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HW7

QN 2 The radial part of the wavefunction is,

$$-\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}-\frac{l(l+1)}{r^2}\right)R(r)+V(r)R(r)=E\,R(r) \qquad \text{—— (10)}$$

put, $R(r)=\frac{u(r)}{r}$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}-\frac{l(l+1)}{r^2}\right)\frac{u(r)}{\cancel{r}}+V(r)\frac{u(r)}{\cancel{r}}=E\frac{u(r)}{\cancel{r}}$$

put $\rho=\frac{r}{\alpha}$ $\quad d\rho=\frac{dr}{\alpha}$ $\Rightarrow dr=\alpha\, d\rho$

$$\frac{1}{\alpha}\frac{d}{d\rho}=\frac{d}{dr}$$

$\Rightarrow r=\alpha\rho$

$\alpha=$ constant

Then,

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\alpha^2\rho^2}\frac{1}{\alpha}\frac{d}{d\rho}\alpha^2\rho^2\frac{1}{\alpha}\frac{d}{d\rho}-\frac{l(l+1)}{\alpha^2\rho^2}\right)u(\rho)+V(\rho)u(\rho)=Eu(\rho)$$

$$-\frac{\hbar^2}{2m}\left(\frac{1}{\alpha^2\rho^2}\frac{d}{d\rho}\rho^2\frac{d}{d\rho}-\frac{l(l+1)}{\alpha^2\rho^2}\right)u(\rho)+V(\rho)u(\rho)=E\,u(\rho)$$

or, $$-\frac{\hbar^2}{2m\alpha^2}\frac{d^2}{d\rho^2}u(\rho)+\left(V(\rho)+\frac{l(l+1)}{\rho^2}\frac{\hbar^2}{2m\alpha^2}\right)u(\rho)=E\,u(\rho) \qquad \text{—— (11)}$$

Take $l = 0$,

Take $V(\rho) = \frac{1}{2} K\alpha^2 \rho^2$

Then,

$$-\frac{\hbar^2}{2m\alpha^2} \frac{d^2}{d\rho^2} u(\rho) + \left[V(\rho) + \cancel{\frac{l(l+1)}{\rho^2}} \stackrel{\to 0}{} \frac{\hbar^2}{2m\alpha^2}\right] u(\rho) = E\, u(\rho)$$

or, $$\underbrace{-\frac{\hbar^2}{2m\alpha^2}}_{\text{divide}} \frac{d^2}{d\rho^2} u(\rho) + \frac{1}{2} K\alpha^2 \rho^2 u(\rho) = E\, u(\rho)$$

or, $$-\frac{d^2}{d\rho^2} u(\rho) + \underbrace{\frac{mK\alpha^4}{\hbar^2}}_{=1\ (\text{choose})} \rho^2 u(\rho) = \underbrace{\frac{2m\alpha^2}{\hbar^2}}_{=\lambda(\text{define})} E\, u(\rho) \quad \text{—— (12)}$$

choose: $\frac{mK\alpha^4}{\hbar^2} = 1$

define: $\frac{2m\alpha^2}{\hbar^2} = \lambda$

Then,

$$\boxed{-\frac{d^2}{d\rho^2} u(\rho) + \rho^2 u(\rho) = \lambda u(\rho)} \quad \text{—— (15)}$$

This is the required equation.