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Question-1

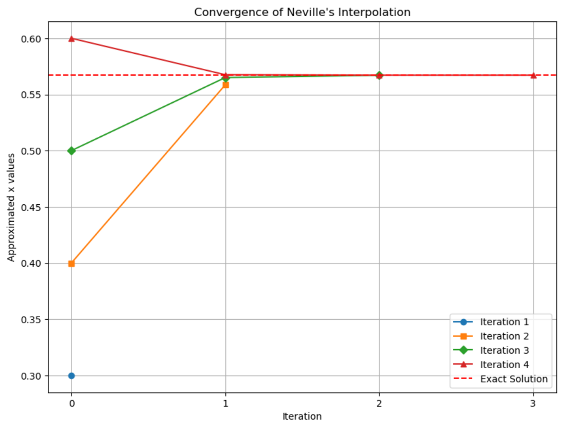

Figure 1: Convergence of Neville's Interpolation

The following table summarizes the results obtained from Neville's algorithm:

x	0	1	2
0.3	0.0000	0.0000	0.0000
0.4	0.55854731	0.0000	0.0000
0.5	0.56504161	0.56711122	0.0000
0.6	0.56754481	0.56714627	0.56714262

Figure 2: Neville's Interpolation Table

Results

The approximated solution for x (where $y = x - e^{-x}$) is given by:

$$\text{Approximated solution for } x : \quad 0.5671426235278707$$

The relative error of the approximation is calculated as follows:

$$\text{Relative error: } 0.000118\%$$

Question-2

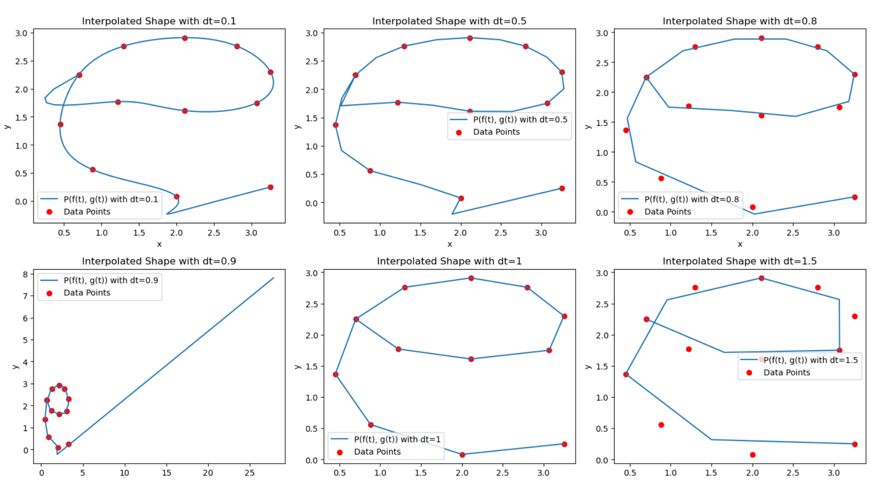

Figure 3: Interpolation of P(x,y) for different dt

Results

Optimal dt Values: $dt = 0.8$ and $dt = 1$ effectively approximate the shape of the letter "E" with minimal distortion, maintaining clarity in the representation but compromising smoothness.

Distortion with Lower dt: Both $dt = 0.1$ and $dt = 0.5$ or below them introduce noticeable distortions, compromising the accuracy of the letter's shape but enhances smothness of the curve.

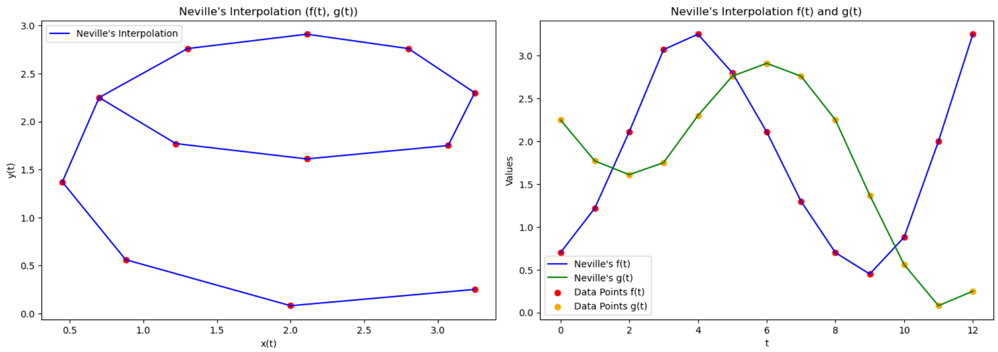

Figure 4: Interpolation of P(x,y)and g(t) and f(t) for dt=1

Question-3

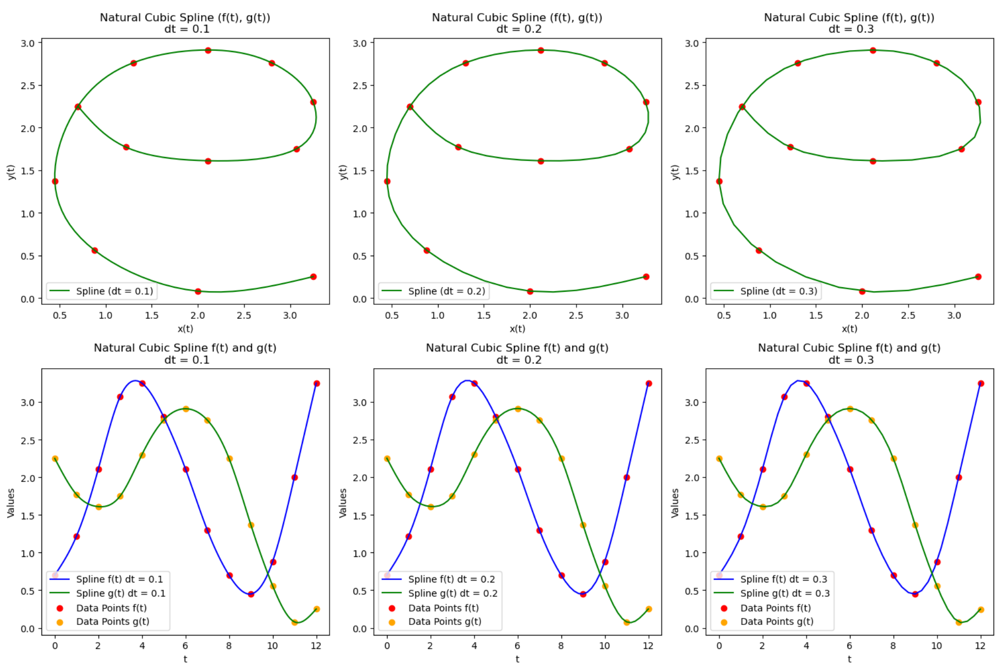

Figure 5: Interpolation of P(x,y) for different dt with Natural cubic spline

Natural Cubic Spline Coefficients

optimal dt = 0.3

Coff	0	1	2	3	4	5	6	7	8	9	10	11
a_x	0.70	1.22	2.11	3.07	3.25	2.80	2.11	1.30	0.70	0.45	0.88	2.00
b_x	0.4379	0.6842	1.0554	0.6442	-0.2123	-0.6051	-0.7874	-0.7453	-0.4614	0.0408	0.8383	1.2562
c_x	0.00	0.2463	0.1250	-0.5362	-0.3204	-0.0724	-0.1099	0.1520	0.1320	0.3702	0.4273	-0.0093
d_x	0.0821	-0.0404	-0.2204	0.0719	0.0826	-0.0125	0.0873	-0.0067	0.0794	0.0190	-0.1455	0.0031

Table 1: Natural Cubic Spline Coefficients for x

Coff	0	1	2	3	4	5	6	7	8	9	10	11
a_y	2.25	1.77	1.61	1.75	2.30	2.76	2.91	2.76	2.25	1.37	0.56	0.08
b_y	-0.5522	-0.3356	-0.0255	0.3775	0.5853	0.3112	-0.0002	-0.3104	-0.7384	-0.9062	-0.7068	-0.1366
c_y	0.00	0.2166	0.0934	0.3096	-0.1018	-0.1722	-0.1392	-0.1709	-0.2571	0.0892	0.1102	0.4599
d_y	0.0722	-0.0411	0.0721	-0.1371	-0.0235	0.0110	-0.0106	-0.0287	0.1154	0.0070	0.1166	-0.1533

Table 2: Natural Cubic Spline Coefficients for y

Smoother Interpolation in Natural Cubic Spline

- The natural cubic spline produces smooth curves with continuous first and second derivatives.
- There are no sharp transitions between data points, resulting in a more flowing, natural curve.

Sharper Transitions in Neville's Interpolation

- Neville's interpolation leads to jagged transitions and sharp corners between the points.
- The piecewise interpolation used does not ensure smoothness, making the curve more angular.

Step Size Differences

- **Natural Cubic Spline(dt = 0.3)**: Finer step size allows for higher precision and more frequent interpolation, creating a curve that closely matches the true data.
- **Neville's Interpolation (dt = 1)**: Larger step size results in fewer interpolated points, causing the curve to appear less precise and more approximate.

Overall Comparison

- **Natural Cubic Spline**: Produces a smoother, more accurate representation of the data with finer detail.
- **Neville's Interpolation**: Yields a rougher, less smooth curve due to both the method and larger step size.

Question-4

To determine a and b from given equation $y = be^{-2\pi a x}$ from given 3 points using linear least square Method

$\rightarrow \quad y = be^{-2\pi a x}$

$\log y = \log(be^{-2\pi a x})$ [Apply log on both sides

$\log y = \log b - 2\pi a x$ [$\therefore \log y = \text{slope} \cdot x + \text{intercept}$]

Given points (0,17), (16,9) (32,5)

$y = mx + c$ $\boxed{m = -2\pi a \ , \ c = \log b}$

Use least square to find parameters

$$m = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$

$$C = \frac{\Sigma Y - m\Sigma x}{n}$$

After calculations :-

$\boxed{a \simeq 0.00608 \qquad b = 16.918}$

Using Non-linear Least square Approach :-

$y = be^{-2\pi a x}$

$S(a,b) = \Sigma \left(y_i^{observed} - be^{-2\pi a x_i}\right)^2$

Goal is to find that a, b which minimize $S(a,b)$

After calculations :-

$$\boxed{\begin{aligned} a &\simeq 0.0654 \\ b &\simeq 17.032 \end{aligned}}$$

Error discussions :-

Although both methods aim to minimize the residual between observed and predicted values. they yield different estimates for parameters.

→ we can see that different a and b for different methods. **Linear error :-**

$$\text{error}_i^{\text{linear}} = \log(y_i^{\text{observed}}) - \log(y_i^{\text{Predicted}})$$

$$\text{error}_i^{\text{linear}} = \log\left(\frac{y_i\,\text{observed}}{y_i\,\text{predicted}}\right)$$

Non-linear error :-

$$\text{error}_i^{\text{non-linear}} = y_i^{\text{observed}} - y_i^{\text{Predicted}}$$

while both methods aim to minimize the overall error the way they treat individual data points differ.

→ Non linear fit minimize absolute difference in y.

→ Linear fit minimize the log difference, which are relative to size of y.

So, this difference in how error are weighted across data points typically results difference in a & b.