

L^AT_EX Mathematics Examples

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September 6, 2015

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1 Delimiters

See how the delimiters are of reasonable size in these examples

$$(a + b) \left[1 - \frac{b}{a + b} \right] = a ,$$

$$\sqrt{|xy|} \leq \left| \frac{x+y}{2} \right|,$$

even when there is no matching delimiter

$$\int_a^b u \frac{d^2 v}{dx^2} dx = u \frac{dv}{dx} \Big|_a^b - \int_a^b \frac{du}{dx} \frac{dv}{dx} dx.$$

2 Spacing

Differentials often need a bit of help with their spacing as in

$$\iint xy^2 \, dx \, dy = \frac{1}{6} x^2 y^3,$$

whereas vector problems often lead to statements such as

$$u = \frac{-y}{x^2 + y^2}, \quad v = \frac{x}{x^2 + y^2}, \quad \text{and} \quad w = 0.$$

3 Arrays

Arrays of mathematics are typeset using one of the matrix environments as in

$$\begin{bmatrix} 1 & x & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 + xy \\ y - 1 \end{bmatrix}.$$

Case statements use cases:

$$|x| = \begin{cases} x, & \text{if } x \geq 0, \\ -x, & \text{if } x < 0. \end{cases}$$

Many arrays have lots of dots all over the place as in

$$\begin{array}{cccccc} -2 & 1 & 0 & 0 & \cdots & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ 0 & 0 & 1 & -2 & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 1 & -2 \end{array}$$

4 Equation arrays

In the flow of a fluid film we may report

$$u_{\alpha} = \epsilon^2 \kappa_{xxx} \left(y - \frac{1}{2} y^2 \right), \quad (1)$$

$$v = \epsilon^3 \kappa_{xxx} y, \quad (2)$$

$$p = \epsilon \kappa_{xx}. \quad (3)$$

Alternatively, the curl of a vector field (u, v, w) may be written with only one equation number:

$$\begin{aligned} \omega_1 &= \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \\ \omega_2 &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \\ \omega_3 &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \end{aligned} \quad (4)$$

Whereas a derivation may look like

$$\begin{aligned} (p \wedge q) \vee (p \wedge \neg q) &= p \wedge (q \vee \neg q) \quad \text{by distributive law} \\ &= p \wedge T \quad \text{by excluded middle} \\ &= p \quad \text{by identity} \end{aligned}$$

5 Functions

Observe that trigonometric and other elementary functions are typeset properly, even to the extent of providing a thin space if followed by a single letter argument:

$$\exp(i\theta) = \cos \theta + i \sin \theta, \quad \sinh(\log x) = \frac{1}{2} \left(x - \frac{1}{x} \right).$$

With sub- and super-scripts placed properly on more complicated functions,

$$\lim_{q \rightarrow \infty} \|f(x)\|_q = \max_x |f(x)|,$$

and large operators, such as integrals and

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{where } n! = \prod_{i=1}^n i,$$
$$\overline{U_\alpha} = \bigcap_{\alpha} U_\alpha.$$

In inline mathematics the scripts are correctly placed to the side in order to conserve vertical space, as in $1/(1-x) = \sum_{n=0}^{\infty} x^n$.

6 Accents

Mathematical accents are performed by a short command with one argument, such as

$$\tilde{f}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx,$$

or

$$\dot{\vec{\omega}} = \vec{r} \times \vec{I}.$$

7 Command definition

The Airy function, $\text{Ai}(x)$, may be incorrectly defined as this integral

$$\text{Ai}(x) = \int \exp(s^3 + isx) ds .$$

This vector identity serves nicely to illustrate two of the new commands:

$$\nabla \times \mathbf{q} = \mathbf{i} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) + \mathbf{j} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + \mathbf{k} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) .$$

8 Theorems et al.

Definition 1 (right-angled triangles) *A right-angled triangle is a triangle whose sides of length a , b and c , in some permutation of order, satisfies $a^2 + b^2 = c^2$.*

Lemma 2 *The triangle with sides of length 3, 4 and 5 is right-angled.*

This lemma follows from the Definition 1 as $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Theorem 3 (Pythagorean triplets) *Triangles with sides of length $a = p^2 - q^2$, $b = 2pq$ and $c = p^2 + q^2$ are right-angled triangles.*

Prove this Theorem 3 by the algebra $a^2 + b^2 = (p^2 - q^2)^2 + (2pq)^2 = p^4 - 2p^2q^2 + q^4 + 4p^2q^2 = p^4 + 2p^2q^2 + q^4 = (p^2 + q^2)^2 = c^2$.