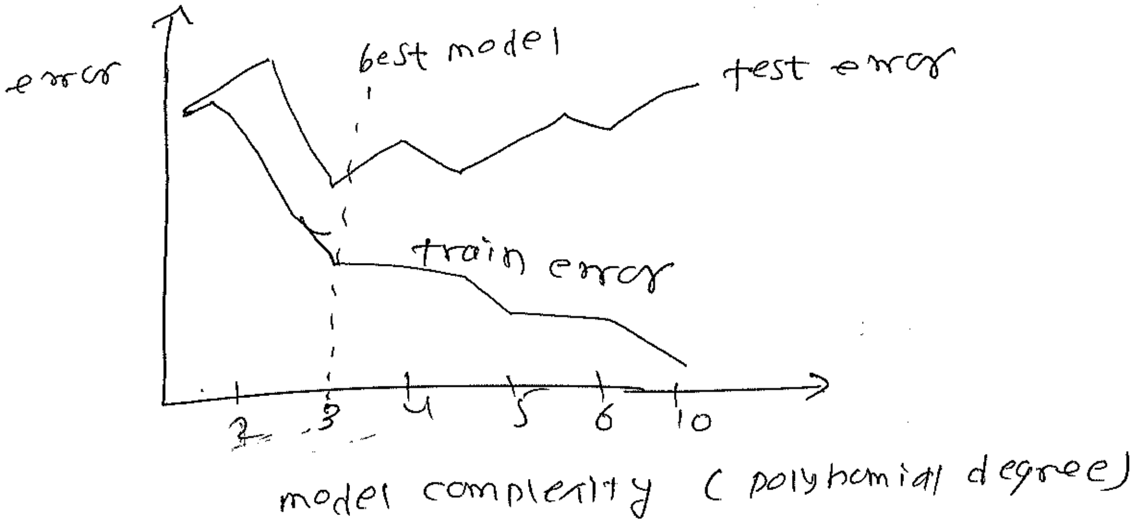
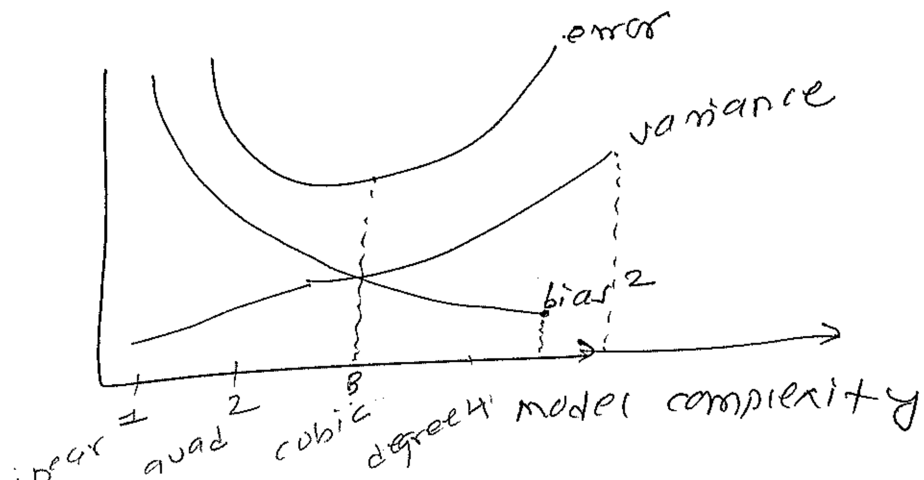
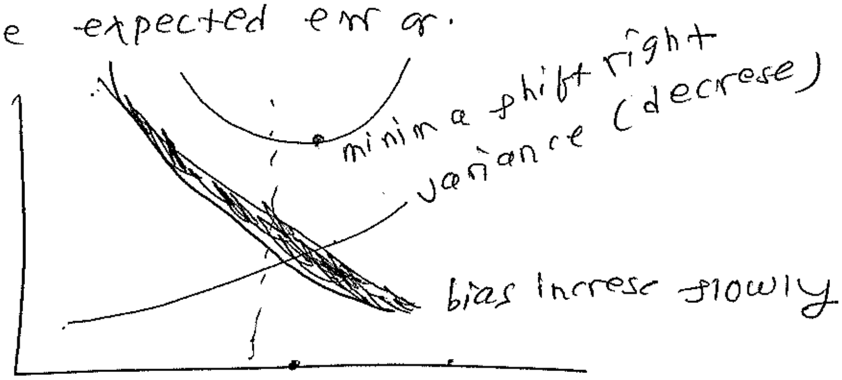
overfitting





when training data increase, variance is reduced and slightly more complicated model minimizes the expected error.



① Show that $X^T A Y$ is a valid kernel if A is sym. ps d.

$\Rightarrow$ $A = Q^T \Lambda Q$ where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$ and $\lambda_1, \lambda_2, \dots, \lambda_n \geq 0$

define $F = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_n})$ then $\Lambda = F^T F$

$\Rightarrow A = Q^T F^T F Q$

$\Rightarrow x^T A y = X^T Q^T F^T F Q Y$

$= (FQX)^T (FQY)$

$= \phi(X)^T \phi(Y)$

where $\phi(X) = FQX$

Aside:

$(AB)^T = B^T A^T$

$(ABC)^T = C^T B^T A^T$

$((AB)C)^T = C^T (AB)^T$

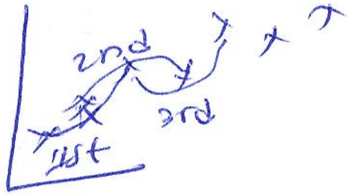
$= C^T B^T A^T$

⑥ cubic spilne smoothing

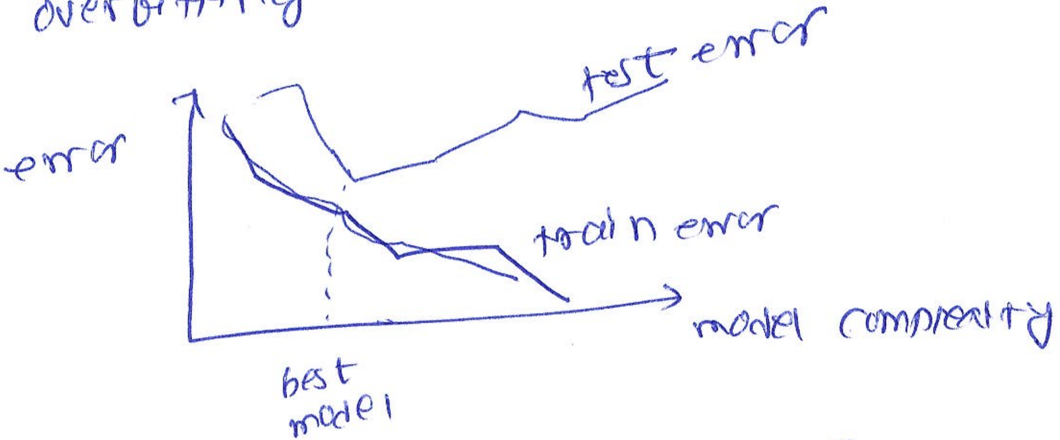
Take 3 points   x   x-xi   x+xi

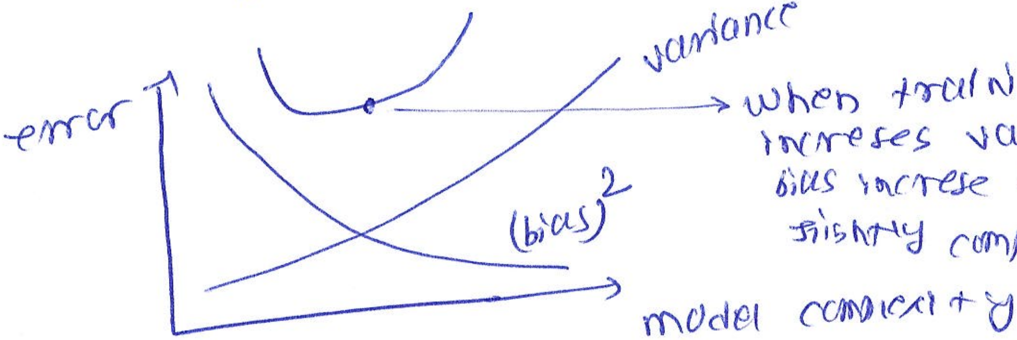
$$s_i(x) = a_i(x-x_i)^3 + b_i(x-x_i)^2 + c_i(x-x_i) + d_i$$

$$\forall x \in [x_i, x_{i+1}]$$



⑦ overfitting





when training data increses variance decrese bias increse and model becomes slightly complex to minize error.

① Likelihood function for Logistic Regression

$$p(t_n|w) = h_n^{t_n} \cdot (1-h_n)^{1-t_n}$$

$$p(t|w) = \prod_{n=1}^{N} h_n^{t_n} (1-h_n)^{1-t_n}$$

$$\text{cost } E = \frac{1}{N} \cdot (-\ln p) = -\frac{1}{N} \sum_{n=1}^{N} \left[t_n \ln h_n + (1-t_n) \ln(1-h_n) \right]$$

$$\text{grad} = \nabla_w J = \frac{1}{N} \cdot \sum_{n=1}^{N} (h_n - t_n) \cdot x_n$$

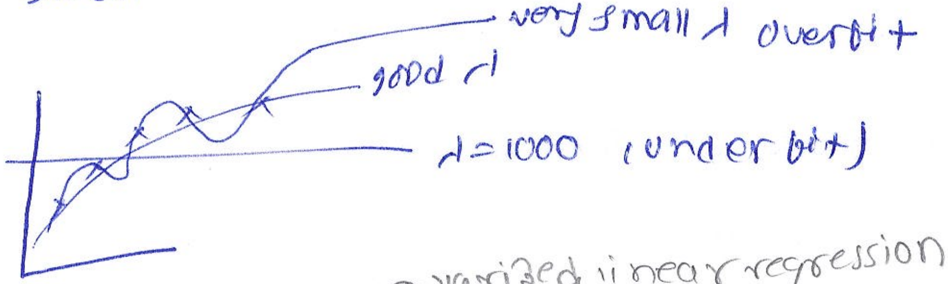
② perceptron criterion

$$\text{minimize} \quad E_p(w) = - \sum_{n \in \text{Misclassified}} t_n w^T x_n$$

```
initialize $w = 0$, $\bar{w} = 0$, $\tau = 1$
for $i = 1:N$
    $h_n = \mathrm{sgn}(w^T x_n)$
    if $h_n \neq t_n$ then
        $w = w + t_n x_n$
        $\tau = \tau + 1$
    $\bar{w} = \bar{w} + w$
    $\tau = \tau + 1$
return $\bar{w} / \tau$
Test: $\mathrm{sgn}(w^T x)$
```

① L2 → more penalty larger weights
does not drive weights to zero
weight vector is <u>NOT</u> sparse.

② λ ↑ ⇒ high bias, low variance, underfit, simpler model
drives weights closer to zero.
small change in training data ⇒ big change in estimate



cost function for L2-regularized linear regression

N - examples
M - features

$$X = \begin{bmatrix} x_1^1 & x_1^2 & x_1^3 & x_1^M \\ x_2^1 & x_2^2 & x_2^3 & x_2^M \\ x_N^1 & x_N^2 & x_N^3 & x_N^M \end{bmatrix}$$

③ $$J = \frac{1}{2N} \sum_{n=1}^{N} (h_n - t_n)^2 + \frac{\lambda}{2} \|w\|^2$$

$$J(w) = \frac{1}{2N} \sum_{n=1}^{N} \left(w_0 + \sum_{j=1}^{M} w_j x_n^j - t_n\right)^2 + \frac{\lambda}{2} \underset{\nearrow}{\sum_{j=1}^{M} w_j^2}$$

(we omit w_0 in regularization)

regularizing w_0 = shifting origin of target
⇒ same change in all target values ⇏ similar change in estimates

④

Batch GD

for num-iters:
$$w^{\tau+1} = w^{\tau} - \underset{\nearrow}{\eta} \nabla J$$
learning rate

stochastic GD

for num-iters:
for sample in data:
$$w^{\tau+1} = w^{\tau} - \eta \nabla J$$

momentum GD $+\gamma v^{\tau}$

$$v^{\tau+1} = \eta \nabla J(w^{\tau})$$
$$w^{\tau+1} = w^{\tau} - v^{\tau+1}$$
$\nearrow$
$\gamma \approx 0.9$

Nesterov Accelerated gradient

$$w^{\tau+1} = w^{\tau} - \eta \nabla J(w^{\tilde{\tau}} - \gamma w^{\tau}) - \gamma w^{\tau}$$

⑤ Gradient Desceent $\rightarrow v^{\tau+1} = \begin{cases} \eta \nabla J(w^{\tau}) & \text{vanilla} \\ \eta \nabla J(w^{\tau}) + \gamma v^{\tau} & \text{with momentum} \\ \eta \nabla J(w^{\tau} - \gamma v^{\tau}) + \gamma v^{\tau} & \text{Nesterov} \end{cases}$

η = learning rate
γ = momentum hyperparameter
$\tilde{w}$ = weight vector at time stamp τ

$$w^{\tau+1} = w^{\tau} - v^{\tau+1}$$

$\gamma \approx 0.9$

Accelerated gradient NAG

(5b) Types of GD based on data used:

Batch GD → use all data
Stochastic GD → use only one example and update $\overset{w}{\text{after}}$ each iteration
minibatch GD → use constant number of examples and update w after each

$$J = \frac{1}{2N} \sum_{n=1}^{N} (h_n - t_n)^2$$

$$\nabla J = \frac{1}{N} \sum_{n=1}^{N} (h_n - t_n) \cdot x_n = \frac{1}{N} \cdot \text{np.sum}((h-t) \cdot x)$$

$$w = w - \eta \, \text{grad}$$

(5c) GD vs Normal eqn

$$w = w - \eta \, \text{grad}$$

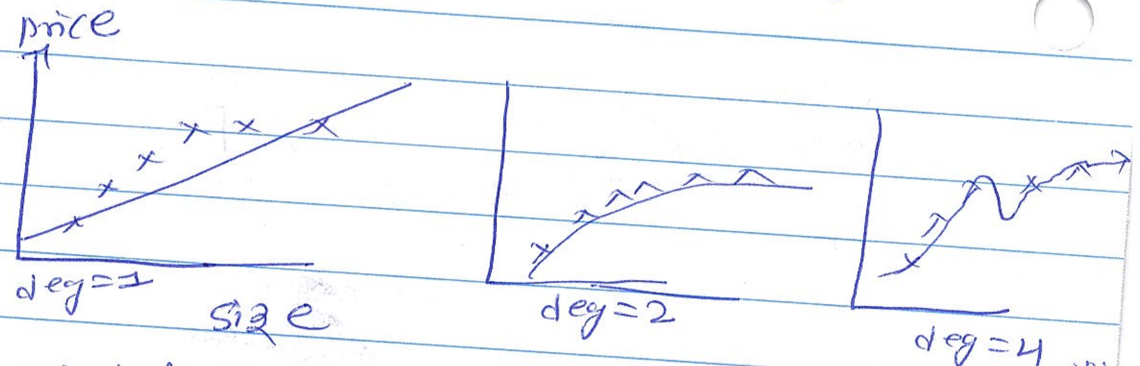
$$w = \underbrace{(X^T X)^{-1}}_{\text{pseudoinverse (pinv)}} X^T t$$

$$w = \underbrace{(X^T X + \lambda N I)^{-1} X^T}_{\text{for L2 regularized}} \; t$$

I is identity matrix of shape $X^T X$

I[0][0] = 0 for not to regularize bias term

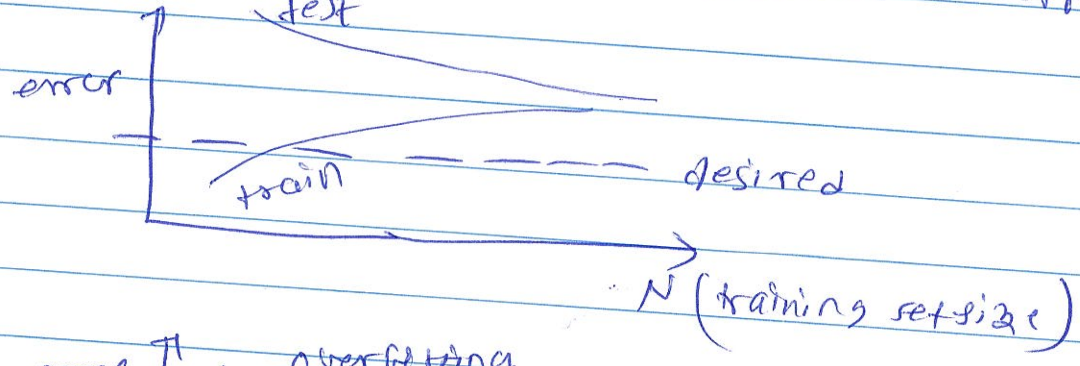
Bias-variance Trade off

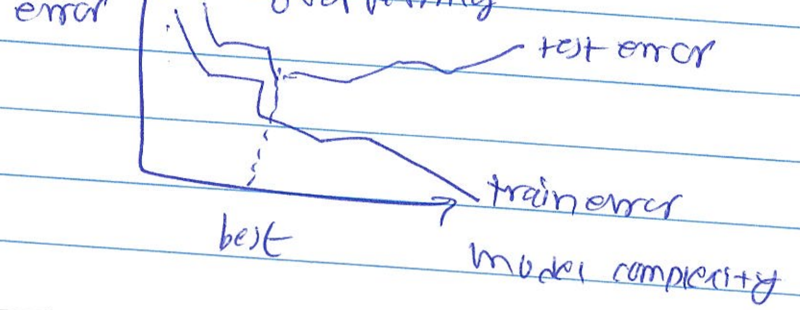


$w_0+w_1 x$	$w_0+w_1x+w_2x^2$	$w_0+w_1x+w_2x^2+w_3x^3+w_4x^4$
high bias	just right	high var.
(underfit)		overfit)
(simple model)		

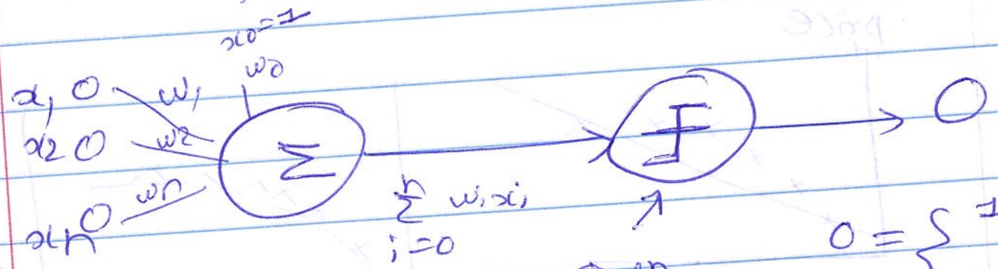
more training eg → fix high σ^2 (overfitting)
smaller feature set → fix high σ^2 (")
larger featureset → fix high bias (underfitting)

Learning curve for high bias (underfit)





perceptron



$$O = \begin{cases} 1 & \text{if } \sum_{i=0}^{n} w_i x_i > 0 \\ -1 & \text{otherwise} \end{cases}$$

sign fn

$$o(x_1, x_2, \ldots, x_n) = \begin{cases} 1 & \text{if } w_0 + w_1 x_1 + \cdots + w_n x_n > 0 \\ -1 & \text{otherwise} \end{cases}$$

output

$$= \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} > 0 \\ -1 & \text{otherwise} \end{cases}$$

① cost for multivariate linear regression

$$J = \frac{1}{2N} \sum_{n=1}^{N} (h_n - t_n)^2 \qquad h = w^T x \qquad h = X1 @ w.T$$

$$J = \frac{1}{2N} \sum_{n=1}^{N} \left(\sum_{j=0}^{M} w_j x_n^j - t_n \right)^2 \qquad h_n = \sum_{j=0}^{M} w_j x_n^j$$

$$h_1 = w_0 + w_1 x_1^{(1)} + w_2 x_1^{(2)} + w_M x_1^{(M)}$$

first row of sample

$$X1 = \begin{bmatrix} 1 & 10 & 100 \\ 1 & 20 & 200 \\ 1 & 30 & 300 \\ 1 & 50 & 500 \end{bmatrix}_{50 \times 4} \qquad t = \begin{bmatrix} 1K \\ 2K \\ 3K \\ 50K \end{bmatrix}_{50 \times 1}$$

$$w = [w_0 \; w_1 \; w_2 \; w_3]_{1 \times 4}$$

bias

2 features age, floorsize

$$h = (50,4) \cdot (4,1) = (50,1)$$

$$e = h - t = \begin{bmatrix} h_1 - t_1 \\ h_2 - t_2 \\ \vdots \\ h_{50} - t_{50} \end{bmatrix}_{50 \times 1}$$

SSE = (h-t) ** 2

$$MSE = \frac{(h-t) ** 2}{N}$$

MSE = np.mean(SSE)

RMSE = np.sqrt(MSE)

J = np.sum((h-t)**2)/2/N

J = 0.5 * np.mean(np.sum(h-t)**2)

$$J = \frac{1}{2} \times MSE \quad (\text{single float})$$

$$\boxed{J = \frac{0.5}{\text{len}(t)} (h-t)^T (h-t)}$$

$$(h-t)^T (h-t) = [h_1 - t_1 \;\; h_2 - t_2 \; \cdots]_{1 \times 50} \begin{bmatrix} h_1 - t_1 \\ h_2 - t_2 \\ h_n - t_n \end{bmatrix}_{50 \times 1} = \left[\sum_n (h_n - t_n)^2 \right]_{1 \times 1}$$

② gradient of J

$$\nabla_w J = \nabla_w \frac{1}{2N} \sum_n (h-t)^2 = \left[\frac{\partial J}{\partial w_0} \;\; \frac{\partial J}{\partial w_1} \;\; \frac{\partial J}{\partial w_2} \;\; \frac{\partial J}{\partial w_M} \right]_{4 \times 1}$$

$$\frac{\partial J}{\partial w_j} \frac{1}{2N} \sum_{n=1}^{N} (Xw_n - t)^2 = \frac{1}{2N} \sum (x_n w_0 + x_n w_1 + \cdots + x_n w_M$$

$$= \frac{1}{2N} \sum_{n=1}^{N} 2 (Xw - t) \cdot X$$

[summation vanishes] bc we use matrix

$$\nabla_w J = \frac{1}{N} (h-t) \cdot X$$

= (h-t).T @ X1 / N

= (1,50).(50,4)

grad = ∇w J = (1,4)

$$\boxed{\text{grad\_ols} = (h-t).T \; @ \; X1}$$

④ X = design matrix $[\ ]_{N \times M}$ N = samples
m = features

$X1$ = biased design matrix $\left[\begin{matrix} 1 \\ 1 \end{matrix}\ \right]_{N, M+1}$

$t = [\ ]_{N \times 1}$ column vector

$w = [w_0\ w_1\ w_2\ w_M]_{1, M+1}$ row vecter (2d array)

w = np.array(w).reshape (1, x1.shape[1]) = (1, M+1)

linear Regression

$h = X1 @ w.T = (N, ~~M+1~~) (~~M+1~~, 1) = (N, 1)$ same as t

$e = h - t = N, 1 = 50, 1$

$$SSE = \sum_{n=1}^{N} (h-t)^2$$

$$MSE = \frac{1}{N} \sum_{n=1}^{N} (h-t)^2$$

$$J = \frac{1}{2N} \sum_{n=1}^{N} (h-t)^2$$ = np.sum((h-t)**2) /2/float(N)

$$RMSE = E\_{RMS} = \sqrt{MSE}$$

Normal eqn: $w = \underline{(X^T X)^{-1} @ X^T} @ t$

moore penrose pseodo inverse of X

np.linalg.pinv(X)

multivarite Linear

bias	X		t
	floorsize	bed	2K
1	100	2	3K
1	200	3	5K
1	500	5	

t ⇓

m features and one bias

$$X = \begin{bmatrix} x_0^{(1)}=1 & x_1^{(1)} & \cdots & x_m^{(1)} \\ x_0^{(2)} & x_1^{(2)} & \cdots & x_m^{(2)} \\ \vdots & \vdots & & \\ x_0^{N} & x_1^{(N)} & & x_m^{(N)} \end{bmatrix}_{N \times m+1} \quad \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ \vdots \\ t_n \end{bmatrix}_{N \times 1}$$

There are m features and one bias total m+1

There are N samples for each feature

$$J = \frac{1}{2N}|xw - t|^2 = \frac{1}{2N}(xw-t)^T(xw-t)$$

$$\nabla_w J = \frac{1}{\cancel{2}} \frac{d}{dx}(xw-t) \cdot \cancel{2}(xw-t)$$

$$\frac{d}{dx}(x^T x) = 2x$$

x is a matrix

$$0 = x^T(xw-t)$$

$$\boxed{\frac{d}{dx}(Ax+b)^T(Ax+b) = 2A^T(Ax+b)}$$

$$x^T x w = x^T t$$

weights $\boxed{w = (x^T x)^{-1}(x^T t)}$

Derivatives of matrix products

L1 vs L2 norm

more used

L2 → more penalty on large weights, but doesn't drive small weights to zero.

L1 → Less penalty ~~for~~ large wt, but leads many weights ~~towards~~ to (or very very close) to zero. leading to weight vector to be sparse.

Bias-variance Tradeoff

ⓐ Ridge: $$\sum_{i=1}^{N}\left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij}\right)^2 + \lambda \sum_{j=1}^{p} \beta_j^2 = RSS + \lambda \sum_{j=1}^{p} \beta_j^2$$

shrinkage parameter which has to be tuned via validation set → regularization strength

Ridge

- $\lambda\uparrow$ → var↓ bias↑
- meaning: small change in training data big change in parameter estimates
- effect will increase with no of parameters

ⓑ Lasso β_j^2 is $|\beta_j|$

Ridge shrinks wj but do not make 0 but lasso makes them zero, makes less no of wts.

Logistic Regression → classification (binary)

linear regression $h = w^T x$

logistic regression $h = \sigma(x) = \frac{1}{1+e^{-w^T x}} = h(x) = y(x)$

likelihood function $p(t|w) = \prod_n h_n^{t_n} (1-h_n)^{1-t_n}$

-ve log likelihood, E or J $= -\ln p(t|w)$

$$= -\sum_{n=1}^{N} [t_n \ln h_n + (1-t_n) \ln(1-h_n)]$$

∴ cost or Error or Loss function

$$\boxed{E_D = -\sum_n [t_n \ln h_n + (1-t_n) \ln(1-h_n)]}$$

where $t_n \in \{0,1\}$ NOT $\{-1,1\}$

add regularizer,

$$E_w = \frac{\lambda}{2} w^T w$$

Then L2-regularized logistic regression cost function

cost

$$\boxed{E = -\frac{1}{N}\sum_n [t_n \ln h_n + (1-t_n) \ln(1-h_n)] + \frac{\lambda}{2} w^T w}$$

λ is regularizer

$$h(x) = \sigma(x) = \frac{1}{1+e^{-w^T x}}$$

$$h_n = h(x_n) = \sigma(x_n) = \frac{1}{1+e^{-w^T x_n}}$$ is sigmoid f^n

Softmax Regression

is classification (multiclass)

Training set: (x_1, t_1), (x_2, t_2), ... (x_n, t_n)

$$X = [1, x_1, x_2, \dots, x_n]$$

$$t_1, t_2, \dots, t_n \in \{1, 2, \dots, K\}$$

$$w_K = [w_{K0}, w_{K1}, w_{K2}, \dots]^T$$

one weight vector per class,

$$p(c_K|x) = \frac{e^{w_K^T x}}{\sum_j e^{w_j^T x}}$$

prob that nth example x_n has class t_n

$$p(t_n|x_n) = \frac{e^{w_{t_n}^T x_n}}{\sum_j e^{w_{t_j}^T x_n}}$$

likelihood is the joint probability of all classes,

$$L(w) = \prod_{n=1}^{N} p(t_n|x_n)$$

MLE

sample: $X_1, X_2, \dots, X_N$

prob. of x is $f(x_n, w)$

joint prob of $X_1, X_2, \dots X_N$ is called likelihood $L(w)$

$$L(w) = p(X_1 = x_1, \dots X_N = x_N)$$

$$= f(x_1, w) \dots\dots$$

pdf of x_1

$$\boxed{L(w) = \prod_{n=1}^{N} f(x_n, w)}$$

cost is the -ve log likelihood,

$$E_D(w) = -\frac{1}{N} \ln L(w)$$

$$= -\frac{1}{N} \ln \prod_{n=1}^{N} p(t_n|x_n)$$

$$= -\frac{1}{N} \sum_n \ln p(t_n|x_n)$$

$$= -\frac{1}{N} \sum_n \ln \frac{e^{w_{t_n}^T x_n}}{Z(x_n)}$$

w_{t_n} is weight vector for nth example

w_K is weight vector for all the examples belonging to class K.

$$\boxed{E_D(w) = -\frac{1}{N} \sum_{n=1}^{N} \sum_{K=1}^{K} \delta_K(t_n) \ln\left(\frac{e^{w_K^T x_n}}{Z(x_n)}\right)}$$

MAP solution for LR

$$p(t|w) = \prod_{n=1}^{N} h_n^{t_n} (1-h_n)^{1-t_n}$$

$$p(w) = \left(\frac{\alpha}{2\pi}\right)^{\frac{m+1}{2}} e^{-\frac{\alpha}{2} w^T w} \quad \propto e^{-\frac{\alpha}{2} w^T w}$$

$$p(w|t) = \frac{p(t|w) \; p(w)}{p(t)} \quad \propto \; p(t|w) \; p(w)$$

now, $\vec{w}_{MAP} = \underset{w}{\arg\max} \; p(w|t)$

$$= \underset{w}{\arg\max} \; p(t|w) \; p(w)$$

$$= \underset{w}{\arg\min} \; -\ln p(t|w) \; - \ln p(w)$$

$$= \underset{w}{\arg\min} \; -\ln \prod_{n} h_n^{t_n} (1-h_n)^{1-t_n} \; - \ln e^{-\frac{\alpha}{2} w^T w}$$

$$= \underset{w}{\arg\min} \; - \sum_{n} \ln \left(h_n^{t_n} (1-h_n)^{1-t_n} \right) + \frac{\alpha}{2} w^T w$$

$$= \underset{w}{\arg\min} \; \underbrace{- \sum_{n} \left[t_n \ln h_n + (1-t_n) \ln (1-h_n) \right]} + \underbrace{\frac{\alpha}{2} w^T w}$$

$$\boxed{\vec{w}_{MAP} = \underset{w}{\arg\min} \; E_D(w) + E_W(w)}$$

Ans

softmax Regression

$$P(C_k|x) = \frac{e^{w_k^T x}}{\sum_j e^{w_j^T x}}$$

$$p(t_n|x_n) = \frac{e^{w_{t_n}^T x_n}}{\sum_{j=1}^{N} e^{w_{t_n}^T x_n}}$$

note

$$h(w) = \frac{1}{\sum_{k=1}^{K} e^{w_k^T x}} \begin{bmatrix} e^{w_1^T x} \\ e^{w_2^T x} \\ \vdots \\ e^{w_k^T x} \end{bmatrix}$$

$$E_D(w) = \frac{1}{N} \cdot -\ln \prod_n p(t_n|x_n)$$

$$= -\frac{1}{N} \ln \prod_n \frac{e^{w_{t_n}^T x_n}}{\sum_j e^{w_{t_n}^T x_n}}$$

$$= -\frac{1}{N} \cdot \sum_n \ln \frac{e^{w_{t_n}^T x_n}}{\sum_j e^{w_{t_n}^T x_n}}$$

$$\boxed{E_D(w) = -\frac{1}{N} \sum_n \sum_k \delta_k(t_n) \ln\left(\frac{e^{w_k^T x_n}}{\sum_k e^{w_k^T x_n}}\right)}$$

Regularizer

$$+ \frac{\alpha}{2} \sum_{k=1}^{K} w_k^T w_k$$

$$\frac{\partial E_w}{\partial w_k} = \alpha w_k$$

(no summation

Let, $E_n = \sum_k \delta_k(t_n) w_k^T x_n - \sum_k \delta_k(t_n) \ln\left(\sum_k e^{w_k^T x_n}\right)$

$$\frac{\partial E_n}{\partial w_j} = \delta_j(t_n) x_n - \delta_j(t_n) \cdot \frac{1}{\sum_j e^{w_j^T x_n}} \cdot e^{w_j^T x_n} \cdot \delta_j(t_n) x_n$$

$$\frac{\partial E_n}{\partial w_k} = \delta_k(t_n) x_n - \frac{e^{w_k^T x_n}}{\sum_k e^{w_k^T x_n}} \cdot x_n$$

prevent overflow

$$p(c_k|x_n) = \frac{e^{w_k^T x_n}}{\sum_k e^{w_k^T x_n}}$$

$$c = \max_{1 \le k \le K} w_k^T x_n$$

$$p(c_k|x_n) = \frac{e^{(w_k^T x_n - c)}}{\sum_k \exp(w_k^T x_n - c)}$$

$$\frac{\partial E_n}{\partial w_k} = \left[\delta_k(t_n) - p(c_k|x_n)\right] x_n$$

$$\Rightarrow \boxed{\frac{\partial E_D(w)}{\partial w_k} = -\frac{1}{N} \sum_n \left[\delta_k(t_n) - p(c_k|x_n)\right] x}$$

there are k-such equations for each classes.

① MLE VS MAP

MLE = maximize P(data|params) by searching over parameters

MAP = maximize P(param|data) by searching over params and accounting for prior over params

MLE → finds ω by maximizing likelihood fn $p(x|w)$.

MAP → maximizes the posterior prob $p(w|x)$

② classification maps inputs to discrete outputs
Regression " continuous ".

③ PCA & feature selection

similarity : reduce the dimension of data

difference : feature selection finds a subset of features,
PCA produces a smaller new set

perceptron

perceptron criterion: $w^T x_n \geq 0$ for $t_n = +1$

$w^T x_n < 0$ for $t_n = -1$

want: $t_n w^T x_n \geq 0$ for all patterns

minimize: $-w^T x_n t_n$ for all misclassified patterns M

$$\boxed{E_p(w) = -\sum_{n \in M} w^T x_n t_n}$$

perceptron

mistakes (misclassified)

Binary perceptron

```
initialize $\vec{w} = 0$
for $n = 1, \dots N$
    $h_n = \mathrm{sgn}(w^T x_n)$
    if $h_n \neq t_n$ then
        $w = w + t_n x_n$
```

repeat until convergence or given number of epochs

① why kernels are symmetric?

Inner products are symmetric by definitions, so, therefore if the kernel function represent an inner product in some Hilbert space, then the kernel function must be symmetric as well.

$$K(x,y) = \langle \phi(x), \phi(y) \rangle$$

$$= \langle \phi(y), \phi(x) \rangle \quad (\because \text{ property of inner product})$$

$$= K(y,x)$$

② Show $K(x,z) = x^T A^T A z$ is a valid kernel.

Let $\phi(x) = Ax$,

then,

$$\langle \phi(x), \phi(z) \rangle = \phi(x)^T \phi(z)$$

$$= (Ax)^T (Az)$$

$$= x^T A^T A z$$

$$= K(x,z)$$

$\therefore$ $K(x,z)$ is an inner product in some Hilbert space.

③ AA^T is PSD matrix (inderjit)

proof Let, $A \in \mathbb{R}^{m\times n}$

$\lambda =$ eigen value of AA^T

$q =$ eigenvector of λ

$(AA^T)q = \lambda q$ (premultiply by q^T)

$\because AA^T = A^TA$

$\Rightarrow q^T AA^T q = q^T \lambda q$

$\Rightarrow \lambda = \dfrac{q^T AA^T q}{q^T q} = \dfrac{q^T A^TA\, q}{q^Tq}$

$= (Aq)^T Aq \,/\, q^Tq$

$= \dfrac{z^T z}{q^T q}$ where $z = A^Tq$

here $z^T z \geq 0$

$q^T q \geq 0$

$\therefore \lambda \geq 0$ so AA^T is PSD.

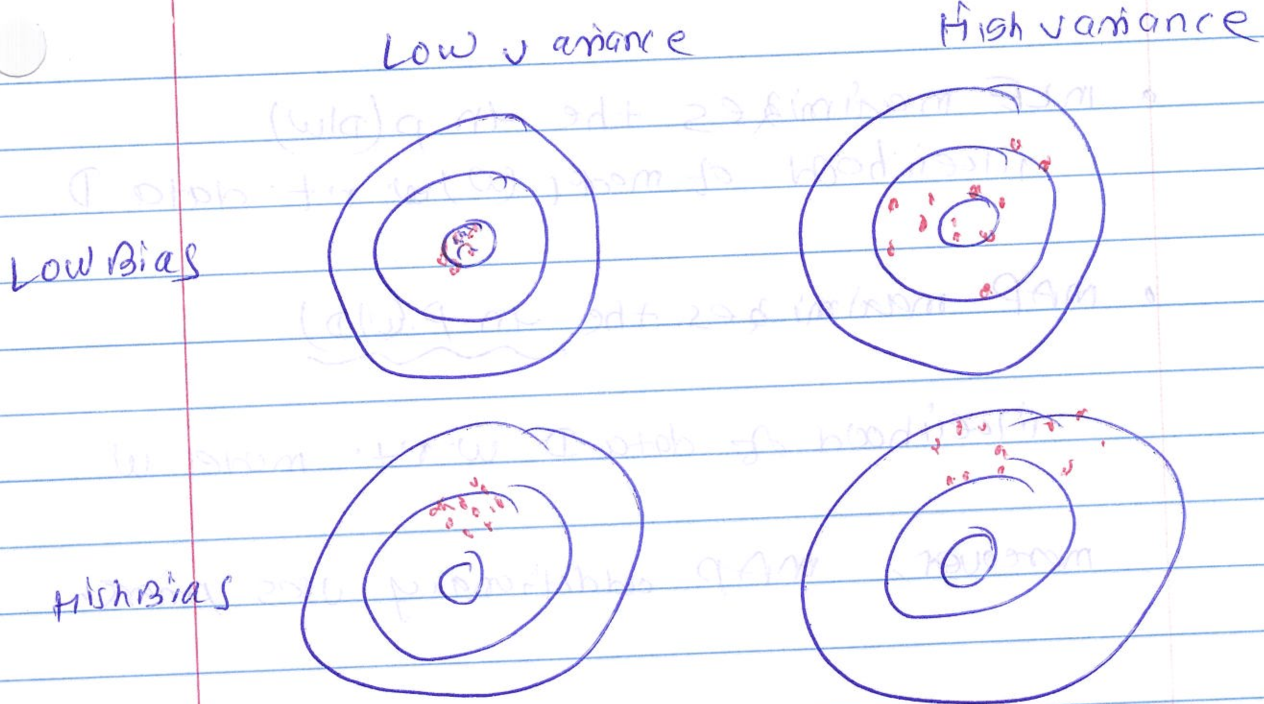
d estimation maximum a priori

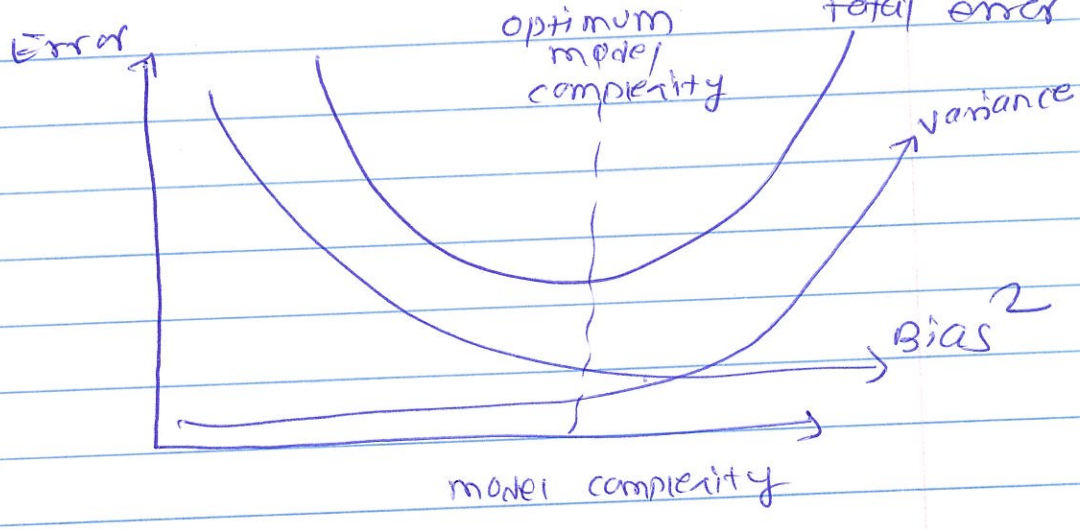
MLE vs MAP

- MLE maximizes the $\ln p(D|w)$ likelihood of model (w) w.r.t. data D
- MAP maximizes the $\underbrace{\ln P.(w|D)}$

  likelihood of data D w.r.t. model w

moreover, MAP additionally uses priors.


Low variance
High variance
Low Bias
High Bias


Error
optimum model complexity
total error
variance
Bias^2
model complexity

* A kernel will be valid if there exist a space such that

$$k(x_1, x_2) = \phi(x_1)^T \phi(x_2) = \sum_{n=1}^{N} \phi_n(x_1)\phi_n(x_2)$$

* consider a quadratic kernel with $D = 2$,

$= \vec{x}\cdot\vec{z}$

$$k(x, z) = (x^T z)^2 = (x_1 z_1 + x_2 z_2)^2$$

$$= (x_1^2 z_1^2 + 2 x_1 z_1 x_2 z_2 + x_2^2 z_2^2)$$

$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

$x^T = (x_1\ x_2$

$x^T z = (x_1\ x_2)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

$1{,}2$

$2{,}1$

This can be expressed as an inner product of space where,

$$\phi(x) = x_1^2 + \sqrt{2}\, x_1 x_2 + x_2^2$$

$$\phi(z) = z_1^2 + \sqrt{2}\, z_1 z_2 + z_2^2$$

this, gives,

$$\phi(x)\,\phi(z) = x_1^2 z_1^2 + 2 x_1 x_2 z_1 z_2 + x_2^2 z_2^2$$

$$k(x, z) = \phi(x)^T \phi(z)$$

* A necessary and sufficient condition for a kernel function to be "valid" is that the Gram matrix be positive and semidefinite for all choices of $\{\vec{x}_m\}$.

A Gram matrix of $\vec{x}$ is $x^T x$.

The linear vector $\vec{x}$ is projected into a quadratic surface.

If all the points in this surface are non-zero then our kernel is valid.

* Fisher's discriminant cost

$$J(W) = Tr\{(W S_W W^T)^{-1}\} \cdot (W S_B W^T)$$

— × — × — × — × —

Tue Oct 31

least square perceptron

$$E(w) = \frac{1}{2} \sum_{n=1}^{N} (h_n - t_n)^2 \swarrow$$ DOES NOT WORK

$$0 \quad \frac{1}{2} \dots \overset{N-1 \text{ mistake}}{\frac{N-2}{2}} \quad \overset{N \text{ mistakes}}{\frac{N}{2}}$$ N+1 values

min $E = 0$

max $E = \frac{N}{2}$

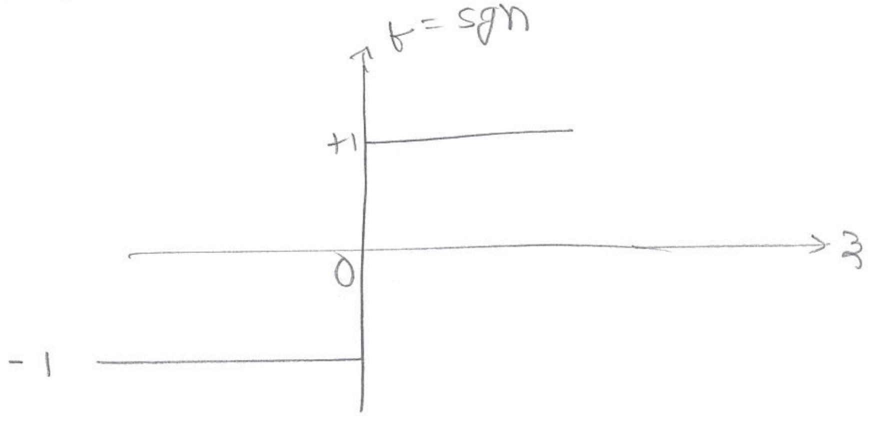
we can not compute gradient, since function is discrete and not continuous.

cost = No. of misclassified patterns

= 0 for no mistake

$\frac{N}{2}$ for N mistakes

$$\text{cost} = [0 \ \frac{1}{2} \ 2 \dots \frac{N-2}{2} \ \frac{N}{2}]$$ discrete set.



Distance metrics

① Euclidean $d(x,y) = \|x-y\|_2 = \sqrt{(x-y)^T (x-y)}$

② Hamming $d(x,y) =$ # of different values in fixed length strings

③ Mahalanobis distance $d(x,y) = \sqrt{(x-y)^T \underset{\nearrow}{S^{-1}} (x-y)}$

S is sample cov. matrix

$S = I \rightarrow$ Euclidean

$S = \mathrm{diag}(\sigma_1^{-2}, \sigma_2^{-2}, \ldots) \rightarrow$ normalized Euclidean dist.

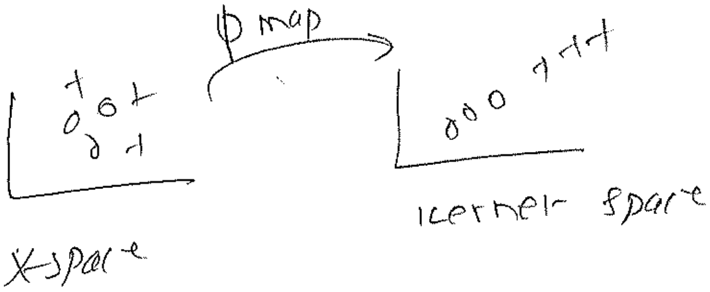
③ cosine similarity

$$d(x,y) = 1 - \cos(x,y) = 1 - \frac{x^T y}{\|x\| \, \|y\|}$$

④ Levenstein distance (edit distance)

min # of basis operations (del, insert, juxtapose) betn two strings

$x =$ 'athens' $y =$ 'hints' $d(x,y) = 4$



① SVM with slack (soft margin svm)

$$\min J(w,b,\xi) = \frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n$$

with constraint

$$t_n (w^T \phi(x_n) + b) \geq 1 - \xi_n$$

$$\xi_n \geq 0 \quad \text{and} \quad \sum_{i=1}^{N} \xi_i \leq Z \qquad \forall n \in \{1, \cdot$$

i.e. $1 - \xi_n - t_n w^T \phi(x_n) - b t_n \leq 0$ —— ① (convex fn)

$-\xi_n \leq 0$ —— ② (convex fn constraint)

$\forall n = 1, 2, \ldots, N$

primal Lagrangian

$$L_p = L(w,b,\xi,\alpha,r) = \frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n + \sum_{n=1}^{N}\alpha_n (1 - \xi_n - t_n w^T \phi_n - b t_n) - \sum_{n=1}^{N} r_n \xi_n$$

dual Lagrangian

$$L_D(\alpha, r) = \inf_{w,b,\xi} L_p(w,b,\xi,\alpha,r)$$

Now,

$$0 = \frac{\partial L_p(w,b,\xi,\alpha,r)}{\partial w} = w - \sum_n \alpha_n t_n \phi(x_n) \Rightarrow \boxed{w = \sum_n \alpha_n t_n \phi_1}$$

$$0 = \frac{\partial L_p}{\partial b} = -\sum_n \alpha_n t_n \Rightarrow \boxed{\sum_n \alpha_n t_n = 0}$$

NO summation!

$$\& \ 0 = \frac{\partial L_p}{\partial \xi_n} = C - \alpha_n - r_n \Rightarrow \boxed{C = \alpha_n + r_n} \quad \forall n \in \{1, 2, \ldots, N\}$$

Then,

$$L_D(\alpha, r) = \frac{1}{2}\|w\|^2 + C \sum_n \xi_n + \sum_n \alpha_n (1 - \xi_n - t_n w^T \phi_n - t_n b) - \sum_n r_n \xi_n$$

$$L_D = \sum_n \alpha_n - \frac{1}{2} \sum_{m,n=1}^{N} \alpha_m \alpha_n t_m t_n \, k(x_m, x_n)$$

Then the optimization problem in dual space is,

$$\underset{\alpha}{\text{maximize}} \quad L_D(\alpha) = \sum_n \alpha_n - \frac{1}{2} \sum_{m,n} \alpha_m \alpha_n t_m t_n \, k(x_m, x_n)$$

with constraints $\quad 0 \leq \alpha_n \leq C \quad \forall n = 1, 2, \cdots, N$

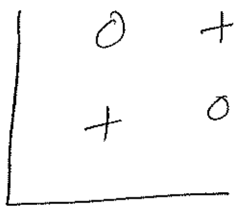
$$\sum_{n=1}^{N} \alpha_n t_n = 0$$

note: $\quad C \sum_n \xi_n - \sum_n \alpha_n \xi_n - \sum_n r_n \xi_n = 0$

since, $\quad C \sum_n \xi_n = \sum_n \alpha_n \xi_n + \sum_n r_n \xi_n$

$$= \sum_n (\alpha_n + r_n) \xi_n$$

①



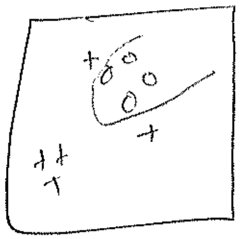
- SVM (quad kernel) can achieve zero training error
- Logis Regr, 3-NN cannot

② If examples are iid,
increase training examples → may increase train error
↳ but decrease test error

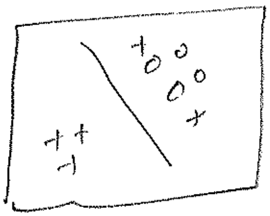
③ SVM effect of 'C'

$$\min J(w,b) = \frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}\xi_n$$

$$\text{s.t.} \quad w^T\phi(x_n)+b \geq 1-\xi_n \quad \forall\, n \in \{0,N\}$$

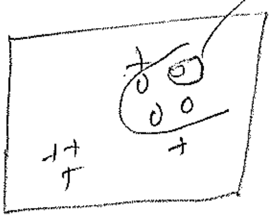


C= 10,000

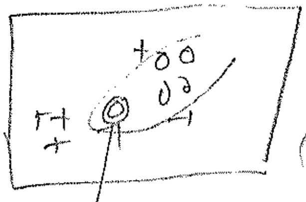


C=0.00005

will not change decision bond.



C= 1



adding this change dec. boun. drastically

between C>>1 and C≈0 choose C≈0 because it maximizes the margin between dominant cloud of points and we can not depend on only few data points which can be noise.

④ Bias variance Tradeoff

	Bias	var
linear regr	high	low
d=2 poly	low	low
d=10 poly	low	high

python

① $X = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$ append ones column to data X.

X = np.array([[1,2,3], [4,5,6]]) = np.arange(1,7).reshape(2,3)

ones = np.ones(X.shape[0]).reshape(-1,1)

X1 = np. $\underset{\text{concatenate}}{\text{append}}$ (ones, X, axis=1).astype(np.int)

X1 = np.c_[np.ones(X.shape[0]).reshape(-1,1), X]

X1 = np.column_stack(("))

X1 = np.c_[np.ones(X.shape[0]) $\underset{\text{.reshape(-1,1)}}{\text{[:, np.newaxis]}}$, X]

= np.c_[np.atleast_2d(np.ones(x.shape[0]).T, X]

= np.c_[np.expand_dims(np.ones(X.shape[0], axis=1), X]

⑤ Given $\phi(x) = [1, x_1, x_2, x_1x_2]$

find the kernel $k(x,x')$.

$\Rightarrow k(x,x') = 1 + x_1x_1' + x_2x_2' + x_1x_2x_1'x_2'$

⑥ L1 vs L2 loss

ⓐ False: L2 is more robust to outlier than L1

gradient of L2 loss can grow without bounds but gradient of L1 loss is bounded, hence influence of outlier is limited.

(Note: L2 gives more values to misclassification than L1)

ⓑ L1 gives sparse solution & used in feature selection.

ⓒ Logistic loss is better than L2 loss in classification task.

⑦ SVM small C,

For linearly separable data, small C can affect training accuracy.

A small C can allow large slacks, thus, the resulting classifier will have smaller w^2 and can have non-zero training error.

① pg① solve the SVM problem without slack using lagrange multiplier method

The optimization problem is

$$\text{minimize} \quad J(w,b) = \frac{1}{2}\|w\|^2$$

$$\text{s.t.} \quad t_n(w^T\phi(x_n)+b) \geq 1 \quad \forall n \in \{1,\dots,N\}$$

$$1 \leq t_n(w^T\phi(x_n)+b)$$

$$1 \leq t_n w^T\phi(x_n) + t_n b$$

$$\underline{1 - t_n w^T\phi(x_n) - t_n b \leq 0} \quad (\text{convex fn constraint})$$

compare $f_i(x) \leq 0$ ~~then~~ $i = 1, \dots, m$

primal Lagrangian,

$$L_p(w,b,\alpha) = \frac{1}{2}\|w\|^2 + \sum_{n=1}^{N}\alpha_n(1 - t_n w^T\phi_n - t_n b)$$

where $\alpha_n \geq 0$ are Lagrange multipliers.

Dual Lagrangian,

$$L_D(\alpha) = \inf_{w,b} L_p(w,b,\alpha)$$

first find the infimum of L_p w.r.t w, b:

$$\frac{\partial}{\partial w}L_p = 0 = w + \sum_n \alpha_n(-)t_n\phi_n \Rightarrow \boxed{w = \sum_n \alpha_n t_n \phi_n} \quad \text{— ①}$$

$$\frac{\partial}{\partial b}L_p = 0 = \sum_n (-)\alpha_n t_n \Rightarrow \boxed{\sum_n \alpha_n t_n = 0} \quad \text{— ②}$$

LOOK LHS

pg ②

Then Dual Lagrangian is,

$$L_D(\alpha) = \frac{1}{2} \sum_{n,m} \alpha_m \alpha_n t_m t_n \phi_m \phi_n + \sum_n \alpha_n - \sum_n \alpha_n t_n \phi_n \cdot \sum_m \alpha_m t_m \phi_m$$

$$- \cancel{\sum_n \alpha_n t_n \overset{=0}{b}}$$

$$\boxed{L_D(\alpha) = \sum_n \alpha_n - \frac{1}{2} \sum_{n,m} \alpha_m \alpha_n t_m t_n K(x_n, x_m)}$$

$$K(x, y) = \vec{\phi}(x) \cdot \vec{\phi}(y) = \phi^T(x)\, \phi(y)$$

Then the optimization problem in Dual space is,

maximize $L_D(\alpha) = \sum_n \alpha_n - \frac{1}{2} \sum_n \sum_m \alpha_m \alpha_n t_m t_n K(x_m, x_n)$

s.t. $\alpha_n \geq 0 \quad \forall n \in \{1, \dots N\}$

$\sum_{n=1}^{N} \alpha_n t_n = 0$

LOOK here

Now, KKT conditions are,

note: we can write $y(x_n) = w^T \phi(x_n) + b$

① primal constraints $1 - t_n(w^T \phi(x_n) + b) \leq 0$ (convex constraint equation) $t_n(w^T \phi(x_n) + b) \geq 1$

② dual constraint $\alpha_n \geq 0 \quad \forall n \in \{1, \dots N\}$

③ complementary slackness $\alpha_n \{1 - t_n w^T \phi(x_n) - t_n b\} = 0$

for any data point, either $\alpha_n = 0$

$1 - t_n w^T \phi(x_n) - t_n b = 0$

these x_n are called support vectors

pg3 here $n = 1, 2, \ldots, N$

Suppose, out of N samples, there are m support vectors then N-m samples will have Lagrange parameter $\alpha = 0$ and m examples will have non-zero Lagrange parameters.

$$1 - t_m w^T \phi(x_m) - t_m b = 0$$

or, $$1 - t_m \phi(x_m) \sum_n \alpha_n t_n \phi(x_n) - t_m b = 0$$

or, $$t_m b = 1 - t_m \phi(x_m) \sum_n \alpha_n t_n \phi(x_n)$$

$$= 1 - t_m \sum_n \alpha_n t_n \phi(x_n) \phi(x_m)$$

$$b t_m = 1 - t_m \sum_n \alpha_n t_n \phi(x_n) \phi(x_m)$$

$$b = \frac{1}{t_m} - \sum_n \alpha_n t_n \phi(x_n) \phi(x_m) = t_m - \sum_n \alpha_n t_n \phi(x_n) \cdot \phi(x_m)$$

$$\boxed{b = t_m - w \cdot \phi(x_m)}$$

$$\boxed{b = t_m - \sum_n \alpha_n t_n k(x_n, x_m)} \quad \because \frac{1}{t_m} = t_m$$

$$1 = \frac{1}{1} \text{ and } -1 = \frac{1}{-1}$$

this is true for all the m examples which have non zero Lagrange parameter α.

For numerical stability we choose value of b as the mean of all b-values, then,

$$\boxed{b = \frac{1}{|S|} \sum_{m \in S} \left[t_m - \sum_{n \in S} \alpha_n t_n k(x_n, x_m) \right]}$$

where S is the subset of all the examples where Lagrange parameter α is non zero.

$$S \subseteq D$$

$$S = \{ n \mid 1 - t_n w^T \phi(x_n) - t_n b = 0 \}$$

then, Linear discriminant function is,

$$\boxed{y(x) = w^T \phi(x) + b = \sum_{m \in S} \alpha_m t_m k(x, x_m) + \frac{1}{|S|} \sum_{m \in S} [t_m - \sum_{n \in S} \alpha_n t_n k(x_n, x_m)}$$

KNN = memory-based (no model to fit)

① find k nearest examples $x_1, x_2 \ldots x_K$ from test x.

$$y(x) = \underset{t \in T}{\arg\max} \sum_{i=1}^{K} {}^{\wedge} \delta_t(t_i)$$

distance-weighted k-NN

w_i gives distance-weighted

$$w_i = \frac{1}{\|x - x_i\|^2}$$

KNN usage

① satellite image classification

② digit recognition

② mahalanobis dis

$$d(x,y) = \sqrt{(x-y)^T \underset{\rightarrow}{S^{-1}} (x-y)}$$

sample covariance matrix

if $S = I$ = Euclidean distance

$S = \text{diag}(\sigma_1^{-2}, \sigma_2^{-2}, \ldots, \sigma_K^{-2})$

normalized Euclidean

③ cosine similary

$$d(x,y) = 1 - \frac{x^T y}{\|x\| \, \|y\|}$$

④ kerner based distance weighted NN

→ binary classification $T \in \{+1, -1\}$

→ $y(x) = \text{sign}\left(\sum_{i=1}^{N} K(x, x_i) \cdot t_i\right)$

wrapper method

① Greedy Forward selection

```
F = all features
S = subset of features
start S = { } empty
    for each feature f in F - S
        find best performing feature $f^*$ and add to S.
    Repeat until performance does not inrrese
        or performance good enough
```

② Recursive Backward Elimination

```
F : {1, 2, . k} is set of features
S = [ ] ranked set of features
    Repeat until F - S is empty
        train $\vec{w}$ using linear SVM and F - S
        find feature f with minimum $|w_f|$
        append f to S
    Return S
```

④ distance-weighted KNN for regression

1. find k nearest points $x_1, x_2 \ldots x_k$

2. $y(x) = \dfrac{\sum_{i=1}^{k} w_i t_i}{\sum_{i} w_i}$ where $w_i = \dfrac{1}{\|x - x_i\|^2}$

K=N → Shepard's method

$y(x) = \sum_{i=1}^{N} K(x,x_i)\, t_i \;/\; \sum_{i=1}^{N} K(x,x_i)$ (kernel-based dist weighted)

⑤ Regression with KNN

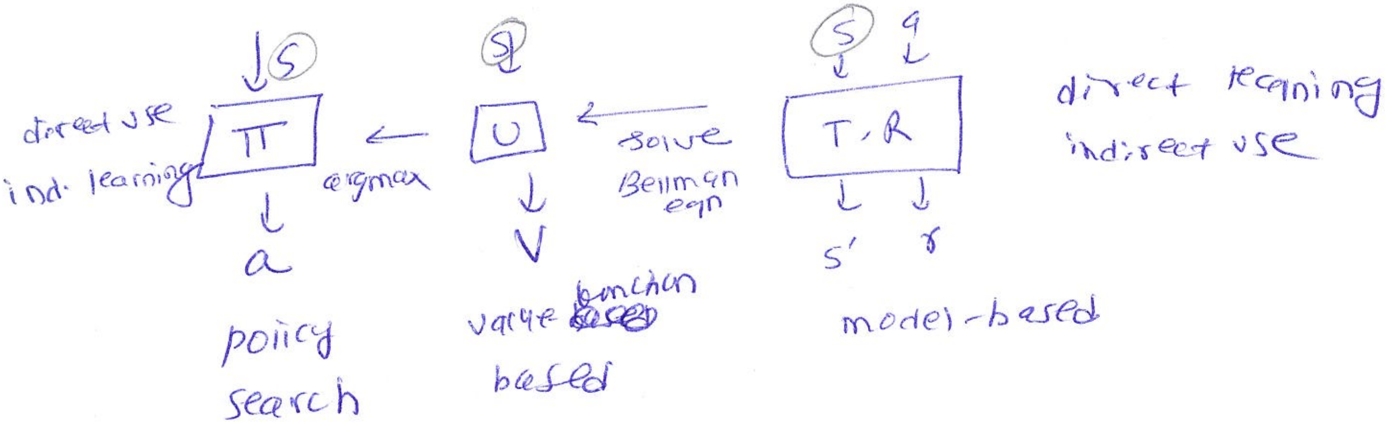
$y(x) = \frac{1}{K} \sum_{i=1}^{k} t_i$ (ti are values, not classes)

⑥ distance weighted KNN (regression)

normal : $y(x) = \sum_{i=1}^{k} w_i t_i \;/\; \sum_{i=1}^{k} w_i$ $w_i = \dfrac{1}{\|x - x_i\|^2}$

kernel based : $y(x) = \sum_{i=1}^{N} K(x,x_i)\, t_i \;/\; \sum_{i=1}^{N} K(x,x_i)$ (ti are values not classes)

3 Approaches to RL



Filter method of feature selection

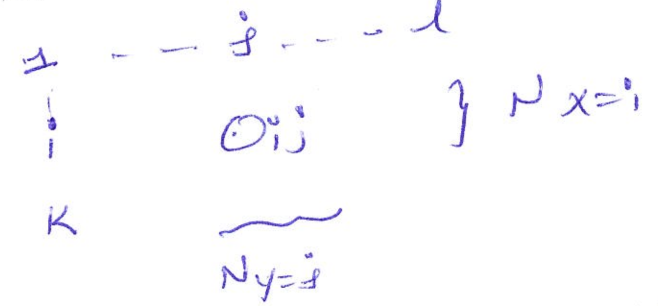
① Mutual Information

$$MI(X,Y) = \sum_X \sum_y p(x,y) \ln \frac{p(x,y}{p(x)\,p(y)}$$

$\left\{\begin{array}{l} =0 \text{ when } x,y \text{ indep} \\ \max \text{ when } x=y \end{array}\right.$

Let there are K examples with l features.

②



O_{ij} = observed value for $x=i$ $y=j$

$$E_{ij} = \frac{N_{x=i}\, N_{y=j}}{N}$$

$$Xe^2 = \sum_{i=1}^{K} \sum_{j=1}^{l} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

classification

③ probabilistic Generative models

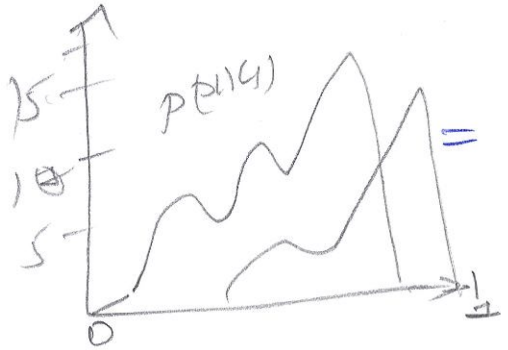
{ Naive Bias
Hidden Markov models

- inf. dec → separate detection
- can use p(x) for outlier or novelty
- need to model dependencies between features

Naive Bias → resilient to noise

- text classification with NB → look HW
- multiple class
  posterior prob of class c_k given data x is,

$$p(c_k|x) = \frac{p(x|c_k) \cdot p(c_k)}{\sum_j p(x|c_j)\, p(c_j)}$$



normalized exponential

$$= \frac{\exp(a_k(x))}{\sum_j \exp(a_j(x))}$$

(softmax fn)

generative

where $a_k(x) = \ln p(x|c_k) \cdot p(c_k)$

① SVM for ranking

optimization problem

minimize $J(w,b) = \frac{1}{2}\|w\|^2 + C\sum \xi_{k,i,j}$

s.t. $w^T \phi(q_k, d_i) \geq w^T \phi(q_k, d_j) + 1 - \xi_{k,i,j}$

$\xi_{k,i,j} \geq 0$

(for a query q_k we wank document d_i be ranked higher than document q_j)

① Add ones column

```
  = np.c-[np.ones(X.shape[0]).reshape(-1,1), X]
X = np.c-[np.ones(X.shape[0])[, np.newaxis].T,  X]
```

②
```
correct = np.sum(y-pred == y-test)
accuracy = correct / len(y-pred)
```

⑥ Gradient Descent (GD or BGD)

vanilla GD

$$v^{\tau+1} = \eta \nabla J(w^{\tau})$$

$$w^{\tau+1} = w^{\tau} - v^{\tau+1}$$

GD with momentum

$$v^{\tau+1} = \gamma v^{\tau} + \eta \nabla J(w^{\tau})$$

$$w^{\tau+1} = w^{\tau} - v^{\tau+1}$$

Batch Gradient Descent (Lect01, p.24)

$$J(w) = \frac{1}{2N} \sum_{n=1}^{N} (h_n - t_n)^2$$

$$w^{\tau+1} = w^{\tau} - \eta \nabla J(w^{\tau})$$

$$= w^{\tau} - \eta \sum_{n=1}^{N} (h_n - t_n)\, x_n$$

$$= w^{\tau} - \text{learninRate}\ (h - t)\ @\ X1$$

confusion matrix

Actual class

$$\begin{array}{lccc} & \text{cat} & \text{not cat} & \\ \text{predicted class cat} & TP & FP & \tilde{P} \\ \text{not cat} & FN & TN & \tilde{N} \\ & & \text{true} & \\ \hline & P & N & \end{array}$$

00 = TN
01 = FP
10 = FN
11 = TP

$$\begin{array}{ccc} & 1 & 0 \\ 1 & TP & FP \\ 0 & FN & TN \\ \hline & P & N \end{array}$$

actual

$$\begin{array}{cccc} & 0 & 1 & \\ 0 & TN & FN & \tilde{N} \\ 1 & FP & TP & \tilde{P} \\ \hline & N & P & \end{array}$$

→ precision $= \frac{TP}{TP+FP}$

recall $= \frac{TP}{P} = \frac{TP}{TP+FN}$

(hit rate)

$\frac{\text{predicted true}}{\text{actual true}}$

F1 score is the Harmonic mean of precision and recall.

$$F1 = \frac{2}{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}}$$

$$= \frac{2\ \text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$$

* prove that the number of elements in x and y is also a kernel

$\Rightarrow$ $k(x,y) = |x \cap y|$ is a kernel.

—— × —— × —— × —— × ——

<u>contd</u> (Nov 11)

$$w^T \phi(x) = \sum_{n=1}^{N} \alpha_n t_n K(x_n, x)$$

$$= \sum_{m \in S} \alpha_m t_m K(x_m, x)$$

$$+ \sum_{m \notin S} \alpha_m t_m K(x_m, x)$$

but if $x_m \notin S$, then $\alpha_m = 0$

strict

since $\downarrow$

$S = n \quad \alpha_n > 0$

$n = 1, 2, \ldots N$

$n \notin S \Rightarrow \alpha_n = 0$

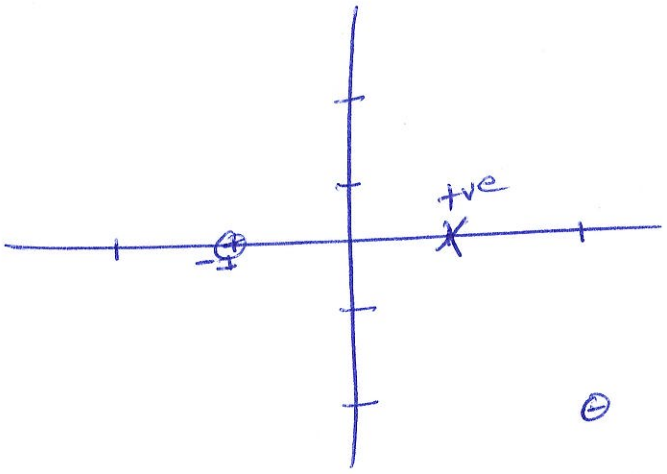
* package libsvm or SVM lite,

input $\rightarrow \{(x_n, t_n)\} \; \forall 1 \leq n \leq N$

output $\rightarrow \{\alpha_m, t_m, x_m\} \quad m \in S$

model.txt $\rightarrow \boxed{\begin{matrix} b \\ \alpha_m \; t_m \; u \; x_m \end{matrix}}$

perceptron update



-1, 0 -ve
2, -2 -ve

1, 0 → +ve

final w = 0 1 2
let's say new point
x = 2, -1.01
xaug = 1 2 -1.01
w xaug = 0 + 2 - 2.02 < 0
∴ -ve label

Augmented data

initial wt $\vec{w}$ = (0 0 0)

1	-1	0	-1
1	2	-2	-1
1	1	0	1
xaug			t

note we can also use $\eta \vec{w} \cdot \vec{x} + b$ original unaugmented x etc

$\eta = 0.9$
$b = 1$

	Test point	hypothesis correct classified?	Updated weights correntexample
eg1	-: (1 -1 0)	wx = 0+0+0 ≮ 0 false	w = w - x = (-1 1 0)
eg2	-: (1 2 -2)	wx = -1+2+0 < 0 false	w = w - x = (-2 -1 2)
eg3	+: (1 1 0)	wx = -2-1+0 ≥ 0 false	w = w + x = (-1 0 2)
eg1	-: (1 -1 0)	wx = -1 + 0 + 0 < 0 true	w = w = (-1, 0 2)
eg2	-: (1 2 -2)	wx = -1 + 0 + (-4) < 0 true	w = w = (-1, 0, 2)
eg3	+: (1 1 0)	wx = -1 + 0 + 0 ≥ 0 false	w = w + x = (0 1 2)
eg1	-: (1 -1 0)	wx = 0 + (-1) + 0 < 0 true	w = w = (0 1 2)
eg2	-: (1 2 -2)	wx = 0 + 2 + (-4) < 0 true	w = w = (0 1 2)
eg3	+: (1 1 0)	wx = 0 + 1 + 0 ≥ 0 true	w = w = 0, 1, 2

final weight

① constrained optimization
(Lagrange multipliers)

maximize $5 - (x_1-2)^2 - 2(x_2-1)^2$

s.t. $x_1 + 4x_2 = 3$

soln: If we ignore constraint we get $x_1 = 2, x_2 = 1$
then $x_1 + 4x_2 = 2 + 4 \cdot 1 = 6$ is too large
for the constraint.
consider
$L = L(x_1, x_2, \lambda) = 5 - (x_1-2)^2 - 2(x_2-1)^2 + \lambda(3 - x_1 - 4x_2)$

Look λ: $\lambda = 0$ $\quad 2, 1$
$\lambda = 1$ $\quad 3/2, 0$
$\lambda = \frac{2}{3}$ $\left(\frac{5}{3}, \frac{1}{3}\right)$ $\quad \frac{5}{3} + 4 \cdot \frac{1}{3} = \frac{5}{3} + \frac{4}{3} = \frac{9}{3} = 3$
✓

formal soln $\quad \frac{\partial L}{\partial x_1} = -2(x_1 - 2) - \lambda = 0$
$= -2x_1 + 4 - \lambda = 0 \Rightarrow 2x_1 = 4 - \lambda$

$2x_1 = 4 - \lambda$
$= 4 - \frac{2}{3}$
$= \frac{4 \cdot 3 - 2}{3} = \frac{10}{3}$
$x_1 = \frac{5}{3}$

$\frac{\partial L}{\partial x_2} = -4(x_2 - 1) - 4\lambda = 0$
$= -4x_2 + 4 - 4\lambda = 0$
$\Rightarrow 4x_2 = 4 - 4\lambda$
$x_2 = 1 - \lambda$

$\frac{\partial L}{\partial \lambda} = 3 - x_1 - 4x_2 = 0$
$\Rightarrow 6 - 2x_1 - 8x_2 = 0$
$\Rightarrow 6 - 4 + \lambda - 8(1 - \lambda) = 0$
$\Rightarrow 6 - 4 + \lambda - 8 + 8\lambda = 0$

$-6 + 9\lambda = 0$
$9\lambda = 6$
$\lambda = 2/3$
~~$2 - 8 + 9\lambda =$~~
~~$-6 + 17\lambda = 0$~~
~~$17\lambda = 6$~~
~~$\lambda = \frac{6}{17}$~~

OP1

$$\min_{w,b} \frac{1}{2}\|w\|^2$$

$$\text{s.t. } t_n(w^T\phi(x_n)+b)\geq 1$$

solution $= w_1^*, b_1^*$

OP2

$$\min_{w,b} \frac{1}{2}\|w\|^2$$

$$\text{s.t. } t_n(w^T\phi(x_n)+b)\geq r$$

$\Updownarrow$ OP3

$$\min_{w,b} \frac{1}{2}\left\|\frac{w}{r}\right\|^2$$

$$\text{s.t. } t_n\left(\left(\frac{w}{r}\right)^T\phi(x_n)+\frac{b}{r}\right)\geq 1$$

OP3 has solution

$w_2^* = w_1^* r$

$b_2^* = b_1^* r$

$\Updownarrow$ OP3 rename

$w' = w/r$

$b' = b/r$

$$\min_{w,b} \frac{1}{2}\|w'\|^2$$

$$\text{s.t. } t_n(w'\phi(x_n)+b')\geq 1$$

Now, decision hyperplanes are,

$$H1 = \{x \mid w_1^{*T}\phi(x)+b_1^* = 0\}$$

$$H2 = \{x \mid w_2^*\phi(x)+b_2^* = 0\}$$

$$= \{x \mid n w_1^*\phi(x)+n b_1^* = 0\}$$

$$= \{x \mid w_1^*\phi(x)+b_1^* = 0\}$$

$H2 = H1$ q.e.d

③ kmcp kernel multi-class perceptron

```
1 define $f(x) = \sum_{i,j} \alpha_{ij} \left[\phi(x_i, t_i)^T \phi(x,t) - \phi(x_i, c_j)^T \phi(x,t)\right]$
      ($f(x) = w^T x = \sum_n \alpha_n t_n x_n^T x = \sum_n \alpha_n t_n K(x_n, x)$)
2 initialize dual params $\alpha_{ij} = 0$
3 for $i = 1..n$
4     $c_j = \arg\max_{t \in T} f(x_i, t)$
          ($h_n = sign(f(x))$)
5     if $c_j \neq t_i$ then    ($h_n \neq t_n$)
6         $\alpha_{ij} = \alpha_{ij} + 1$    ($\alpha_n = \alpha_n + 1$)
} Repeat
```

Testing: $t^* = \arg\max_{t \in T} f(x,t)$ ($h(x) = sign(f(x))$)

② mcp (concept of kernel comes from here)

```
initialize parameters $w = 0$
for $i = 1..N$
    $c_j = \arg\max_{t \in T} w^T \phi(x_i, t)$
    if $c_j \neq t_i$ then
        $w = w + \phi(x_i, t_i) - \phi(x_i, c_j)$
}
```

w is loop invariant and is the weighted average

$$w = \sum_{i,j} \alpha_{ij} \left(\phi(x_i, t_i) - \phi(x_i, c_j) \right)$$

$$f(x) = w^T \phi(x,t) = \sum_{i,j} \alpha_{ij} \left(\phi(x_i, t_i)^T \phi(x,t) - \phi(x_i, c_j)^T \phi(x,t) \right)$$

① KP

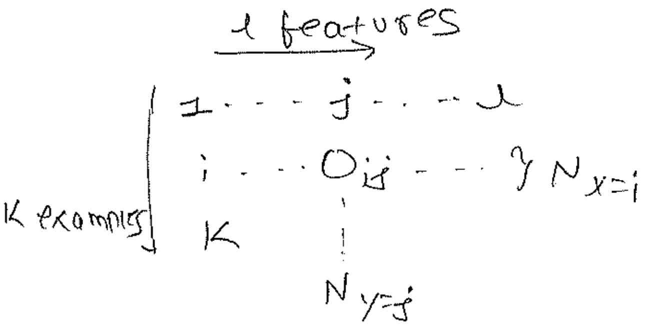
```
define $f(x) = w^T x = \sum_n \alpha_n t_n K(x_n, x)$
initialize dual parameter $\alpha_n = 0$
for $i = 1..N$
    $h_n = sign(f(x))$
    if $h_n \neq t_i$ then
        $\alpha_n = \alpha_n + 1$
} Repeat
```

Test: $y(x) = sign(f(x))$

$$MI(X;Y) = \sum_x \sum_y p(x,y) \cdot \ln \frac{p(x,y)}{p(x) \cdot p(y)} = \begin{cases} 0 & \text{when } x,y \text{ indp} \\ \max & \text{when } x=y \end{cases}$$

① mutual information
- → works with nominal
- → biased towards high arity feature
- → may choose redundant feature

② Chi square



$$E_{ij} = \frac{N_{x=i} \cdot N_{y=j}}{N}$$

$$\chi_0^2 = \sum_{i,j} \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

③ pearson corr. coeff

$$\rho(x,y) = \frac{E[(x-\mu_x)(y-\mu_y)]}{\sigma_x \sigma_y} = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

④ SNR

$$\mu(x,\overset{\text{binary}}{y}) = \frac{|\mu_+ - \mu_-|}{\sigma_x + \sigma_-}$$

⑤ T Test

$$T(x,y) = \frac{|\mu_+ - \mu_-|}{\sqrt{\frac{\sigma_+^2}{\mu_+} + \frac{\sigma_-^2}{\mu_-}}}$$

Filter vs	wrapper
1 much faster since no need to train the model	- computationally expen.
2 use statistical method of evaluation	- uses cross-vaiidation
3 might fail to find best subset	- finds best subset
4 less prone to verfitting	- more prone to overfitting

① SVM for regression

min $J(w,b) = \frac{1}{2}\|w\|^2 + C\sum_{n=1}^{N}(\xi_n + \hat{\xi}_n)$

s.t. $t_n \le t_n(w^T\phi(x_n)+b) + \epsilon + \xi_n$

$t_n \ge t_n(w^T\phi(x_n)+b) - \epsilon - \hat{\xi}_n$

$\xi_n, \hat{\xi}_n \ge 0 \quad \forall\ 1 \le n \le N$

② SVM for rancing

min $J(w,b) = \frac{1}{2}\|w\|^2 + C\sum \xi_{k,i,j}$

s.t. $w^T\phi(q_k, d_i) \ge w^T(\phi q_k, d_j) + 1 - \xi_{k,i,j}$

$\xi_{k,i,j} \ge 0$

MDP markov Decision process

① policy

argmax

$$\pi^* = \underset{\pi}{\text{argmax}}\ E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \mid \pi\right]$$

(policy expectation)

$$\pi^* = \underset{a}{\text{argmax}} \sum_{s'} T(s,a,s')\, U(s')$$

(best action policy)

R = Reward

$\gamma^t R$ = discounted Reward

② utility

expectation & max only

$$U(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t)\ )\pi\ ,\ s_0 = s\right]$$

(expectation)

$$U(s) = R(s) + \gamma \max_{a} \sum_{s'} T(s,a,s')\ U(s')$$ ← Bellman eqn

* Naive Bias

3 Boolean input vectors $x_1\ x_2\ x_3$ and output y

$p(y=1)$

$p(x_1=1 \mid y=0)$ $p(x_1=1 \mid y=1)$

$p(x_2=1 \mid y=0)$ $p(x_2=1 \mid y=1)$

$p(x_3=1 \mid y=0)$ $p(x_3=1 \mid y=1)$

- \# of parameters $= 2n+1 = 2\times3+1 = 7$
- \# of parameters if no conditional independence $= 1+2(2^n-1)$ $= 1+2(8-1$
  $= 1+2(2^3-1)$ $= 1+14 = 1$

Example

category label

	DOC	words (wi)	class	Total
Train				
3 documents for C1	1	Chinese Beiging Chinese	C1	n1=8
	2	Chinese Chinese Shanghai	C1	
	3	Chinese Macao	C1	
1 document for C2	4	Tokyo Japan Chinese	C2	n2 = 3
			?	11 words 6 unique
Test	5			
Total	$\lvert D\rvert = \lvert D_1\rvert + \lvert D_2\rvert = 3+1 = 4$			

1 Chinese 卌|
2 Beiging 1
3 Shanghai 1
4 Macao 1
} ignore
5 Tokyo 1
6 Japan 1

① vocabulary, V = { Chinese, Beiging, Shanghai, Macao, Tokyo, Jap

$|V| = 6$

②

category C1 = c

- prior $P(C_1) = \frac{|D_1|}{|D|} = \frac{3}{4}$
- # of words in class C1, $n_1 = 8$

conditional probabilities

$$p(w_1|c_1) = p(\text{Chinese}|c) = \frac{5+1}{8+6} = \frac{6}{14} = \frac{3}{7}$$

$n_{ki} = n_{11} = 5$ → Laplace smoothing

n_k → 8, $|V|$ → 6

$p(w_2|c_1) = p(\text{Beiging}|c) = \frac{1+1}{14} = \frac{2}{7}$

$p(w_3|c_1) = p(\text{shanghai}|c) = 2/7 = P$

$$P(w_2|c_1) = P(\text{Tokyo}|c) = \frac{0+1}{8+6} = \frac{1}{14}$$

$$p(w_3|c_1) = p(\text{Japan}|c) = \frac{0+1}{8+6} = \frac{1}{14}$$

category C2 = j (Japan)

- prior $p(c_2) = \frac{|D_2|}{|D|} = \frac{1}{4}$
- # of words in class C2, $n_2 = 3$

$$p(w_1|C_2) = p(\text{Chinese}|j) = \frac{1+1}{3+6} = \frac{2}{9}$$

$$p(w_2|c_2) = p(\text{Tokyo}|j) = \frac{1+1}{3+6} = \frac{2}{9}$$

$$p(\;3|c_2) = p(\text{Japan}|j) = \frac{1+1}{3+6} = \frac{2}{9}$$

③ choosing a class

prior, Chinese, Japan, Tokyo

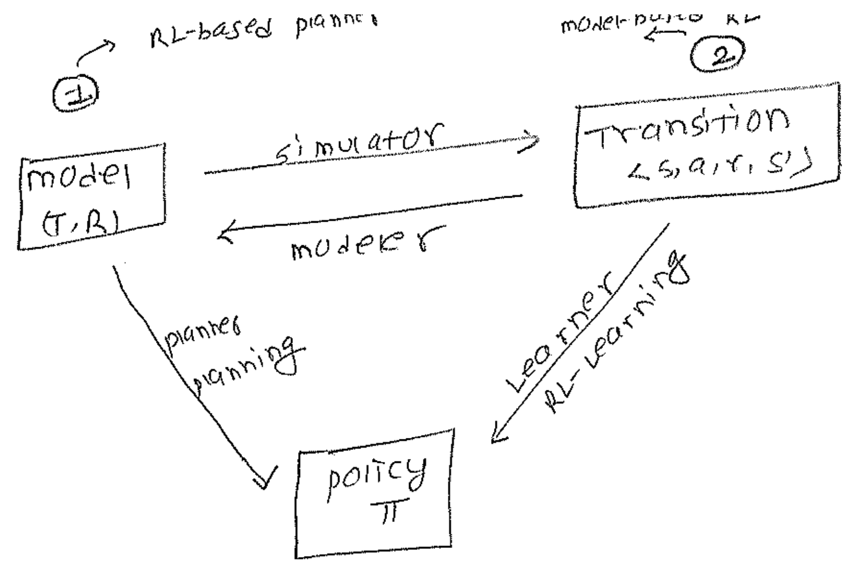
$$p(c_1|x) \propto p(c_1) \prod_i p(w_i|c_1) = \frac{3}{4} \cdot \left(\frac{3}{7}\right)^3 \cdot \frac{1}{14} \cdot \frac{1}{14} \cong 0.0003$$

$$p(c_2|x) \propto p(c_2) \prod_i p(w_i|c_2) = \frac{1}{4} \cdot \left(\frac{2}{9}\right)^3 \cdot \frac{2}{9} \cdot \frac{2}{9} \approx 0.0001$$

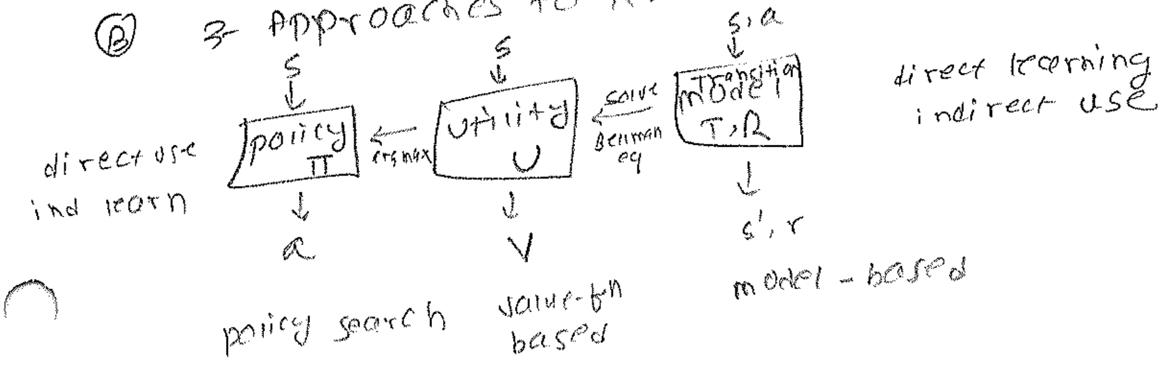
$$c^* = \underset{c_k}{\operatorname{argmax}}\; p(c_k) \prod_{j=1}^{n} p(w_j|c_k) = \underset{c_k}{\operatorname{argmax}} \{0.0003, 0.0001\}$$

= [illegible] 0.0003 = C1

Ⓐ



Ⓑ 3 Approaches to RL



Ⓒ Q function

$$U(s) = R(s) + \gamma \max_{a} \sum_{s'} T(s,a,s')\, U(s')$$

change u to Q

(Bellman eqn)

utility is a scalar

$$\pi(s) = \arg\max_{a} \sum_{s'} T(s,a,s')\, U(s')$$

(policy gives an action)

$$Q(s,a) = R(s) + \gamma \sum_{s'} T(s,a,s') \max_{a'} Q(s',a')$$

quiz ⇒ $U(s) = \max_{a} Q(s,a)$

⇒ $\pi(s) = \arg\max_{a} Q(s,a)$

4NB

① mutual information

$$MI(X,Y) = \sum_{X} \sum_{y} p(x,y) \cdot \ln\left(\frac{p(x,y)}{p(x)\,p(y)}\right) \quad = 0 \text{ when } x,y \text{ indp}$$

$$= KL\left[p(x,y) \,\|\, p(x)\,p(y)\right] \quad = \max \text{ when } x = y$$

bad → biased towards high arity features

bad → may choose redundant features

bad → works only with nominal features + labels

* 3 parametric Approaches to

① Discriminant function

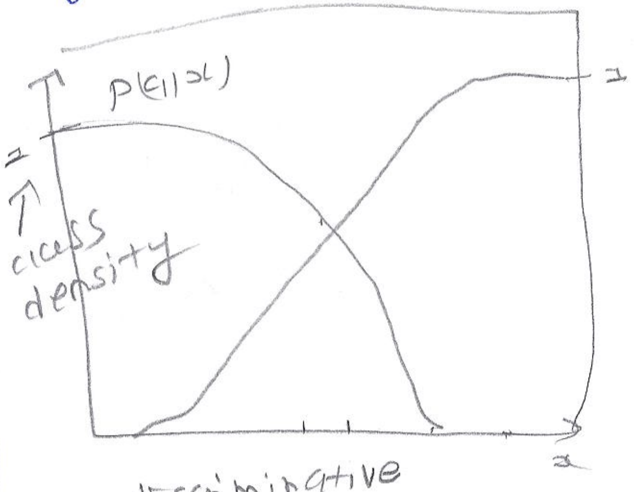
- Fisher's linear disc
- perceptron
- SVM

inference and decision are combined as single learning problem

② probabilistic discriminative models

- logistic regression
- conditional random fields

- inf & dec → separate
- less data need to compute $p(C_k|x)$ than $p(x|C_k)$
- can accommodate many overlapping features



discriminative

③ pearson corr coeff

$$\rho(X,Y) = \frac{E[(X-\mu_x)(Y-\mu_y)]}{\sigma_x \, \sigma_y}$$ (population)

(sample)

$$\rho(x,y) = \frac{\sum_{i=1}^{n} (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i-\bar{x})^2} \cdot \sqrt{\sum_{i=1}^{n} (y_i-\bar{y})^2}}$$

$$-1 \le \rho(x,y) \le 1$$

④ SNR

binary classes = $\{+,-\}$
mean = $\{\mu_+, \mu_-\}$

examples are binary
$y \in \{+1,-1\}$

$$\mu(x,y) = \frac{|\mu_+ - \mu_-|}{\sigma_+ + \sigma_-}$$

μ_+, σ_+ = mean & std for +ve class samples

③ T-test

$$T(x,y) = \frac{|\mu_+ - \mu_-|}{\sqrt{\frac{\sigma_+^2}{\mu_+} + \frac{\sigma_-^2}{\mu_-}}}$$

CS 4900/5900: Machine Learning
Fall 2017

Class Meetings: Tue, Thu 10:30–11:50am, ARC 212
Instructor: Razvan Bunescu
Office: Stocker 341
Office Hours: Tue, Thu 12:00–12:30pm, or by email appointment
Email: bunescu @ ohio edu
Class Website: `http://ace.cs.ohio.edu/~razvan/courses/ml4900`

Prerequisites:

The students are expected to be comfortable with programming and familiar with basic concepts in linear algebra and statistics.

Textbook:

There is no textbook for this class. Slides and supplementary materials will be made available on the course website.

Supplementary Texts:

Machine Learning: The Art and Science of Algorithms that Make Sense of Data
  by Peter Flach, Cambridge University Press, 2012
A Course in Machine Learning [free online]
  by Hal Daume III
Machine Learning
  by Tom Mitchell. McGraw Hill, 1997
Pattern Recognition and Machine Learning
  by Christopher Bishop. Springer, 2007
Pattern Classification
  by Richard O. Duda, Peter E. Hart, & David G. Stork. Wiley-IS, 2001
The Elements of Statistical Learning: Data Mining, Inference, and Prediction
  by T. Hastie, R. Tibshirani, & J. H. Friedman. Springer Verlag, 2009

Course Description:

This course will give an overview of the main concepts, techniques, and algorithms underlying the theory and practice of machine learning. The course will cover the fundamental topics of classification, regression and clustering, and a number of corresponding learning models such as perceptrons, logistic regression, linear regression, Naive Bayes, nearest neighbors, and Support Vector Machines. The description of the formal properties of the algorithms will be supplemented with motivating applications in a wide range of areas including natural language processing, computer vision, bioinformatics, and music analysis. The topics covered in this course will also prepare students for taking more advanced courses in data mining and deep learning.

Grading:

50%: Homework Assignments
20%: Midterm Exam (Oct 12, in class) 1 hr 20 min
30%: Final Exam (Dec 12, 10:10am – 12:10pm)

Grading Scale:

A (> 92%) A−(> 90%) B+(> 87%) B(> 83%) B−(> 80%)
C+(> 77%) C(> 73%) C−(> 70%) D+(> 67%) D(> 63%) D−(> 60%)

Important Dates:

Friday, Sep 1: Last day to add class.
Tuesday, Oct 10: Reading Day, no class.
Friday, Nov 3: Last day to drop class.
Thursday, Nov 23: Thanksgiving break, no class.
Thursday, Dec 7: Last day of this class.

Course and Attendance policies:

Assignments: `All homework assignments are due before the class.` No late submissions will be accepted without prior approval.

Attendance: `It is in your best interest to attend the lectures.` Some of the material will not be found in the supplementary text or on the slides. Extra credit will be awarded for class activity. Also, be sure to check your OU email for important announcements on a regular basis.

Academic Dishonesty Policy:

`All work must be the student's own.` All external references used in reports must be properly cited. No credit will be given for duplicate or plagiarized work. Additional measures may be imposed by the University Judiciaries, when conditions warrant. Students may appeal academic sanctions through the grade appeal process. The OU Student Code of Conduct Policy is available online at:

`http://www.ohio.edu/communitystandards/academic/students.cfm`

Disability-based Accommodation:

Any student who suspects s/he may need an accommodation based on the impact of a disability should contact the class instructor privately to discuss the students specific needs and provide written documentation from the Office of Student Accessibility Services. If the student is not yet registered as a student with a disability, s/he should contact the Office of Student Accessibility Services.

Other Policies:

Be sure to notify the professor of any exam conflicts or other extenuating circumstances well in advance. `No missed exams will be made up without prior approval.` Medical excuse forms need to explicitly mention that the student could not have attended the exam at the specified time due to health concerns.