DS 288 (AUG) 3:0 Numerical Methods
Homework–5 [1]

`Due date: November 19, 2024 (Tuesday); 10:00 A.M.`

1. Derive Simpson's Rule with error term by using

$$\int_{x_0}^{x_2} f(x)dx = a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2) + k f^{(4)}(\xi) \tag{1}$$

   Find a_0, a_1, and a_2 from the fact that Simpson's rule is exact for $f(x) = x^n$ when $n = 0, 1, 2, and 3$. Then find k by applying the integration formula to $f(x) = x^4$. [`3 points`]

2. Apply Romberg Integration to the following integrals until $R_{n-1,n-1}$ and $R_{n,n}$ agree to within 10^{-5}. Report the value of n and the number of function evaluations. Also, compute the result to that obtained from the Trapezoidal rule for the same number n (note that you already calculate this value to get $R_{n,n}$). [`3.5 points`]

$$(a). \int_0^1 x^{1/3} dx; \qquad (b). \int_0^1 x^2 e^{-x} dx \tag{2}$$

3. Approximate the integrals in Problem 2(a) and 2(b) using Gaussian Quadrature with n=2, 3, 4, and 5. Report the number of function evaluations and compare the results with those obtained using Romberg Integration in Problem-2. [`3.5 points`]

[1] Posted on: `October 30, 2024`.