Quiz 2 ($4 \times 5 = 20$ marks)

1. Solve the following LP using the simplex tableau method.

$$\begin{array}{lrcrl}
\text{maximize} & x_1 & + & x_2 & \\
\text{subject to} & -x_1 & + & x_2 & \leq 2 \\
 & & & x_2 & \leq 4 \\
 & x_1 & + & x_2 & \leq 9 \\
 & x_1 & & & \leq 6 \\
 & x_1 & - & x_2 & \leq 5 \\
 & & & & \\
 & x_1 & , & x_2 & \geq 0
\end{array}$$

2. Consider a primal-dual pair of LPs: 1) maximize $c^T x$ subject to $Ax \leq b, x \geq 0$ (primal) and 2) minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$ (dual).

   Prove the following statement (of the strong duality theorem) using Farkas' lemma: if the primal is feasible and bounded, the dual is feasible and bounded, and moreover the optima of the primal and dual coincide.

3. Suppose in an instance of LP, we have n variables that are unconstrained in sign. Show how they can be replaced by $n+1$ variables that are constrained to be non-negative.

4. Prove or dispove: if A and B are totally unimodular matrices, then the composed matrix $(A \mid B)$ is totally unimodular.