Chennai Mathematical Institute

Masters Thesis

Random Spanning Trees

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in the

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Declaration of Authorship

I, Bhishmaraj S, declare that this thesis titled, “Random Spanning Trees” and the work presented in it are my own. I confirm that:

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- I have acknowledged all main sources of help.
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Abstract

Faculty Name
Computer Science at CMI

Master of Science

Random Spanning Trees

by Bhishmaraj S

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too. . . .
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Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor…

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WSF	**W**hat (it) **S**tands **F**or

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Naive chain rule algorithm

Input: $G = (V, E)$ and L_G^+
Output: Set of edges corresponding to a random spanning tree
for $e = (u, v) \in E$ **do**
    $R_e^{\text{eff}} = (\chi_u - \chi_v)^T \; L_G^+ \; (\chi_u - \chi_v)$;
    if $(X \sim Bernoulli(R_e^{\text{eff}})) = 1$ **then**
        Add edge e to the spanning tree;
        $G = G/e$;
    else
        $G = G \setminus e$;
    end
    Update L_G^+ ;
end

Algorithm 1: Sampling uniform spanning tree using chain rule

5 Laplacian Paradigm

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