DS 288 (AUG) 3:0 Numerical Methods
Homework–3 [1]

`Due date: October 10, 2024 (Thursday); 10:00 A.M.`

1. Exercise Set 3.2, Problem#12 in Text (Page-124). Be sure to read the note about inverse interpolation directly above the problem. Solve this problem using an iterated interpolation approach (i.e., Neville's algorithm). Report the relative error of your result and what value you used for the exact solution. [`1.5 points`]

2. In some applications one is faced with the problem of interpolating points which lie on a curved path in the plane, for example in computer printing of enlarged letters. Often the complex shapes (i.e., alphabet characters) cannot be represented as a function of x because they are not single-valued. One approach is to use *Parametric Interpolation*. Assume that the points along a curve are numbered $P_1, P_2, ..., P_n$ as the curved path is traversed and let d_i be the (straight-line) distance between P_i and P_{i+1}. Then define $t_i = \Sigma_{j=1}^{i=1} d_j$, for for i =1, 2, ..., n (i.e., $t_1 = 0$, $t_2 = d_1$, $t_3 = d_1 + d_2$, etc). If $P_i = (x_i, y_i)$, one can consider two sets of data (t_i, x_i) and (t_i, y_i) for i = 1, 2, ...n which can be interpolated independently to generate the functions $f(t)$ and $g(t)$, respectively. Then $P(f(t), g(t))$ for $0 \leq t \leq t_n$ is a point in the plane and as t is increased from 0 to t_n, $P(t)$ interpolates the desired shaped (hopefully!). Interpolation of the data given below via this method should produce a certain letter. Adapt your algorithm from problem (1) to perform the interpolations on $f(t)$ and $g(t)$ where t is increased from 0.0 to 12.0 in steps of dt (see data below). Report the value of dt you use to achieve 'reasonable' results. Turn in a plot of your interpolated shape (not the numeric values of $P(t)$) as well as plots of $f(t)$ and $g(t)$ individually. [`3 points`]

t_i	x_i	y_i
0.0	0.70	2.25
1.0	1.22	1.77
2.0	2.11	1.61
3.0	3.07	1.75
4.0	3.25	2.30
5.0	2.80	2.76
6.0	2.11	2.91
7.0	1.30	2.76
8.0	0.70	2.25
9.0	0.45	1.37
10.0	0.88	0.56
11.0	2.00	0.08
12.0	3.25	0.25

3. Repeat problem (2) with a natural cubic spline. Also report all four coefficients for each of the cubics which comprise the interpolants for both $f(t)$ and $g(t)$. How does your letter compare with that produced in problem (2)? Explain any differences. [`3.5 points`]

4. Consider the oscillograph record of the free-damped vibrations of a structure. From vibration theory, it is known that for viscous damping (damping proportional to velocity),

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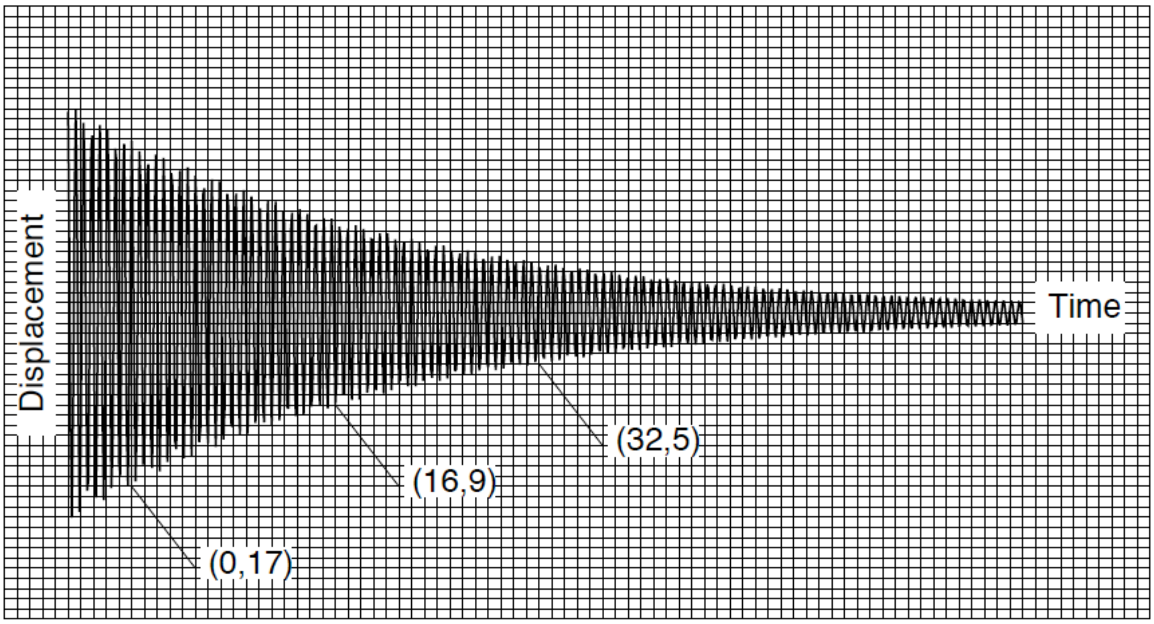
the envelop of such a vibration (i.e., the curve through the peaks of the oscillations) is an exponential function of the form

$$y = be^{-2\pi a x}$$

where x is the cycle number, y is the corresponding amplitude and a is a damping factor. Using the three data points shown in the figure, determine a and b that result from a best fit based on the least-squares criterion. Use a linear least squares approach by suitable change of variable. You may solve this problem "by hand" if you wish.

An alternate approach to this problem would be to construct a nonlinear least-squares fit using the data directly as given. Would this approach lead to exactly the same a and b values you determine above (assuming perfect math, i.e., no rounding errors in either case)?. [`2 points`]
Hint: see if you can relate the errors at the i^{th} data point between the two fits.



viscous damping