

## Midsem : CC2019

28 Feb 2019

### Instruction

Answer as much as you can. Maximum you can score is 100. All questions carry equal marks.

### 1 Problems

1. A language is unary if every string in it of the form  $1^i$  for some integer  $i$ . Show that if any unary language is NP-complete, then  $P=NP$ .
2. Suppose that  $L_1, L_2 \in NP \cap coNP$ . Define the following language  $L_1 \oplus L_2 = \{x : x \text{ is in exactly in } L_1 \text{ or } L_2\}$ . Show that  $L_1 \oplus L_2$  is in  $NP \cap coNP$ . ✓
3. Prove that  $DSAPCE(n) \neq NP^1$ .
4. Show that  $2SAT$  is in  $NL$ . ✓
5. Show that if  $3SAT \leq_p \overline{3SAT}$ , then  $PH = NP$ .
6. A language  $L$  is SPARSE if there exist a polynomial  $p$  such that for each  $n$ , the number of strings of length  $n$  in  $L$  is at most  $p(n)$ . Argue that  $L \in P/poly$ . ✓

<sup>1</sup>Hint: Use padding argument. Although we do not know whether one is contained in the other.