Random Spanning Trees

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Overview

1. Background
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2. Second Section

Paragraphs of Text

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Naive algorithm using effective resistance

Input: $G = (V, E)$ and L_G^+
Output: Set of edges corresponding to a random spanning tree
for $e = (u, v) \in E$ **do**
    $R_e^{\text{eff}} = (\chi_u - \chi_v)^T \; L_G^+ \; (\chi_u - \chi_v)$;
    if $(X \sim Bernoulli(R_e^{eff})) = 1$ **then**
        Add edge e to the spanning tree;
        $G = G/e$;
    else
        $G = G \setminus e$;
    end
    Update L_G^+ ;
end

Algorithm 1: Sampling uniform spanning tree using chain rule

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

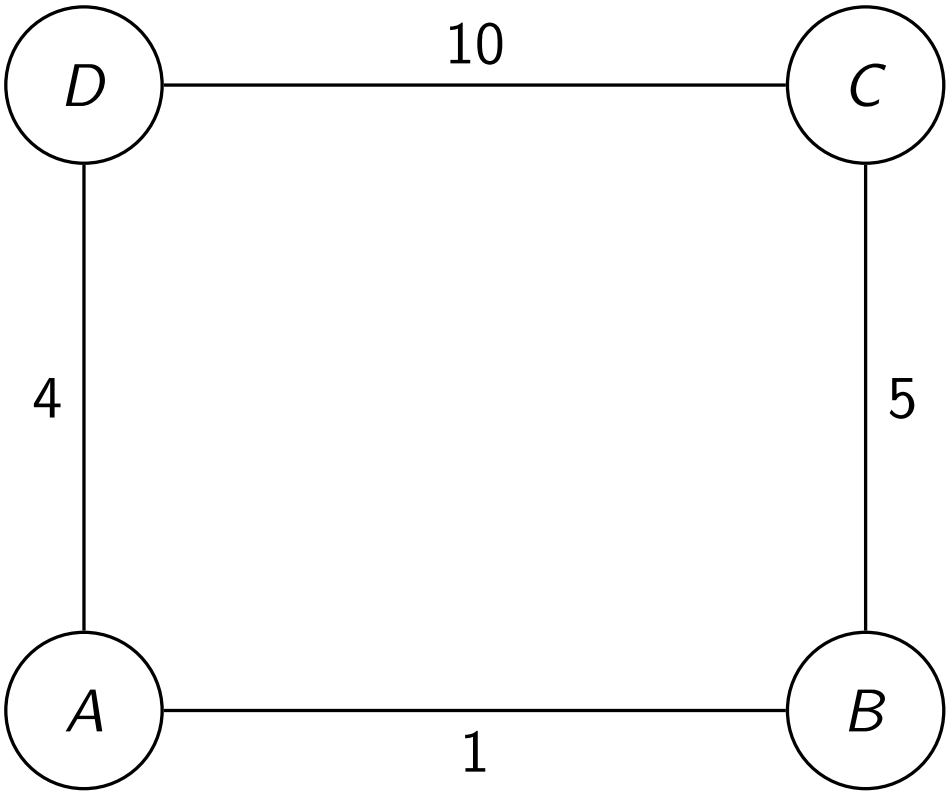
$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

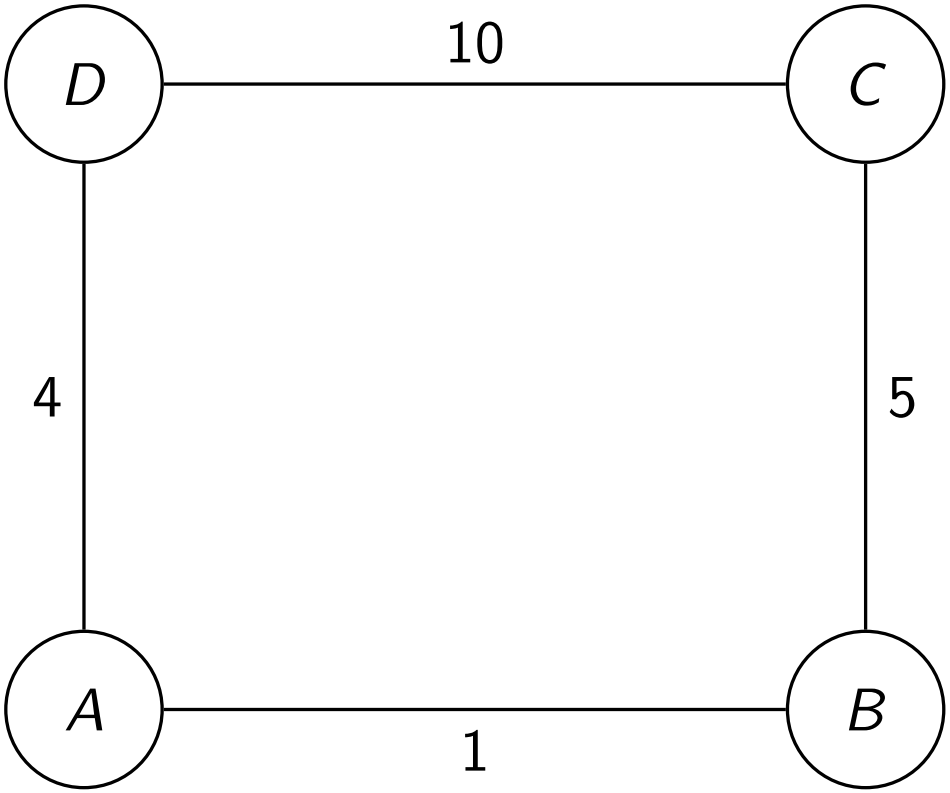
Figure



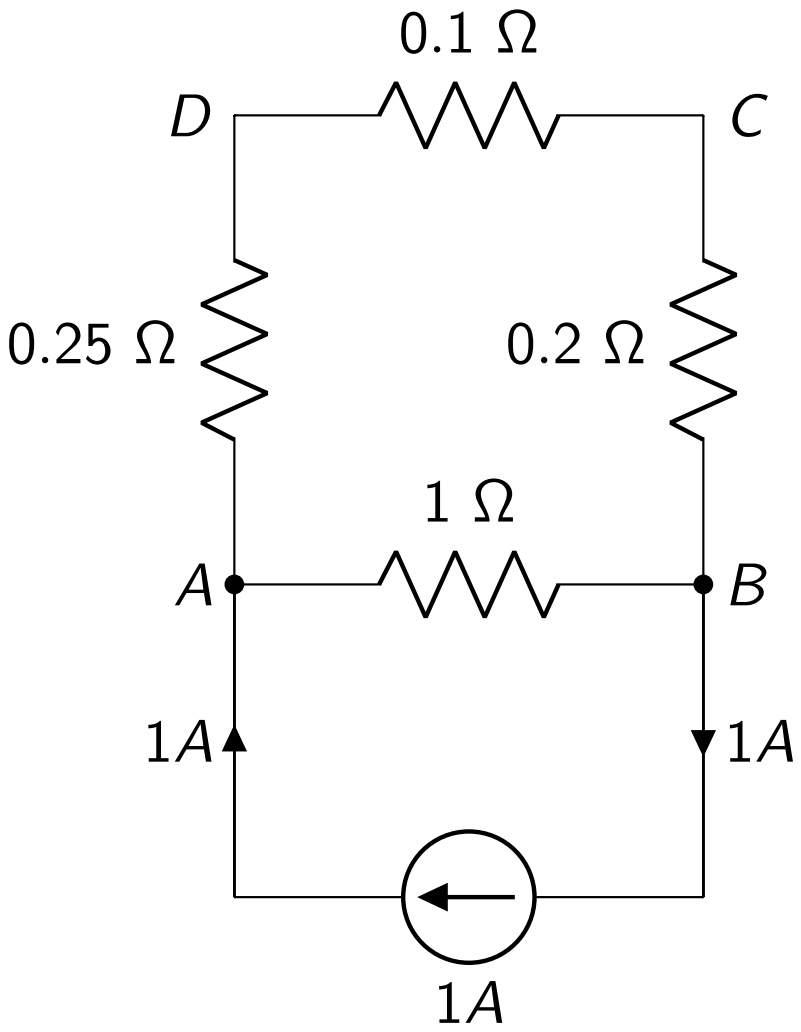
(a) The original graph G

(b) The electric network version of G

Figure

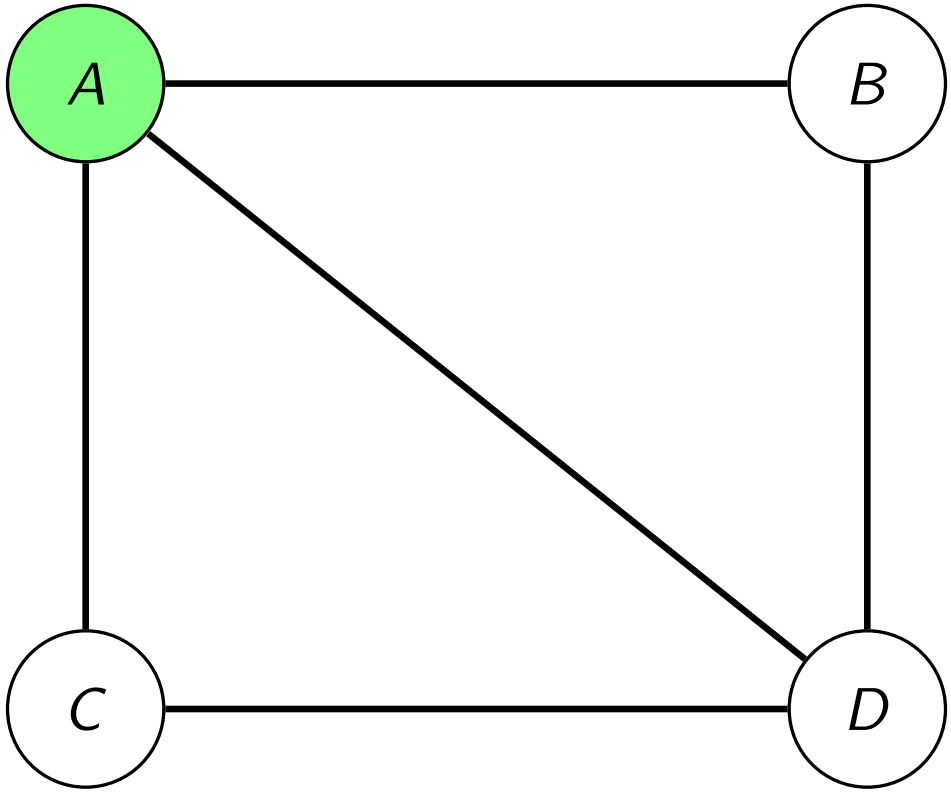


(a) The original graph G

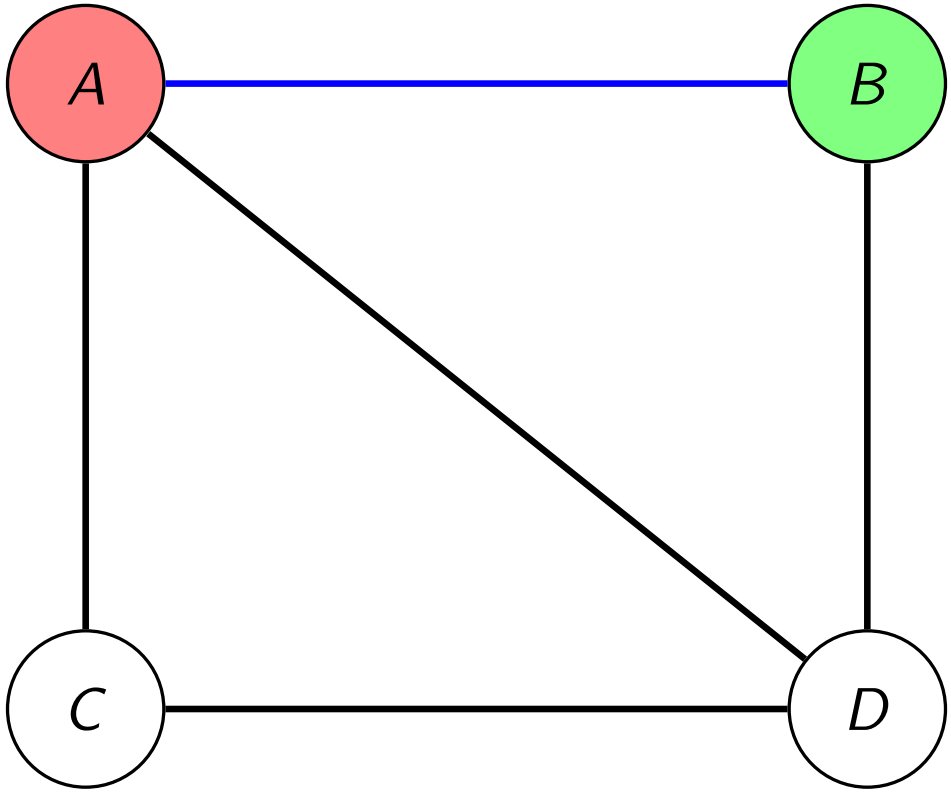


(b) The electric network version of G

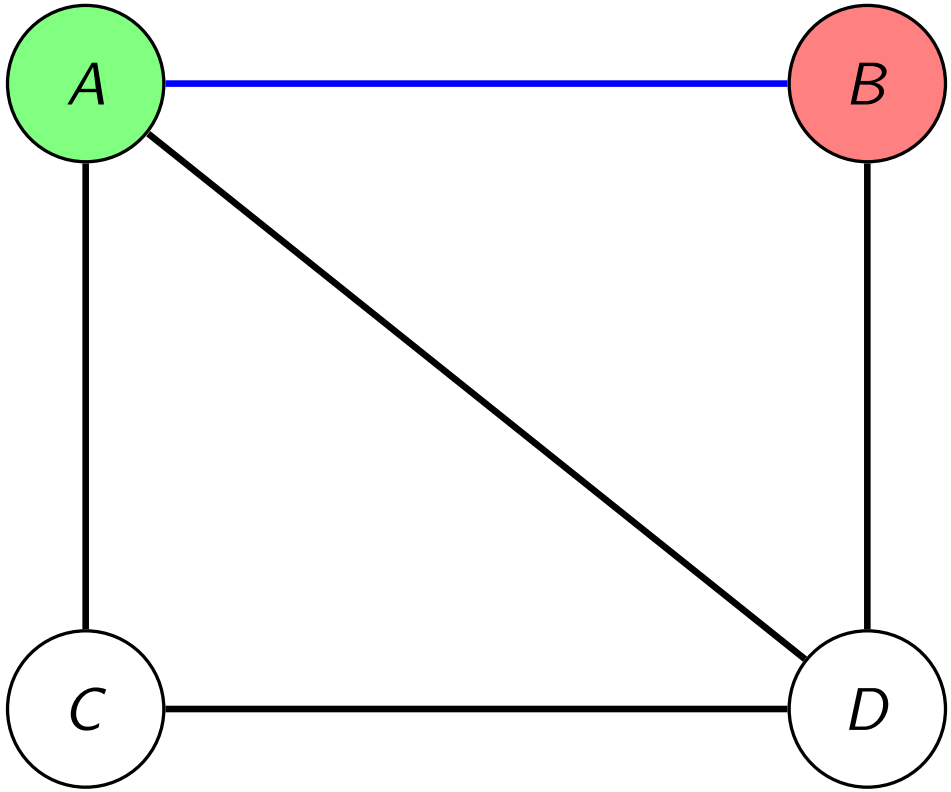
Aldous-Broder Algorithm example



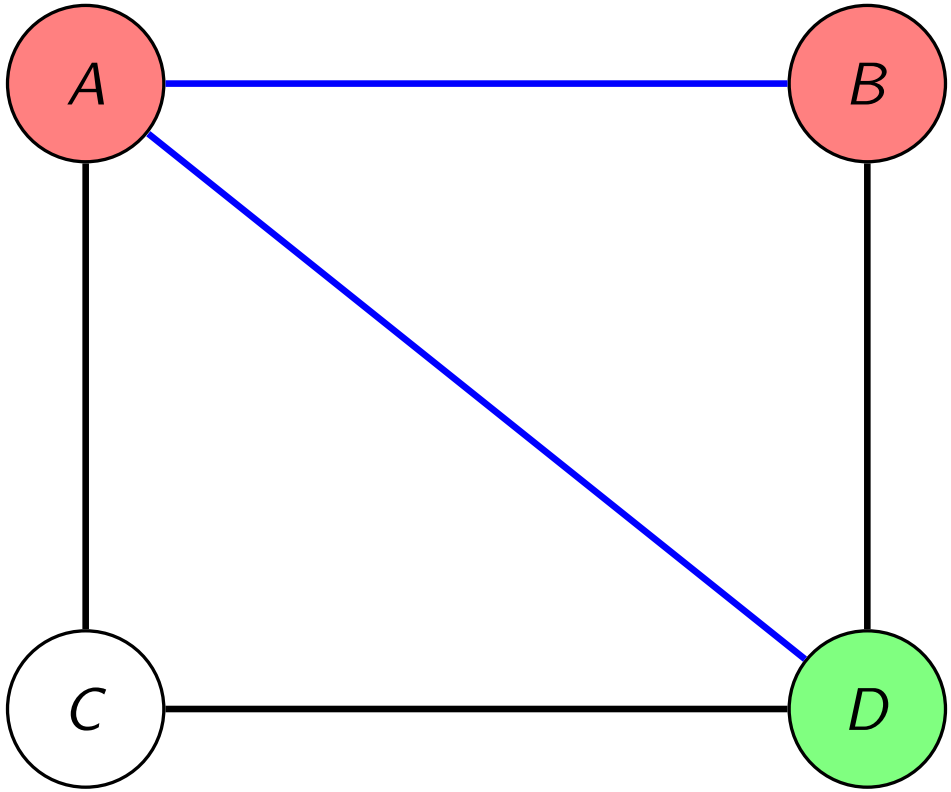
Aldous-Broder Algorithm example



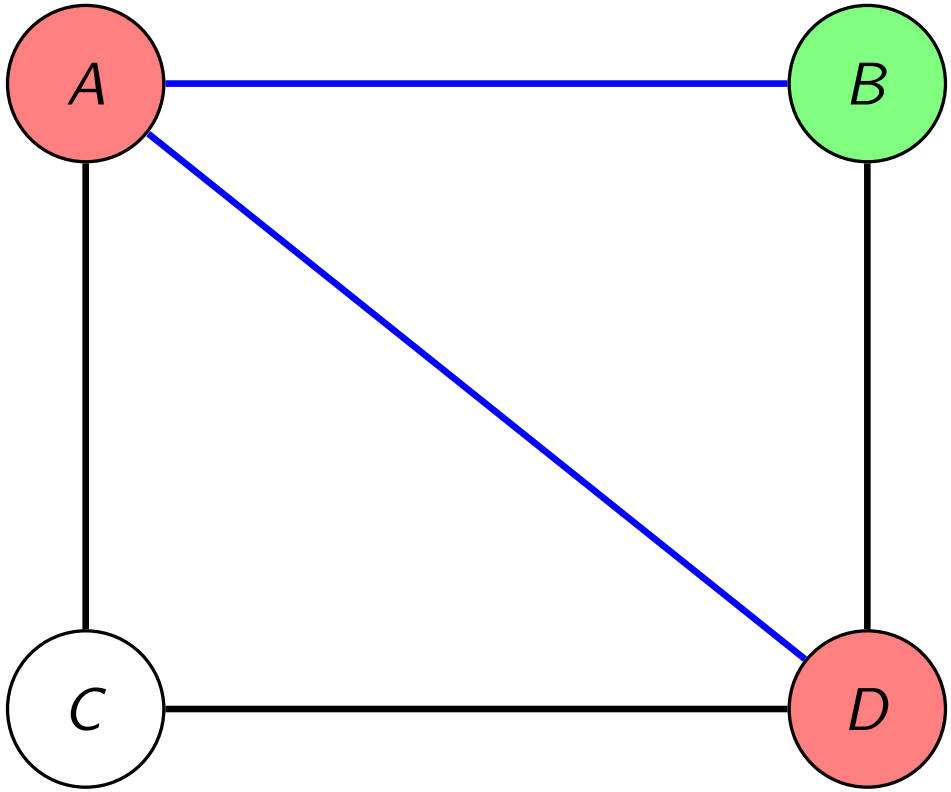
Aldous-Broder Algorithm example



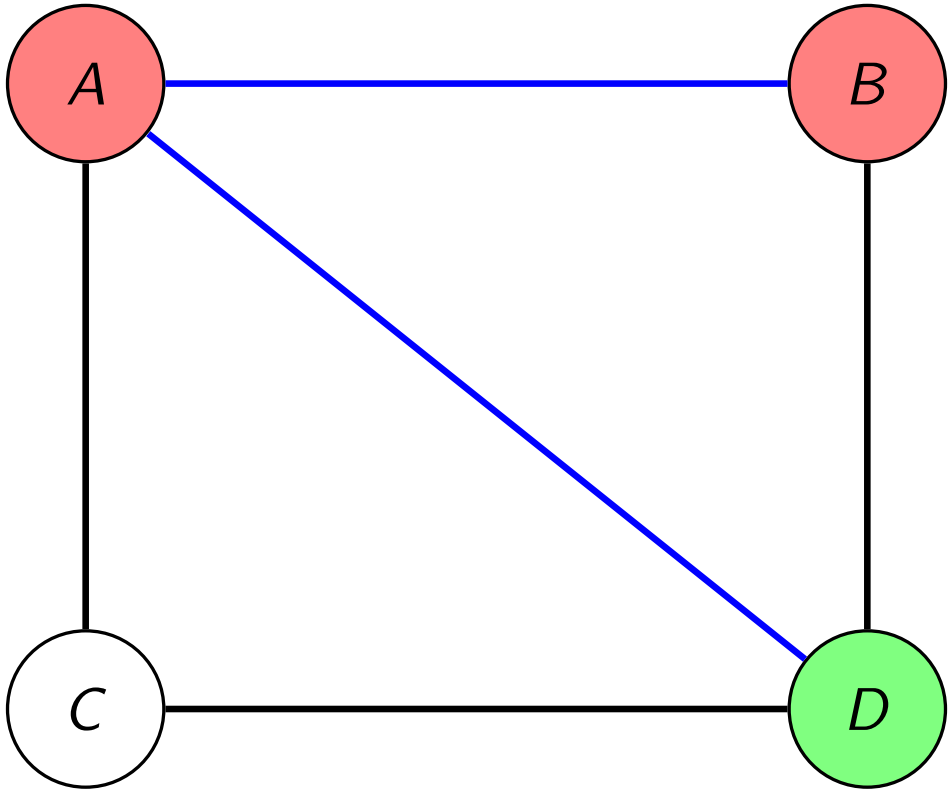
Aldous-Broder Algorithm example



Aldous-Broder Algorithm example



Aldous-Broder Algorithm example



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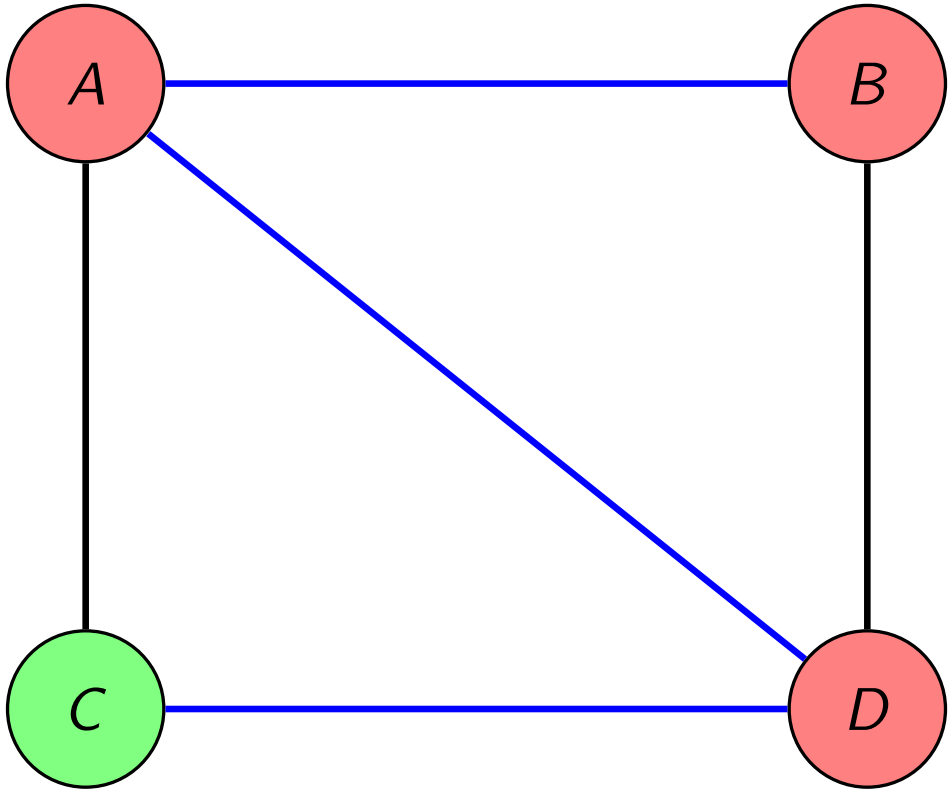
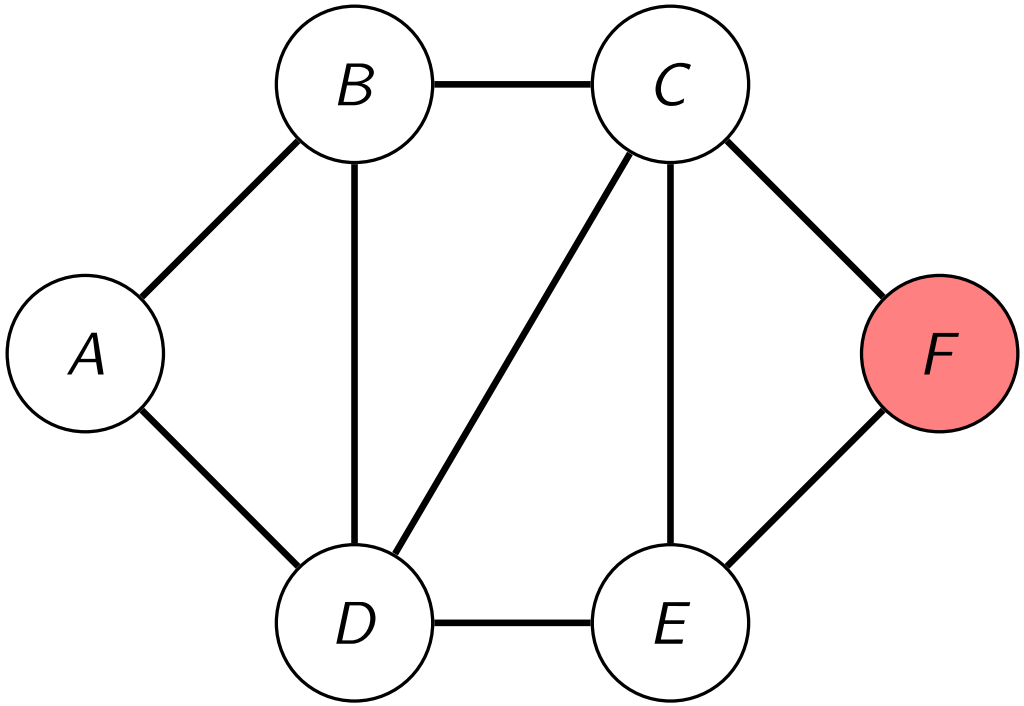
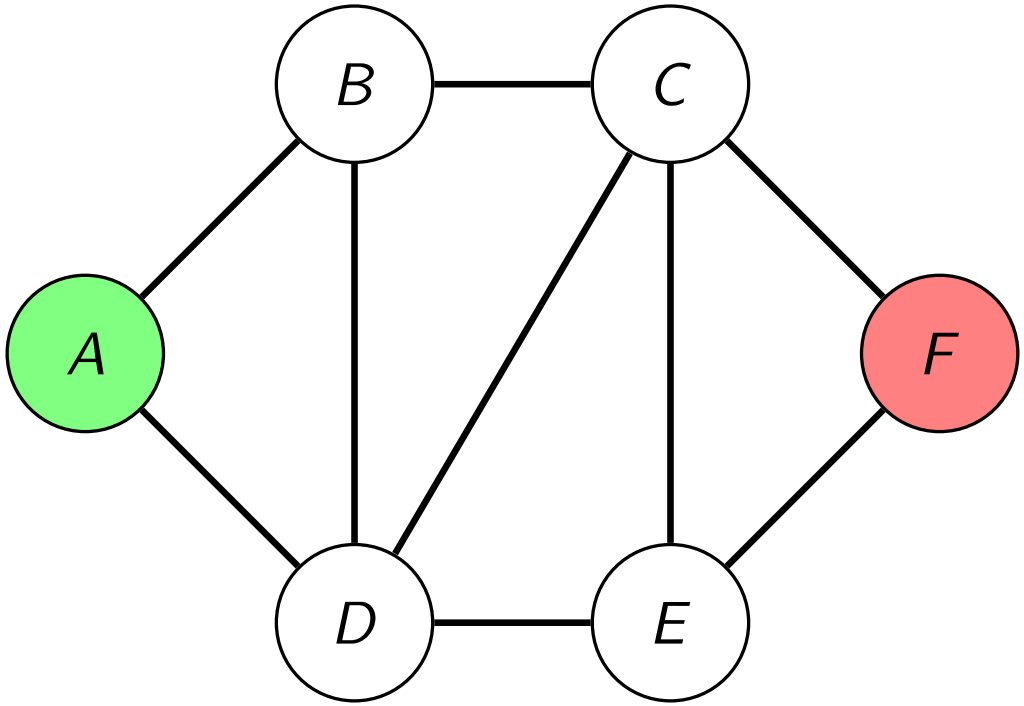


Wilson's algorithm example

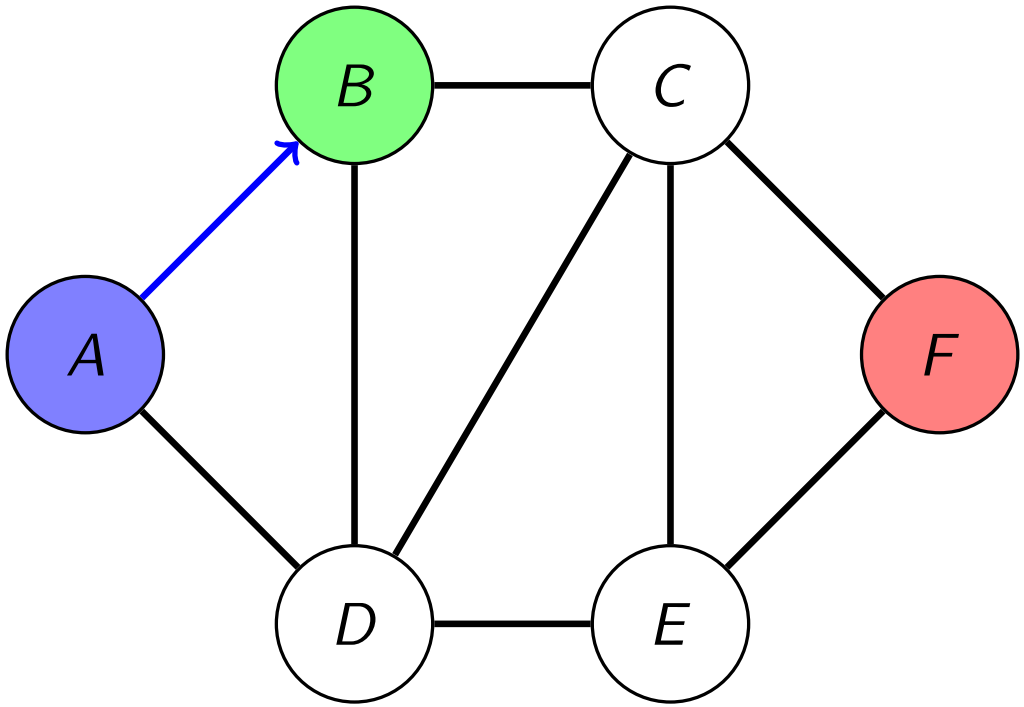


Wilson's algorithm example

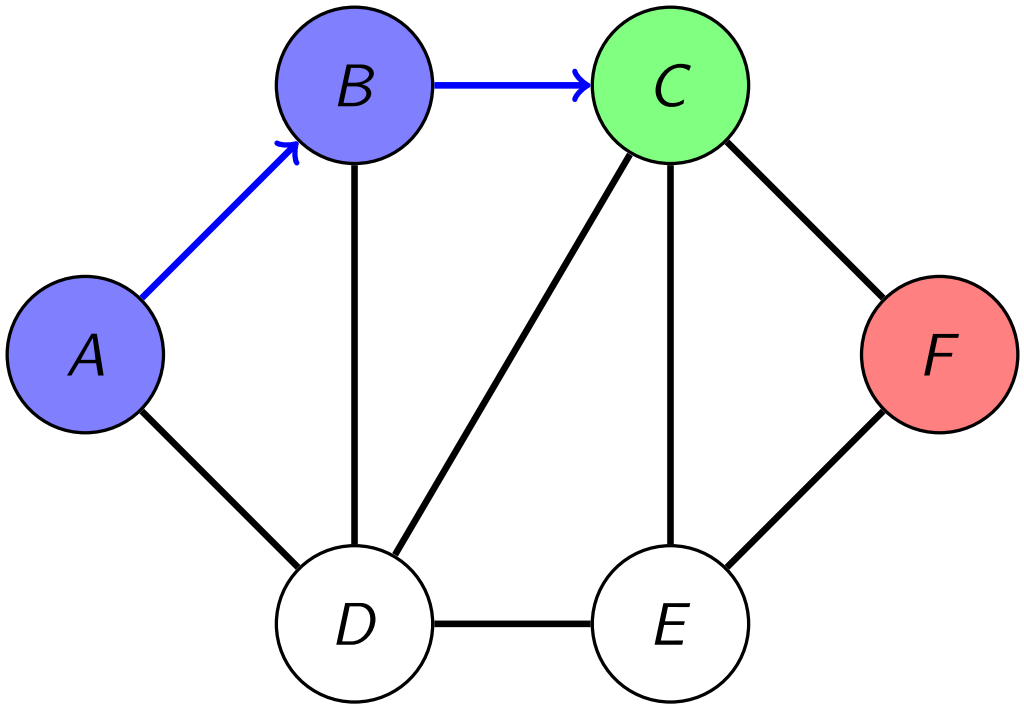
Start at A



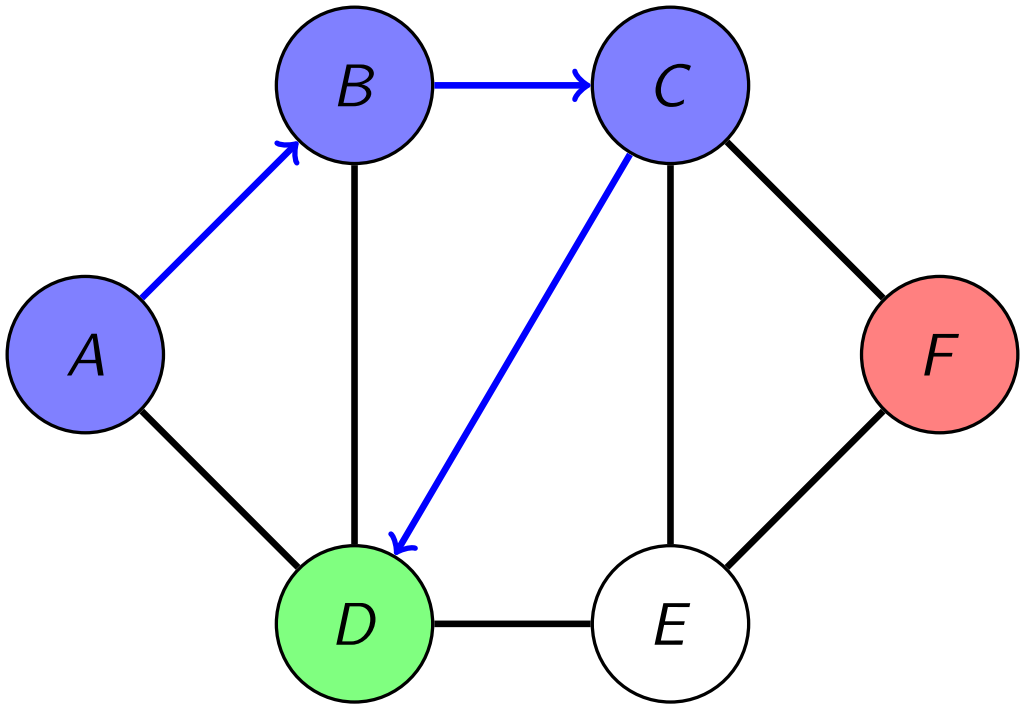
Wilson's algorithm example



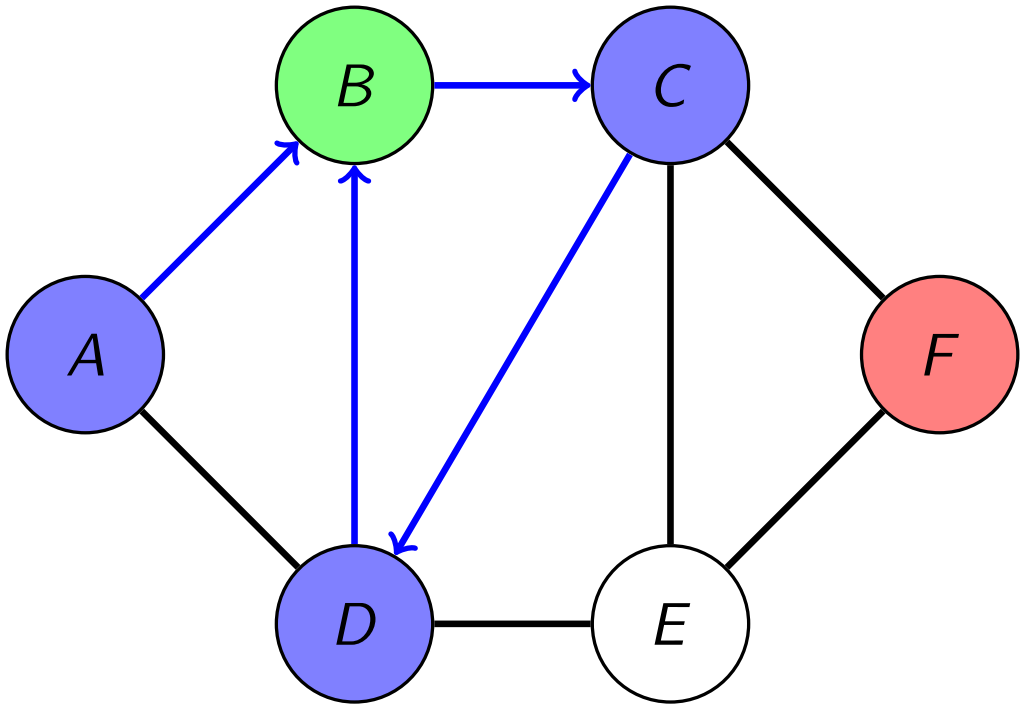
Wilson's algorithm example



Wilson's algorithm example



Wilson's algorithm example



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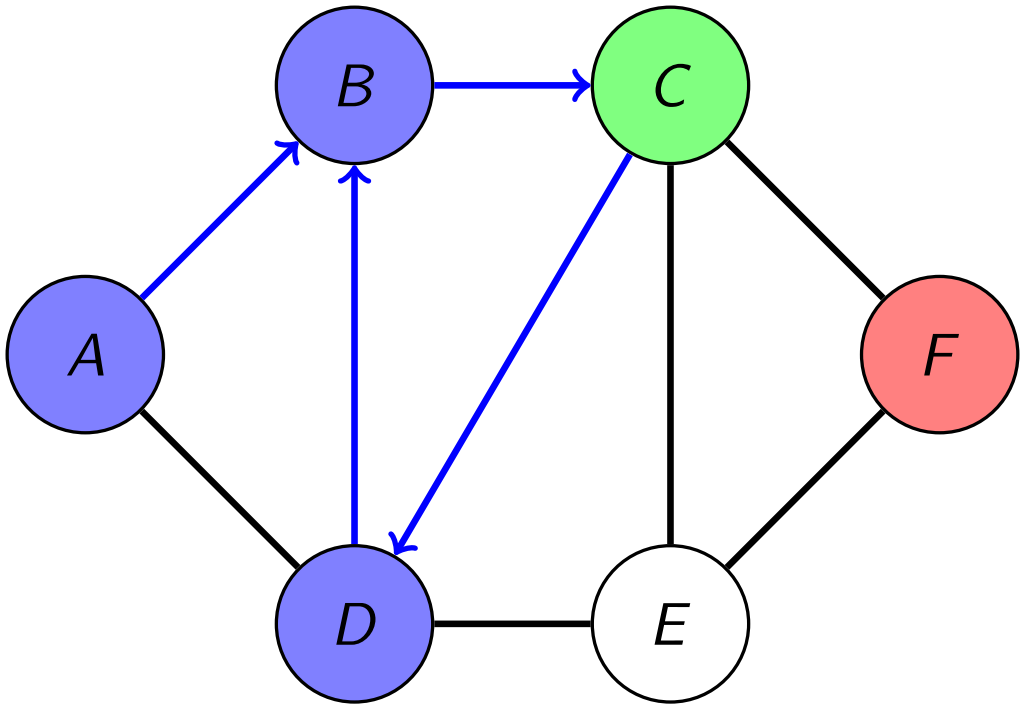
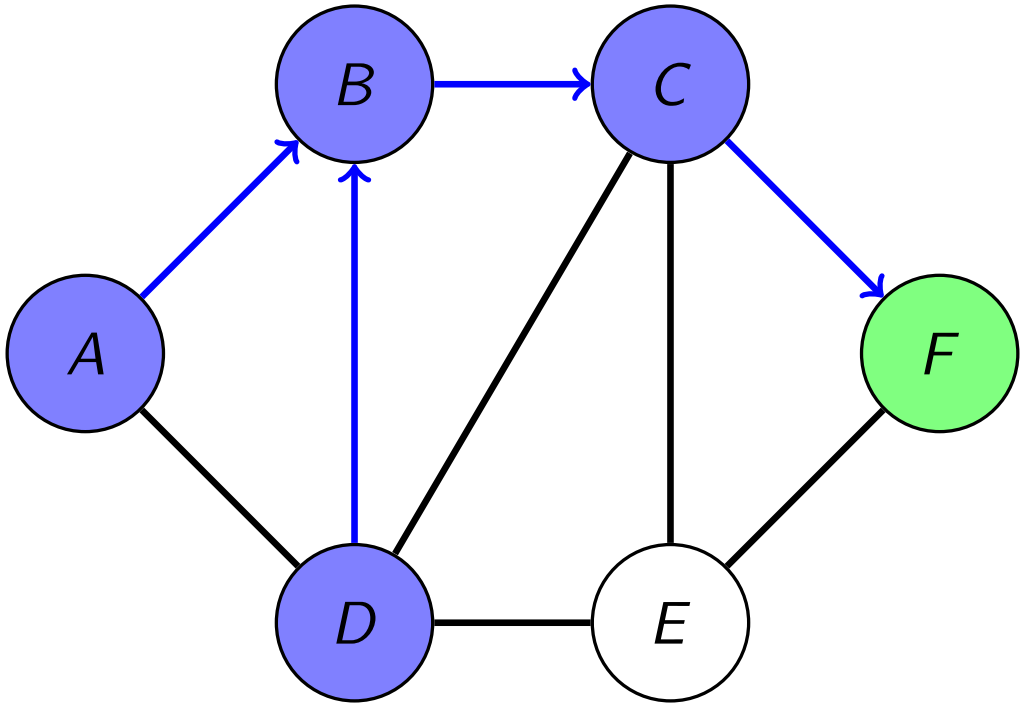


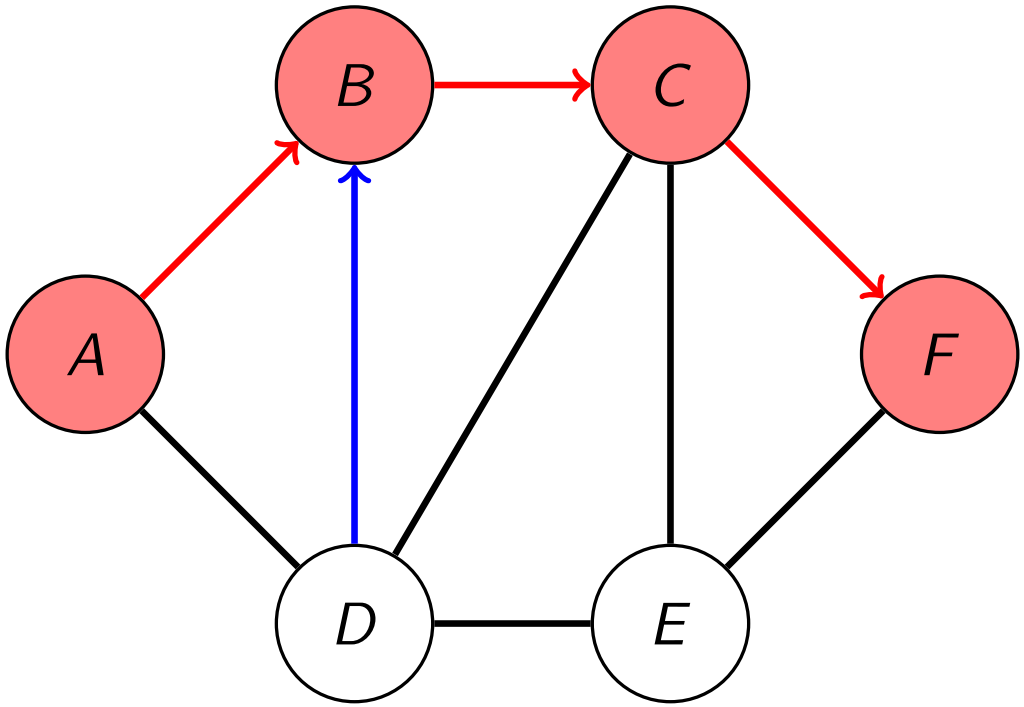
Wilson's algorithm example

Notice the **next(C)** has changed from D to F.



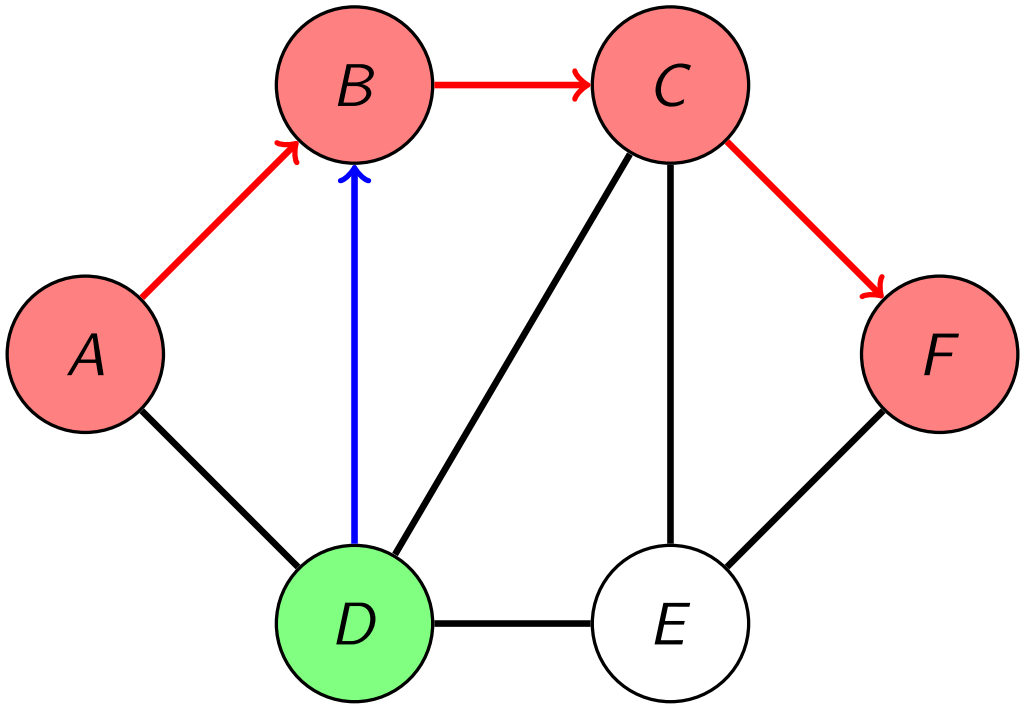
Wilson's algorithm example

Since a vertex already in the tree has been reached (namely F), starting from A we trace the successors and set their **inTree** value to *True*

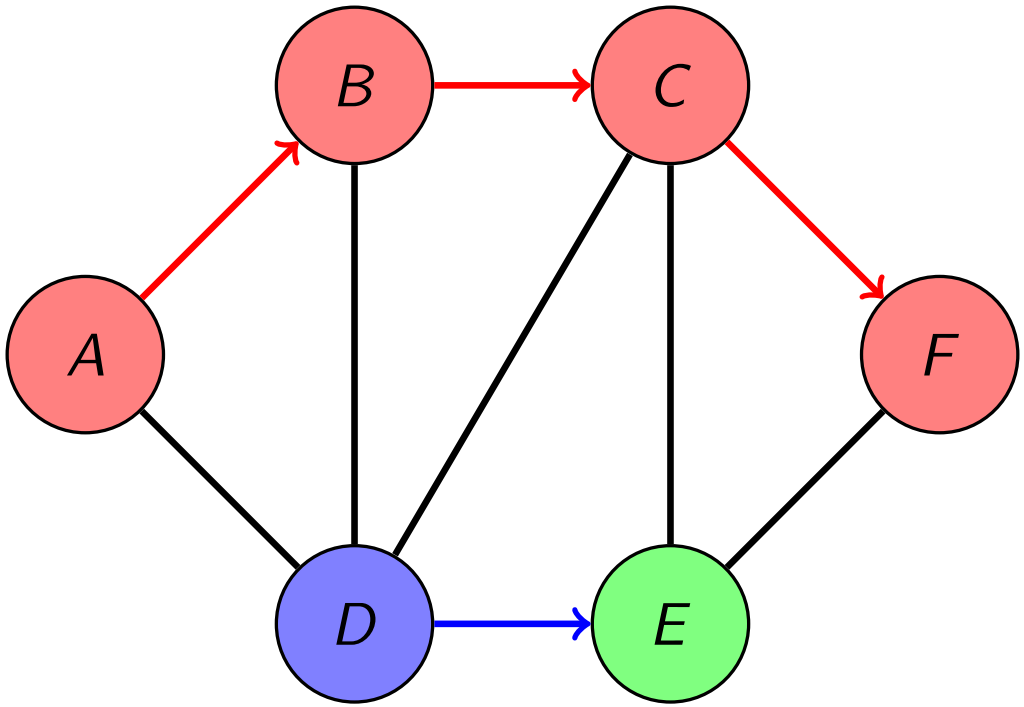


Wilson's algorithm example

Since B, C are already in the tree they will be skipped and now will start at D

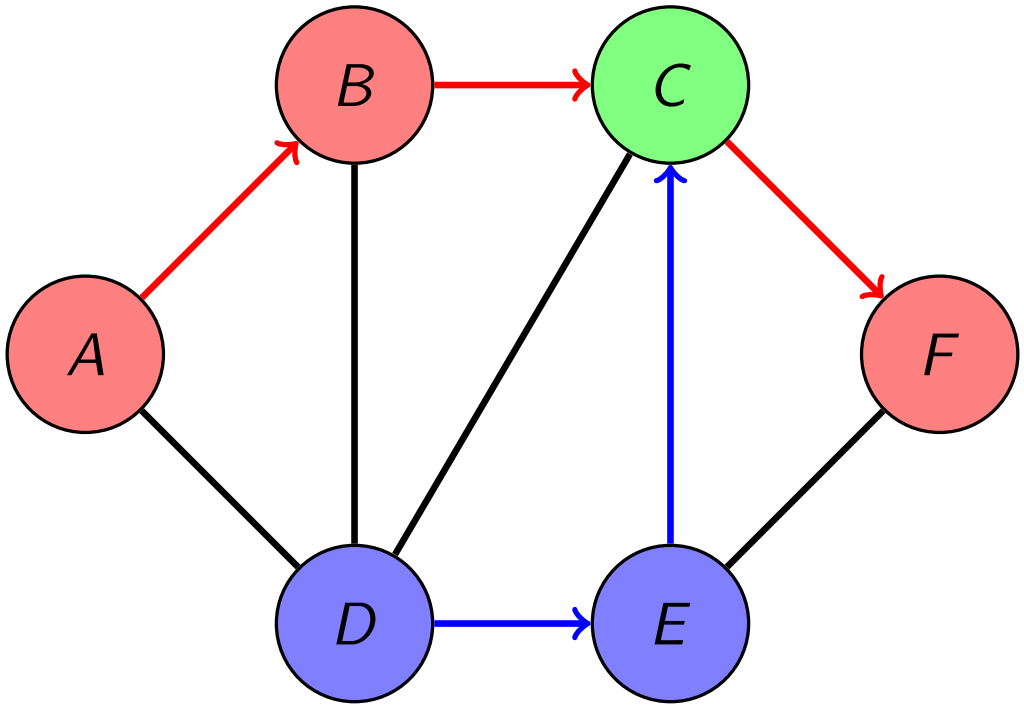


Wilson's algorithm example



Wilson's algorithm example

Since C is already in the tree, the random walk stops and the algorithm retraces from D and includes the vertices into the tree



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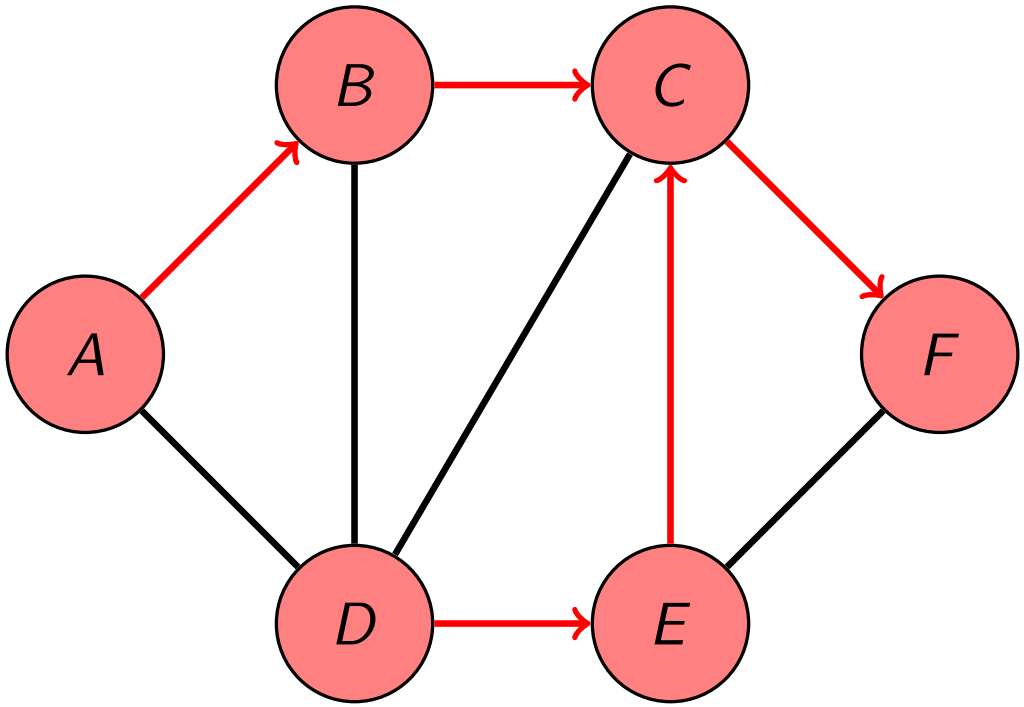

References

John Smith (2012)

Title of the publication

Journal Name 12(3), 45 – 678.

The End