Theory of Computation

Mid-semester Exam — 24/09/2018

Maximum marks: 40. Duration: 3 hours.

1. *(2 marks)* Let $L_1, L_2, L_3 \subseteq \{a, b\}^*$. Either prove or disprove the following:

   (a) $L_1 \cdot (L_2 \cup L_3) = L_1 \cdot L_2 \cup L_1 \cdot L_3$

   (b) $L_1 \cdot (L_2 \cap L_3) = L_1 \cdot L_2 \cap L_1 \cdot L_3$

2. Give regular expressions for the following languages. The alphabet $\Sigma = \{a, b\}$.

   (a) *(1 mark)* $L_1 = \{w \mid w \text{ has even number of } b\}$

   (b) *(2 marks)* $L_2 = \{w \mid w \text{ does not contain } bb \text{ as a factor/infix}\}$

3. *(1 mark)* Give a DFA for L_2 defined in Question 2b.

4. *(3 marks)* Write down the Myhil-Nerode relation $\equiv_L$ for $L = \{abb, bb\}$. (Recall that $\equiv_L \subseteq \Sigma^* \times \Sigma^*$. Write down which pairs from $\Sigma^* \times \Sigma^*$ belong to $\equiv_L$.)

5. *(3 marks)* Can two different regular languages have the same Myhil-Nerode relations? If yes, give an example. If not, argue why.

6. *(4 marks)* Let $f(i)$ denote the ith Fibonacci number. We have $f(1) = 1, f(2) = 1$ and $f(i) = f(i-1) + f(i-2)$ for $i > 2$. Consider the language $L_{\text{fib}} = \{a^n b^m \mid m = f(n) \text{ and } n > 0\}$. Prove that L_{fib} is not context-free.

7. Consider the context-free grammar G given by

   $$S \to SaS \mid bSb \mid \epsilon$$

   (a) *(2 marks)* Prove that G is ambiguous.

   (b) *(5 marks)* Either prove, or give a counter example for the following claim.
   Claim: $L(G) = \{w \mid w \text{ has even number of } b\}$.

   (c) *(2 marks)* Give an unambiguous grammar for $L(G)$.

8. Let $\Sigma = \{a, b\}$. Two words $u, v \in \Sigma^*$ are said to be conjugates if there exist $w_1, w_2 \in \Sigma^*$ such that $u = w_1 w_2$ and $v = w_2 w_1$. In other words, v is obtained from u by a cyclic permutation of letters.

   (a) *(4 marks)* Prove that u and v are conjugates if and only if there exists $w \in \Sigma^*$ such that $uw = wv$.

   (b) *(6 marks)* For $L \subseteq \Sigma^*$, let $\text{cl}(L) = \{v \mid \text{ there is } u \in L \text{ such that } u \text{ and } v \text{ are conjugates}\}$. Prove that, if L is regular then so is $\text{cl}(L)$.

9. *(5 marks)* Describe an algorithm to check whether a *non-deterministic* finite state automaton A over $\Sigma = \{a, b\}$ accepts a word of every length. (The input is an NFA. If for each $n \geq 0$ there is at least one word of length n in $L(A)$, then the algorithm outputs *yes* and it outputs *no* otherwise.)

b ab b a b

26×25

52×51

2

$26 \times 25 \times 50$

bbb a bb a b

$u \in L$

$f(i-n) < f(i) - n$

$n' = n + n$

$f(n+1) - f(n)$

$f(n+n)$

$f(n) +$

u v w x y