DS 298: Work Assignment - 3

Due April 4, 2025

The goal of this work is to analyze the errors E in the Algorithms (1) and (2) presented in the class to evaluate the top-k singular value components, for various values n in matrix dimensions and number of singular components k, in each case of the two different classes of matrices described.

Let relative errors E_1 and E_2 be given by $\frac{\sum_1^k(\sigma_i'-\sigma_i)^2}{\sum_1^k \sigma_i^2}$ and $|\frac{\|A-\hat{U}\hat{U}^T A\|_F^2}{\sum_{k+1}^n \sigma_i^2} - 1|$ respectively, where σ_i and σ_i' are the input and the estimated singular values respectively as ordered sets, and matrix $\hat{U}$ represents the estimated left singular vectors. Let $A^{n\times n}$ be considered with values of n (x-axis in plots) as 100, 200, 400, 800, 1600. The relative errors are to be traced (in the y-axis) for three different values of $k = \log_2 n, (\log_2 n)^2, \frac{n}{10}$ rounded to the nearest integer in a single plot. E_1 and E_2 can be traced in two separate plots. The number of columns sampled in algorithm (1) can be fixed as $c = 2k$. Average over 10 runs of the algorithms for each data point in the plot. These plots, two each for algorithms (1) and (2), are to be further replicated for the two matrix classes I and II.

The trial matrices are to be generated using the singular value decomposition $U\Sigma V^T$. The singular values given by Σ_{kk} should be fixed as i) $e^{-\frac{10(k-1)}{n}}$ for Class-I matrices, and ii) as $\frac{n-k+1}{n}$ for Class-II matrices. The algorithm for generating the trial matrices is briefly described below.

Algorithm: Generation of a trial matrix in a class

Inputs : $\Sigma \in \mathcal{R}^{n\times n}$; diagonal matrix of singular values.
Outputs : $A \in \mathcal{R}^{n\times n}$; trial matrix.
Initialize : $M_1 \longleftarrow rand[n, n]$ and $M_2 \longleftarrow rand[n, n]$; Initialize two random matrices.
Generate singular vectors : $Q_1R_1 \longleftarrow M_1$ and $Q_2R_2 \longleftarrow M_2$; QR factorization of the two matrices.
Build trial matrices : $A \longleftarrow Q_1\Sigma Q_2^T$; generate test matrix of a given singular value distribution.

Note : Submit the responses with the plots, and the codes, as separate files all zipped into a single folder identified by your name in full, and upload it on the MS-Teams channel for DS 298.