QUIZ -1

Theory of Computation

26/08/2017

The number of stars next to a question is a rough indication of its difficulty.

1. Let $\Sigma = \{0, 1\}$. A string w is thought of as a binary number, with the most significant bit on the left. We denote its decimal value by $(w)_2$. For example, $(1101)_2 = 13$. We set the convention that $(\epsilon)_2 = 0$.

   (a) (**) Design a DFA recognising the language $L = \{w \mid (w)_2 \equiv 1 \mod 5\}$.

   (b) (**) Let L_p be the set of prime numbers in binary. $L_p = \{w \mid (w)_2$ is a prime $\}$. Construct an NFA for $L_p^{-1}L$.

   (c) (*) Give a DFA for $L_p^{-1}L$. What can you say about this language?

2. (* * *) Let A be a complete DFA whose language is non-empty. We obtain the NFA A' by making all states of A initial. The transitions and accepting states remain unchanged. Prove/disprove the following claim.

   Claim: $L(A') = \Sigma^*$.

3. (***) A term $n+m$ is denoted as a string 0^n10^m. Let $T \subseteq 0^*10^*$ be a set of such terms. The set $S(T) \subseteq 0^*$ consists of all the answers of evaluating the terms of T. That is, $S(T) = \{0^\ell \mid$ there exists a string $0^n10^m \in T$ such that $\ell = m + n\}$. Show that if a set of terms T is recognizable, then the set of answers $S(T)$ is also recognizable.

4. (* * * * *) Now suppose we use the string 0^n10^m to denote the term $|n - m|$. Let $T \subseteq 0^*10^*$ be a set of such terms. The set $D(T) \subseteq 0^*$ consists of all the answers of evaluating the terms of T. That is, $D(T) = \{0^\ell \mid$ there exists a string $0^n10^m \in T$ such that $\ell = |m - n|\}$. Show that if a set of terms T is recognizable, then the set of answers $D(T)$ is also recognizable.