CHENNAI MATHEMATICAL INSTITUTE

MASTERS THESIS

Random Spanning Trees

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in the

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Declaration of Authorship

I, Bhishmaraj S, declare that this thesis titled, “Random Spanning Trees” and the work presented in it are my own. I confirm that:

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Abstract

Faculty Name
Computer Science at CMI

Master of Science

Random Spanning Trees

by Bhishmaraj S

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too. . . . I see

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor…

Contents

List of Figures

List of Tables

List of Abbreviations

LAH	**L**ist **A**bbreviations **H**ere
WSF	**W**hat (it) **S**tands **F**or

1 Introduction

2 Background

2.1 Markov Chains

2.1.1 Fundamental theorem of Markov chain

2.1.2 Markov chain tree theorem

2.2 Results from Spectral Graph Theory

2.2.1 Kirchoff Matrix Tree Theorem

2.2.2 Some properties of Laplacian

2.3 Electric Networks

3 Random Walk Approach

3.1 Aldous, Broder

3.2 Wilson

4 Matrix Approach

4.1 Colbourn, Dey, Nel

4.2 Harvey, Xu

4.2.1 Techniques used

Naive chain rule algorithm

Input: $G = (V, E)$ and L_G^+
Output: Set of edges corresponding to a random spanning tree
for $e = (u, v) \in E$ **do**
  $R_e^{\text{eff}} = (\chi_u - \chi_v)^T \; L_G^+ \; (\chi_u - \chi_v)$;
  if $(X \sim \mathit{Bernoulli}(R_e^{\text{eff}})) = 1$ **then**
    Add edge e to the spanning tree;
    $G = G/e$;
  else
    $G = G \setminus e$;
  end
  Update L_G^+ ;
end

Algorithm 1: Sampling uniform spanning tree using chain rule

5 Laplacian Paradigm

5.1 Kelner, Madry

6 Conclusion