Assignment 5

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1 Question 1: Integral of sine x

In this problem we calculated the integral of $sin(x)$ from 0 to pi. The exact value is 2.0. We calculated numerical integral values using Trapezoid,Simpsons,and Gauss-Legendre method. The template integ1.f90 was provided and further modification was done in the template. errors are absolute errors.

1.1 part a

In this part we prepared a table for the values of the given integral.
source code is inside qn1: single and double folders. sp is single precision dp is double precision.
source codes are: hw5qn1sp.f90 and hw5qn1dp.f90
outputs are : hw5qn1sp.dat and hw5qn1dp.dat

for the plot:
source codes are: hw5qn1spplot.f90 and hw5qn1dpplot.f90
outputs are : hw5qn1spplot.dat and hw5qn1dpplot.dat

1.2 part b

In this part, we plotted the graph of $log_{10}(e)$ vs. $log_{10}(N)$ for 3 methods for single and double precisions.
The graphs looks like this:

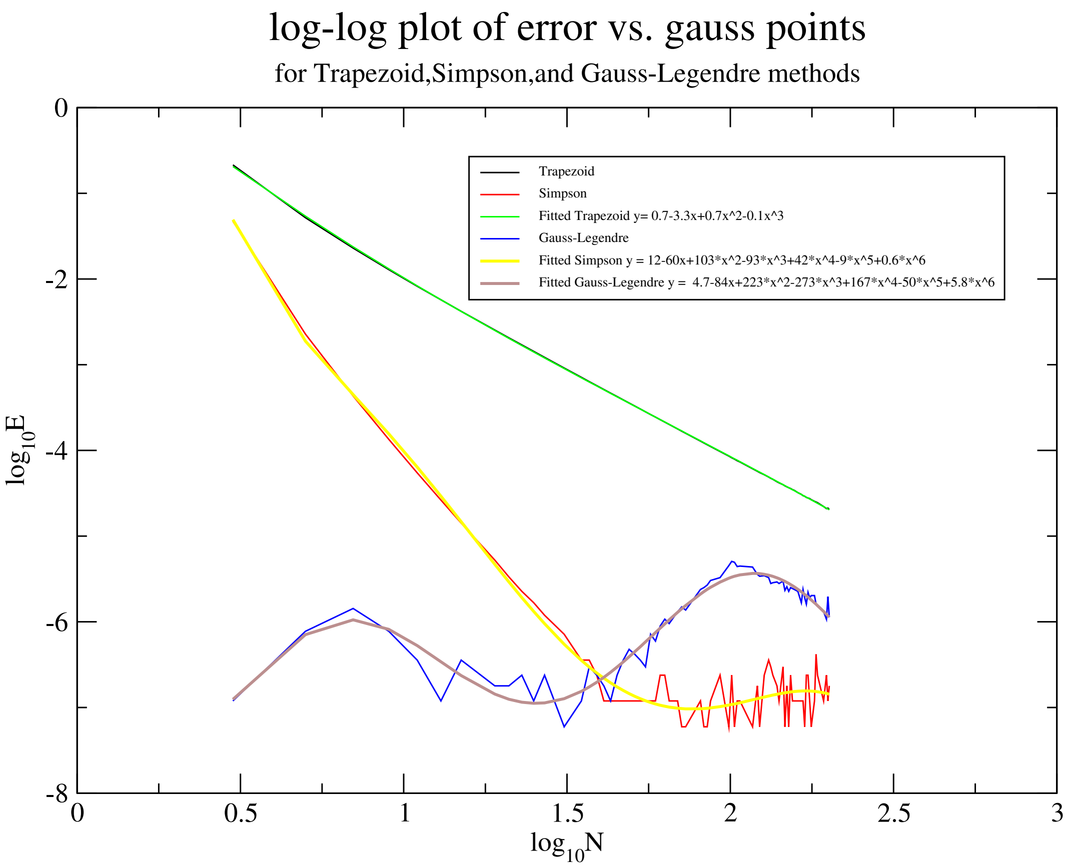

Figure 1: Plot for Qn1 single precision

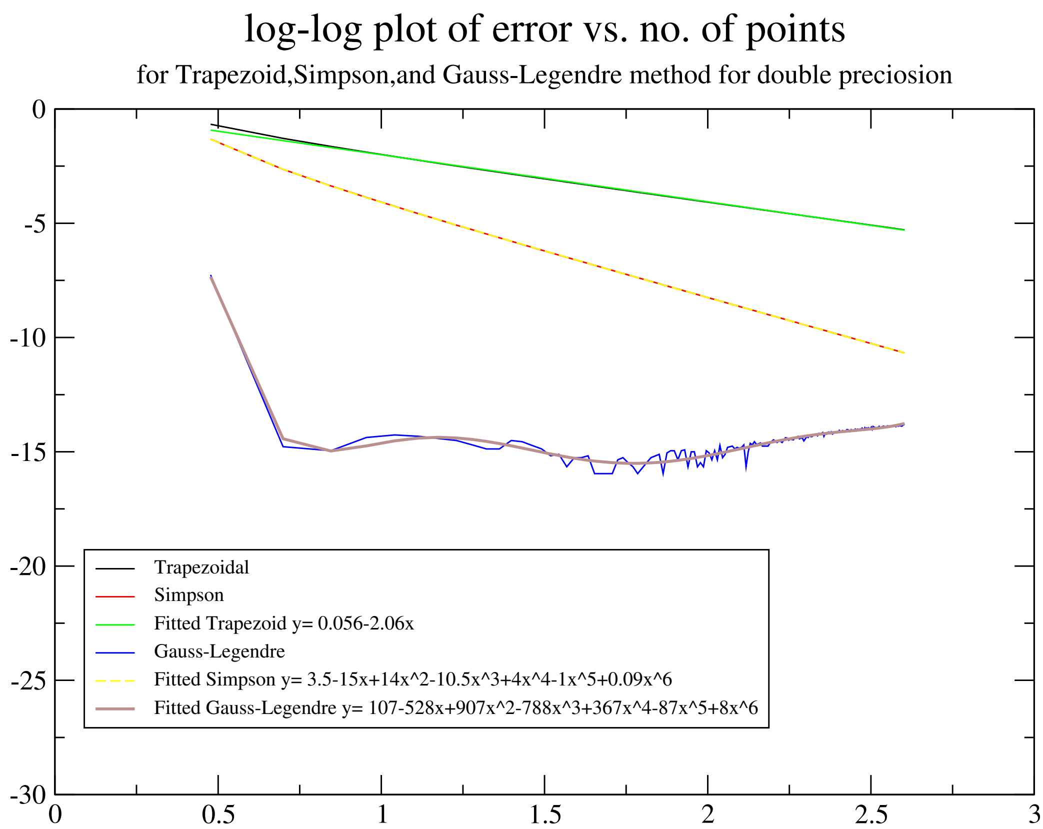

Figure 2: Plot for Qn1 double precision

1.3 part c

In part b, I have plotted the graph to determine the power-law dependence of the error on the number of points N.

1.4 part d

The plot for both single and double precision were drawn.

1.5 part e

Here, I used Trapezoid method to calculate the integral value of the given function. In this case we don't know the exact value of the integral. The integrals are calculated as the sum of the terms. We can choose some tolerance value such as:
modulus(term/sum) is greater than tolerance.

2 Question 2: Integral of sine squared x

The given integrals are:

$$I = \int_{-1}^{1} \sqrt{1 - x^2}\, dx$$

and,

$$I = \int_{-1}^{1} sin^2\theta \, d\theta$$

The exact value $= \Pi/2 = 0.157079632679E + 01$
I used Trapezoidal,Simpson,and Gauss-Legendre method to calculate the numerical interal value of the given integrals separately.
I used double precision to solve the problem.
The source codes are : hw5qn2a.f90 and hw5qn2b.f90
The outputs are : hw5qn2a.dat,and hw5qn2b.dat
To get the precision $= 0.157E + 01$
for square root integral:
Trapezoid iteration $= 233$
Simpson iteration $= 17$
Gauss iteration $= 5$
To get the precision $= 0.157079632679E + 01$
for sine squared integral:
Trapezoid iteration $= 85$
Simpson iteration $= 5$
Gauss iteration $= 13$

3 Question 3: Plank's black body radiation

3.1 part a: Gauss-Laguerre

In this part we calculated numerical value using Gauss-Laguerre method for 2,4,6,8,and 10 points. The error plot was also drawn. The given inegral is :

$$I = \int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

Comparing to the standard format for generalized Gauss-Laguerre quadrature:

$$I = \int_0^\infty e^{-x} x^\alpha\, f(x)\, dx$$

we get: $\alpha = 3$ and

$$f(x) = \frac{1}{1 - e^{-x}}$$

The source code is: hw5qn3a.f90
The plot is : hw5qn3.eps

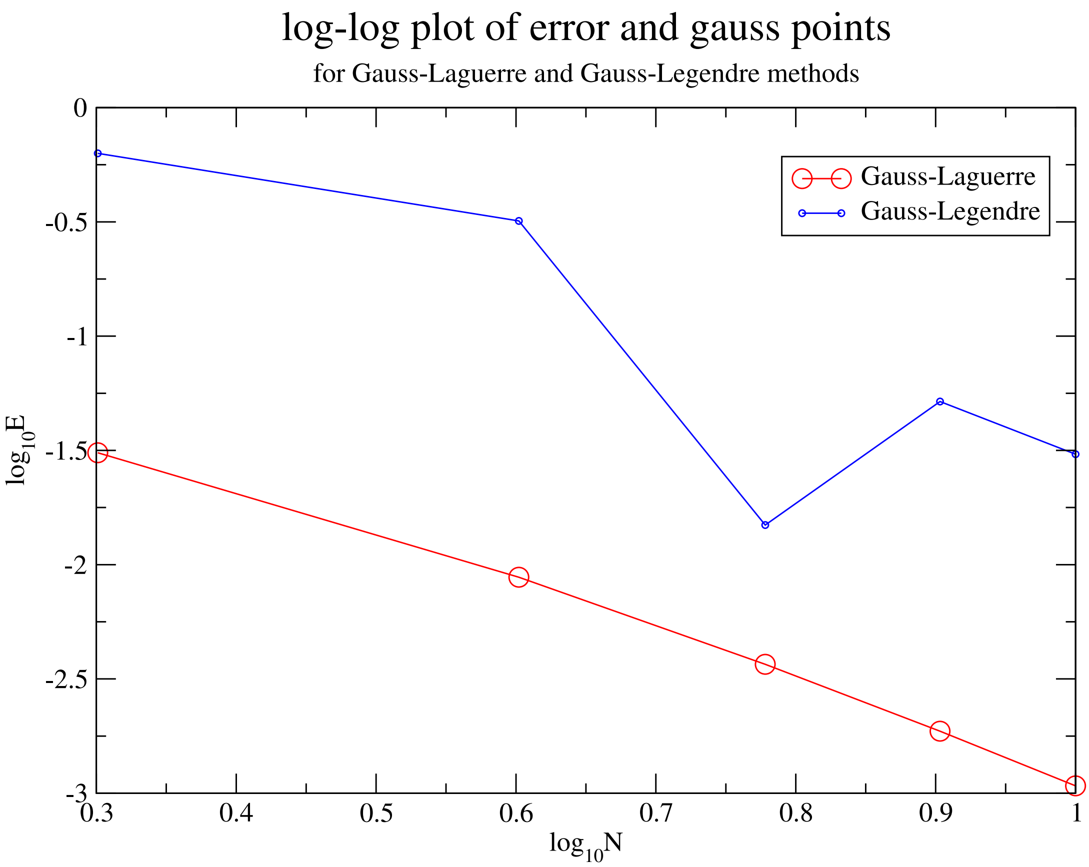

Figure 3: Plot for Qn3

3.2 part b: Gauss-Legendre

Here, to map the given interval (0 to infinity) into (0 to 1)
I substituted : $x = \tan \frac{pi*y}{2}$
Then, I calculated the value of f(y) for the integral of the form:

$$I = \int_0^1 f(y)\, dy$$

The source code is: hw5qn3b.f90
The output is : hw5qn3b.dat

4 Question 4 Sine Integral

In this question the sample code integ3.f90 was provided for the integral:

$$I = \int_{1}^{1000} \frac{sin(x)}{x}\, dx$$

We have to solve numerically the integral:

$$I = \int_{1}^{100} \frac{sin(40x)}{x}\, dx$$

So, I substituted $y = 40x$, then, I got the integral:

$$I = \int_{40}^{4000} \frac{sin(y)}{y}\, dy$$

The exact solution is : $Si(4000) - Si(40)$
Where Sine integral 'Si' is an entire function defined as:

$$Si(z) = \int_{0}^{z} \frac{sin(t)}{t}\, dt$$

The source code is : hw5qn4.f90 and hw5qn4plot.f90
The output is : hw5qn4.dat and hw5qn4plot.dat
The plot is : hw5qn4.eps

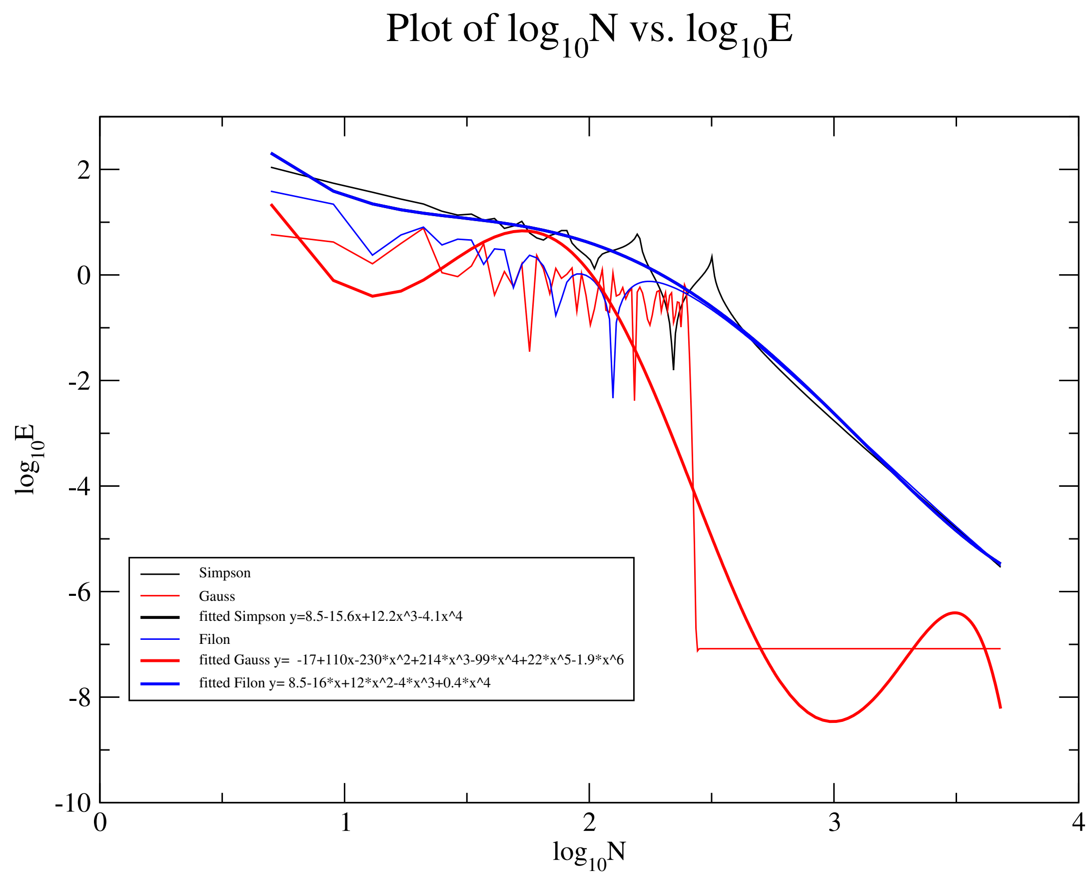

Figure 4: Plot for Qn4