

# Theory of Computation

Mid-semester Exam — 24/09/2018

Maximum marks: 40. Duration: 3 hours.

- (2 marks) Let  $L_1, L_2, L_3 \subseteq \{a, b\}^*$ . Either prove or disprove the following:
  - $L_1 \cdot (L_2 \cup L_3) = L_1 \cdot L_2 \cup L_1 \cdot L_3$
  - $L_1 \cdot (L_2 \cap L_3) = L_1 \cdot L_2 \cap L_1 \cdot L_3$
- Give regular expressions for the following languages. The alphabet  $\Sigma = \{a, b\}$ .
  - (1 mark)  $L_1 = \{w \mid w \text{ has even number of } b\}$
  - (2 marks)  $L_2 = \{w \mid w \text{ does not contain } bb \text{ as a factor/infix}\}$
- (1 mark) Give a DFA for  $L_2$  defined in Question 2b.
- (3 marks) Write down the Myhill-Nerode relation  $\equiv_L$  for  $L = \{abb, bb\}$ . (Recall that  $\equiv_L \subseteq \Sigma^* \times \Sigma^*$ . Write down which pairs from  $\Sigma^* \times \Sigma^*$  belong to  $\equiv_L$ .)
- (3 marks) Can two different regular languages have the same Myhill-Nerode relations? If yes, give an example. If not, argue why.
- (4 marks) Let  $f(i)$  denote the  $i$ th Fibonacci number. We have  $f(1) = 1, f(2) = 1$  and  $f(i) = f(i-1) + f(i-2)$  for  $i > 2$ . Consider the language  $L_{\text{fib}} = \{a^n b^m \mid m = f(n) \text{ and } n > 0\}$ . Prove that  $L_{\text{fib}}$  is not context-free.
- Consider the context-free grammar  $G$  given by

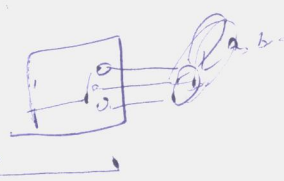
$$S \rightarrow SaS \mid bSb \mid \epsilon$$

- (2 marks) Prove that  $G$  is ambiguous.
  - (5 marks) Either prove, or give a counter example for the following claim.  
Claim:  $L(G) = \{w \mid w \text{ has even number of } b\}$ .
  - (2 marks) Give an unambiguous grammar for  $L(G)$ .
- Let  $\Sigma = \{a, b\}$ . Two words  $u, v \in \Sigma^*$  are said to be conjugates if there exist  $w_1, w_2 \in \Sigma^*$  such that  $u = w_1 w_2$  and  $v = w_2 w_1$ . In other words,  $v$  is obtained from  $u$  by a cyclic permutation of letters.
    - (4 marks) Prove that  $u$  and  $v$  are conjugates if and only if there exists  $w \in \Sigma^*$  such that  $uw = wv$ .
    - (6 marks) For  $L \subseteq \Sigma^*$ , let  $\text{cl}(L) = \{v \mid \text{there is } u \in L \text{ such that } u \text{ and } v \text{ are conjugates}\}$ . Prove that, if  $L$  is regular then so is  $\text{cl}(L)$ .
  - (5 marks) Describe an algorithm to check whether a non-deterministic finite state automaton  $A$  over  $\Sigma = \{a, b\}$  accepts a word of every length. (The input is an NFA. If for each  $n \geq 0$  there is at least one word of length  $n$  in  $L(A)$ , then the algorithm outputs *yes* and it outputs *no* otherwise.)

26 x 25  
51 x 51  
2



26 x 25 x 50  
bb a b b a b  
u ∈ L



$f(n+1) = f(n) + f(n-1)$   
 $f(0) + f(n-1) = f(n)$

$f(n+n)$

$n' = n + n$   
 $u v w n y$

$f(i-n) \times f(i) - n$   
 $f(n) +$