Algorithms & Data Structures
Lecture 1

Outline

Lecture I

1. Motivation
2. Sorting algorithms
3. Complexity analysis
4. Linear data structures

Lecture II

5. Nonlinear data structures
6. Abstract data types
7. Dijkstra's algorithm
8. Summary

Attention!

Lecture aimed at non-computer scientists.

Focus is on explaining concepts,
rather than technical correctness.

1. Motivation

Motivation

- Algorithms
- Data Structures

1. **Everything** running on your computer is an **algorithm**
2. Analysing them is paramount to **writing**, **maintaining** and **improving** them
3. Several **tools** exist to help achieve this

Motivation

- Algorithms
- Data Structures

1. Data Structures define how data is **stored** in **RAM**
2. Many variations, each with **advantages** and **disadvantages**
3. Strongly coupled to algorithmic **complexity**

2. Sorting algorithms

Sorting

- Suppose we have some **unsorted list**
- We want to make it **sorted**

6	5	3	1	8	7	2	4

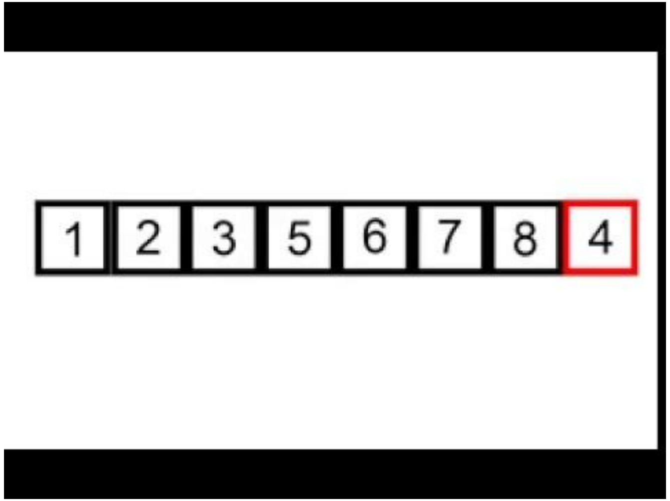
↓

1	2	3	4	5	6	7	8

Insertion sort

- In pseudocode:
- “Naive” sorting algorithm
- One-by-one, take each element and move it
- When all elements have been moved, list is sorted!

```
i ← 1
while i < length(A)                              N steps
    j ← i
    while j > 0 and A[j−1] > A[j]                N steps
        swap A[j] and A[j−1]
        j ← j − 1
    end while
    i ← i + 1
end while
```



Total: ***N*²** **steps**

Bubble sort

- Traverse the list, taking **pairs** of elements
- **Swap** if order incorrect
- **Repeat** N times
- Now it's **sorted**!

***N* steps**

***N* steps**

Total: ***N^2* steps**

Intermezzo: Divide and conquer

- Generic algorithm **strategy**
  - **Divide** the problem into smaller parts
  - **Solve** (conquer) the problem for each part
  - **Recombine** the parts
- Straightforward to **parallelise**
- Closely related to **map-reduce**
- Has been advocated by Caesar, Machiavelli, Napoleon...

Merge sort

- Much **smarter** sort
  - **Split** the dataset into chunks
  - **Sort** each chunk
  - **Merge** the chunks back together
- Example of **divide-and-conquer**
- Splitting & sorting takes $\mathbf{\log_2(\mathit{N})}$ **steps**
- Merging takes ***N*** **steps**

Total: ***N*** **log(*N*) steps**

3. Complexity analysis

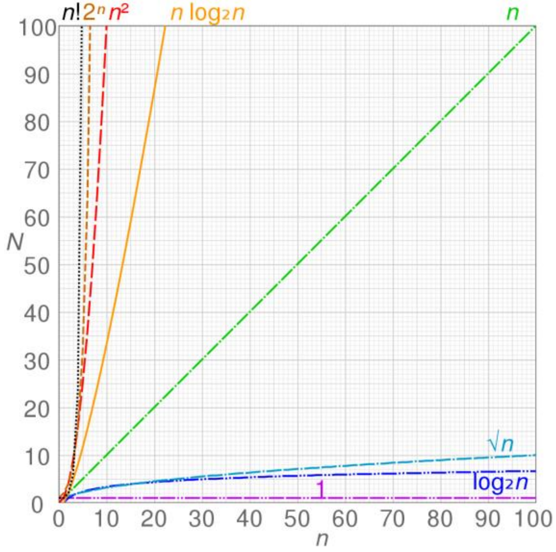
Complexity – O

- We use "**Big-O**" notation
- Represents an **upper bound**
- Ignores **constants**
- Only shows **dominant term**
- In these examples: N is amount of **data**

- Linear: **O(*N*)**
  - $O(N) = O(2*N) = O(k*N + m)$ <u>for any constants *k*, *m*</u>
- Quadratic: **O(*N*²)**
  - $O(N^2) = O(a*N^2 + b*N + c)$ <u>for any constants *a*, *b*, *c*</u>

Complexity – O

- Roughly **three categories**, in **decreasing order**:
  - Exponential – O(k^N)
  - Polynomial – O(N^k)
  - Polylogarithmic – O($\log(N)^k$)
- This is an **abstraction**!
  - Does not *directly* relate to runtimes
  - A good O(N^2) algorithm may be faster than a bad O($\log(N)$) one
  - Depends on your input data!



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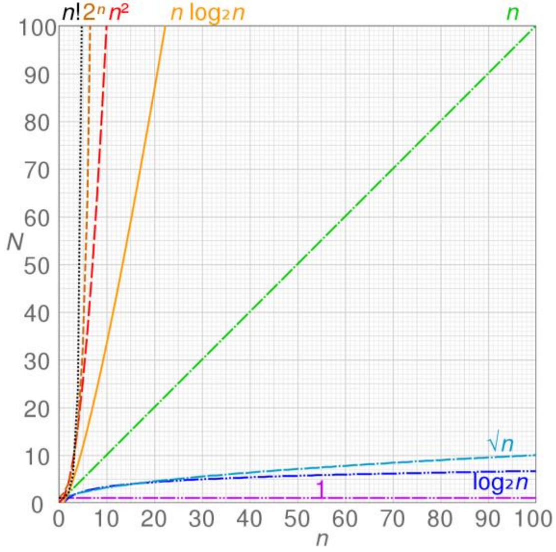
Complexity – O

- Remember: it's an **upper bound**!
- And it's a **property** (**not** an equivalence relation)!
  - $O(N) = O(N^2)$
  - But $O(N^2) \neq O(N)$

- $O(N) = O(2N)$
- $2N = O(N)$
- $N^2 + N = O(N^2)$
- $O(N^2) + O(N) = O(N^2)$
- $O(\log(N)) + O(N) = O(N)$
- $O(1) \rightarrow$ used for **constant time**

Complexity – O

- Formal definition:
  $f(N) = O(g(N)) \leftrightarrow$
  $\exists\, N_0, M : \forall\, N > N_0 : f(N) \leq M\, g(N)$
- In words: starting from N_0, f is bounded from above by g (up to some constant M)



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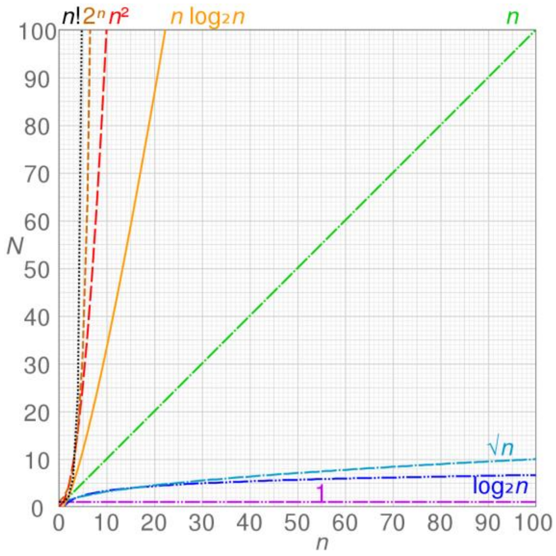
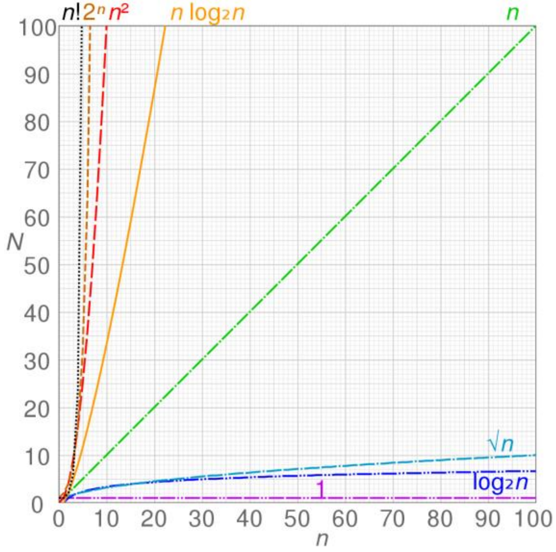
Complexity – Ω

- Ω: **lower** bound
  - Formal definition:
    $f(N) = \Omega(g(N)) \leftrightarrow$
    $$\exists\, N_0, M : \forall\, N > N_0 : f(N) \geq M\, g(N)$$
  - In words: starting from N_0, f is bounded from below by g (up to some constant M)
- Examples:
  - $N = \Omega(\log(N))$
  - $N^2 = \Omega(N)$



Complexity – Θ

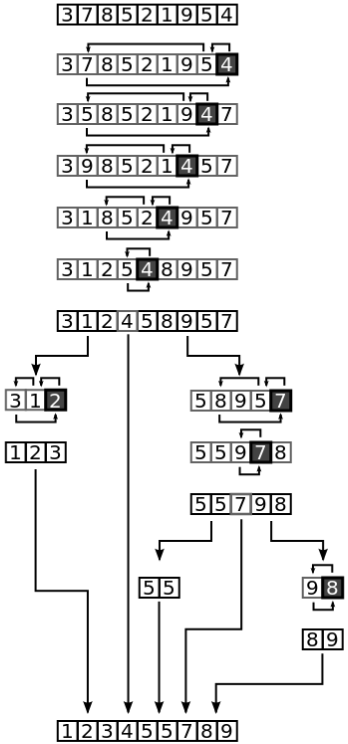
- Θ: **exact** bound
  - Formal definition:
    $f(N) = \Theta(g(N)) \leftrightarrow$
    $f(N) = O(g(N))$ and $f(N) = \Omega(g(N))$
- Examples:
  - $N = \Theta(N)$
  - $2N = \Theta(N)$
  - $3N^2+5N+1 = \Theta(N^2)$
  - $N \neq \Theta(N^2)$ (even though $N = O(N^2)$)



One more sorting example: Quicksort

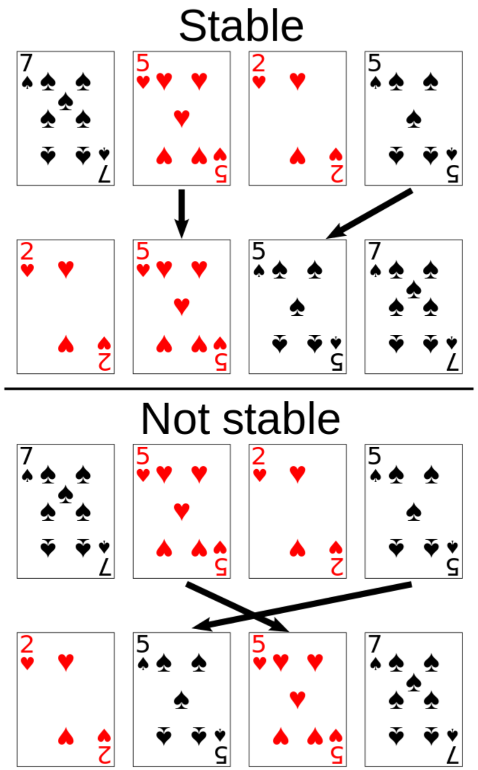
- Pick an element, called **pivot**
- **Partitioning**: reorder the array so that the pivot is in the correct place
- **Recursively** apply the above steps to the sub-arrays on either side of the pivot

- **Randomised-quicksort**: select the pivot **randomly**



Stable sorting

- A sorting algorithm is **stable** iff it **conserves the order** of equal elements

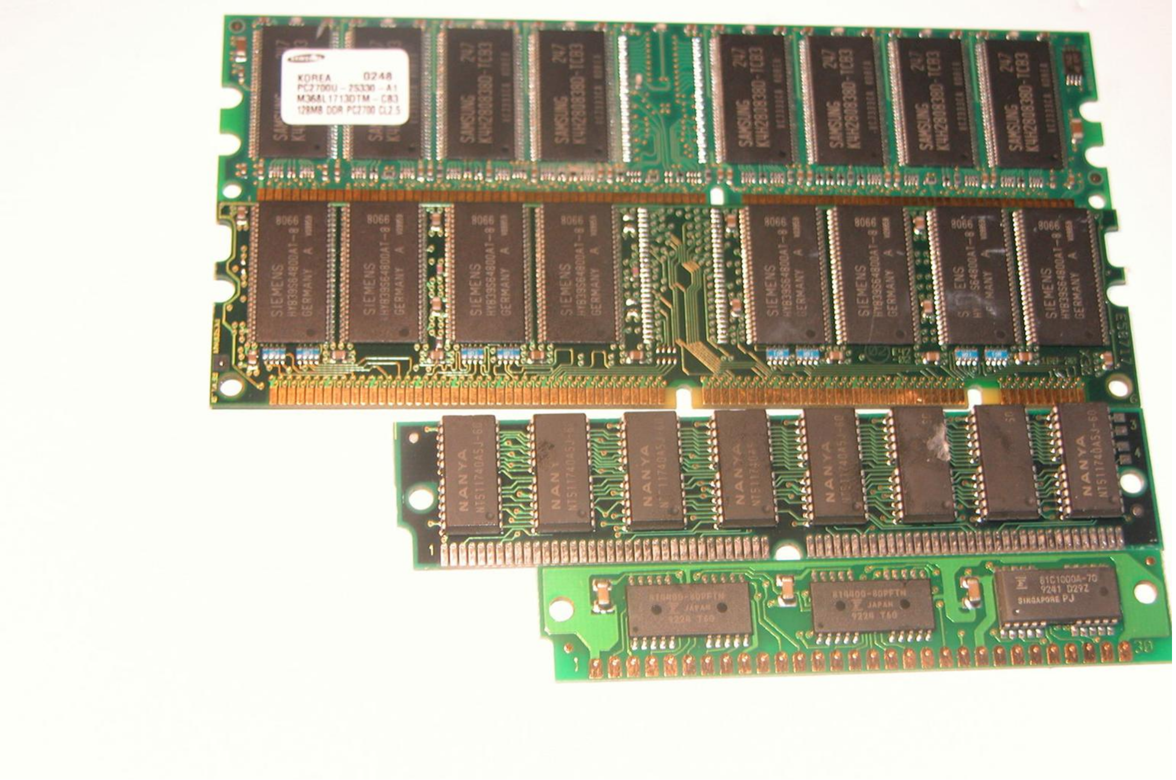


Comparison of algorithms

Algorithm	**Stable?**	**Complexity**
Insertion sort	✅	$O(N^2)$
Bubble sort	✅	$O(N^2)$
Merge sort	✅	$O(N \log(N))$
Quicksort	❌	?

4. Linear data structures

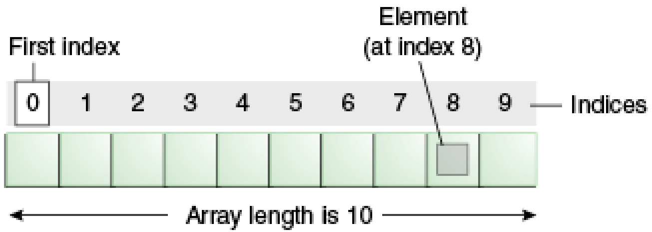
Memory


KOREA 0248
PC2700U - 25330 - A1
M368L1713DTM - CB3
128MB DDR PC2700 CL2.5
SAMSUNG
SIEMENS
GERMANY
NANYA

Arrays

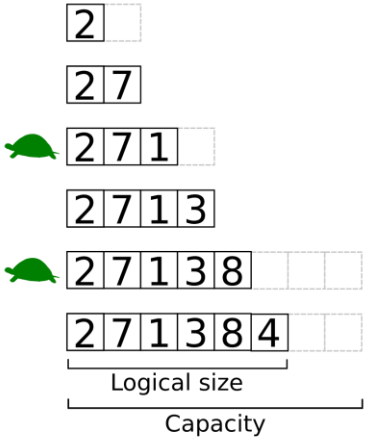
- **Linear**, **contiguous** list of data
- Accessible by **index**
- **Fixed-size**
  - *N * d*
- **Supported** by all major systems

- Back-insert/remove: **O(1)**
- Random insert/remove: **O(*N*)**
- Index-lookup: **O(1)**
- Lookup: **O(*N*)**



Dynamic arrays

- **Linear**, **contiguous** list of data
- Accessible by **index**
- **Resizable**

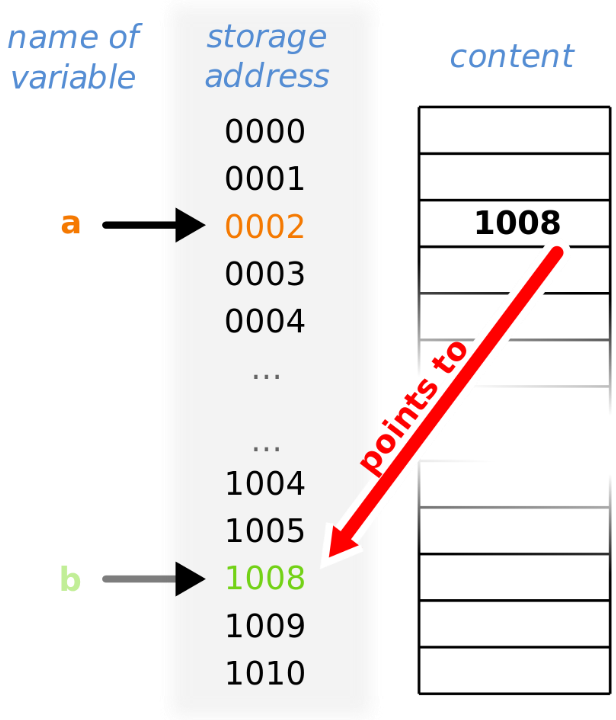


- Back-insert/remove: **O(1)***
- Random insert/remove: **O(*N*)**
- Index-lookup: **O(1)**
- Lookup: **O(*N*)**

*Amortised.

C++: `std::vector`
Python: `list`
C#: `System.Collections.ArrayList`
Java: `java.util.ArrayList`

Pointers



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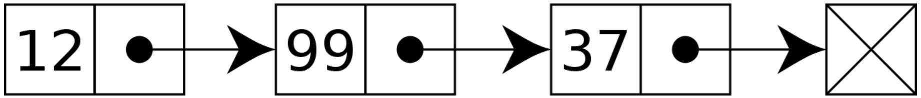
Linked list

- **Linear**, **contiguous** list of data
- Accessible by **iteration**
- **Resizable**

- Back-insert/remove: **O(1)**
- Random insert/remove: **O(*N*)**
- Index-lookup: **O(*N*)**
- Lookup: **O(*N*)**

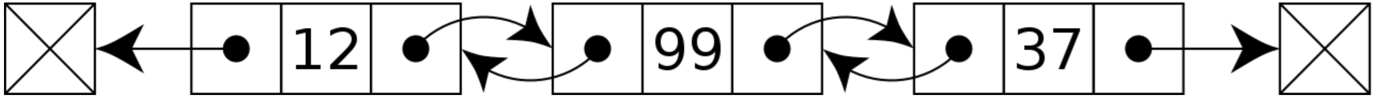


C++: **`std::forward_list`**

Doubly Linked list

- Pointers **both ways**
- Uses **more memory**, but allows **iteration** both ways

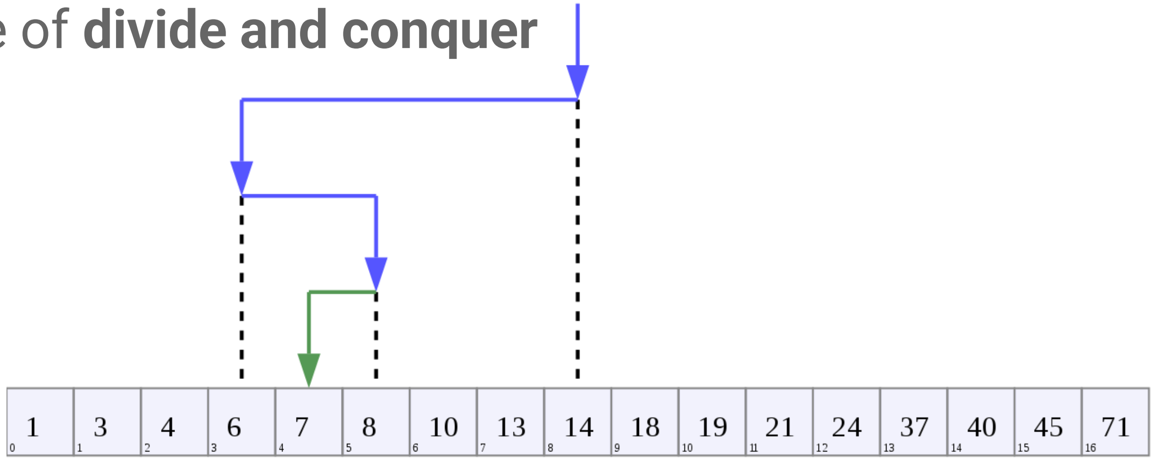
- Back-insert/remove: **O(1)**
- Random insert/remove: **O(*N*)**
- Index-lookup: **O(*N*)**
- Lookup: **O(*N*)**



C++: **`std::list`**
C#: **`System.Collections.Generic.LinkedList`**
Java: **`java.util.LinkedList`**

Binary search

- **Searches** a sorted linear data structure
- Takes **Θ(log(*N*))**
- Example of **divide and conquer**



Algorithms & Data Structures
Lecture 2

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iCSC 2018

Outline

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1. Motivation
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Lecture II

5. Nonlinear data structures
6. Abstract data types
7. Dijkstra’s algorithm
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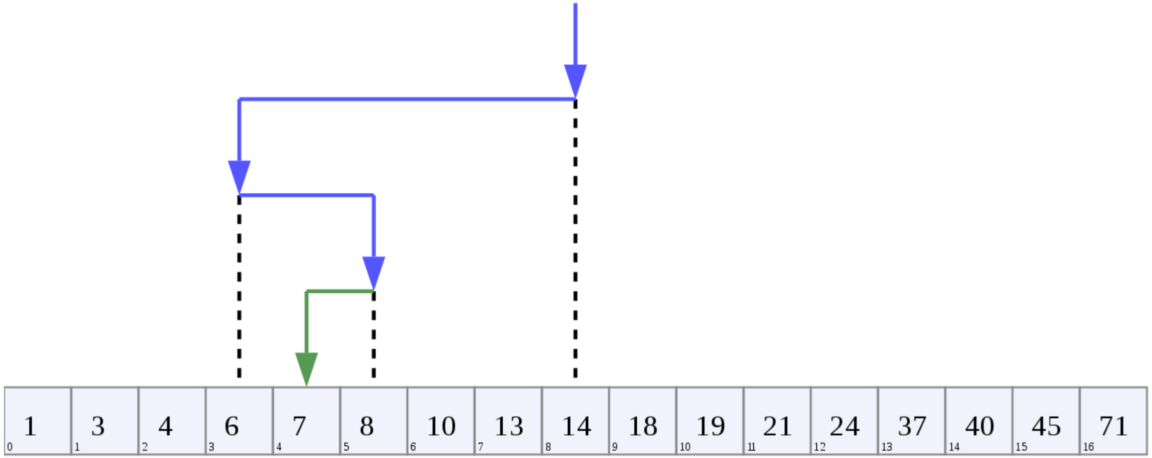
5. Nonlinear data structures

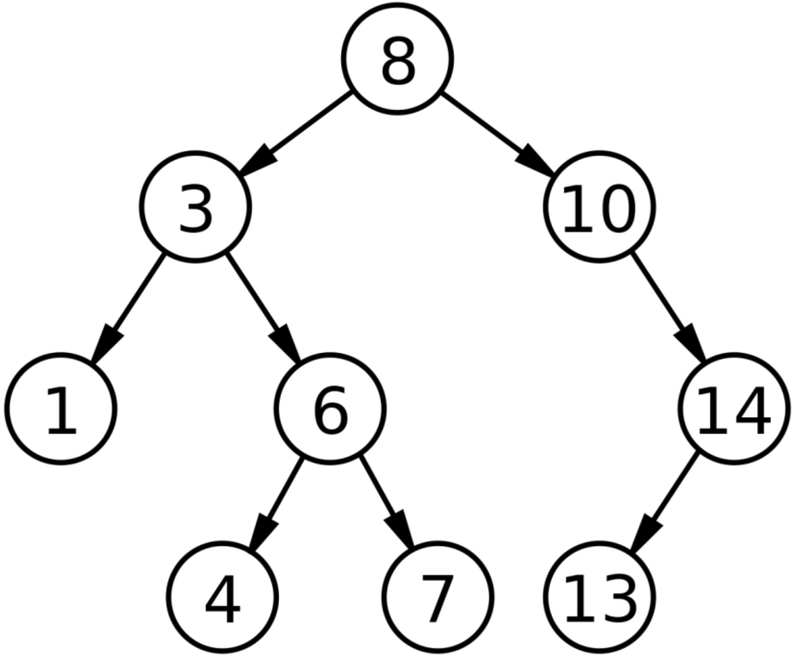
Recall: Binary search

- **Searches** a sorted linear data structure
- Takes **Θ(log(*N*))**
- … let's use this as inspiration for a **data structure**!



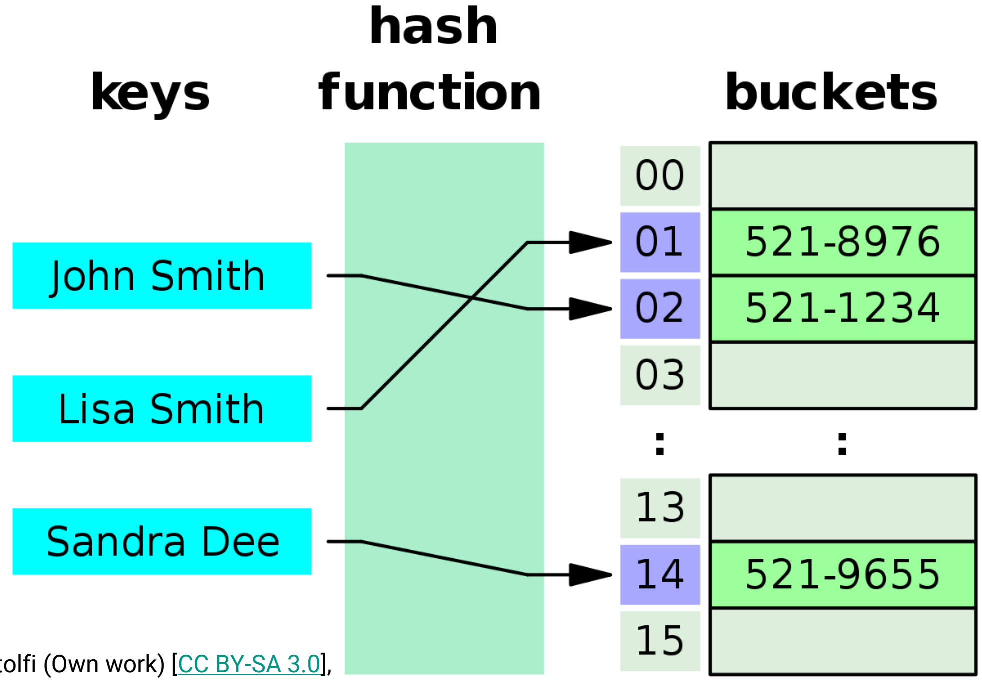
Binary search trees

- **Tree** structure
- **Pointers** between nodes
  - To the right: only **larger**
  - To the left: only **smaller**
- Allows easy **sorted iteration**
- Search/insert/delete:
  all **O(log(*N*))**



Hash tables

- Idea: create **buckets** numbered 1 to B
- For each item, **compute** in which bucket it belongs
- **Put** the item in that bucket
- Search/insert/delete: all **O(1)**



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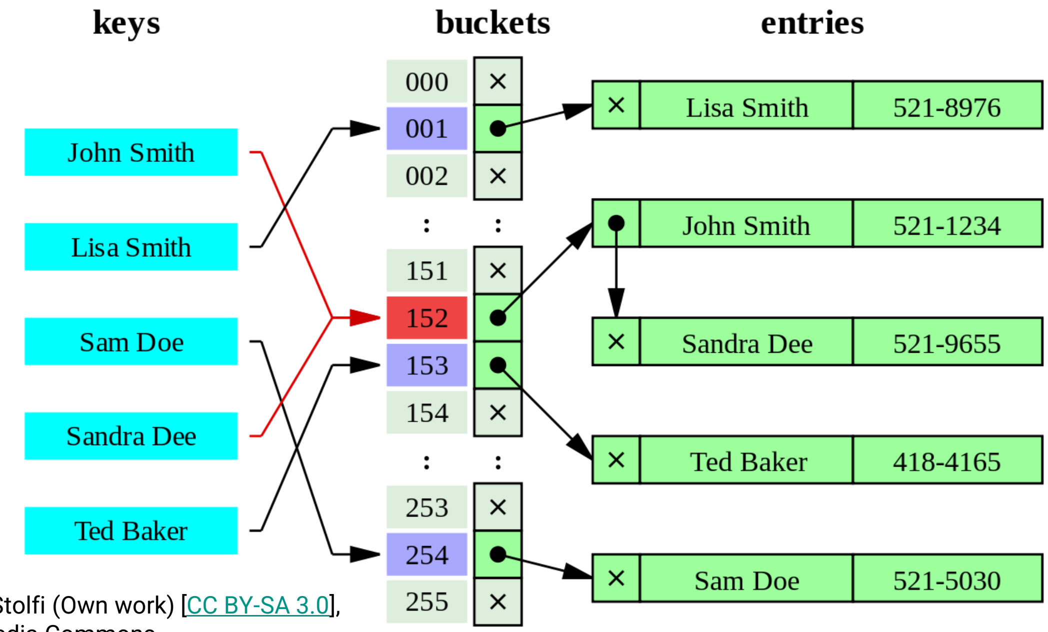
Hash tables

- Problem: **clashing** hashes!
- Solution: replace entry with **linked list** (chaining)

- New problem: **load factor** can become **too high**!
- Solution: **copy** to new table with **more buckets**



Comparing data structures

Data structure → Operation ↓	Dynamic array	Linked list	Binary search tree	Hash table
Lookup	O(*N*)	O(*N*)	O(log(*N*))	O(1)
Indexed lookup	O(1)	O(*N*)	N/A	N/A
Back-insert	O(1)*	O(1)	O(log(*N*))	O(1)*
Random insert	O(*N*)	O(*N*)	N/A	N/A
Remove	O(*N*)	O(*N*)	O(log(*N*))	O(1)*

*Amortised.

6. Abstract data types

Why “Abstract”?

- Abstract Data Type (**ADT**) does **not** define a real data structure
  - Only defines an **interface**
  - **Implemented** using one of the “real” data structures
- Usually **limits** operations compared to actual DS
- Enhances **flexibility**

Related to several **core programming principles**:

- Program against the **interface**, not the **implementation**!
- Use **high cohesion**, **loose coupling**
- **Separate** the **concerns**

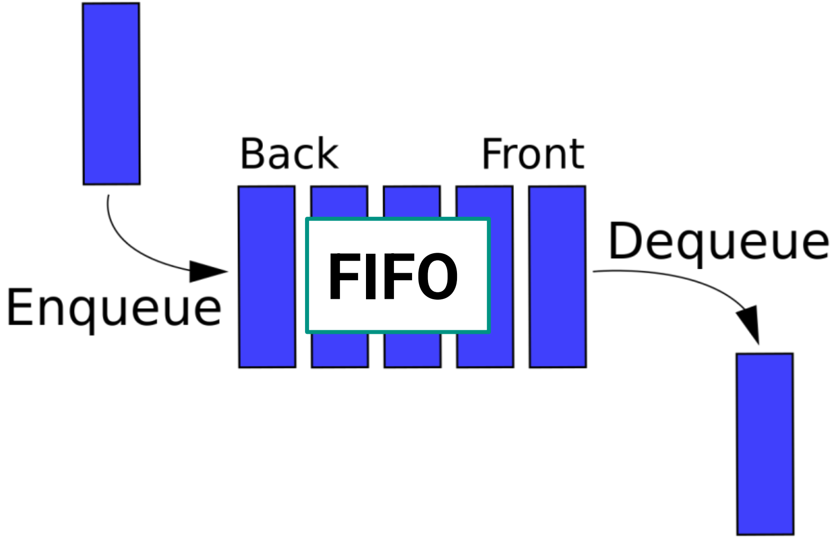
Queue

Operations:

- **Enqueue**: add item to beginning of queue
- **Dequeue**: retrieve and remove item from end of queue

Typical underlying data structure:

- **Linked list**
- Dynamic array



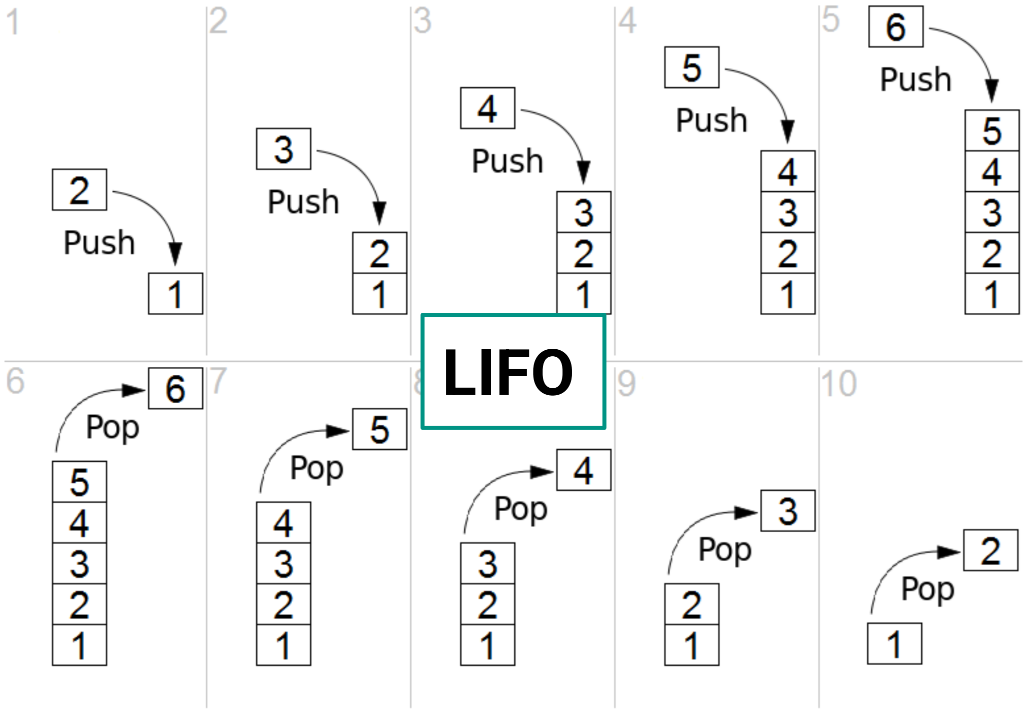
Stack

Operations:

- **Push**: add item to top of stack
- **Pop**: retrieve and remove item from top of stack

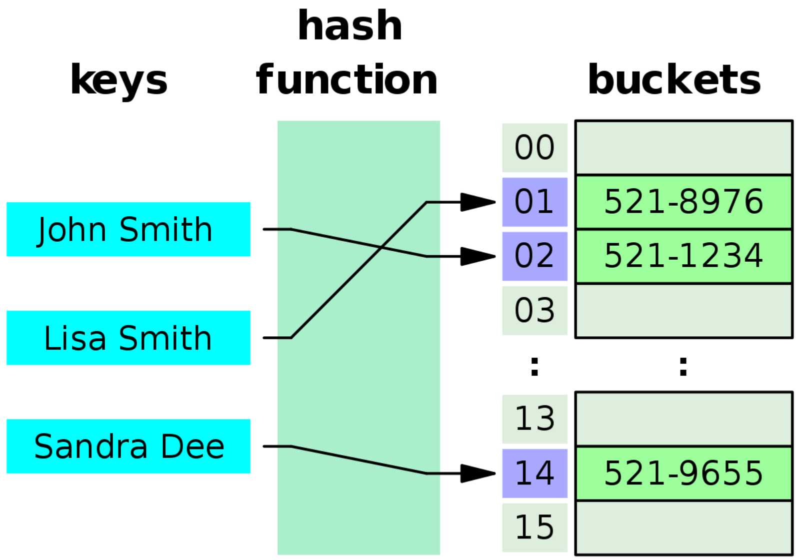
Typical underlying data structure:

- **Linked list**
- Dynamic array



Map

- Map: dataset that maps (associates) **keys** to **values**
- Keys are **unique** (values need not be)
- Values can be **retrieved** by key
- Not indexed...
  - ...although an array could be seen as a map with **integer keys**!



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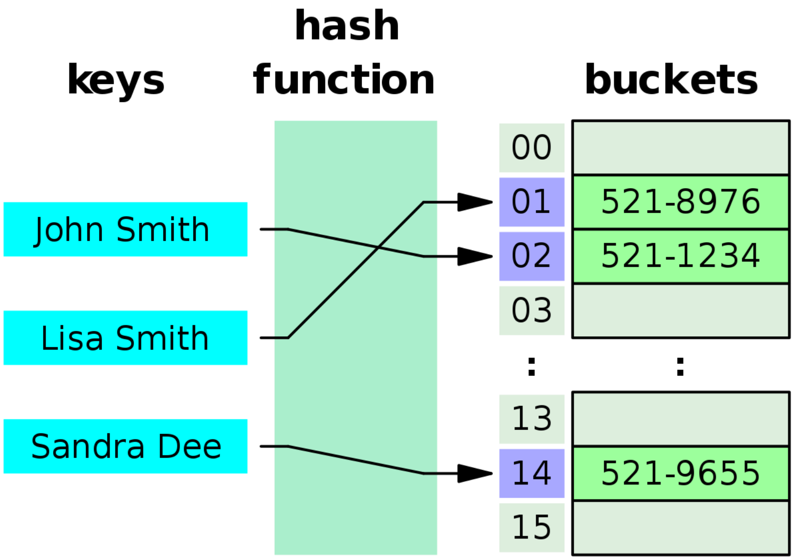
Map

Operations:

- **Lookup**: retrieve value for a key
- **Insert**: add key-value pair
- **Replace**: replace value for a specified key
- **Remove**: remove key-value pair



Map

Typical implementations:

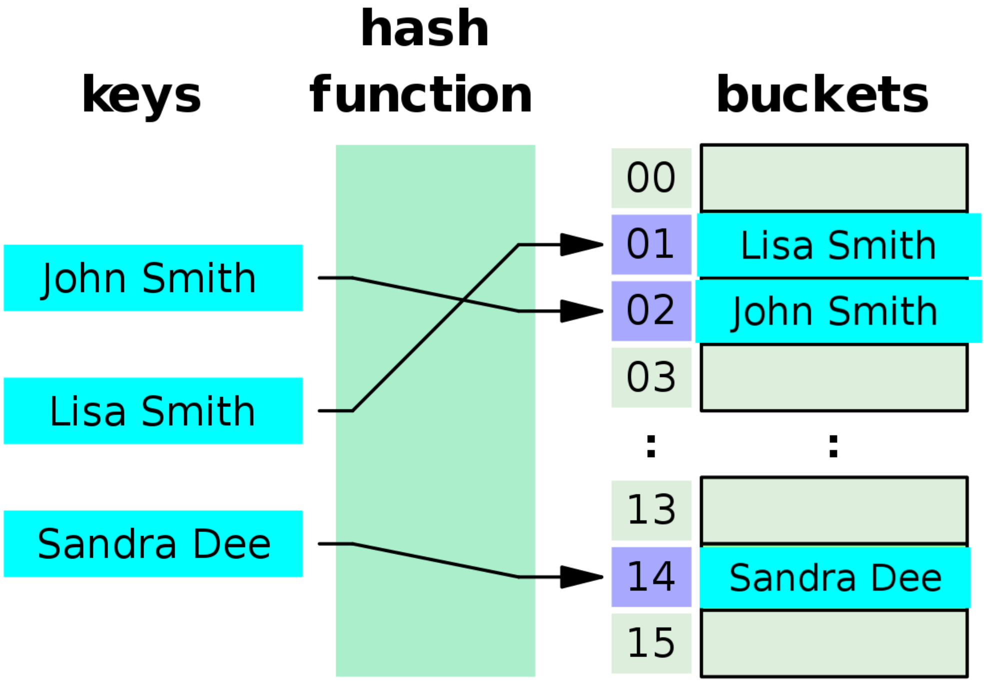
- Binary Search Tree
  - Requires **sortable keys**
  - Can do **indexed/range** queries!
  - Fast with many **insertions**
- Hash Table
  - Generally very **fast**
  - **Space-efficient**
  - Need to keep **load factor** under control...

C++: **`std::map`**
C#: **`System.Collections.Generic.SortedSet`**
Java: **`java.util.TreeMap`**

Python: **`dict`**
C++: **`std::unordered_map`**
Java: **`java.util.HashMap`**

Set

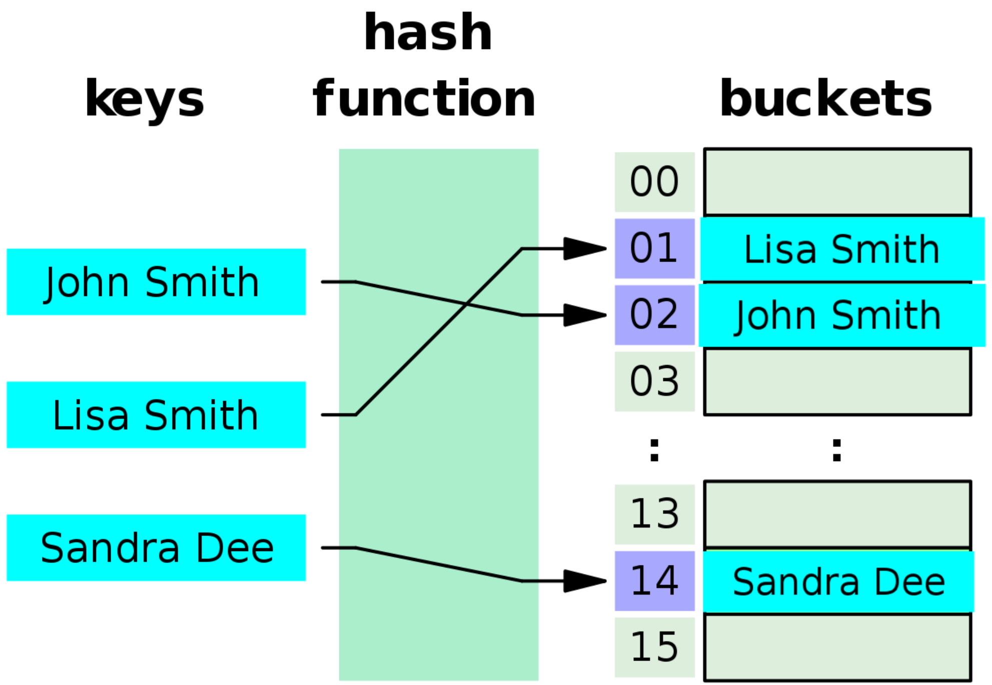
- Set: dataset that contains certain **values**
- No **ordering**, no **multiplicity**
- A value is either **present** or **not**



Set

Operations:

- **Contains**: check whether a value is present
- **Add**: add a value
- **Remove**: remove a value



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Set

Typical implementations:

- Binary Search Tree
- Hash Table
- *Bloom filter*

C++: **`std::set`**
C#: **`System.Collections.Generic.SortedSet`**
Java: **`java.util.TreeSet`**

Python: **`set`** (and **`frozenset`**)
C++: **`std::unordered_set`**
C#: **`System.Collections.Generic.HashSet`**
Java: **`java.util.HashSet`**

Comparing ADTs

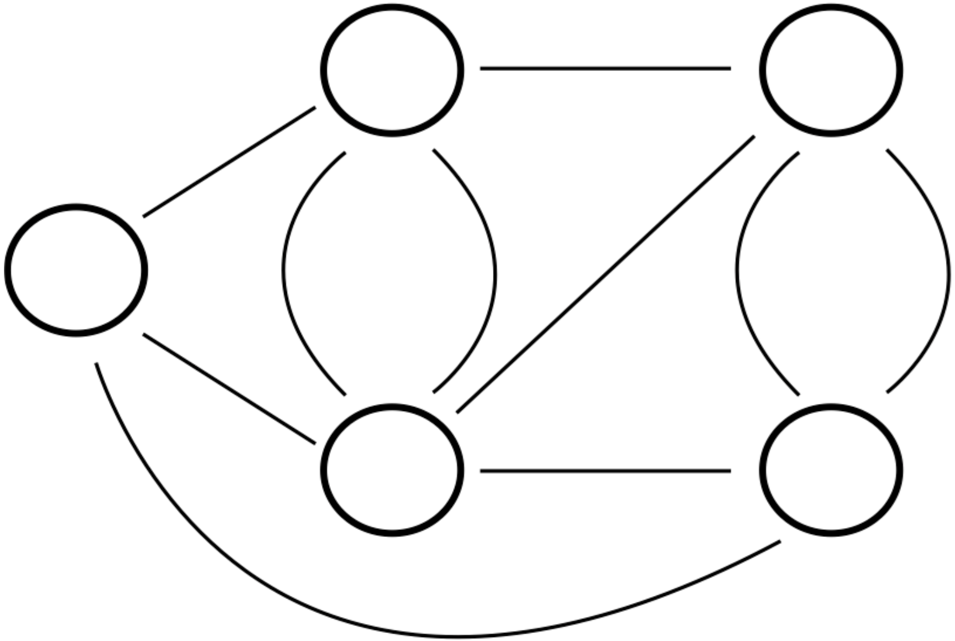
Abstract Data Type → Operation ↓	Queue	Stack	Map	Set
Lookup	N/A*	N/A*	By key	Contains
Add	Enqueue	Push	Key + value	Add
Replace	N/A	N/A	By key	N/A
Remove	Dequeue	Pop	By key	Remove

*Only by removing element
(some may support *peek*)

7. Dijkstra’s algorithm

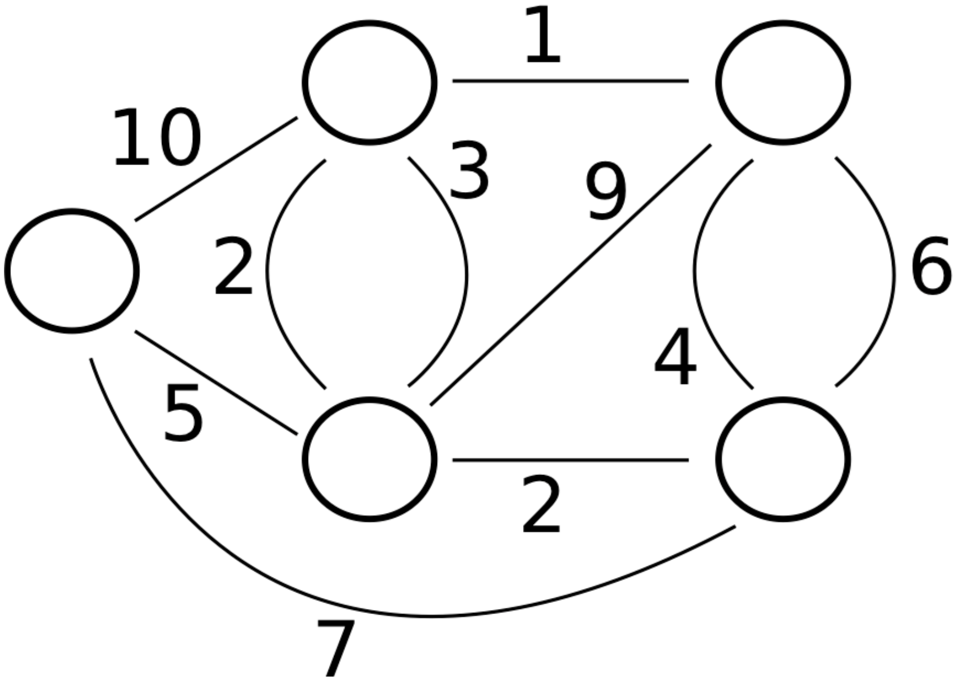
Graphs

- Another data structure!
- Consists of **vertices** (*V*) and **edges** (*E*)



Graphs

- Another data structure!
- Consists of **vertices** (*V*) and **edges** (*E*)
- Edges may carry a **weight**

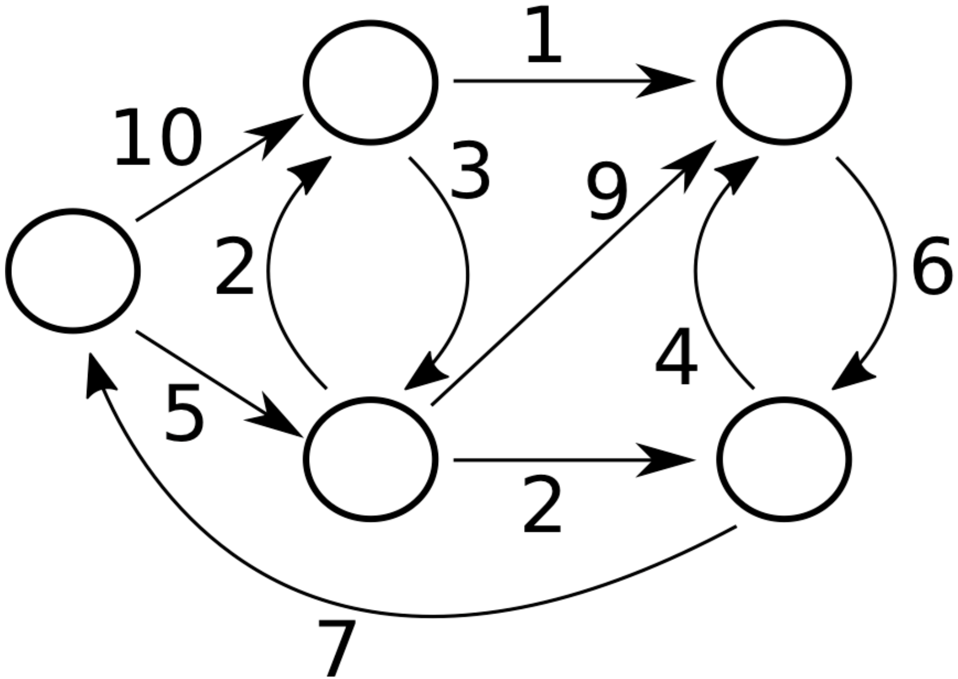


Graphs

- Another data structure!
- Consists of **vertices** (*V*) and **edges** (*E*)
- Edges may carry a **weight**

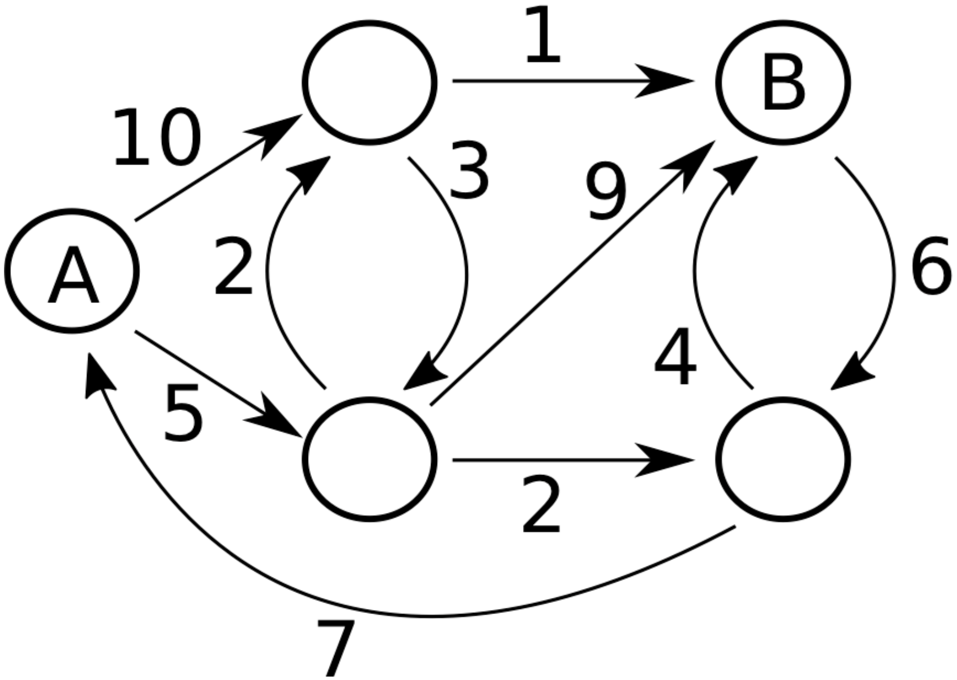
Directed graph:

- Edges are **directed**



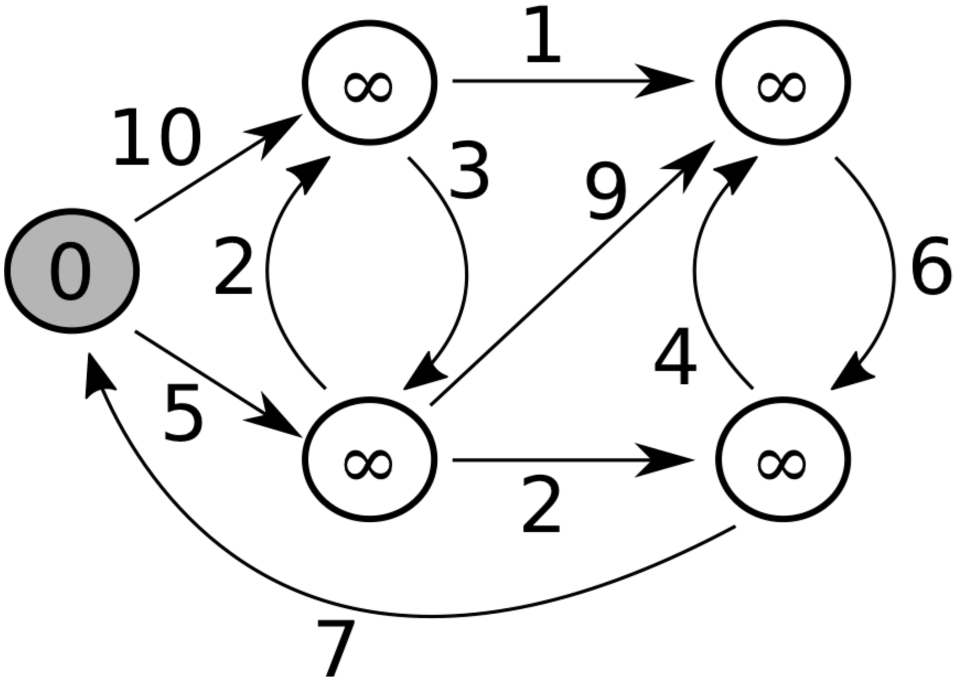
Pathfinding

- Problem: find **shortest path** from A to B
- Shortest is defined as **lowest** total **edge weights**



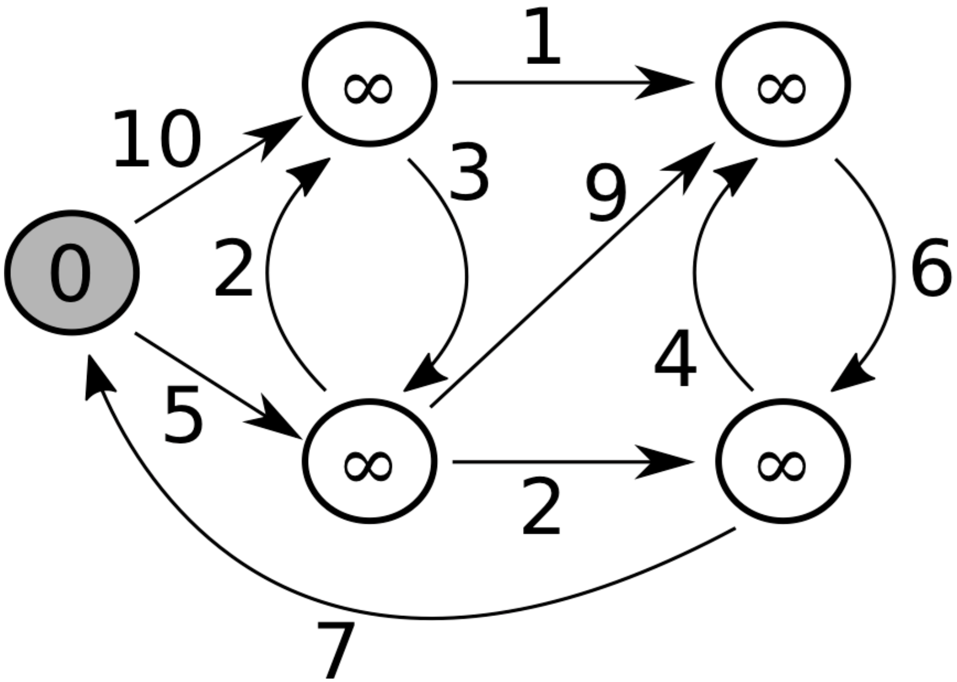
Dijkstra’s algorithm

- Algorithm to obtain **shortest path** from a given vertex to **any other** vertex
- Example of **greedy** algorithm
- Initially: set shortest-path **estimates** to **0** for start vertex and **∞** for the others



Dijkstra’s algorithm

- Repeat the following:
  - Select unvisited vertex with **lowest** estimate
  - Look at paths to **unvisited** nodes
  - Update estimates if **lower** than previous estimate



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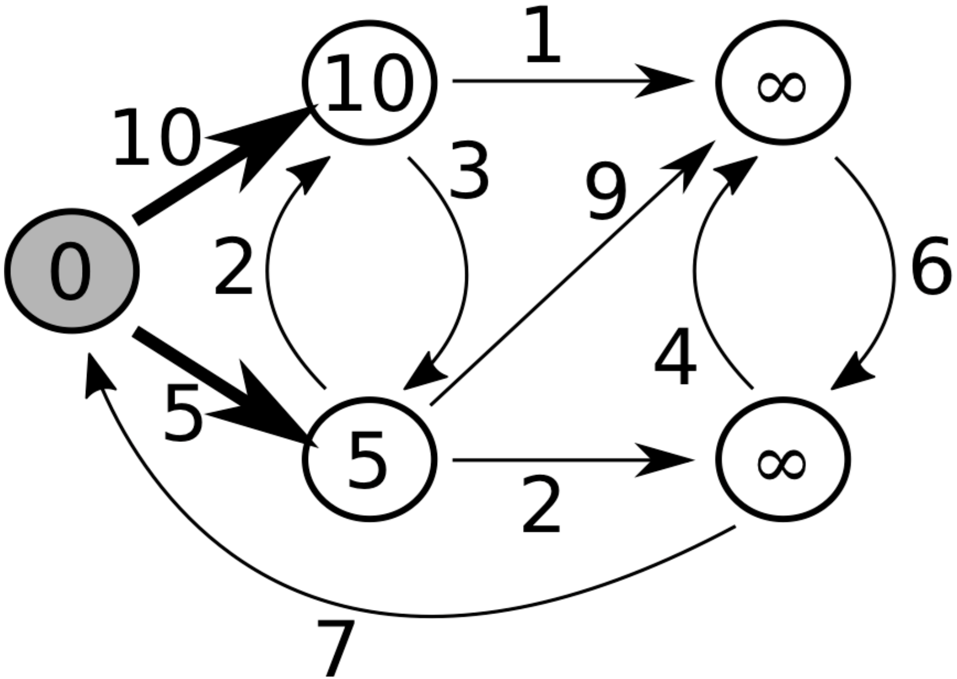

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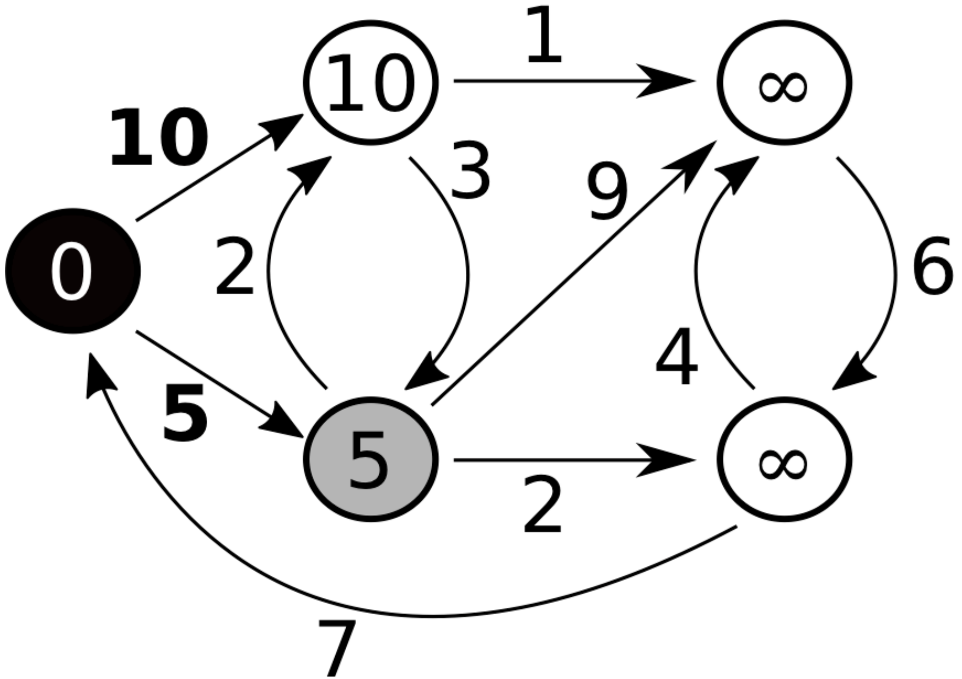

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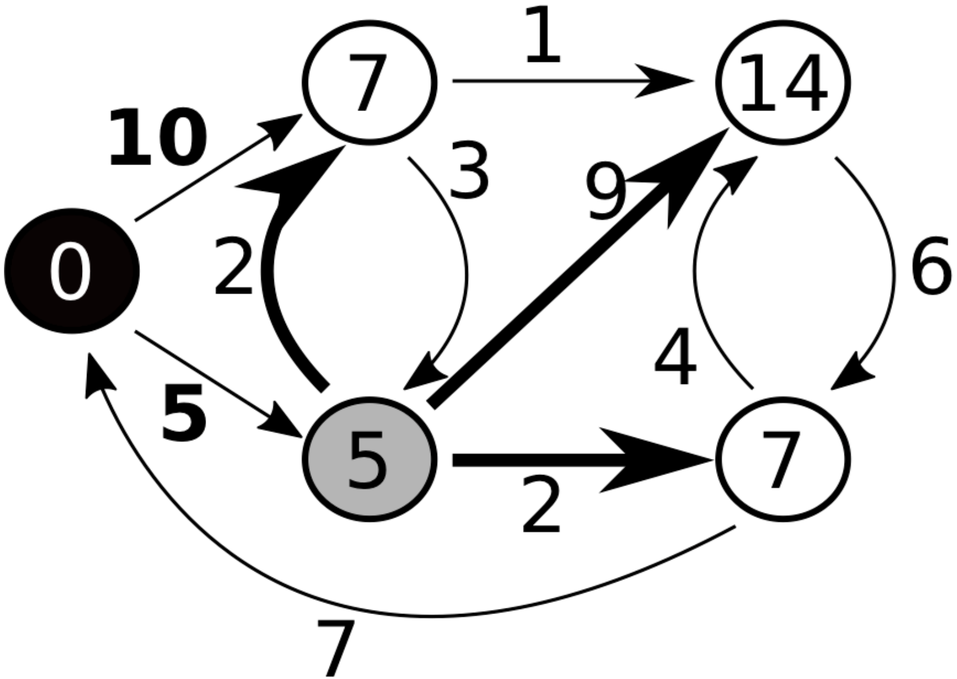

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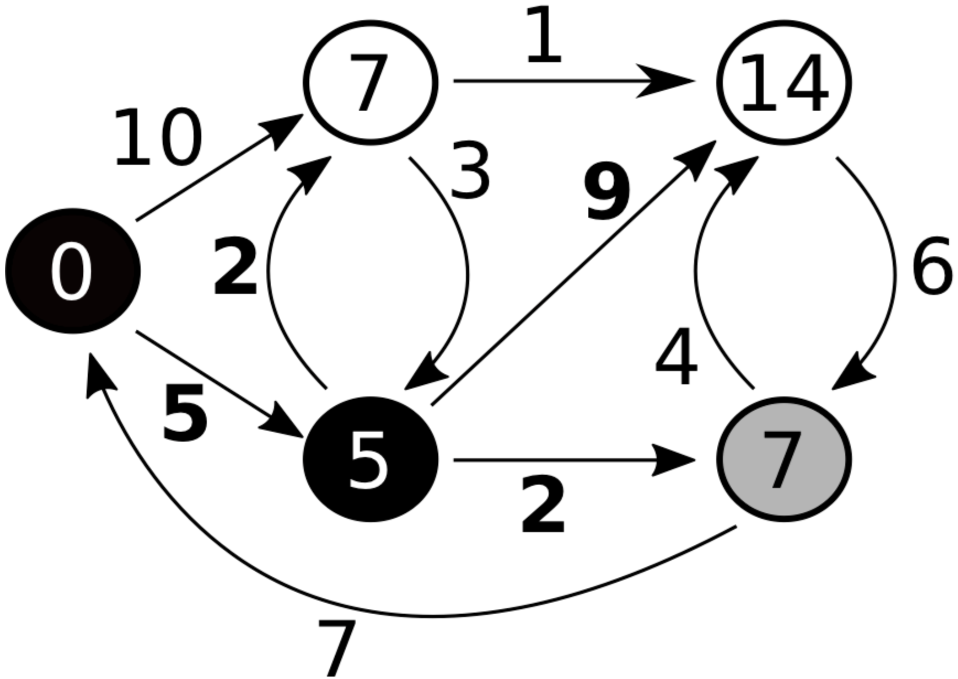

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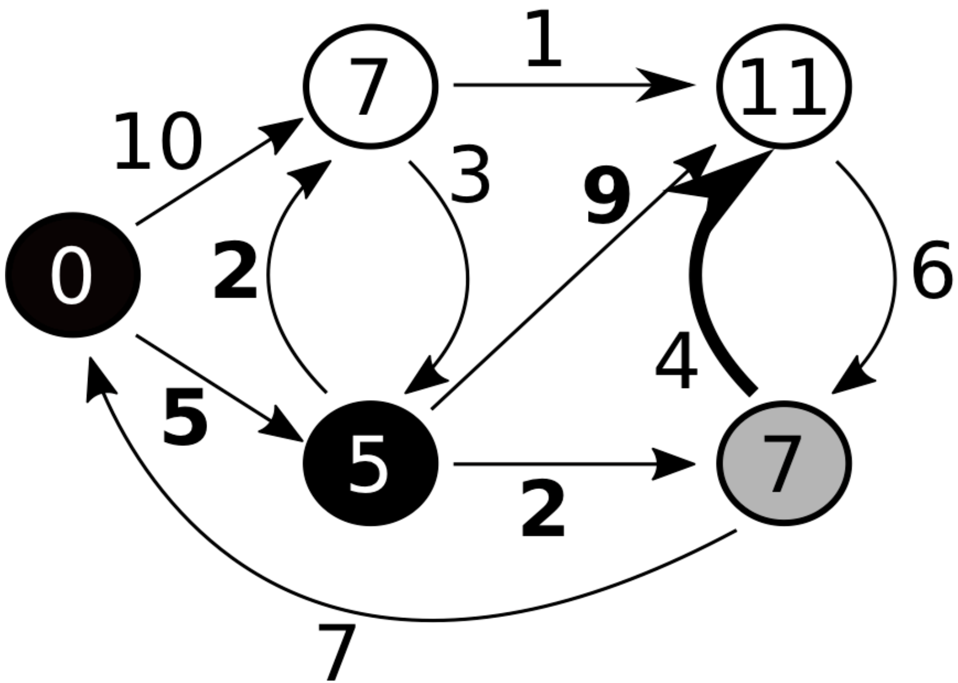
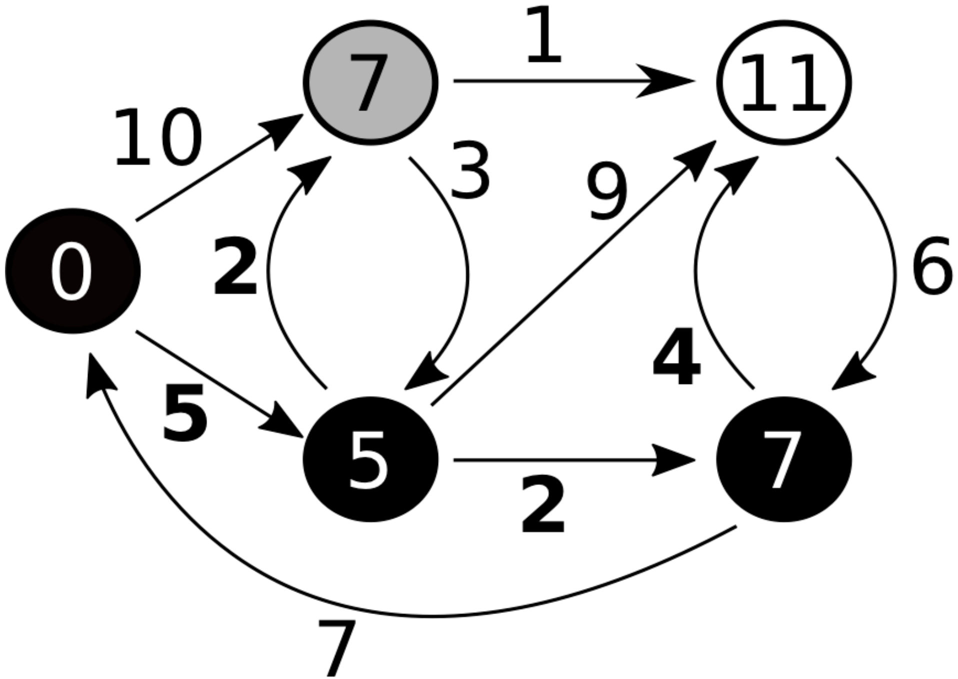


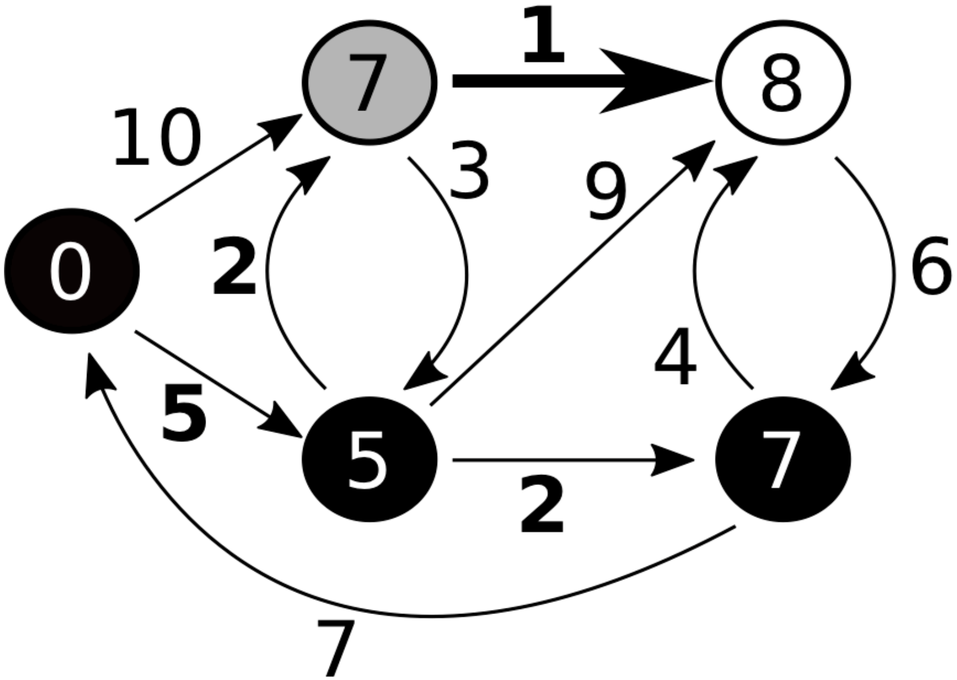
Dijkstra’s algorithm

- Repeat the following:
    - Select unvisited vertex with **lowest** estimate
    - Look at paths to **unvisited** nodes
    - Update estimates if **lower** than previous estimate



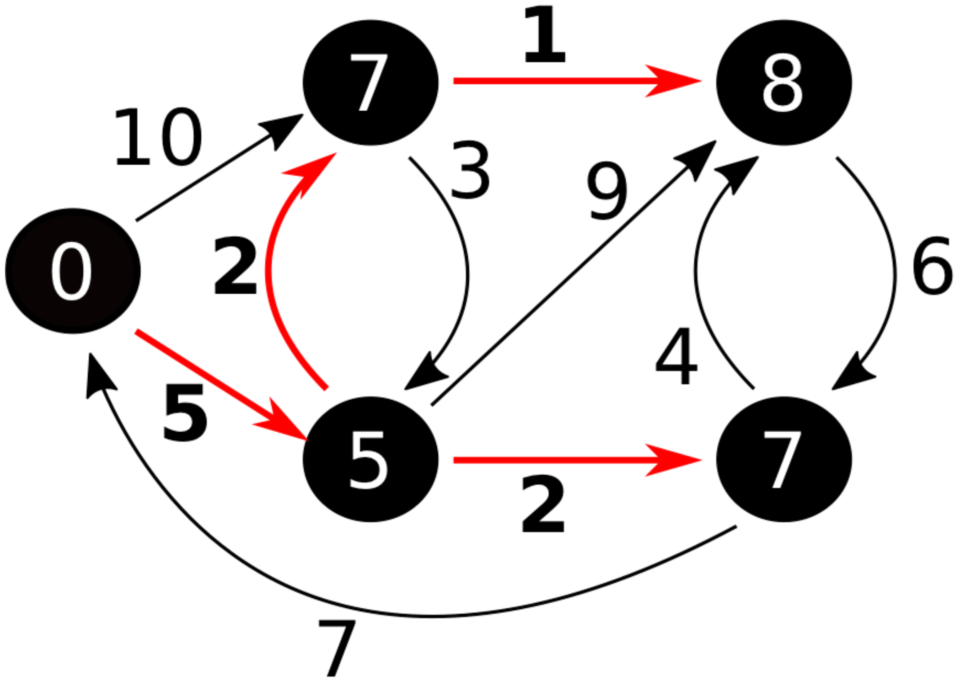
Dijkstra’s algorithm

- Repeat the following:
  - Select unvisited vertex with **lowest** estimate
  - Look at paths to **unvisited** nodes
  - Update estimates if **lower** than previous estimate



Dijkstra's algorithm

- Repeat the following:
  - Select unvisited vertex with **lowest** estimate
  - Look at paths to **unvisited** nodes
  - Update estimates if **lower** than previous estimate
- Shortest paths indicated in red
- Complexity: **O(*E* + *V* log *V*)**



8. Summary

Summary

- Concepts

- Divide and conquer
- Complexity
    - O, Θ, Ω
- (Un)stable sorting
- Pointers

Summary

- Concepts
- Sorting algorithms

- Insertion sort
- Bubble sort
- Merge sort
- Quicksort

Summary

- Concepts
- Sorting algorithms
- Data structures

- Arrays
- Dynamic arrays
- (Doubly) linked lists
- Binary search trees
- Hash tables
- (Directed) graphs

Summary

- Concepts
- Sorting algorithms
- Data structures
- Abstract data types

- Queues
- Stacks
- Maps
- Sets

Summary

- Concepts
- Sorting algorithms
- Data structures
- Abstract data types
- Algorithms

- Binary search
- Dijkstra’s algorithm