Assignment 4

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1 Question 1

This program computes the first derivative of functions

$$
\begin{aligned}
f(x) &= cos(x) && (1)\\
f(x) &= exp(x) && (2)\\
f(x) &= \sqrt(x) && (3)\\
& && (4)
\end{aligned}
$$

at points $x = 0.1$,$x = 1$,$x = 30$

1.1 part a

The source code is:
assign4/qn1/hw4qn1.f90

The outputs are :

```
for single precision:
cos01sp.dat,cos1sp.dat,cos30sp.dat for x=0.1,1.0,and30.0 respectively
exp01sp.dat,exp1sp.dat,exp30sp.dat
sq01sp.dat,sq1sp.dat,sq30sp.dat

For double precision:
cos01dp.dat,cos1dp.dat,cos30dp.dat for x=0.1,1.0,and30.0 respectively
sq01dp.dat,sq1dp.dat,sq30dp.dat

Note: double precision for exp(x) was not required.
```

I chose $h = 40.0$ for $cos(x)$ and $\exp(x)$ and $h = 0.1$ for $\sqrt{x}$
The step size was reduced upto the machine precison upto $1e-6$ for single precision and $1e-14$ for double precision.

1.2 part b

The source code is:
hw4qn1.f90

output dat files are:
cos01sp.dat,cos01dp.dat etc.

I plotted modulus of logE vs logH for 3 functions for single and double precisions.
The number of decimal places obtained agrees with the estimates in the text.
The graphs looks like this:

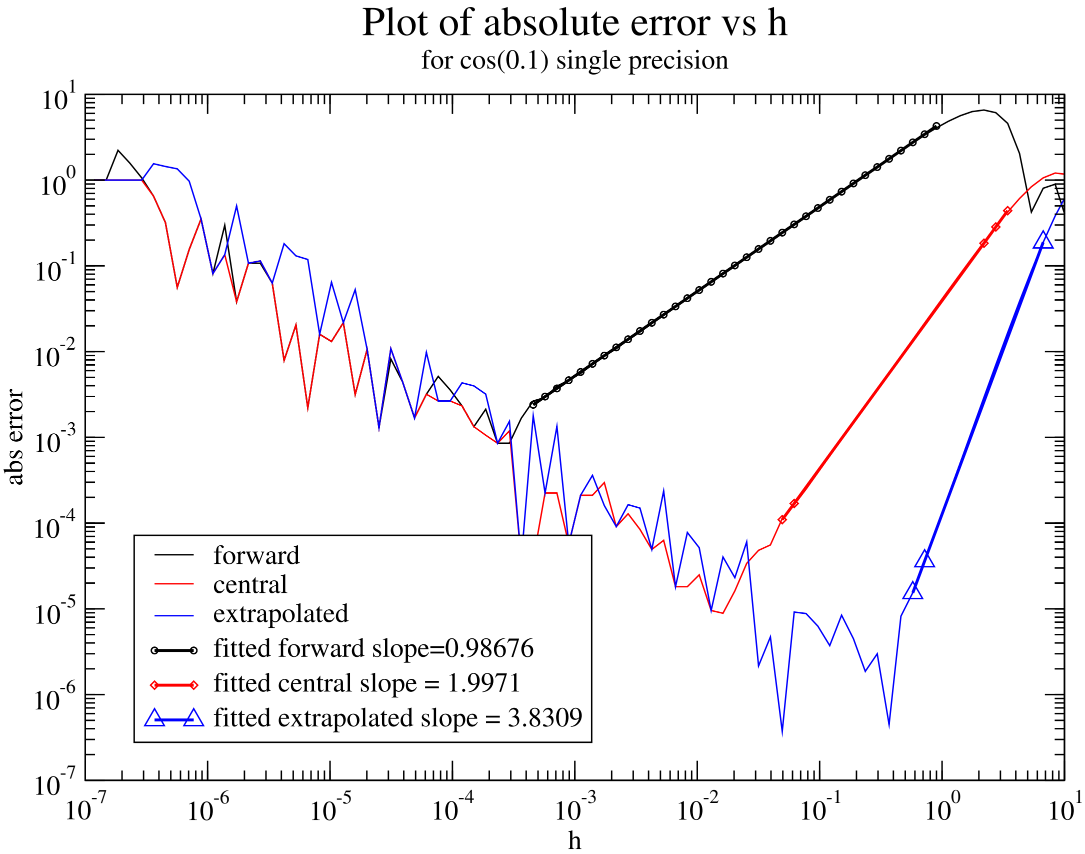

Figure 1: Plot of cos(0.1) single precision

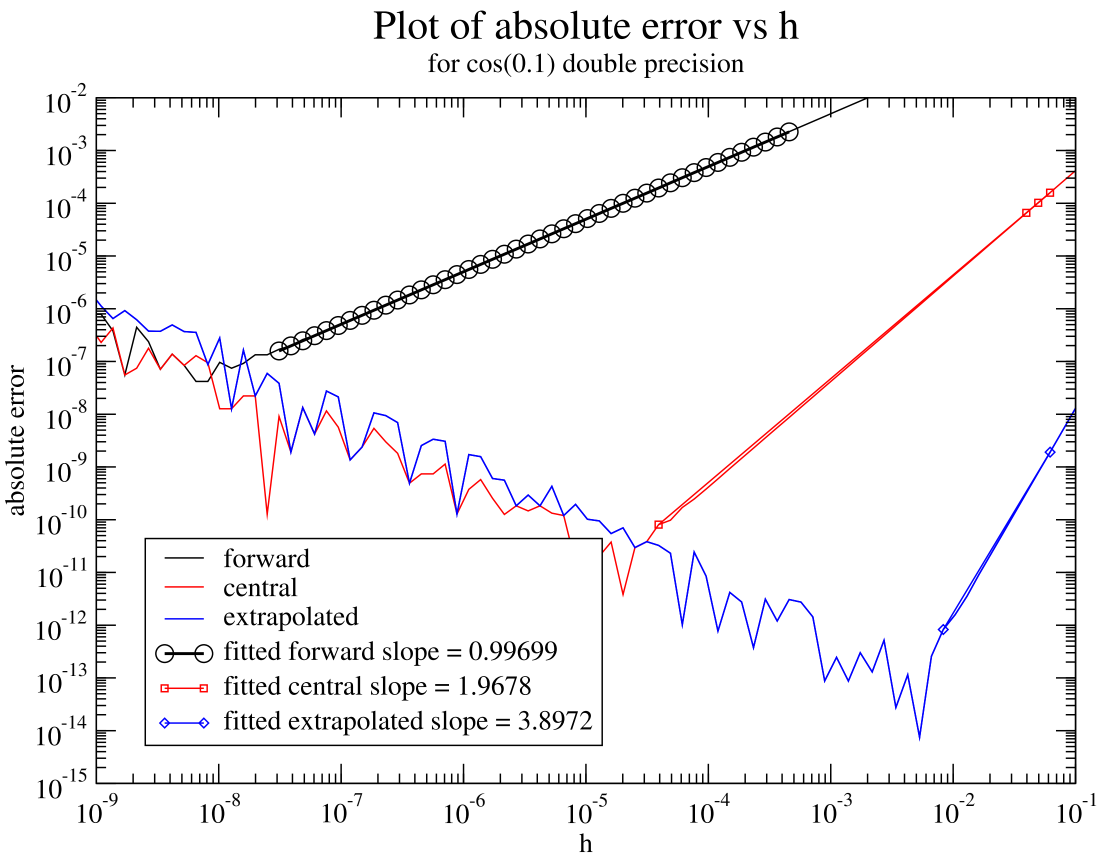

Figure 2: Plot of cos(0.1) double precision

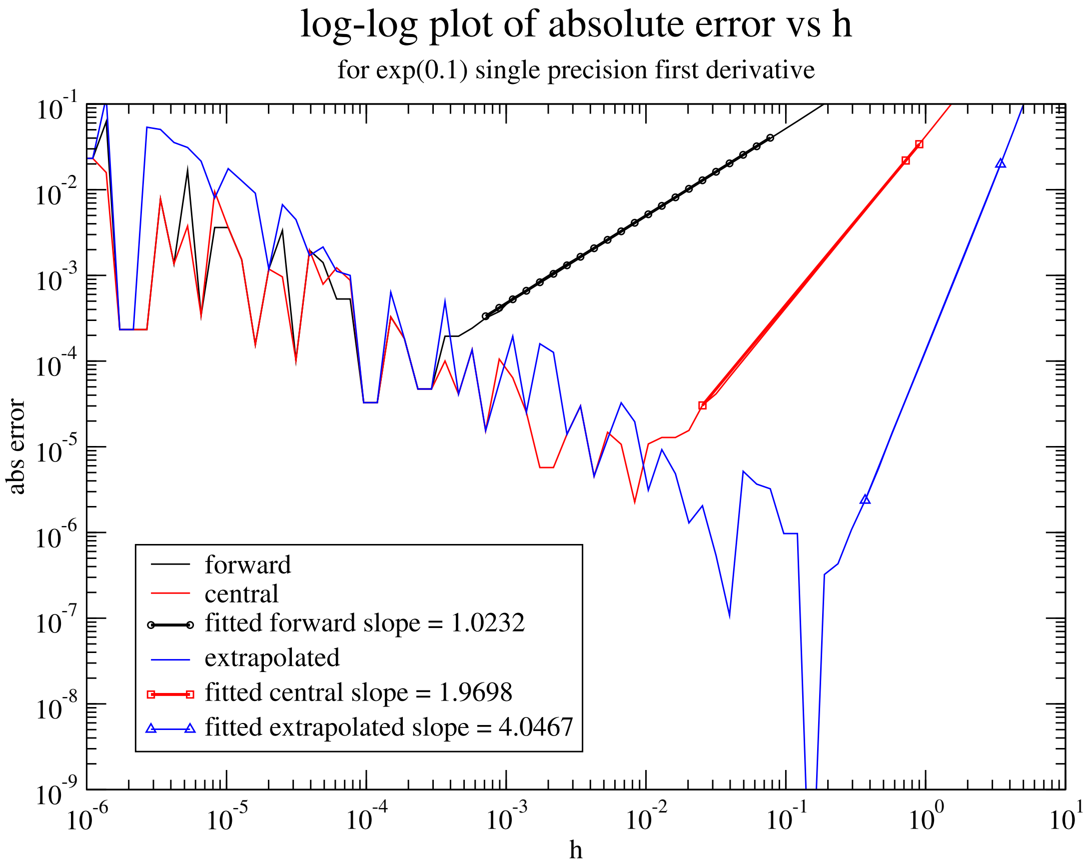

Figure 3: Plot of exp(0.1) single precision

1.3 part c

I plotted the best fit of the graph with log-log fit from xmgrace. Appropriate range for X and Y axes were chosen. From the plots we know that for cosine function in double precision:
slope for forward differentiation is nearly 1.
slope for central differentiation is nearly 2.
slope for extrapolation differentiation is nearly 4.

The source code is:
assign4/qn1/hw4qn1.f90

The output plots are:
cos01dp.eps,cos1dp.eps,etc

1.4 part d

I repeated the analysis for $\cos(x)$ and $\sqrt{x}$ in double precision and compared to the single precision.
The source code is:
assign4/qn1/hw4qn1.f90

The output data files are:
cos01dp.dat,cos1dp.dat,etc

The output plots are:
cos01dp.eps,cos1dp.eps,etc

1.5 part e

In the above plots I paid special attention to the algorithmic errors. The best fit was plotted for the algorithmic error part. The best fit eps files are inside qn1 folder.

2 Question 2

In this question we studied three-point and five point formula for second order derivative of functions.

2.1 part a

The three-point and five-point formula was derived and the pdf can be found inside: writeup/hw4qn2a.pdf

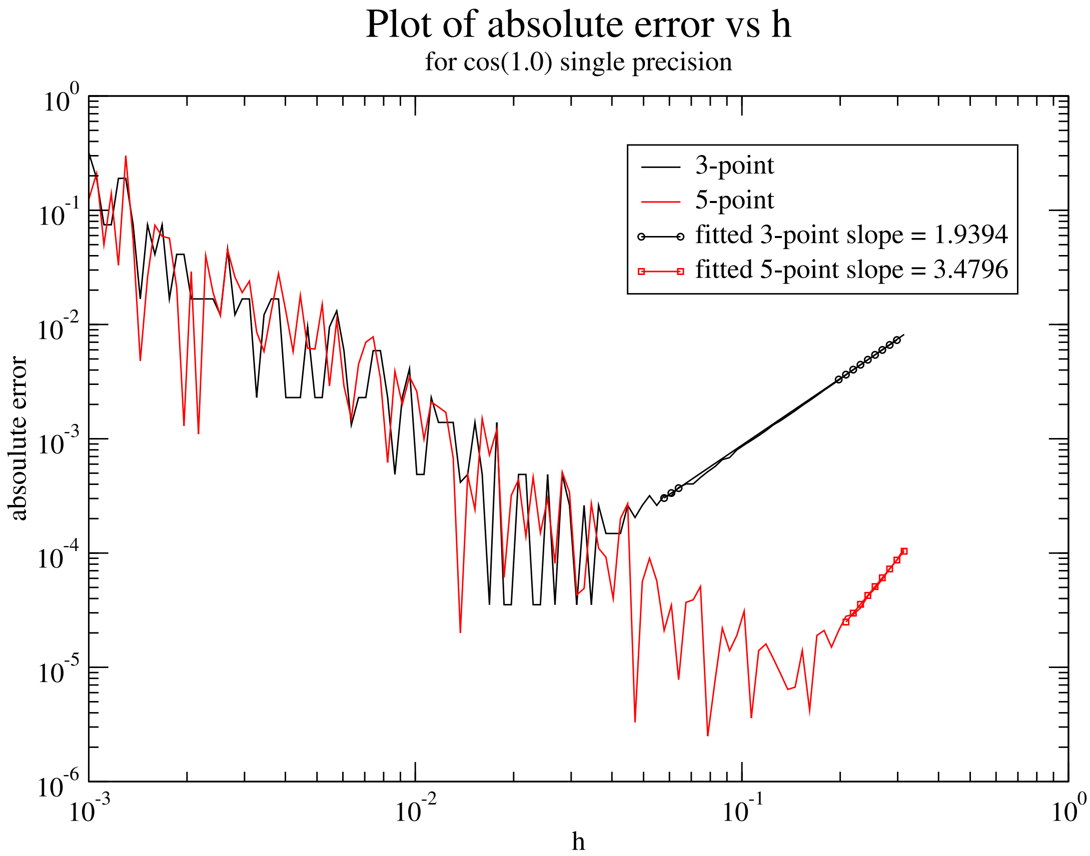

Figure 4: Plot of cos(0.1)

or, qn1/hw4qn2a.pdf

2.2 part b

I wrote the code to calculate 2nd order derivative for $cos(x)$ in single precision for the 3 values of $x = 0.1$,$x = 1$,$x = 30$. I started with h=0.314 and keep going down upto machine precision 1e-6.
While calculating derivative special attention was given in grouping the terms. Similar terms are grouped together.
The grouping can be seen in the source code.
The source code is:
qn2/hw4qn2.f90

and the outpuf files are:
$hw4qn2\_1.dat, hw4qn2\_01.dat, and, hw4qn2_30.dat.$

2.3 part c

The derivative and its relative errors were produced. I reduced step size upto 1e-6 for single precision.

2.4 part d

The log-log plot of logE versus logH was created. The plots are:
$hw4qn2\_1.eps, hw4qn2\_01.eps, and, hw4qn2_30.eps.$
The number of decimal places obtained agrees with the estimates in the text.

2.5 part e

We can see truncation error at large h and roundoff error at small h in the graph.

3 Question 3: Population Growth Problem

In this problem we solved population growth equation both numerically and analytically.

3.1 part a: when b=0

First we took, $b = 0$ and solved the equation. The source code is:
$qn3/hw4qn3a.f90$

The output is:
$qn3/hw4qn3a.dat$

The plot is:
$qn3/hw4qn3a.eps$

3.2 part b: when b=3

Here, we took a =10 and b=3. The source code is:
qn3/hw4qn3b.f90

The output is:
qn3/hw4qn3b.dat

the plot is :
qn3/hw4qn3b.eps

Initially the population decreases with time since the $-bN^2$ term dominates and as the time passes by population growth seems to be constant. We can see this in the plots.