-\hat{D}_2^{n-1.n}\phi_2^{n-1} + \left[\Sigma_{a2}^n\Delta + \hat{D}_2^{n-1.n} + \hat{D}_2^{n,n+1}\right]\phi_2^n - \hat{D}_2^{n,n+1}\phi_2^{n+1} = \Sigma_{s12}^n \Delta\phi_1^n$$

Boundary Conditions for Finite-Difference Equations





$$\phi(\vec{r}_s) = 0 \qquad \phi(\vec{r}_s + d_{extrap}) = 0$$

For zero flux b.c. on the right had side:

$$J_g^N = -D_g^N \frac{\left[-\phi_g^N\right] - \phi_g^N}{\Delta} \Rightarrow J_g^N = \left[\frac{2D_g^N}{\Delta}\right]\phi_g^n$$

note: net current is positive on right surface

For zero incoming flux b.c.

$$J^-(\vec{r},E) = \frac{1}{4}\phi(\vec{r},E) - \frac{1}{2}J_n(\vec{r},E)$$

which implies

$$J_g^N = -D_g^N \frac{\phi_g^s - \phi_g^N}{\Delta/2} = \frac{\phi_g^s}{2} \quad \Rightarrow \quad \phi_g^s\left(\frac{1}{2} + \frac{2D_g^N}{\Delta}\right) = \frac{2D_g^N}{\Delta}\phi_g^N \quad \Rightarrow\Rightarrow J_g^N = -\frac{2D_g^N}{\Delta}\left[\frac{\frac{2D_g^N}{\Delta}}{\left(\frac{1}{2} + \frac{2D_g^N}{\Delta}\right)} - 1\right]\phi_g^N$$

$$J_g^N = \left[\frac{1}{1 + \frac{4D_g^{N+1}}{\Delta}}\right]\left[\frac{2D_g^N}{\Delta}\right]\phi_g^N$$

∞

For zero current

2-Group 1-D Finite-Difference Matrix Equations

If we number our cells consecutively,

1	2	3	4	5	6	7	8	...	...	...	N

We can express the finite difference equations in matrix form as:

$$\left[L+D+U\right]_1\left[\phi_1\right]=[M]_1\left[\phi_1\right]+[M]_2\left[\phi_2\right]$$
$$\left[L+D+U\right]_2\left[\phi_2\right]=[T]_2\left[\phi_1\right]$$

Defining a vector of group fluxes of group one followed by group two fluxes:

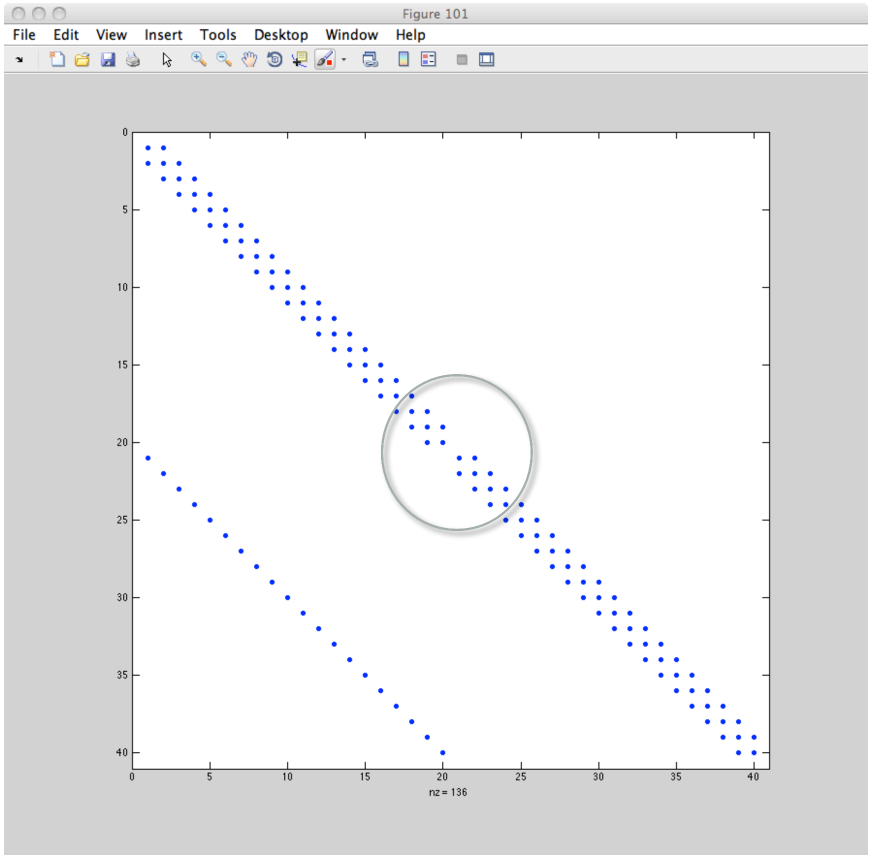
$$\begin{bmatrix} \left[L+D+U\right]_1 & [0] \\ -[T]_2 & \left[L+D+U\right]_1 \end{bmatrix}\begin{bmatrix}\phi_1 \\ \phi_2\end{bmatrix}=\begin{bmatrix}[M]_1 & [M]_2 \\ [0] & [0]\end{bmatrix}\begin{bmatrix}\phi_1 \\ \phi_2\end{bmatrix}$$

Or, in compressed matrix notation:

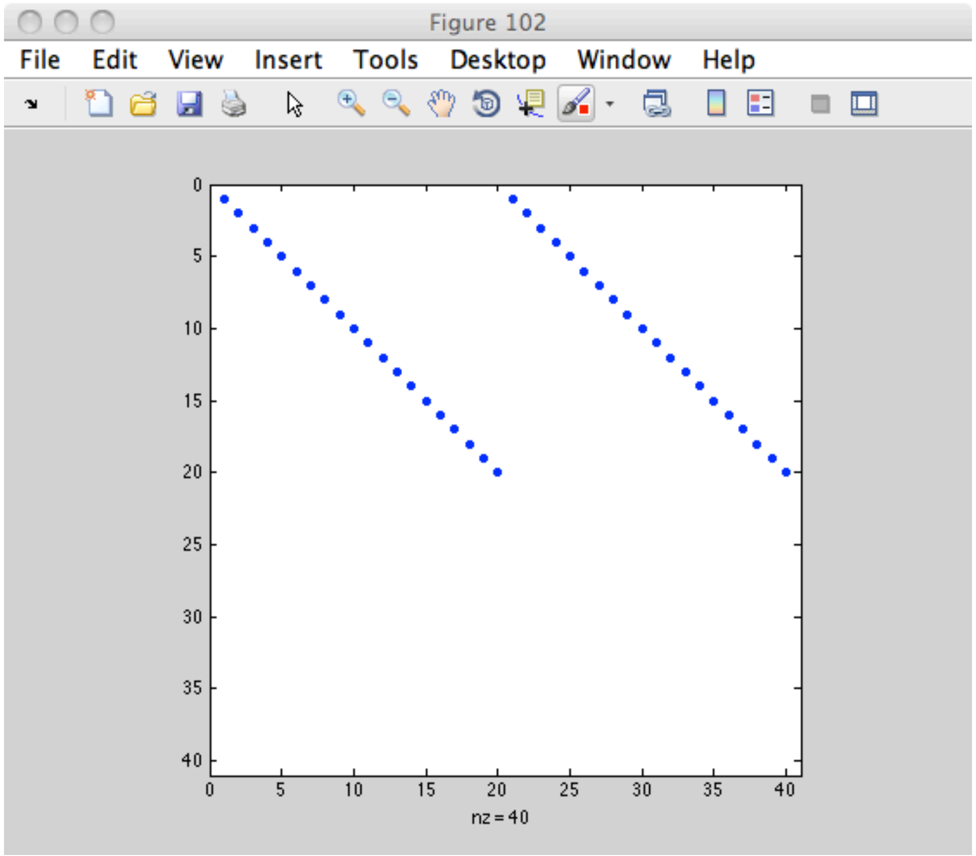
$$\left[A\right]\left[\phi\right]=\left[M\right]\left[\phi\right]$$

Note: vector length is 2N

Constructing Destruction and Production Matrix



$[A]$



$[M]$

$$\begin{bmatrix} [L+D+U]_1 & [0] \\ -[T]_2 & [L+D+U]_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} [M]_1 & [M]_2 \\ [0] & [0] \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

Power Iteration with P-J and Gauss-Seidel Iterative Flux Matrix Inversion

Point-Jacobi

$$[A][\phi]^{i+1} = \frac{1}{k_{eff}}[M][\phi]^{i} \qquad k_{eff}^{i+1} = \frac{[M][\phi]^{i+1}}{[M][\phi]^{i}}$$

"i" is fission source iteration index

$$[A]=[D]+[O] \quad \Rightarrow \quad [D][\phi]^{m+1} = \frac{1}{k_{eff}}[M][\phi]^{i} - [O][\phi]^{m}$$

$$[\phi]^{m+1} = [D]^{-1}\left\langle \frac{1}{k_{eff}}[M][\phi]^{i} - [O][\phi]^{m} \right\rangle$$

"m" is flux iteration index

Gauss-Seidel

$$[A][\phi]^{i+1} = \frac{1}{k_{eff}}[M][\phi]^{i} \qquad k_{eff}^{i+1} = \frac{[M][\phi]^{i+1}}{[M][\phi]^{i}}$$

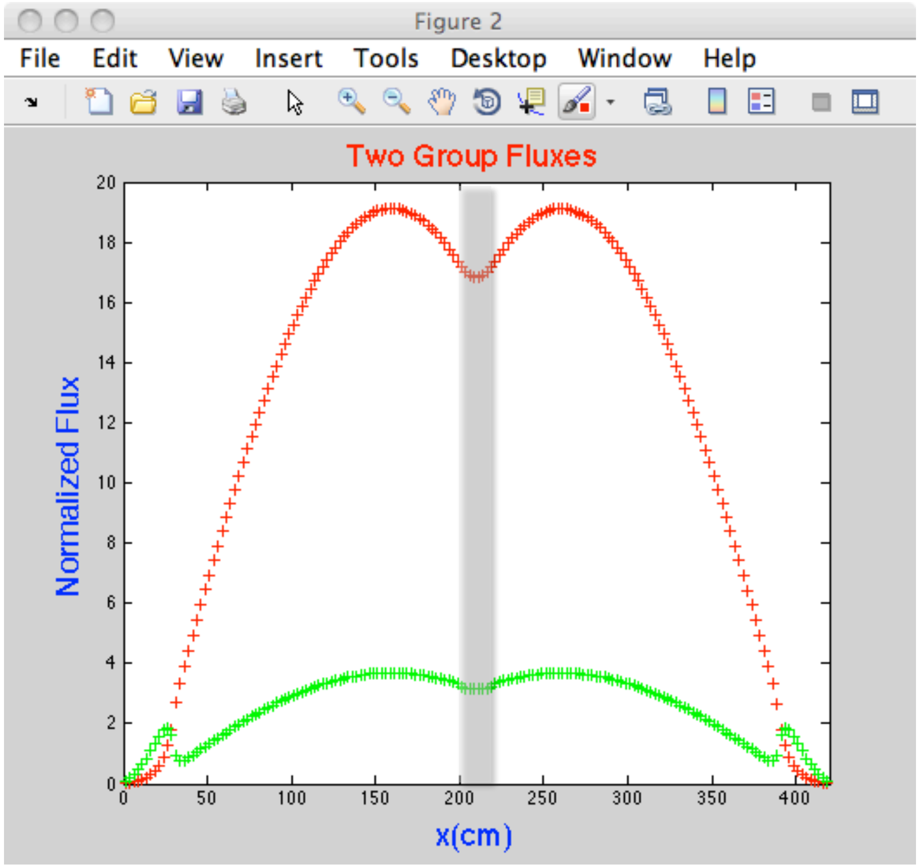
"i" is fission source iteration index

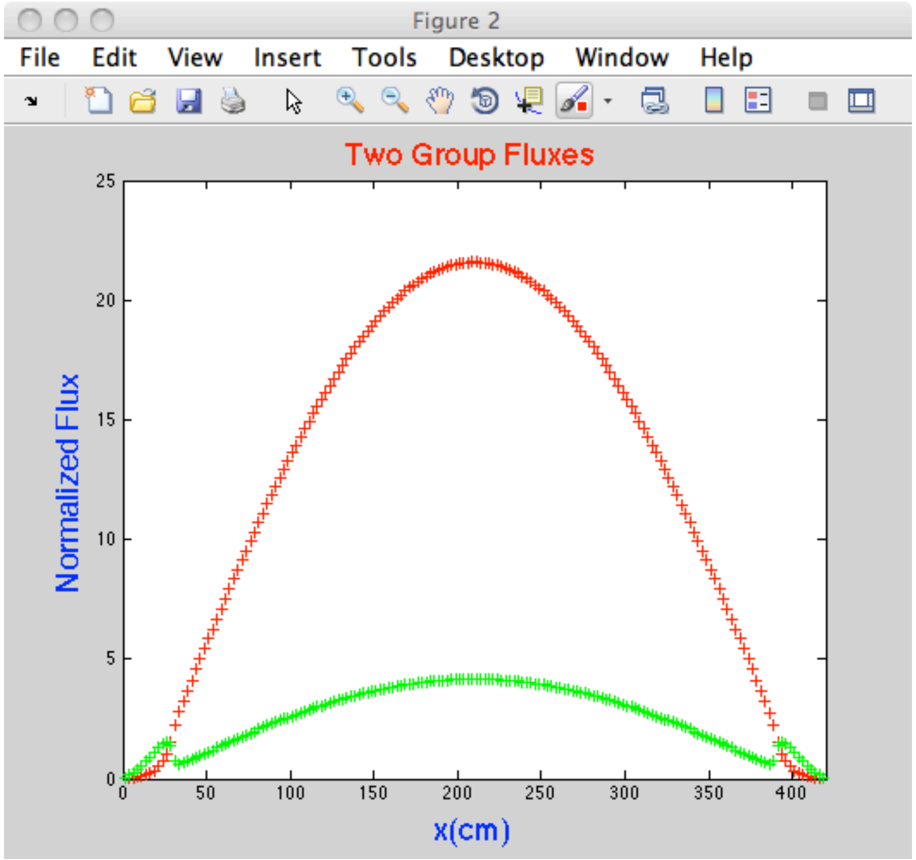
$$[A]=[L]+[D]+[U] \quad \Rightarrow \quad \left\langle [L]+[D] \right\rangle [\phi]^{m+1} = \frac{1}{k_{eff}}[M][\phi]^{i} - [U][\phi]^{m}$$

$$[\phi]^{m+1} = \left\langle [L]+[D] \right\rangle^{-1}\left\langle \frac{1}{k_{eff}}[M][\phi]^{i} - [U][\phi]^{m} \right\rangle$$

"m" is flux iteration index

Transient Control Rod Withdrawal in1-D Reflected Core





Simulations in 1-D

Static Solutions

Point Kinetics Equations from Lecture 2

Assume flux can be separated into space/energy and time-dependent terms: $\phi(\vec{r},E,t)=S(\vec{r},E)T(t)$

If one integrates over all space and energy and normalize $\int dE \int d\vec{r}\,\frac{1}{v}S(\vec{r},E)\equiv 1.0$,

$$\frac{d}{dt}\big[T(t)\big]=\int dE\int d\vec{r}\left[\begin{array}{l}\nabla\cdot D(\vec{r},E,t)\nabla\, S(\vec{r},E)-\Sigma_t(\vec{r},E,t)\, S(\vec{r},E)+\int_0^\infty \Sigma_s(\vec{r},E'\to E,t)\, S(\vec{r},E')\, dE' \\ +\sum_j\left\{\chi_p^j(E)(1-\beta^j)+\sum_i \chi_d^i(E)\beta_i^j\right\}\int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE'\end{array}\right]T(t)$$

$$-\int dE\int d\vec{r}\left[\sum_j \chi_d^j(E)\beta^j\int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE'\right]T(t)$$

$$+\int dE\int d\vec{r}\left[\sum_i \chi_d^i(E)\lambda_i C_i(\vec{r},t)\right]$$

$$+\int dE\int d\vec{r}\left[Q(\vec{r},E,t)\right]$$

and

$$\frac{d}{dt}\left[\int dE\int d\vec{r}\; C_i(\vec{r},t)\right]=\int dE\int d\vec{r}\left[\sum_j \beta_i^j\int_0^\infty \nu\Sigma_p^j(\vec{r},E',t)\, S(\vec{r},E')\, dE'\right]T(t)-\lambda_i\int dE\int d\vec{r}\; C_i(\vec{r},t)$$

Point Kinetics Equations: Slightly Simplified

If one integrates over all space and energy and normalize $\int dE \int d\vec{r}\, \frac{1}{v} S(\vec{r},E) \equiv 1.0$,

$$\frac{d}{dt}\left[T(t)\right] = \int dE \int d\vec{r} \left[\begin{array}{l} \nabla \cdot D(\vec{r},E,t)\nabla\, S(\vec{r},E) - \Sigma_t(\vec{r},E,t)\, S(\vec{r},E) + \int_0^\infty \Sigma_s(\vec{r},E' \rightarrow E,t)\, S(\vec{r},E')\, dE' \\ + \sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \end{array} \right] T(t)$$

$$- \int d\vec{r} \left[\sum_j \beta^j \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right] T(t)$$

$$+ \int dE \int d\vec{r} \left[\sum_i \chi_d^i(E)\lambda_i C_i(\vec{r},t) \right]$$

$$+ \int dE \int d\vec{r} \left[Q(\vec{r},E,t) \right]$$

and

$$\frac{d}{dt}\left[\int d\vec{r}\, C_i(\vec{r},t)\right] = \int d\vec{r} \left[\sum_j \beta_i^j \int_0^\infty \nu\Sigma_p^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right] T(t) - \lambda_i \int d\vec{r}\, C_i(\vec{r},t)$$

More Simplify PKEs

$$\frac{d}{dt}\left[T(t)\right] = \int dE \int d\vec{r} \begin{bmatrix} -\Sigma_t(\vec{r},E,t)\, S(\vec{r},E) + \int_0^{\infty} \Sigma_s(\vec{r},E' \to E,t)\, S(\vec{r},E')\, dE' \\ +\left\{\chi_p(E)(1-\beta) + \chi_d(E)\beta\right\} \int_0^{\infty} \nu\Sigma_f(\vec{r},E',t)\, S(\vec{r},E')\, dE' \end{bmatrix} T(t)$$

$$-\beta \int d\vec{r} \left[\int_0^{\infty} \nu\Sigma_f(\vec{r},E',t)\, S(\vec{r},E')\, dE'\right] T(t)$$

$$+\int dE \int d\vec{r} \left[\chi_d(E) \sum_i \lambda_i C_i(\vec{r},t)\right]$$

$$\frac{d}{dt}\left[\int d\vec{r}\, C_i(\vec{r},t)\right] = \beta_i \int d\vec{r} \left[\int_0^{\infty} \nu\Sigma_f(\vec{r},E',t)\, S(\vec{r},E')\, dE'\right] T(t) - \lambda_i \int d\vec{r}\, C_i(\vec{r},t)$$

What are the simplifying assumptions?

- There is no net current (leakage) out of the reactor model
- There is no external source of prompt or delayed neutrons
- Prompt fission emission spectrum is independent of fissioning species
- The delayed neutron yield is independent of fissioning species
- Delayed neutron emission spectrum is independent of delayed neutron group
- There is no spatial dependence to delayed neutron yield

Further Simplifications

$$\frac{d}{dt}\left[T(t)\right]=\int dE\int d\vec{r}\begin{bmatrix}-\Sigma_t(\vec{r},E,t)\,S(\vec{r},E)+\int_0^\infty \Sigma_s(\vec{r},E'\to E,t)\,S(\vec{r},E')\,dE' \\ +(1-\beta)\chi_p(E)\int_0^\infty \nu\Sigma_f(\vec{r},E',t)\,S(\vec{r},E')\,dE'\end{bmatrix}T(t)\;+\int dE\int d\vec{r}\left[\chi_d(E)\sum_i \lambda_i C_i(\vec{r},t)\right]$$

and

$$\frac{d}{dt}\left[\int d\vec{r}\;\;C_i(\vec{r},t)\right]=\beta_i\int d\vec{r}\left[\int_0^\infty \nu\Sigma_f(\vec{r},E',t)\,S(\vec{r},E')\,dE'\right]T(t)-\lambda_i\int d\vec{r}\;\;C_i(\vec{r},t)$$

Or

$$\frac{d}{dt}\left[T(t)\right]=\int dE\int d\vec{r}\begin{bmatrix}-\Sigma_t(\vec{r},E,t)\,S(\vec{r},E)+\int_0^\infty \Sigma_s(\vec{r},E'\to E,t)\,S(\vec{r},E')\,dE' \\ +(1-\beta)\chi_p(E)\int_0^\infty \nu\Sigma_f(\vec{r},E',t)\,S(\vec{r},E')\,dE'\end{bmatrix}T(t)\;+\int d\vec{r}\left[\sum_i \lambda_i C_i(\vec{r},t)\right]$$

and

$$\frac{d}{dt}\left[\int d\vec{r}\;\;C_i(\vec{r},t)\right]=\beta_i\int d\vec{r}\left[\int_0^\infty \nu\Sigma_f(\vec{r},E',t)\,S(\vec{r},E')\,dE'\right]T(t)-\lambda_i\int d\vec{r}\;\;C_i(\vec{r},t)$$

Traditional PKEs: Weight by Adjoint Flux, not Unity

Assume that the flux can be separated into a space/energy term and a time-dependent term: $\phi(\vec{r},E,t)=S(\vec{r},E)T(t)$, and one multiples by the adjoint flux $\phi^*(\vec{r},E)$ and integrates over all space and energy

and normalize $\int dE \int d\vec{r}\ \phi^*(\vec{r},E)\frac{1}{v(\vec{r})}S(\vec{r},E)\equiv 1.0$,

$$\frac{d}{dt}\left[T(t)\right]=\int dE\int d\vec{r}\ \phi^*(\vec{r},E)\begin{bmatrix}-\Sigma_t(\vec{r},E,t)\ S(\vec{r},E)+\int_0^\infty \Sigma_s(\vec{r},E'\to E,t)\ S(\vec{r},E')\ dE' \\ +\left[1-\beta(\vec{r})\right]\chi_p(E)\ \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\ S(\vec{r},E')\ dE'\end{bmatrix}T(t)$$

$$+\int dE\int d\vec{r}\ \ \phi^*(\vec{r},E)\left[\chi_d(E)\sum_i \lambda_i C_i(\vec{r},t)\right]$$

and

$$\frac{d}{dt}\left[\int d\vec{r}\ \ C_i(\vec{r},t)\right]=\int d\vec{r}\ \beta_i(\vec{r})\left[\int_0^\infty \nu\Sigma_f(\vec{r},E',t)\ S(\vec{r},E')\ dE'\right]T(t)-\lambda_i\int d\vec{r}\ \ C_i(\vec{r},t)$$

Adjoint only shows up in amplitude function equation - not delayed neutron equations

Which adjoint flux do we use?

We will come back to that in a few minutes

Multi-group PKEs Diffusion Equation

Assume that the flux can be separated into a space/energy term and a time-dependent term: $\phi_g(\vec{r},t) = S_g(\vec{r})T_g(t)$, and one multiples by the adjoint flux $\phi_g^*(\vec{r})$ and integrates over all space and energy

and normalize $\sum_g \left\langle \int d\vec{r}\, \phi_g^*(\vec{r}) \frac{1}{v_g(r,t)} S_g(\vec{r}) \right\rangle \equiv 1.0,$

$$\frac{d}{dt}\left[T_g(t)\right] = \int d\vec{r}\, \phi_g^*(\vec{r}) \begin{bmatrix} -\Sigma_{t,g}(\vec{r},t)\, S_g(\vec{r})T_g(t) + \left[\sum_{g'} \Sigma_{s,g'\to g}(\vec{r},t)\, S_{g'}(\vec{r})T_{g'}(t)\right] \\ +\left[1-\beta(\vec{r})\right]\chi_{p,g}\left[\sum_{g'} \nu\Sigma_{f,g'}(\vec{r},t)\, S_{g'}(\vec{r})T_{g'}(t)\right] \end{bmatrix}$$

$$+\int d\vec{r}\;\; \phi_g^*(\vec{r})\left[\chi_{d,g}\sum_i \lambda_i C_i(\vec{r},t)\right], \qquad g = 1,G$$

and

$$\frac{d}{dt}\left[\int d\vec{r}\;\; C_i(\vec{r},t)\right] = \int d\vec{r}\, \beta_i(\vec{r})\left[\sum_g \nu\Sigma_{f,g}(\vec{r},t)\, S_g(\vec{r})T_g(t)\right] - \lambda_i \int d\vec{r}\;\; C_i(\vec{r},t), \qquad i = 1,I$$

2-group PKEs Diffusion Equations

$$\frac{d}{dt}\left[T_1(t)\right]=\int d\vec{r}\ \phi_1^*(\vec{r})\left\langle -\hat{\Sigma}_{r,1}(\vec{r},t)\ S_1(\vec{r})T_1(t)+\left[1-\beta(\vec{r})\right]\left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r})T_1(t)+\nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r})T_2(t)\right]\right\rangle$$

$$+\int d\vec{r}\ \ \phi_1^*(\vec{r})\left[\sum_i \lambda_i C_i(\vec{r},t)\right]$$

$$\frac{d}{dt}\left[T_2(t)\right]=\int d\vec{r}\ \phi_2^*(\vec{r})\left[-\Sigma_{a,2}(\vec{r},t)\ S_2(\vec{r})T_2(t)+\Sigma_{s,1\to 2}(\vec{r},t)\ S_1(\vec{r})T_1(t)\right]$$

What are the simplifying assumptions?

- No leakage out of the reactor
- All fission neutrons are born in the fast group
- All delayed neutrons are born in the fast group
- Downscatter is replace by effective downscatter
- Upscatter is zero

$$\frac{d}{dt}\left[\int d\vec{r}\ \ C_i(\vec{r},t)\right]=\int d\vec{r}\ \beta_i(\vec{r})\left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r})T_1(t)+\nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r})T_2(t)\right]-\lambda_i\int d\vec{r}\ \ C_i(\vec{r},t)\qquad i=1,I$$

Matrix 2-group PKE Diffusion Equations

$$\frac{d}{dt}\begin{bmatrix} T_1(t) \\ T_2(t) \end{bmatrix} = \begin{bmatrix} \int d\vec{r}\ \phi_1^*(\vec{r})\left[-\hat{\Sigma}_{r,1}(\vec{r},t) + \left[1-\beta(\vec{r})\right]\nu\Sigma_{f,1}(\vec{r},t)\ \right]S_1(\vec{r}) & \int d\vec{r}\ \phi_1^*(\vec{r})\left[\ \left[1-\beta(\vec{r})\right]\nu\Sigma_{f,2}(\vec{r},t)\right]S_2(\vec{r}) \\ \int d\vec{r}\ \phi_2^*(\vec{r})\Sigma_{s,1\to 2}(\vec{r},t)\ S_1(\vec{r}) & \int d\vec{r}\ \phi_2^*(\vec{r})\left[\qquad -\Sigma_{a,2}(\vec{r},t)\right]S_2(\vec{r}) \end{bmatrix}\begin{bmatrix} T_1(t) \\ T_2(t) \end{bmatrix}$$

$$+\begin{bmatrix} \int d\vec{r}\ \ \phi_1^*(\vec{r})\left[\sum_i \lambda_i C_i(\vec{r},t)\right] \\ 0 \end{bmatrix}$$

$$\frac{d}{dt}\left[\int d\vec{r}\ \ C_i(\vec{r},t)\right] = \int d\vec{r}\ \beta_i(\vec{r})\left[\nu\Sigma_{f,1}(\vec{r},t)S_1(\vec{r})T_1(t) + \nu\Sigma_{f,2}(\vec{r},t)S_2(\vec{r})T_2(t)\right] - \lambda_i\int d\vec{r}\ \ C_i(\vec{r},t)$$

Point Kinetics Parameters

From Lecture 2 we had simple PKE parameters (without leakage) of :

$$\rho(t) \equiv \frac{\int dE \int d\vec{r} \left[-\Sigma_t(\vec{r},E,t)\, S(\vec{r},E) + \int_0^\infty \Sigma_s(\vec{r},E' \to E,t)\, S(\vec{r},E')\, dE' + \sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}$$

reactivity $\cong \dfrac{k_{eff} - 1.0}{k_{eff}}$

$$\beta_i(t) \equiv \frac{\int dE \int d\vec{r} \left[\sum_j \chi_d^i(E)\beta_i^j(\vec{r}) \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}$$

beta $\cong \beta_i$

$$\Lambda(t) \equiv \frac{\int dE \int d\vec{r} \frac{1}{v} S(\vec{r},E)}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E)\beta_i^j \right\} \int_0^\infty \nu\Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E')\, dE' \right]}$$

promt neutron lifetime $\cong \dfrac{1}{v\, \upsilon\Sigma_f} \cong \dfrac{1}{v\, \Sigma_a}$

$$C_i(t) \equiv \int d\vec{r}\, C_i(\vec{r},t)$$

i-th precursor

In 2-group form:

$$\rho(t) \equiv \frac{\int d\vec{r}\, \phi_1^*(\vec{r}) \left\langle -\hat{\Sigma}_{r,1}(\vec{r},t)\, S_1(\vec{r})T_1(t) + \left[\nu\Sigma_{f,1}(\vec{r},t)\, S_1(\vec{r})T_1(t) + \nu\Sigma_{f,2}(\vec{r},t)\, S_2(\vec{r})T_2(t) \right] \right\rangle + \int d\vec{r}\, \phi_2^*(\vec{r}) \left\langle -\hat{\Sigma}_{a,2}(\vec{r},t) S_2(\vec{r})T_2(t) + \hat{\Sigma}_{1\to 2}(\vec{r},t)\, S_1(\vec{r})T_1(t)) \right\rangle}{\int d\vec{r}\, \phi_1^*(\vec{r}) \left[\nu\Sigma_{f,1}(\vec{r},t)\, S_1(\vec{r}) + \nu\Sigma_{f,2}(\vec{r},t)\, S_2(\vec{r}) \right]}$$

reactivity

$$\beta_i(t) = \frac{\int d\vec{r}\, \phi_1^*(\vec{r})\beta_i(\vec{r},t) \left[\nu\Sigma_{f,1}(\vec{r},t)\, S_1(\vec{r}) + \nu\Sigma_{f,2}(\vec{r},t)\, S_2(\vec{r}) \right]}{\int d\vec{r}\, \phi_1^*(\vec{r}) \left[\nu\Sigma_{f,1}(\vec{r},t)\, S_1(\vec{r}) + \nu\Sigma_{f,2}(\vec{r},t)\, S_2(\vec{r}) \right]}$$

beta-effective $\cong \beta_i$

$$\Lambda(t) = \frac{1}{\int d\vec{r}\, \phi_1^*(\vec{r}) \left[\nu\Sigma_{f,1}(\vec{r},t)\, S_1(\vec{r}) + \nu\Sigma_{f,2}(\vec{r},t)\, S_2(\vec{r}) \right]}$$

promt neutron lifetime

$$C_i(t) \equiv \int d\vec{r}\, C_i(\vec{r},t)$$

i-th precursor

2-group PKE Parameters

$$\rho(t) \equiv \frac{\begin{array}{c}\int d\vec{r}\ \phi_1^*(\vec{r},t)\left\langle -\hat{\Sigma}_{r,1}(\vec{r},t)\ S_1(\vec{r},t)T_1(t) + \left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r},t)T_1(t) + \nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r},t)T_2(t)\right]\right\rangle \\ + \int d\vec{r}\ \phi_2^*(\vec{r},t)\left\langle -\hat{\Sigma}_{a,2}(\vec{r},t)S_2(\vec{r})T_2(t) + \hat{\Sigma}_{1\to 2}(\vec{r},t)\ S_1(\vec{r},t)T_1(t))\right\rangle\end{array}}{\int d\vec{r}\ \phi_1^*(\vec{r},t)\left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r}) + \nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r})\right]}$$

$$\beta_i(t) = \frac{\int d\vec{r}\ \phi_1^*(\vec{r})\beta_i(\vec{r},t)\left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r},t) + \nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r},t)\right]}{\int d\vec{r}\ \phi_1^*(\vec{r},t)\left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r},t) + \nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r},t)\right]}$$

$$\Lambda(t) = \frac{1}{\int d\vec{r}\ \phi_1^*(\vec{r},t)\left[\nu\Sigma_{f,1}(\vec{r},t)\ S_1(\vec{r},t) + \nu\Sigma_{f,2}(\vec{r},t)\ S_2(\vec{r},t)\right]}$$

$$C_i(t) \equiv \int d\vec{r}\ C_i(\vec{r},t)$$

Are they still "point" kinetics equations?

$$\frac{d}{dt}T(t) = \frac{\rho(t) - \sum_i \beta_i(t)}{\Lambda(t)}T(t) + \sum_i \lambda_i C_i(t) + Q(t)$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i(t)}{\Lambda(t)}T(t) - \lambda_i C_i(t)$$

Adjoint Fluxes for Critical Reactor Systems

Expressing the transport (or diffusion) equation in operator notation:

$$A\psi = \frac{1}{k} M\psi \qquad \Rightarrow A\psi - \frac{1}{k} M\psi = 0$$

$$A^*\psi^* = \frac{1}{k^*} M^*\psi \quad \Rightarrow A^*\psi^* - \frac{1}{k^*} M^*\psi = 0$$

Multiplying first equation by adjoint flux/integrate over phase space, multiplying second equation by real flux/integrate over phase space, subtract later from former:

$$\left\langle \psi^*, A\psi \right\rangle - \left\langle \psi, A^*\psi^* \right\rangle - \frac{1}{k}\left\langle \psi^*, M\psi \right\rangle + \frac{1}{k^*}\left\langle \psi, M^*\psi^* \right\rangle = 0$$

But recalling the definition of the adjoint operating on any operator f

$$\boxed{\left\langle \psi^*, f\psi \right\rangle = \left\langle \psi, f^*\psi^* \right\rangle}$$

Consequently,
$$\left\langle \frac{1}{k} - \frac{1}{k^*} \right\rangle \left\langle \psi^*, M\psi \right\rangle = 0$$

The fundamental mode solution (in which we are interested) has everywhere positive real and adjoint fluxes, and since the fission operator is an everywhere positive operator, we find that the real and adjoint eigenvalues must be identical:

$$\boxed{k^* = k}$$

Perturbation Theory Expression for Reactivity

If we start from an unperturbed critical reactor system

$$A_0\psi_0 - \frac{1}{k_0}M_0\psi_0 = 0 \qquad A_0^*\psi_0^* = \frac{1}{k_0^*}M_0^*\psi_0^*$$

and perturb the operators such that

$$A = A_o + \delta A \qquad M = M_o + \delta M \qquad \psi = \psi_o + \delta\psi$$

Then,

$$(A_0 + \delta A)(\psi_0 + \delta\psi) - \frac{1}{(k_0 + \delta k)}(M_0 + \delta M)(\psi_0 + \delta\psi) = 0$$

Expanding

$$\frac{1}{(k_0 + \delta k)} = \frac{1}{k_0(1 + \delta k / k_0)} = \frac{1}{k_0}(1 - \delta k / k_0) + O(\delta k)^2$$

and ignoring terms $O(\delta k)^2$

$\boxed{0}$

$$-\frac{\delta k}{k_0^2}M_0\psi_0 = (A_0 - \frac{1}{k_0}M_0)\psi_0 + (\delta A - \frac{1}{k_0}\delta M)\psi_0 + (A_0 - \frac{1}{k_0}M_0)\delta\psi + O(\delta)^2$$

Multiplying by ψ_0^* and integrating over phase space

$$-\frac{\delta k}{k_0^2}\left\langle \psi_0^*, M_0\psi_0 \right\rangle = \left\langle \psi_0^*, (\delta A - \frac{1}{k_0}\delta M)\psi_0 \right\rangle + \left\langle \psi_0^*, (A_0 - \frac{1}{k_0}M_0)\delta\psi \right\rangle + O(\delta)^2$$

First-Order Perturbation Theory

But since the definition of adjoint operators is for any operator

$$\left\langle \psi_0^*, (A_0 - \frac{1}{k_0} M_0)\delta\psi \right\rangle = \left\langle \delta\psi, (A_0^* - \frac{1}{k_0} M_0^*)\psi_0^* \right\rangle$$

0, for any $\delta\psi$

we can obtain

$$-\frac{\delta k}{k_0^2} = \frac{\left\langle \psi_0^*, (\delta A - \frac{1}{k_0}\delta M)\psi_0 \right\rangle}{\left\langle \psi_0^*, M_0 \psi_0 \right\rangle} + O(\delta)^2$$

There are only second order errors in reactivity

Which is clearly much more accurate than our previous expression:

$$-\frac{\delta k}{k_0^2} = \frac{(\delta A - \frac{1}{k_0}\delta M)\psi_0 + (A_0 - \frac{1}{k_0} M_0)\delta\psi}{M_0 \psi_0} + O(\delta)^2$$

Without knowing $\delta\psi$ there is a first order error in reactivity

Finally, making use of definition of reactivity perturbed from critical

$$\rho = \frac{k-1}{k} \quad \Rightarrow \rho - \rho_0 = \frac{k-1}{k} - \frac{k_0 - 1}{k_0} = \frac{\delta k}{k k_0} \Rightarrow \delta\rho = \frac{\delta k}{(1)^2} + O\left(\delta k\right)^2$$

one obtains the first order perturbation (FOP) expression for reactivity:

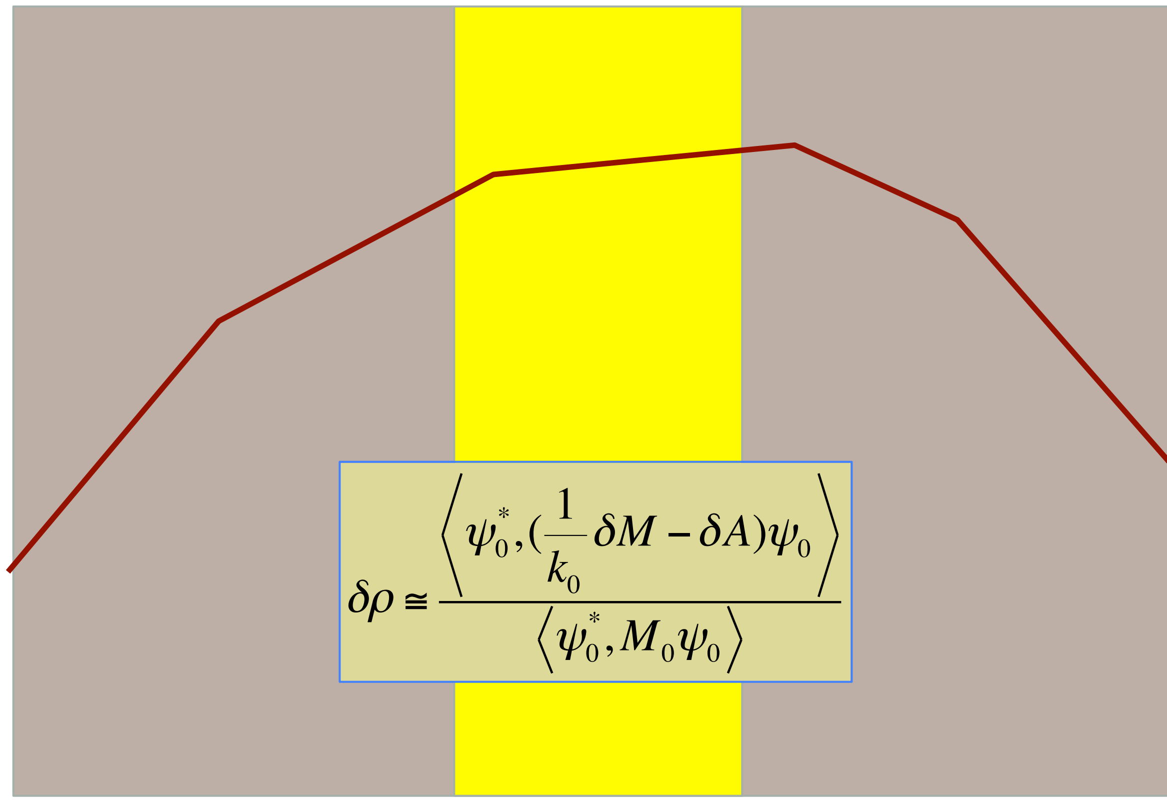
$$\delta\rho \cong \frac{\left\langle \psi_0^*, (\frac{1}{k_0}\delta M - \delta A)\psi_0 \right\rangle}{\left\langle \psi_0^*, M_0 \psi_0 \right\rangle}$$

First-Order Perturbation Theory

- One can evaluate reactivity resulting from any changes in operator by simply evaluating delta cross sections convoluted on unperturbed real and adjoint fluxes e.g. fuel temperature, coolant density, boron concentration

- Reactivity effects from perturbations in different spatial regions are superimposable

- Reactivity effects from perturbations of different phenomenon are superimposable (e.g. coolant density and temperature)



$$\delta\rho \cong \frac{\left\langle \psi_0^*, \left(\frac{1}{k_0}\delta M - \delta A\right)\psi_0 \right\rangle}{\left\langle \psi_0^*, M_0 \psi_0 \right\rangle}$$

- Each reactivity perturbation can be computed without need for solving for perturbed spatial flux distributions.

Adjoint Flux in a Critical reactor

Recall from point kinetics that the neutron population is given by

$$\frac{d}{dt}N(t)=\frac{\rho(t)-\beta(t)}{\Lambda}N(t)+\sum_i \lambda_i C_i(t)+Q$$

$$\frac{d}{dt}C_i(t)=\frac{\beta_i}{\Lambda}N(t)-\lambda_i C_i(t)$$

And for the reactivity equals zero case

$$\frac{d}{dt}N(t)=Q\Rightarrow N(t)=N_0+Q(t-t_0)$$

Thus neutron population grows linearly in time with an external source (Q=neutrons sec^{-1})

But consider introducing Q neutrons: just at time zero at position, energy, and direct $(\vec{r},\vec{\Omega},E)$ then

$$\frac{d}{dt}N(t)=Q\delta(t-t_0)\Rightarrow N(\infty)=N_0+Q\cdot X$$

Or

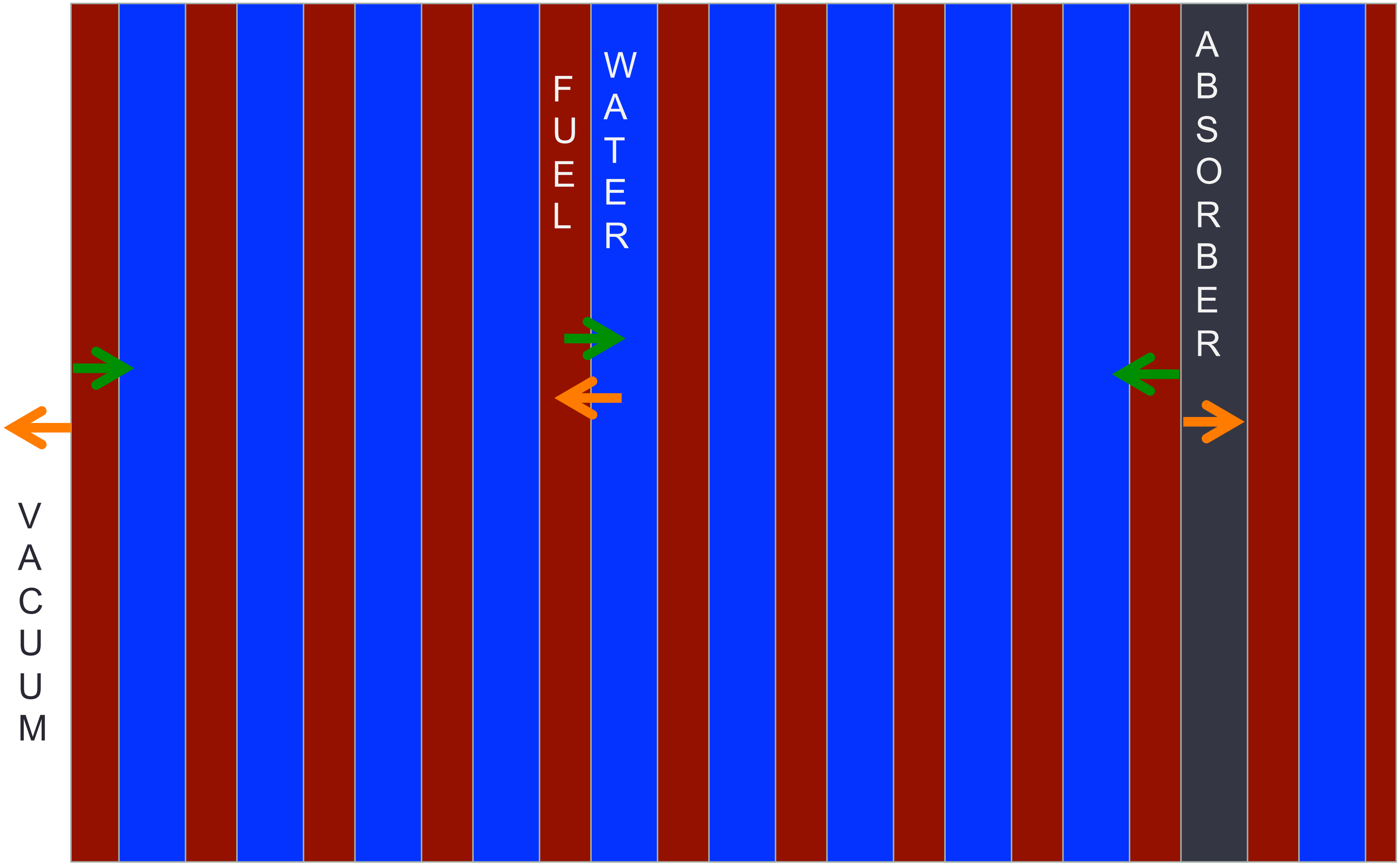
$$\frac{d}{dt}N(t)=Q\delta(t-t_0)\Rightarrow N(\infty)=N_0+Q\psi^*(\vec{r},\vec{\Omega},E)$$

$$\Rightarrow \frac{N(\infty)-N_0}{Q}=\psi^*(\vec{r},\vec{\Omega},E)$$

The adjoint flux is defined as the asymptotic increase in total neutron population of a critical reactor for one neutron introduced a position r, direction omega, and energy E.

Always remember this definition!

Thermal Adjoint Flux in an LWR Lattice



Multi-Group Real and Adjoint Flux Equations

$$-\nabla\cdot D_g(\vec{r})\nabla\phi_g(\vec{r})+\Sigma_{tg}(\vec{r})\phi_g(\vec{r})=\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{G}\nu\Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r})+\sum_{g'=1}^{G}\Sigma_{sg'\to g}(\vec{r})\phi_{g'}(\vec{r})$$

$$-\nabla\cdot D_g(\vec{r})\nabla\phi_g^*(\vec{r})+\Sigma_{tg}(\vec{r})\phi_g^*(\vec{r})=\frac{\chi_{g'}}{k_{eff}^*}\sum_{g'=1}^{G}\nu\Sigma_{fg}(\vec{r})\phi_{g'}^*(\vec{r})+\sum_{g'=1}^{G}\Sigma_{sg\to g'}(\vec{r})\phi_{g'}^*(\vec{r})$$

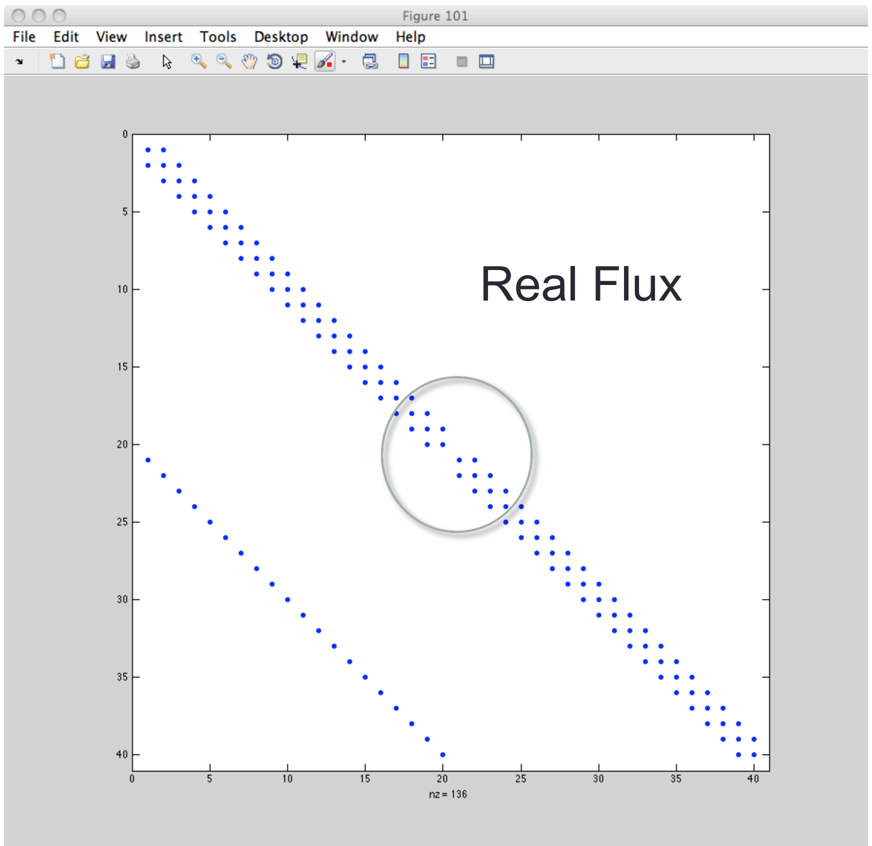
For 2-group, infinite medium case (with effective downscatter only):

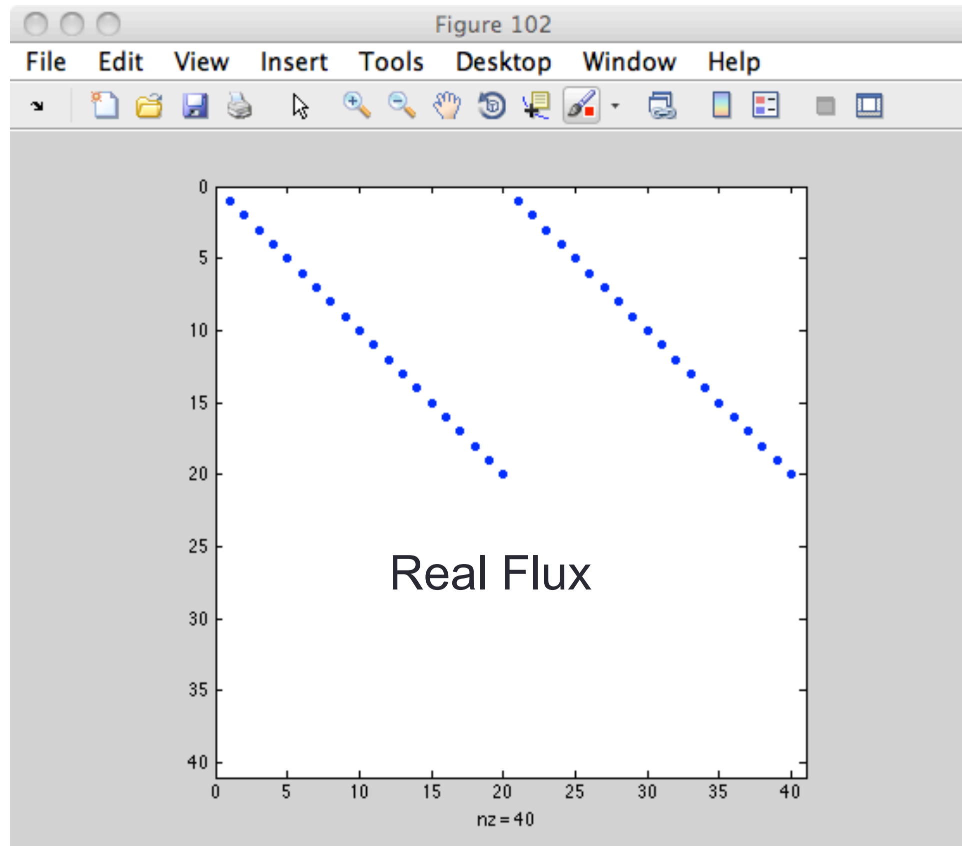
$$\begin{bmatrix}\Sigma_{a1}+\Sigma_{1\to2}-\frac{1}{k_{\inf}}\upsilon\Sigma_{f1} & -\frac{1}{k_{\inf}}\upsilon\Sigma_{f2}\\ -\Sigma_{1\to2} & \Sigma_{a2}\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix}=0 \qquad \frac{\phi_2}{\phi_1}=\frac{\Sigma_{1\to2}}{\Sigma_{a2}} \qquad k_{\inf}=\frac{\upsilon\Sigma_{f1}\phi_1+\upsilon\Sigma_{f2}\frac{\Sigma_{1\to2}}{\Sigma_{a2}}}{\Sigma_{a1}+\Sigma_{1\to2}}$$

$$\begin{bmatrix}\Sigma_{a1}+\Sigma_{1\to2}-\frac{1}{k_{\inf}^*}\upsilon\Sigma_{f1} & -\Sigma_{1\to2}\\ -\frac{1}{k_{\inf}^*}\upsilon\Sigma_{f2} & \Sigma_{a2}\end{bmatrix}\begin{bmatrix}\phi_1^*\\ \phi_2^*\end{bmatrix}=0 \qquad \frac{\phi_2^*}{\phi_1^*}=\frac{\frac{1}{k_{\inf}^*}\upsilon\Sigma_{f2}}{\Sigma_{a2}} \qquad k_{\inf}^*=\frac{\upsilon\Sigma_{f1}\phi_1+\upsilon\Sigma_{f2}\frac{\Sigma_{1\to2}}{\Sigma_{a2}}}{\Sigma_{a1}+\Sigma_{1\to2}}$$

If k_{inf}= 1.2, thermal fission =.15 cm^{-1}, and thermal absorption=.10 cm^{-1} $\quad \frac{\phi_2^*}{\phi_1^*}=1.25$

Constructing Adjoint Destruction and Production Matrix?



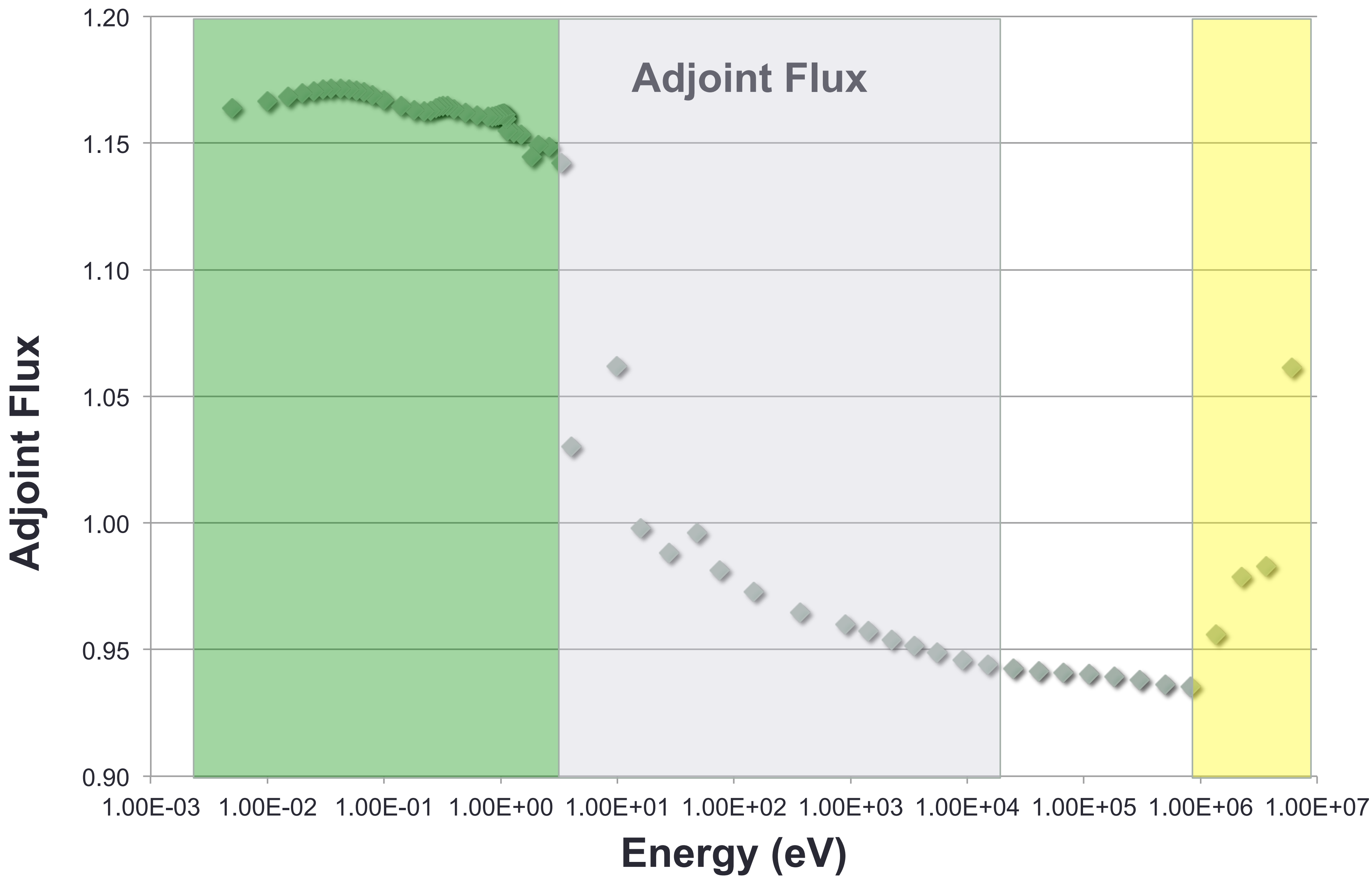


$[A]$?

$[M]$?

$$\begin{bmatrix} [L+D+U]_1 & [0] \\ -[T]_2 & [L+D+U]_2 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} [M]_1 & [M]_2 \\ [0] & [0] \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$$?

PWR Lattice Adjoint Spectrum (70 Energy Groups)



Adjoint-Weighted Reactivity

$$\rho(t) \equiv \frac{\int dE \int d\vec{r} \left[\nabla \cdot D(\vec{r},E,t) \nabla S(\vec{r},E,t) - \Sigma_t(\vec{r},E,t)\, S(\vec{r},E,t) + \int_0^\infty \Sigma_s(\vec{r},E' \to E,t)\, S(\vec{r},E',t)\, dE' + \sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E) \beta_i^j \right\} \int_0^\infty \nu \Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E',t)\, dE' \right]}{\int dE \int d\vec{r} \left[\sum_j \left\{ \chi_p^j(E)(1-\beta^j) + \sum_i \chi_d^i(E) \beta_i^j \right\} \int_0^\infty \nu \Sigma_f^j(\vec{r},E',t)\, S(\vec{r},E',t)\, dE' \right]}$$

$$\rho(t) \equiv \frac{\left\langle 1, \left[-A(\vec{r},E,t) + M(\vec{r},E,t) \right] S(\vec{r},E,t) \right\rangle}{\left\langle 1, M(\vec{r},E,t)\, S(\vec{r},E,t) \right\rangle}$$

If we knew the exact shape function, the weight function does not matter

$$\rho(t) \cong \frac{\left\langle 1, \left[-A_0(\vec{r},E) + M_0(\vec{r},E) \right] S_0(\vec{r},E) \right\rangle}{\left\langle 1, M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle} + \frac{\left\langle 1, \left[-\delta A_0(\vec{r},E) + \delta M_0(\vec{r},E) \right] S_0(\vec{r},E) \right\rangle}{\left\langle 1, M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle} + \frac{\left\langle 1, \left[-A_0(\vec{r},E) + M_0(\vec{r},E) \right] \delta S(\vec{r},E,t) \right\rangle}{\left\langle 1, M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle} + O(\delta)^2$$

$$\rho(t) = \frac{\left\langle 1, \left[-\delta A_0(\vec{r},E) + \delta M_0(\vec{r},E) \right] S_0(\vec{r},E) \right\rangle}{\left\langle 1, M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle} + \frac{\left\langle 1, \left[-A_0(\vec{r},E) + M_0(\vec{r},E) \right] \delta S(\vec{r},E,t) \right\rangle}{\left\langle 1, M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle}$$

If we use the initial shape function and a unity weight function, there is a first-order error if the true shape function changes with time

$$\rho(t) = \frac{\left\langle S_0^*(\vec{r},E), \left[-\delta A_0(\vec{r},E) + \delta M_0(\vec{r},E) \right] S_0(\vec{r},E) \right\rangle}{\left\langle S_0^*(\vec{r},E), M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle} + O(\delta)^2$$

If we use the initial shape function and the initial adjoint as the weight function, there is no first-order error if the true shape functions changes with time

REMEMBER: if flux shapes changes a lot in a transient, even second order errors may be important!

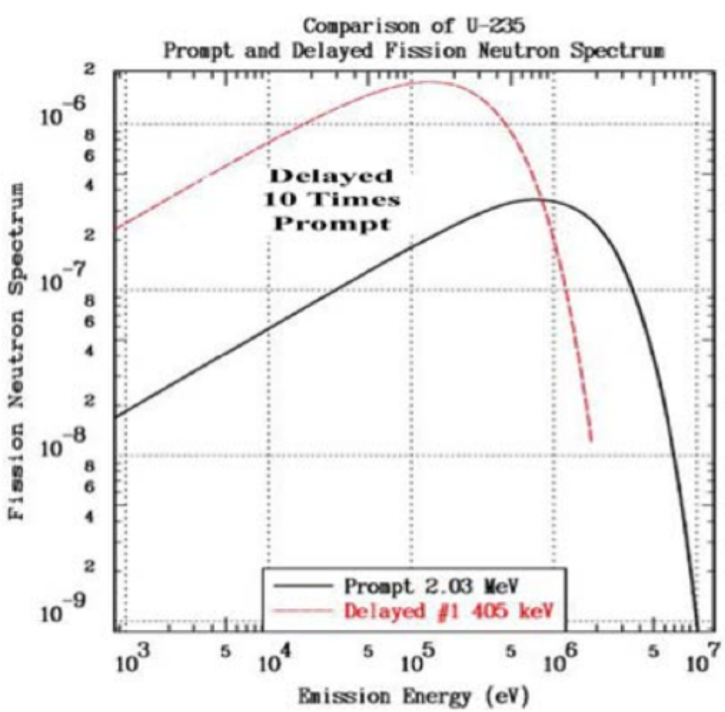
Adjoint Weighted Beta-effective

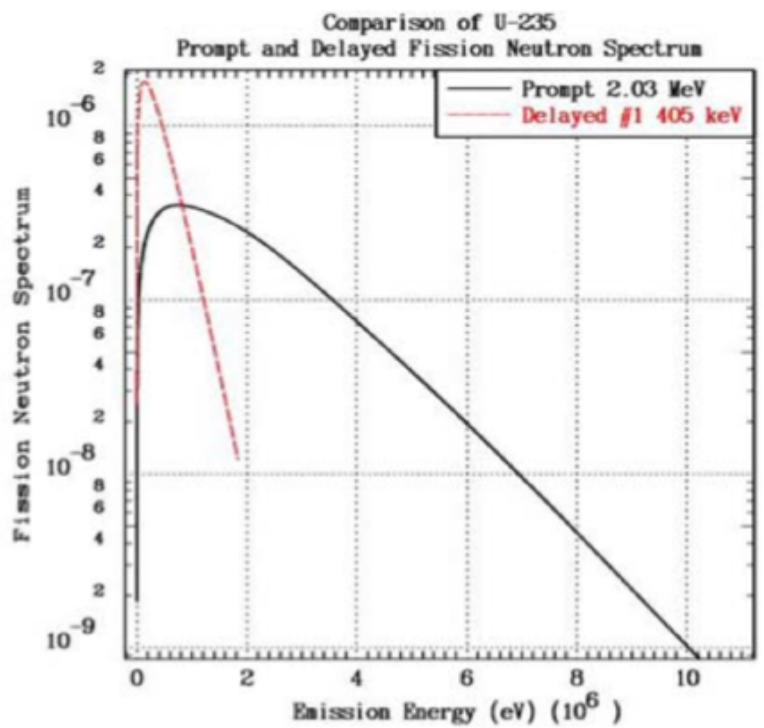
$$\beta_i^{eff}(t) \cong \frac{\beta_i \left\langle W_0^*(\vec{r},E), \chi_d(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle}{\left\langle W_0^*(\vec{r},E), M_0(\vec{r},E)\, S_0(\vec{r},E) \right\rangle}$$

Assume here a single fission species, that all delayed neutrons have the same emission spectra, and the delay spectra impact on the denominator is insignificant

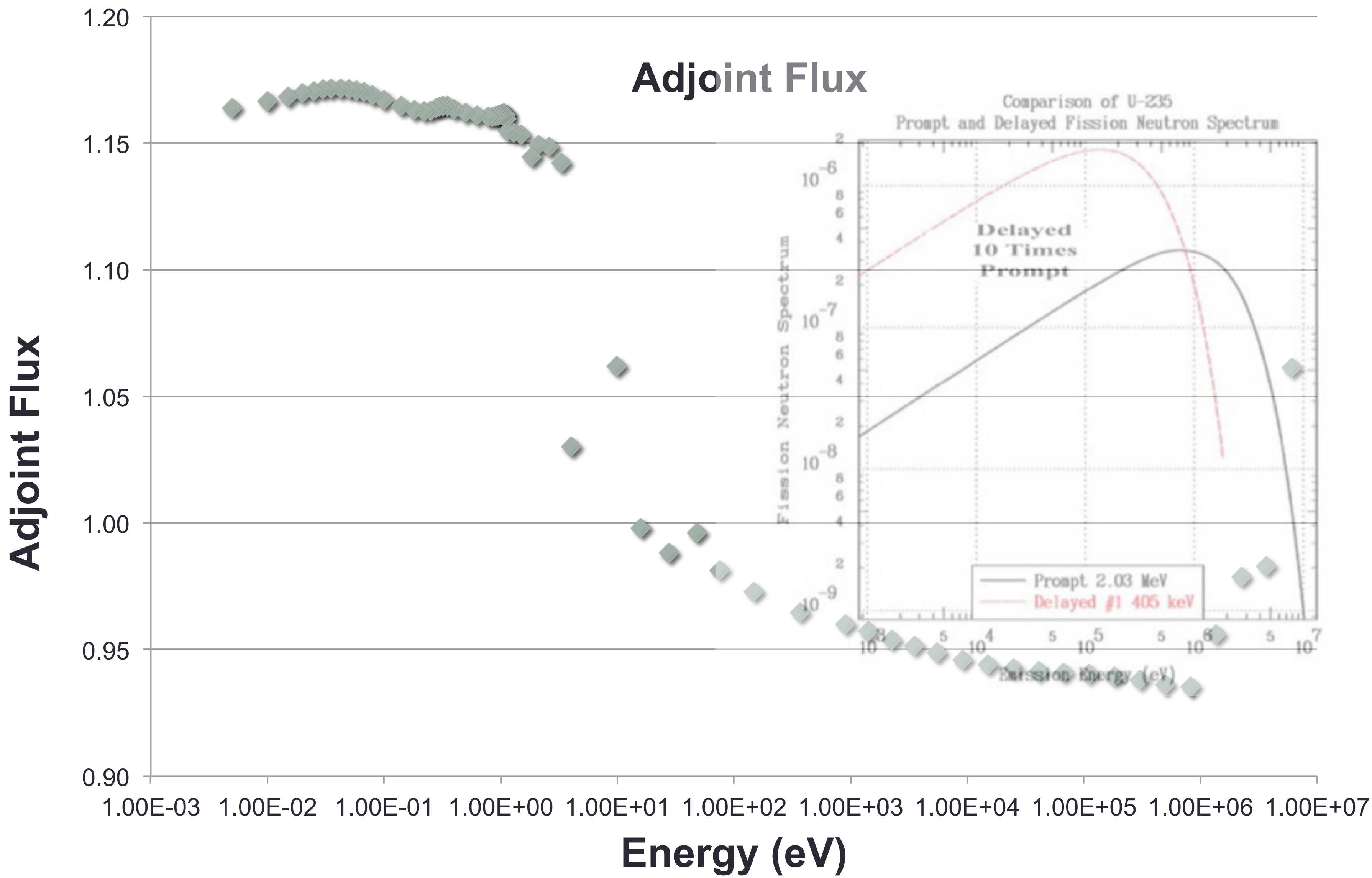
$$\beta_i^{eff}(t) \cong \frac{\beta_i \left\langle 1, \chi_d(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle}{\left\langle 1, \chi_p(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle} = \beta_i$$

$$\beta_i^{eff}(t) \cong \frac{\beta_i \left\langle S_0^*(\vec{r},E), \chi_d(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle}{\left\langle S_0^*(\vec{r},E), \chi_p(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle} \cong \frac{\beta_i \left\langle S_0^*(E), \chi_d(E) \right\rangle}{\left\langle S_0^*(E), \chi_p(E) \right\rangle}$$





PWR Lattice Adjoint Spectrum (70 Energy Groups)



LWR Beta-Effective

$$\beta_i^{eff}(t) \cong \frac{\beta_i \left\langle S_0^*(\vec{r},E), \chi_d(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle}{\left\langle S_0^*(\vec{r},E), \chi_p(E) \int_0^\infty \nu\Sigma_f(\vec{r},E',t)\, S_0(\vec{r},E')\, dE' \right\rangle} \cong \frac{\beta_i \left\langle S_0^*(E), \chi_d(E) \right\rangle}{\left\langle S_0^*(E), \chi_p(E) \right\rangle}$$

Often this ratio of adjoint weighted delayed chi to prompt chi is called “I-Bar” and it has a value of about 0.97

For example in an LWR at BOL, beta-effective is approximately:

{.0066*(0.95) + .0157*(0.05)} = 0.00705

.00705 x 0.97 = 0.00684

Don’t get confused as beta effective is greater that beta U235 because of the U238 fissions, not because of I-Bar.

I-Bar is not always 0.97 – be sure to compute if for the spectrum you have.
I-Bar cannot be computed directly with few group calculations

Remember, from Fuchs-Nordheim model, the importance of beta in RIA power and fuel temperatures

$$P^{peak} = P_0 + \frac{C_p\left(\rho_{rod} - \beta\right)^2}{2\Lambda\alpha}$$

$$\Rightarrow T_{fuel}^{peak} = \frac{(\rho_{rod} - \beta)}{\alpha}$$

Two-group equations need beta-effective, as prompt and delayed group chis are both 1.0/0.0

Next Few Lectures/Assignments

- Solve true time-dependent problem by direct solution at each time step and compare to:
  - PKE solution using unity-weighted initial steady-state fluxes
  - PKE solution using unity-weighted final steady-state fluxes
  - PKE solution using adjoint-weighted initial steady-state fluxes
  - PKE solution using adjoint-weighted final steady-state fluxes
  - 2-group PKE solution using unity-weighted initial fluxes
  - 2-group PKE solution using unity-weighted final fluxes
  - Geometrically-interpolated shape- and weight-function PKEs
  - Quasi-Static solution
  - Improved Quasi-Static solution

Assignment for Next Class

- Read references on Point-Jacobi and Gauss-Seidel iterations
- Prepare for solving 1-D, 2-group adjoint finite-difference equations
- Get Started on Pset 2: solving 1-D, 2-group finite-difference problems
- Think about how you will compute generalized PKE parameters from your 1-D, 2-group solutions.
- If you get bored, write solver for 2-D, 2-group steady-state LRA problem and compute PKE parameters from that steady-state solution.