DS 288 (AUG) 3:0 Numerical Methods
Homework–1 [1]

`Due date: September 3, 2024 (Tuesday); 10:00 A.M.`

A Microwave engineer is interested in developing a hazardous waste treatment facility based on microwave exposure to the hazardous material. The design is based on cylindrical microwave cavity and requires computing of various modes of the electromagnetic fields that will exist in this structure. The modes of the systems are described by the Bessel functions $J_i(x)$ for i = 1,2,..., n. As a numerical methods expert, your job is to help the engineer to compute these Bessel functions using the recurrence relation

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x) \tag{1}$$

1. Compute the recursion in the forward direction, i.e., compute $J_2(x)$ from $J_1(x)$ and $J_0(x)$ with starting values taken from the table-1. Use only the first 5 digits given in the table for each quantity when supplying the starting values to your program. For x = 1,5, and 50, how accurate is $J_{10}(x)$?. Compute both the absolute and relative errors of these values taking the tabulated values (table-1) as truth. [`3 points`]

2. Compute the recursion backward, i.e. start with $J_10(x)$ from $J_9(x)$ compute $J_8(x)$. Again use only first 5 digits and for x = 1,5, and 50, how accurate is $J_0(x)$ in this backward approach?. Compute both the absolute and relative errors of these values taking the tabulated values (table-1) as truth. Is the last value computed by the recurrence relation is having less or more error compared to the forward approach?. [`3 points`]

3. Explain your finding. Can the error propagation be formally analyzed using the difference equation analysis we performed in class?. Can the error behavior be understood by this analysis?. Defend your answer to these questions. In case your answers are yes, do the analysis. If the answer is no, how will you explain the error propagation?. [`4 points`]

n	$J_n(1)$	$J_n(5)$	$J_n(50)$
0	7.6519768656e-01	-1.7759677131e-01	5.5812327669e-02
1	4.4005058574e-01	-3.2757913759e-01	-9.7511828125e-02
2	1.1490348493e-01	4.6565116278e-02	-5.9712800794e-02
3	1.9563353983e-02	3.6483123061e-01	9.2734804062e-02
4	2.4766389641e-03	3.9123236046e-01	7.0840977282e-02
5	2.4975773021e-04	2.6114054612e-01	-8.1400247697e-02
6	2.0938338002e-05	1.3104873178e-01	-8.7121026821e-02
7	1.5023258174e-06	5.3376410156-02	6.0491201260e-02
8	9.4223441726e-08	1.8405216655e-02	1.0405856317e-01
9	5.2492501799e-09	5.5202831385e-03	-2.7192461044e-02
10	2.6306151237e-10	1.4678026473e-03	-1.1384784915e-01

Table 1: Bessel functions of integer order (n = 0-10) for x = 1, 5, and 50.

[1] Posted on: `August 6, 2024.`