Design and Analysis of Algorithms

Quiz 1

The allotted time is 75 minutes. Solve any **two** problems. All problems carry equal marks.

Problems

1. Consider the searching problem:

   - Input: A sequence of n numbers $A = \langle a_1, a_2, \ldots, a_n \rangle$ and a value v.
   - Output: An index i such that $A[i] = v$, or the special value NIL if v does not appear in A.

   (i) Write pseudocode for *linear search*, which scans through the sequence looking for v. Using a loop invariant, prove that your algorithm is correct. Make sure that your loop invariant fulfills the three necessary properties.

   (ii) Derive a good upper bound on the running time of your algorithm, assuming that each array operation and the comparison of two numbers takes constant time.

2. We can express insertion sort as a recursive procedure as follows. In order to sort $A[1 \ldots n]$, we recursively sort $A[1 \ldots (n-1)]$ and then insert $A[n]$ into the sorted array $A[1 \ldots (n-1)]$.

   (i) Write pseudocode for this recursive version of insertion sort. You *don't* have to provide a proof of correctness.

   (ii) Write a recurrence for the worst-case running time of this procedure and solve it to get a good upper bound.

3. State the basic definition of the Θ notation, in terms of the existence of certain constants etc., which was taught in class. Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition, prove that $\max(f(n), g(n)) = \Theta(f(n) + g(n))$.

4. Solve the following recurrences. In each case your solution should be of the form $T(n) = \Theta(f(n))$. Applying the Master Theorem will not, by itself, count as a solution.

   (i) $T(n) = 4T(n/3) + n$

   (ii) $T(n) = 4T(n/2) + n^2$

   (iii) $T(n) = 3T(\sqrt{n}) + \log n$

$A\left(\frac{n^2}{4}\right)$.

2 min. $\leq \frac{1}{2}(f(n) + g(n)) \leq$ 2 max (...)

$c_1 g h(n) \leq f(n) + g(n) \leq c_2 h(n)$.

insert_sort (arr, index)

if index <= 0

return

else

sort (arr, index - 1)

val = arr[index]

while (arr[pos] > val)

arr[pos+1] = arr[pos]

$\sum_{i=0}^{\log_3 n} 4^i \cdot \frac{n}{3^i}$

$\sum (\frac{4}{3})^i$

$\log_a b$

min = g

f+g ≥ g.

$f(n) \in \Theta(g ...)$

f+g ≥ m

$O(n^{\epsilon - \epsilon})$