DS 288 (AUG) 3:0 Numerical Methods
Homework–2 [1]

`Due date: September 24, 2024 (Tuesday); 10:00 A.M.`

1. Using Newton's method, Secant method, and Modified Newton's method, find the solution of $f(x) = 0$ for the functions listed. For the Newton methods start with an initial guess of $p_0 = 0$. For the Secant method start with initial guesses (or interval) of $p_0 = 0$ and $p_1 = 1$. Iterate until you reach a relative tolerance of 10^{-6} between successive iterates. Report the root found and the number of iterations needed for each method.
   (a) $f(x) = x + e^{-x^2}cosx$.
   (b) $f(x) = (x + e^{-x^2}cosx)^2$.
   Comment on the observed convergence rates in these cases. Does your results agree with the analysis we did in class?. [`4 points`]

2. Develop the functional form for a cubicly convergent fixed point iteration function $g(p_n)$ to solve the problem $f(x) = 0$ by writing

$$g(x) = x - \phi(x)f(x) - \psi(x)f^2(x)$$

   and determining $\phi(x)$ and $\psi(x)$. Specify the asymptotic order of convergence (α) and write the asymptotic error constant (λ). Write all expressions in terms of $f(p)$ and its derivatives and *simplify* your answers. You are allowed to scan the hand-written derivation for this part alone. *Hint*: Extend the approach we used in class to derive Newton's method. The scheme you will produce is often referred to as "Cubic Newton's Method". [`3 points`]

3. The figure below shows a four bar linkage where θ_4 is the input angle and the output angles θ_2 and θ_3 are to be determined. The relationships among the linkages can be expressed in terms of the two nonlinear equations

$$f_1(\theta_2, \theta_3) = r_2cos\theta_2 + r_3cos\theta_3 + r_4cos\theta_4 - r_1 = 0$$
$$f_2(\theta_2, \theta_3) = r_2sin\theta_2 + r_3sin\theta_3 + r_4sin\theta_4 = 0$$

   Assume $r_1 = 10$, $r_2 = 6$, $r_3 = 8$, $r_4 = 4$, $\theta_4 = 220^o$, and solve for θ_2 and θ_3 using Newton's method for systems of nonlinear equations. Compute to a relative tolerance of 10^{-4} and report the number of iterations required to reach this level of convergence. Start with initial guesses of $\theta_2 = 30^o$ and $\theta_3 = 0^o$. Think about whether θ should be specified in radians or degrees in your code. Invert the 2x2 Jacobian for this problem. [`3 points`]

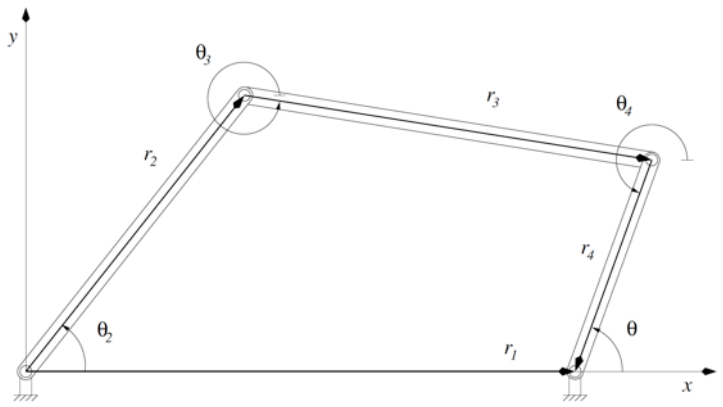

Figure 1: Four-bar linkage.

[1] Posted on: `August 27, 2024`.