Parameterized Algorithms

Quiz 1

The allotted time is 180 minutes. Solve all problems. All problems carry equal marks.

Problems

1. Recall that a language L is *decidable* if there is an algorithm which (i) halts on every input, and (ii) correctly reports whether an arbitrary input string belongs to L.

   (i) Define the following:

   - A parameterized language
   - A fixed-parameter tractable (FPT) algorithm
   - A kernelization algorithm, and a kernel

   (ii) Let L be a decidable parameterized language. Prove that L is fixed-parameter tractable if and only if L has a kernel.

2. Recall that a subset $S \subseteq V(G)$ of vertices of a graph G is a *vertex cover* of G if every *edge* in G has at least one of its two end-vertices in S. The standard parameterization of the VERTEX COVER problem is as follows:

VERTEX COVER	Parameter: k
Input: A graph G and an integer k.	
Question: Does G have a vertex cover of size at most k?	

   (i) Write pseudocode for the $O^*(2^k)$ FPT algorithm for VERTEX COVER which we discussed in class.

   Your pseudocode should consist of a function `vertexCover()`, invoked as `vertexCover(G,k)`, which returns **true** if graph G has a vertex cover of size at most k, and **false** otherwise. You should write *what* the algorithm should do, and omit the fine details of *how* it should be done. In particular, you should assume that you have access to a Graph API using which you can manipulate and query graphs; you should *not* implement a graph object of your own from scratch. Your pseudocode for `vertexCover(G,k)` should not be much more than ten or fifteen lines.

   (ii) Prove that your algorithm correctly solves VERTEX COVER, and that it runs in $O^*(2^k)$ time.

3. In this problem we improve on the previous algorithm.

(i) Write the pseudocode for an algorithm which solves VERTEX COVER in $O^*(1.5^k)$ time.

Once again, your pseudocode should consist of a function `vertexCover()`, invoked as `vertexCover(G,k)`, which returns true if graph G has a vertex cover of size at most k, and false otherwise. It should also have the other properties of pseudocode as stated in Problem (2), except that your function may have more than fifteen lines this time.

(ii) Prove that your algorithm correctly solves VERTEX COVER, and that it runs in $O^*(1.5^k)$ time.

4. Recall that a subset $S \subseteq V(G)$ of vertices of a graph G is a *feedback vertex set* (FVS) of G if every *cycle* in G has at least one of its vertices in S. The standard parameterization of the FEEDBACK VERTEX SET problem is as follows:

FEEDBACK VERTEX SET	Parameter: k
Input: A graph G and an integer k.	
Question: Does G have a feedback vertex set of size at most k?	

(i) Write pseudocode for the $O^*(5^k)$ FPT algorithm for FEEDBACK VERTEX SET which we discussed in class.

As for previous problems, you should assume that you have access to a Graph API using which you can manipulate and query graphs.

(ii) Prove that your algorithm correctly solves FEEDBACK VERTEX SET, and that it runs in $O^*(5^k)$ time.