

DS 288 (AUG) 3:0 Numerical Methods

Homework-4

Question-1

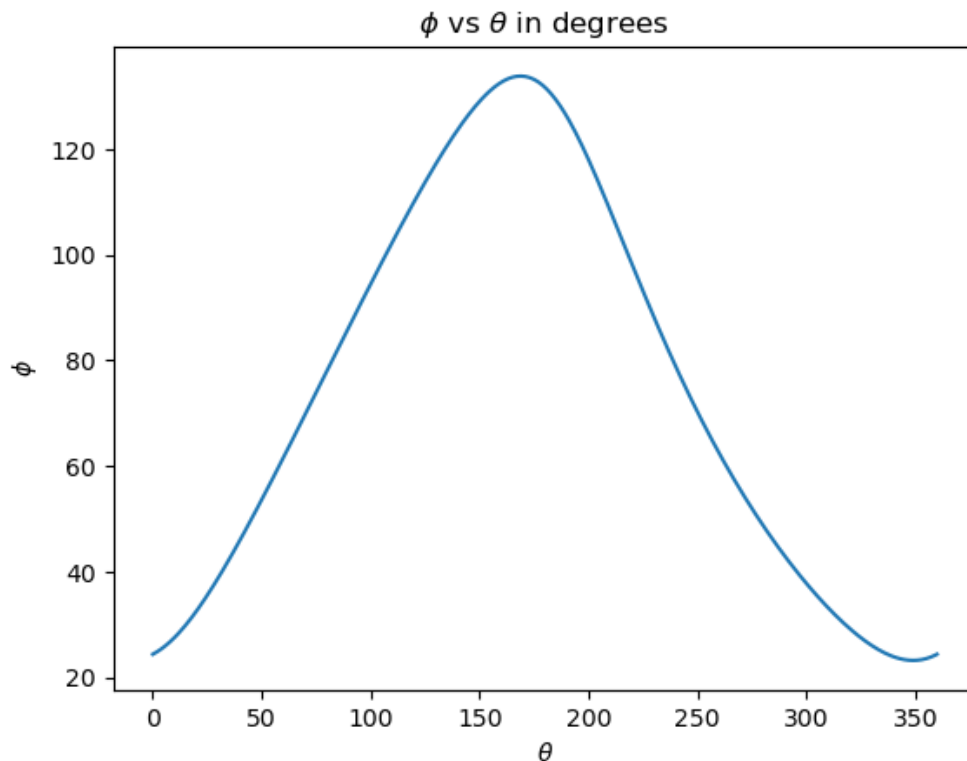
- We incremented θ from 0° to 360° in 1° steps and calculated ϕ at each step using Newton's method, with each solution of ϕ using the previous value as an initial guess to improve convergence.
- The first derivative of ϕ with respect to θ was computed using both forward and centered difference approximations.
- The forward difference has an error of $O(\Delta\theta)$, leading to higher truncation error and less accuracy compared to the centered difference.
- The centered difference has an error of $O(\Delta\theta^2)$, providing smoother, more accurate results, as seen in a plot where it yields a more precise derivative approximation than the forward difference.

Forward Difference First Derivative:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Central Difference First Derivative:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Figure 1: ϕ as a function of θ

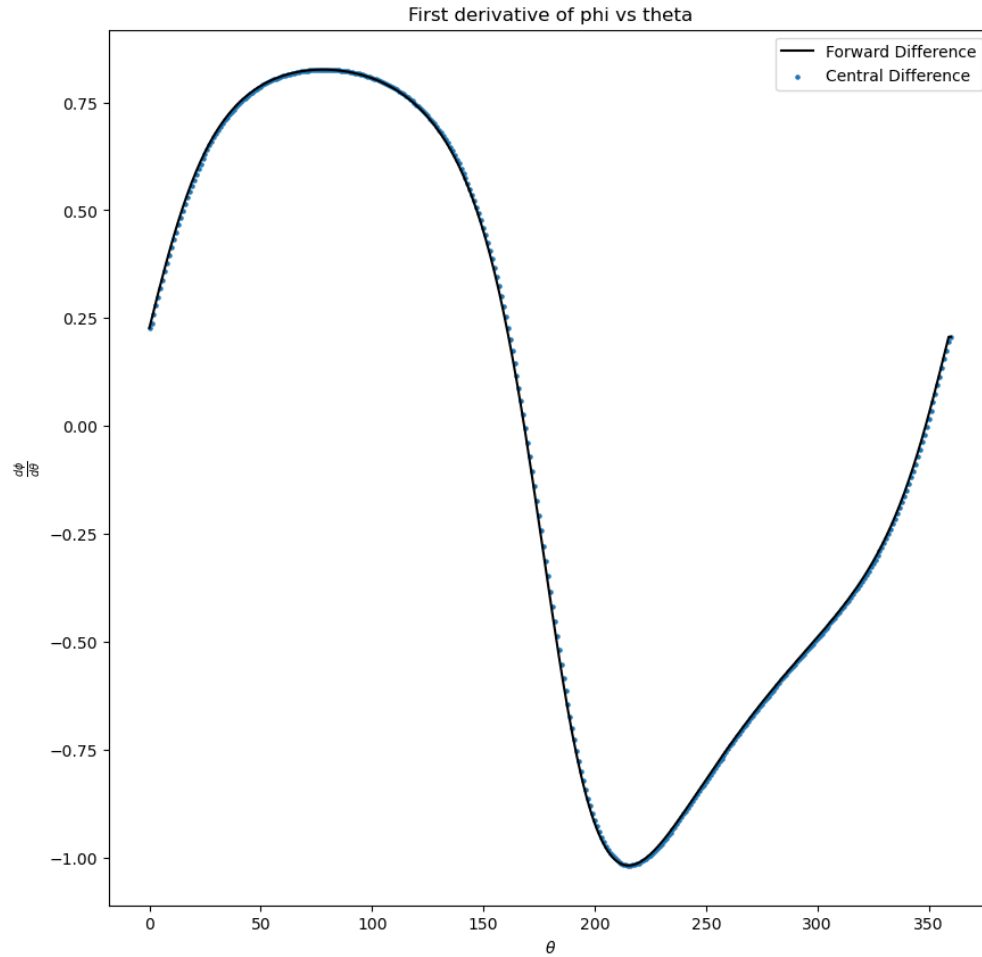


Figure 2: $\frac{d\phi}{d\theta}$ vs θ

Question-2

Forward Difference First Derivative:

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Central Difference First Derivative:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Forward Difference Second Derivative:

$$f''(x) \approx \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$$

Central Difference Second Derivative:

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

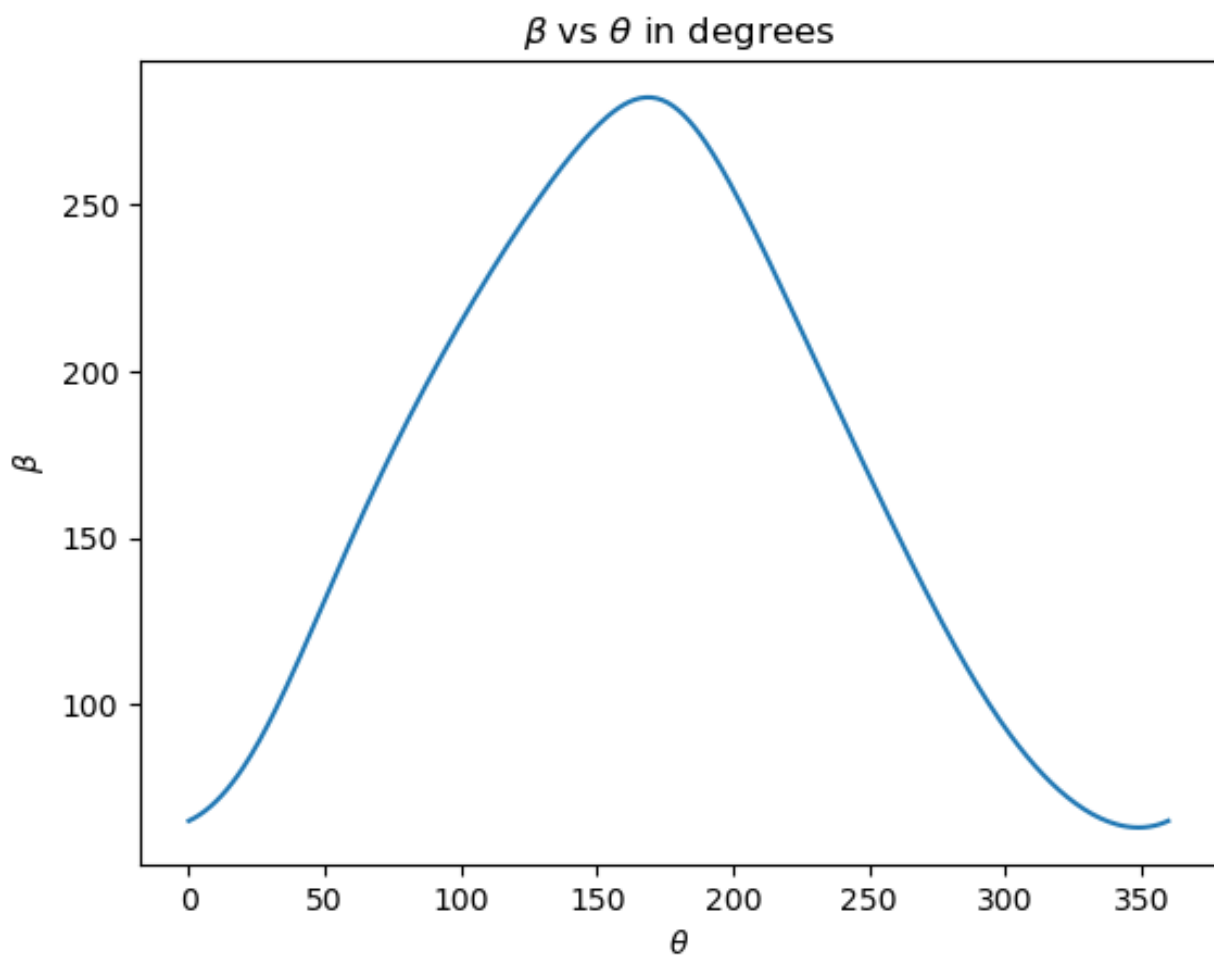


Figure 3: β vs θ in degrees.

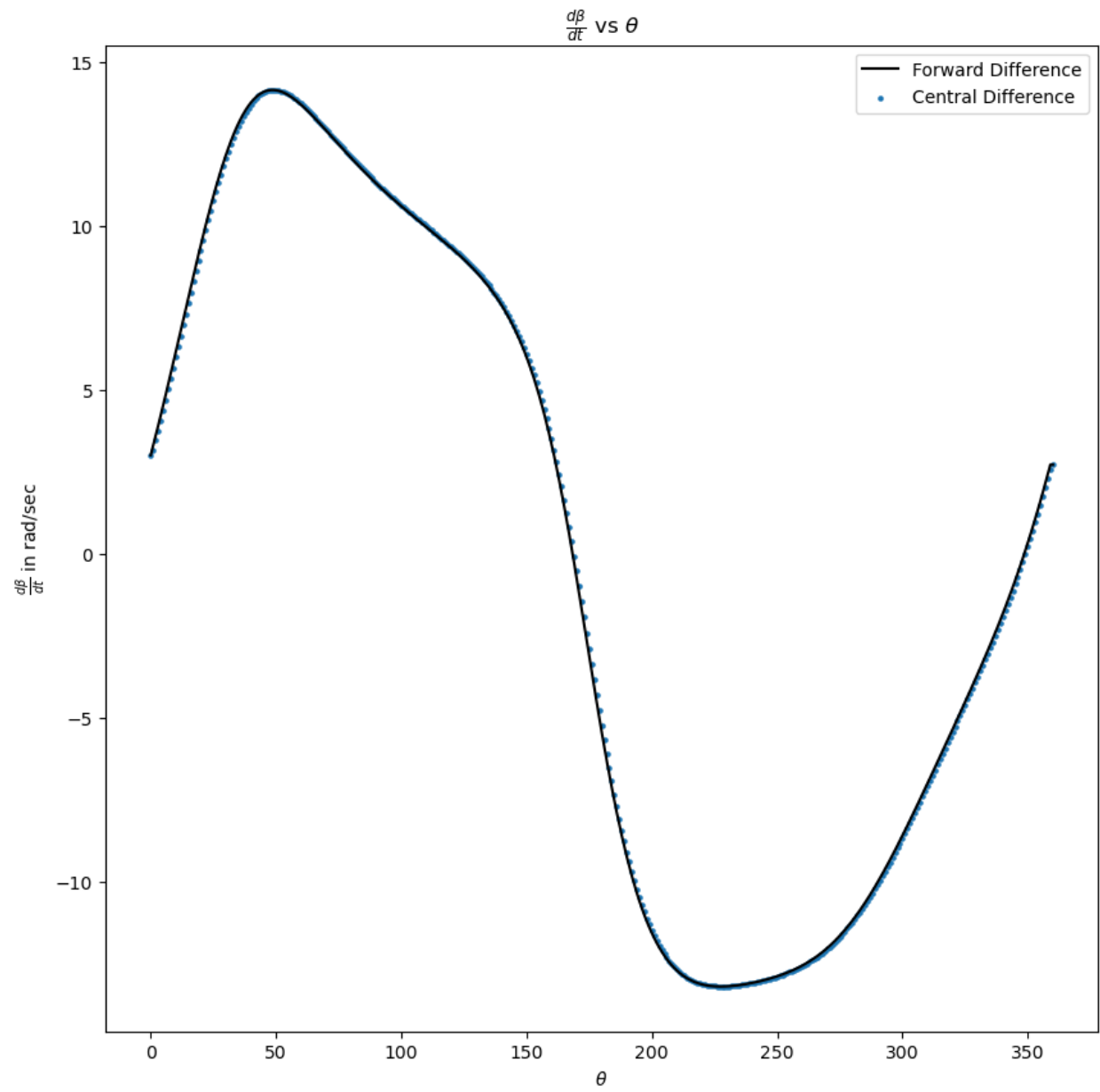


Figure 4: First Derivative of β vs θ

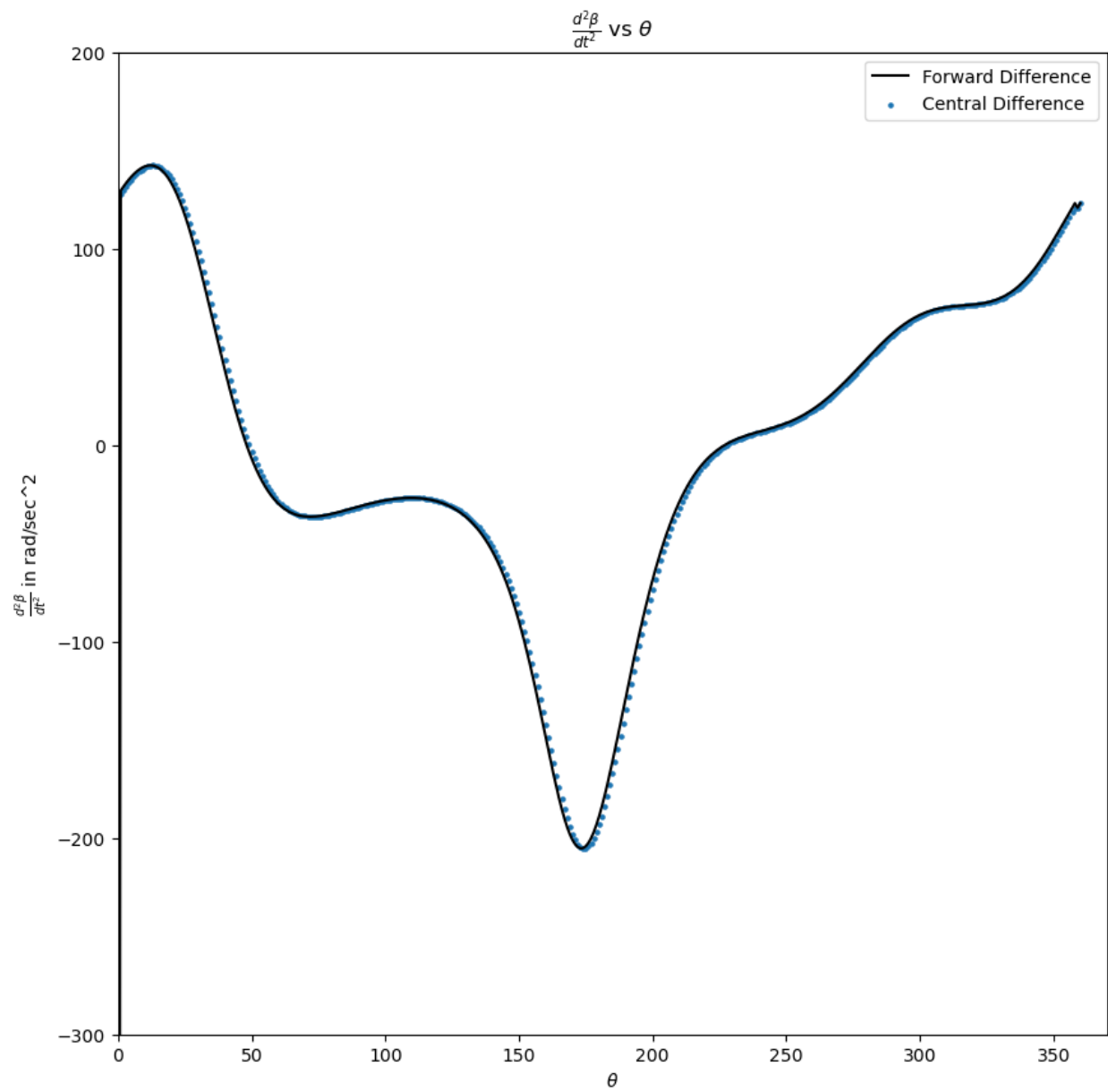


Figure 5: Second Derivative of β

- Angular velocity at $\theta = 100^\circ$: 10.60 rad/sec
- Angular acceleration at $\theta = 100^\circ$: -28.13 rad/sec^2