Theoretical Foundations of Computer Science (Test 2)

Time: 1 hr 15 min | October 31, 2018 | Marks: 30

Questions:

1. Find out whether the following statements are true or false. Justify. 9 marks

   (a) Every subgroup of a cyclic group is cyclic.

   (b) No group can be written as a union of two proper subgroups. (A subgroup is *proper* if it is neither the whole group nor just identity.)

   (c) If G is a cyclic group then every element of G generates G.

2. Let V be a k-dimensional vector space over a finite field $\mathbb{F}$ of size q. Find the number of distinct bases of V. 12 marks

3. Let D be a square-free integer (i.e. no perfect square divides D). Is the set $\mathbb{Z}[\sqrt{D}] = \{a + b\sqrt{D} \mid a, b \in \mathbb{Z}\}$ a ring? Is it a field? What about the sets $\mathbb{Z}[\sqrt[3]{D}]$ and $\mathbb{Q}[\sqrt{D}]$ defined analogously? 9 marks