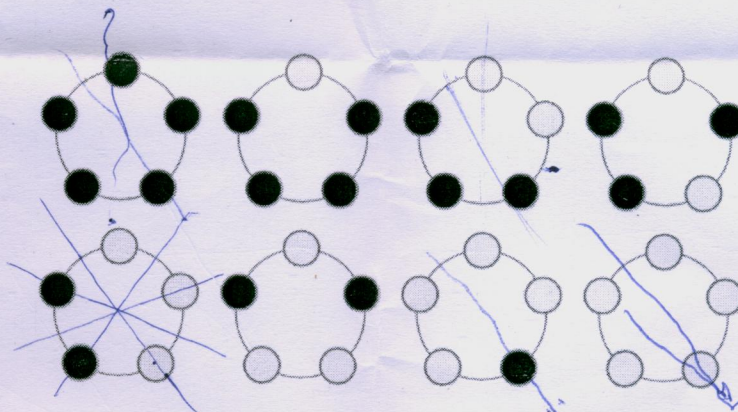


Theoretical Foundations of Computer Science (Endsem)

November 27, 2018

1. Prove that a simple graph with no odd-degree vertex has no cut-edge. For each $k \geq 1$, construct a $2k + 1$ -regular simple graph having a cut-edge. 6 marks
2. Prove or disprove: No group can be written as a union of two proper subgroups. 7 marks
3. Prove that the product of all non-zero elements of \mathbb{F}_p is -1 . 10 marks
4. Let T_i be the random variable defined as the number of balls that need to be thrown so that exactly i bins are non-empty, when balls are thrown into bins at random. Find $\mathbb{E}[T_n]$ and $\Pr[T_n \geq n^2]$. 12 marks
5. Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6. For each even value of n with $n \geq 8$, construct an n -vertex simple planar graph that has exactly four vertices with degree less than 6. 9 marks
6. You have a large number of black beads and white beads, and you are trying to make a n -bead necklace. Two necklaces are considered to be the "same" if one can be rotated or flipped in space so that it becomes the other. For example, for $n = 5$, there are following 8 necklaces: Find a closed



formula for the total number of different necklaces that holds for every prime number $n \geq 3$. 16 marks

7. Show that there is a 2-coloring of K_n with at most $\binom{n}{a} 2^{1-\binom{a}{2}}$ -many monochromatic K_a 's in it. 10 marks