**Assignment IX: Monte Carlo Applications**

Due 11/2/2015

## 1. Multi-Dimensional Integration

Evaluate the 10-dimensional integral

$$I = \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_{10} (x_1 + x_2 + \cdots + x_{10})^2. \tag{1}$$

Check your numerical answer against the analytic result 155/6.

**(a)** Conduct 16 trials and take the average as your answer.

**(b)** Try sample sizes of $N$= 2, 4, 8, .... 8192.

**(c)** Plot the absolute value of the error versus $1/\sqrt{N}$ and try to identify linear behavior.

## 2. 3D Integration with Importance Sampling

Consider the integral defined in a box with length 1

$$\int d^x x \rho(\mathbf{x}) \equiv \int_0^1 dx \int_0^1 dy \int_0^1 dz \rho(x, y, z) \tag{2}$$

with

$$\rho(x, y, z) = \frac{1}{1 + x^2}\, y e^{-y^2}\, \frac{e^{-z}}{\sqrt{z}} \tag{3}$$

**(a)** Evaluate this integral in a similar fashion as in Problem **2**, i.e. use the built in random number generator. Conduct at least 8 runs and plot your answer as function of $1/\sqrt{N}$. Estimate the correct answer.

**(b)** Employ importance sampling and use the built in random number generator to evaluate the integral. Use logarithmic maps in $y$ and $z$ direction and a tangent map in $x$ direction. Use that

$$\int dx \frac{1}{1 + x^2} = \arctan x \tag{4}$$

$$\int dy e^{-y} = e^{-y} \tag{5}$$

Proceed analogously as demonstrated in the code *log_car_sob.f90*. Conduct at least 8 runs and plot your answer as function of $1/\sqrt{N}$. Assess how much the importance sampling improves your calculation, i.e. determine if a smaller number $N$ is sufficient to reach the same results as **(a)**. Plot your answer as function of $1/\sqrt{N}$.

**(c)** Employ importance sampling and use the quasi-random numbers generated by the Sobol sequence to obtain the result (code *log_car_sob.f90*). Plot your answer as function of $1/N$. Assess the difference in computational effort between the Sobol sequence and the built in random number generator.

**(d)** Calculate the same integral using Gaussian quadrature. Estimate the error as function of $x$, $y$, and $z$, and find your optimum set of points. Compare with the Monte Carlo integration.

**(e)** Compare the cpu-time needed to obtain the results for (a), (b), (c), and (d) for an accuracy of you choice, but at east 2 significant figures.