DS298 – HW3 Report

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Observations

Error analysis plots of Algorithm 1 (Sampling-based SVD) and Algorithm 2 (Randomized SVD), applied to Class-I and Class-II matrices.

- **E1**: Relative squared error.
- **E2**: Projection error.

Class-I Matrices (Exponential Singular Value Decay)

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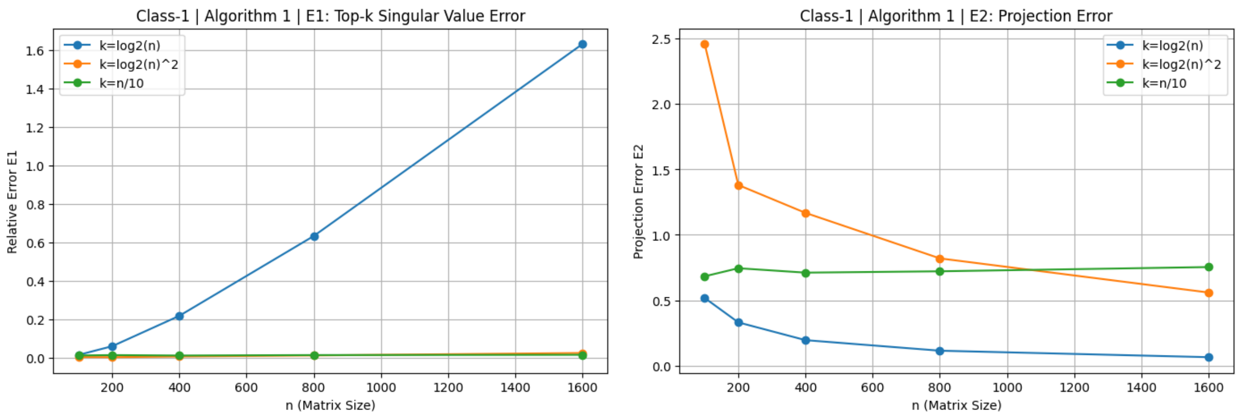

Figure 1: Algorithm 1 – Class I: E1 (left) and E2 (right)

- For small $k = \log_2(n)$, E1 increases as the matrix size grows. However, for larger values like $k = (\log_2(n))^2$ and $k = n/10$, E1 remains relatively low.
- E2 gradually decreases with increasing n for all values of k, indicating that Algorithm 1 becomes more effective at capturing the dominant singular subspaces as the matrix size expands.
- In general, using larger k values results in better approximations for both E1 and E2.

- Algorithm 2 shows very low E1 across all k and n, indicating accurate singular value approximation.
- E2 is high for $k = (\log_2(n))^2$ especially at small n, but decreases as n increases.
- The randomized projection performs well on singular value estimation, but less so on subspace projection at small k.

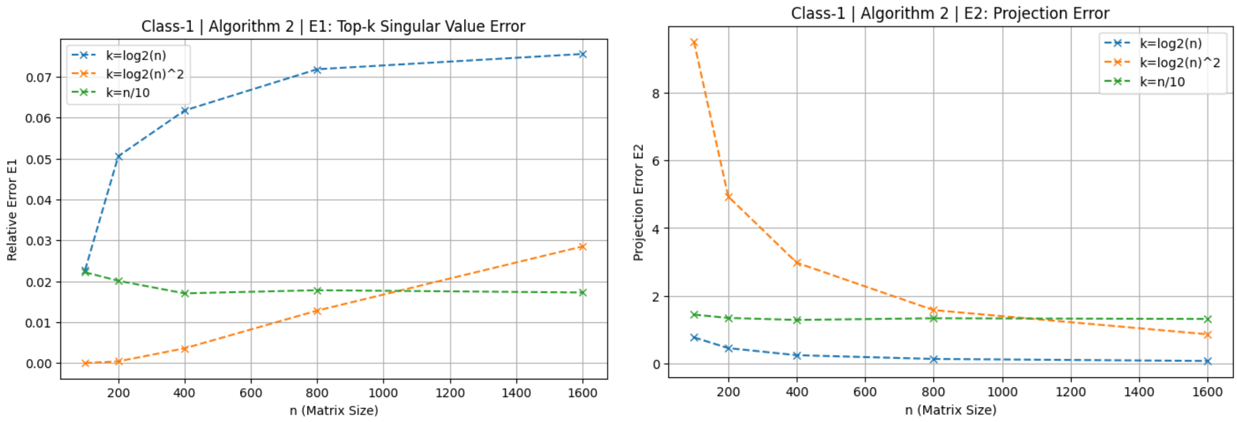

Figure 2: Algorithm 2 – Class I: E1 (left) and E2 (right)

Class-II Matrices (Linear Singular Value Decay)

- For small k values, especially $k = \log_2(n)$, E1 rises sharply due to the slower decay of singular values.
- E2 remains relatively small and stable across different k values, showing improved projection performance as k increases.
- Algorithm 1 effectively handles projection but requires a higher k to accurately estimate singular values in this class.
- Algorithm 2 keeps E1 consistently low across all k, demonstrating its ability to capture singular values well, even in more challenging matrices.
- E2 stays low and decreases with n, indicating that randomized methods perform better as matrix size grows.
- Overall, the algorithm is reliable across both metrics, though increasing k enhances subspace accuracy.

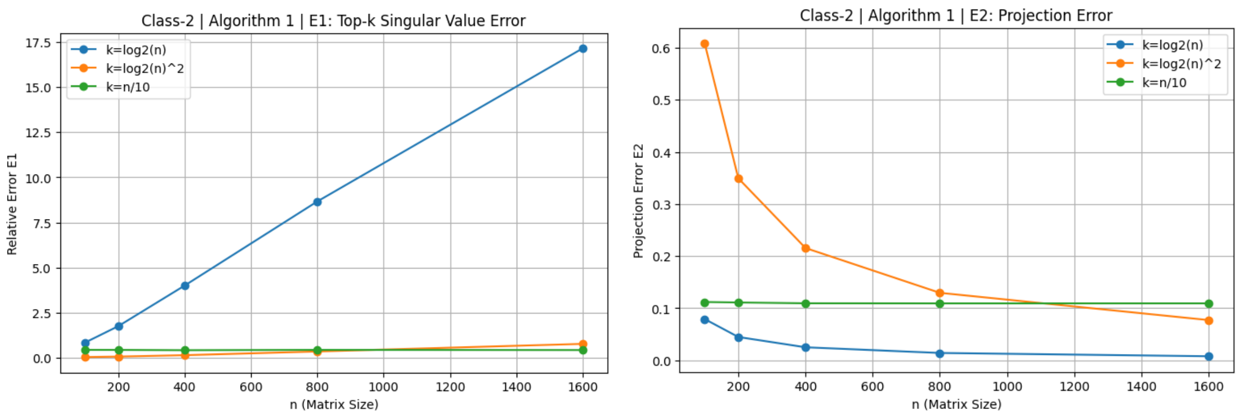

Figure 3: Algorithm 1 – Class II: E1 (left) and E2 (right)

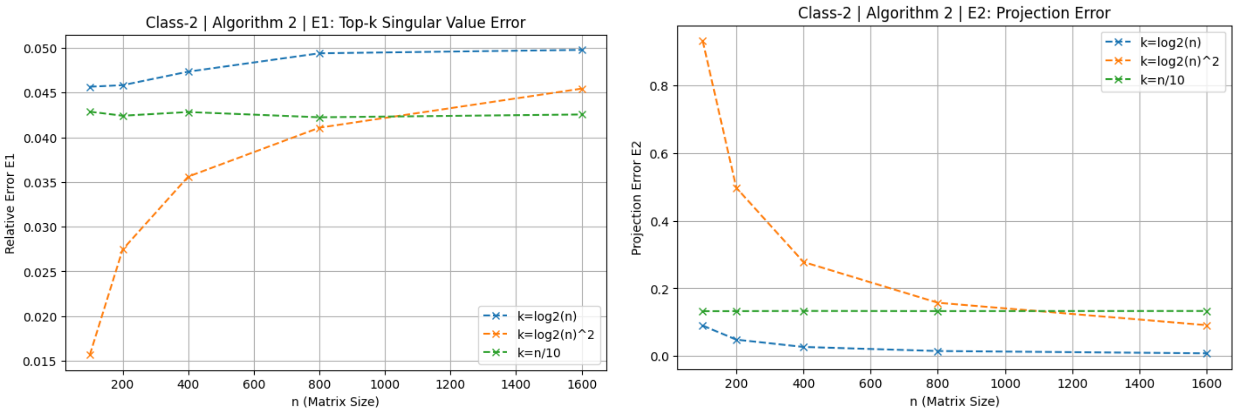

Figure 4: Algorithm 2 – Class II: E1 (left) and E2 (right)

Overall Insights:

- Algorithm 1 keeps the projection more accurate (lower E2), especially for small k.
- Algorithm 2 estimates singular values better (lower E1), especially for Class II matrices.
- For Class I (exponential decay), both work well at large k, but Algorithm 1 is more stable.
- For Class II (linear decay), a higher k is needed to estimate singular values well.