Project II : Bound States in Momentum Space

Bhishan Poudel

Dec 9,2015

Contents

List of Figures

1 Qn 1: Legendre Polynomials

In this part I wrote a code to calculate the legendre polynomials of orders from 0 to 8 . I used the recursion formula:

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x) \tag{1}$$

to calculate the polynomials. The calculated polynomials are accurate up to seven significant digits. I used Wolfram Alpha online to check the accuracy of the legendre polynomials. For example I took the value $x = 0.8$ and compared the values.

n	x	calculated p(n,0.8)	Wolfram p(n,0.8)
0	0.8	1.0000000	1
1	0.8	0.8000000	0.8
2	0.8	0.4600000	0.46
3	0.8	0.0800000	0.08
4	0.8	-0.2330000	-0.233
5	0.8	-0.3995200	-0.39952
6	0.8	-0.3917960	-0.391796
7	0.8	-0.2396512	-0.2396512
8	0.8	-0.0166553	-0.0166553

I also checked the accuracy of my results by comparing the result of integral:

$$\int_{-1}^{1} P_n^2(x)\, dx \tag{2}$$

obtained from my code to find polynomials with the orthogonality relation:

$$\frac{2}{2n+1} \tag{3}$$

The values are true up to seven significant figures, which we can see in the file '*legendre_compare.dat*'

The solution directory is :

```
location             : Pr2/Part1
source code          : Legendre_Polynomials.f90 and legendre_compare.f90
plots                : legendre_polynomials.eps
datafiles            : legendre_polynomials.dat and legendre_compare.dat
provided subroutines : gauleg.f90
```

The figures are shown below:

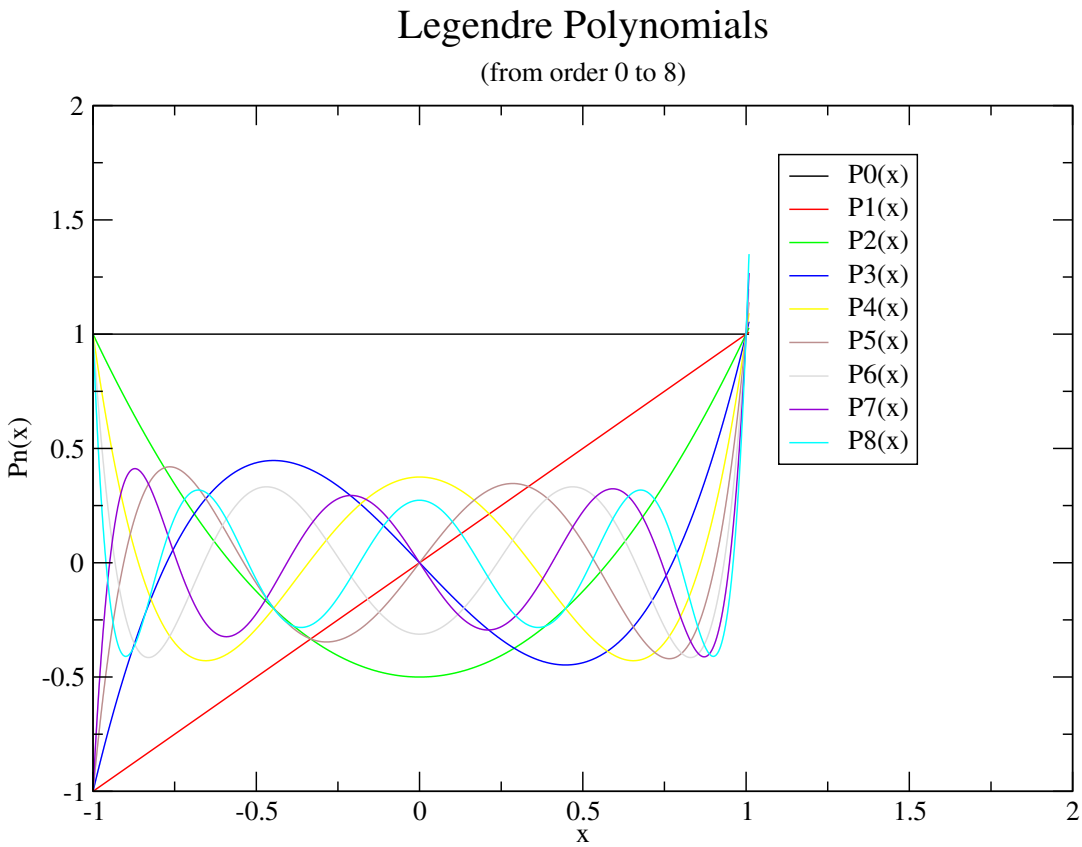


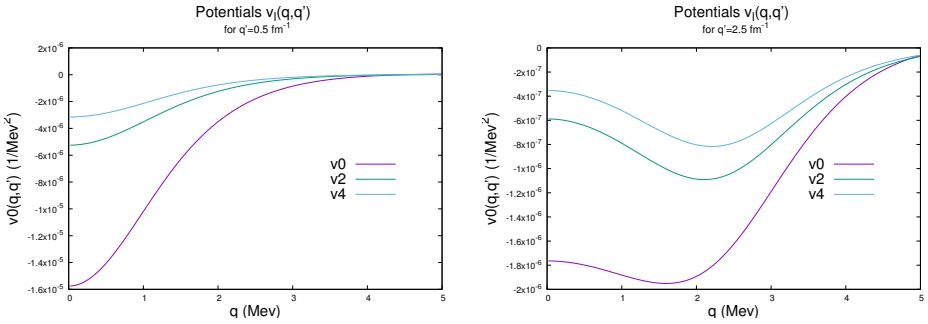
Figure 1: Legendre Polynomials

2 Qn 2: Partial Waves Potentials

In this Qn I calculated the partial wave potentials v_0, v_2, v_4. Where, v_l is the partial wave potential given by:

$$v_l(q,q') = 2\pi \int_{-1}^{1} P_l(cos\theta) V(q,q')\, d(cos\theta) \tag{4}$$

Where, $V(q,q')$ is the interaction potential between neutron and proton given in the question. P_l is the Legendre polynomial of order l and q,q' are relative momenta for the proton-neutron system.
I plotted the two graphs for v_0,v_2,v_4 for constant value of $q' = 0.5fm^{-1}$ and $q' = 2.5fm^{-1}$



(a) for $q' = 0.5fm^-1$ (b) for $q' = 2.5fm^-1$

Figure 2: deuteron potentials for different q'

3 Qn 3: Schrodinger equation using matrix

The Unit analysis is presented at the end of this document.
The solution algorithm is like this:
First, I solved the momentum space Schrodinger equation for two (neutron and proton) of reduced mass $\mu = m/2$given by:

$$\frac{q^2}{2\mu}\Psi_{nl}(q) + \frac{2}{\pi}\int_0^{\infty} p''^2 v_l(q,p')\Psi_{nl}(p')\, dp' = E_n \Psi_{nl}(q) \tag{5}$$

This is a second order differential equation. To solve this equation, first I wrote the integral equation as the matrix equation and calculated the determinant of the matrix using LAPACK algorithm DGEEV. Then I found the value of binding energy using condition that the values must be less than

zero. I followed the process given in Morten Hjorth-Jensen Lecture Notes Fall 2007 Chapter 12.8.

4 Qn 4: Gauss-Legendre mapping for the integration

To solve the integral part of $v_l(q, q')$ I used the Gauss-Legendre method. I used the given subroutine 'gauleg' to solve the integral. To solve an integral first of all we have to convert it to summation.

$$\int_a^b f(x)\,dx = \sum_{i=1}^{N} w_i f(x_i) \tag{6}$$

Where, N is Number of mess points, w_i are weights, x_i are points and $f(x_i)$ is the function to integrate. Here, also I have to use the mapping to $(0, \infty)$. So i used following mapping:

$$p_i = tan\frac{\pi}{4}(1 + x_i) \tag{7}$$

$$w_i = \frac{\pi}{4}\frac{w_i}{cos^2(\frac{\pi}{4}(1 + x_i))} \tag{8}$$

5 Qn 5: Binding energy and errors

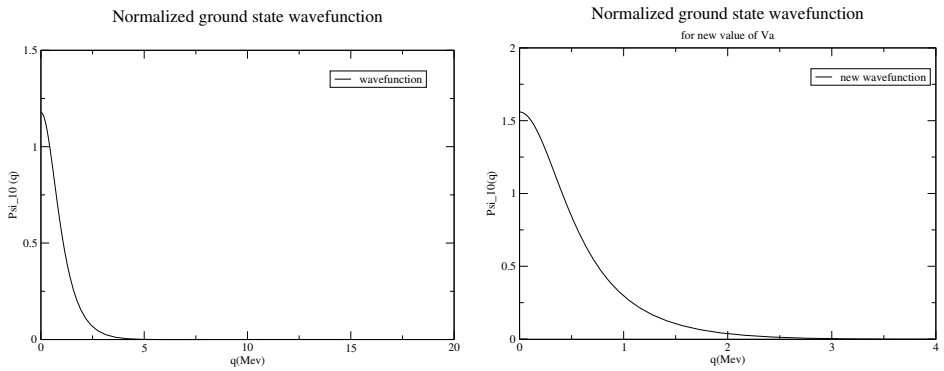
In this part I evaluated the binding energy of the deuteron.
I used 50 number of gauss points to find the integral and the final value of binding value obtained is -56.3 Mev. The source code is 'pr2qn5.g90'

6 Qn 6: Normalized ground state wavefunction

In this part I evaluated the normalized ground state wavefunction. The code is 'pr2qn6.f90'. The plot is shown below:

7 Qn 7: Variation of constant V_A

Here, The potential of the deuteron depends on the parameter V_A, which gives the strength of the attraction between neutron and proton. I varied the value of this parameter and got new potentials and wavefunction.



(a) given Va (b) smaller Va

Figure 3: ground state wavefunctions at different parameter Va

8 Qn 8: New normalized ground state wavefunction

Here, for new value of V_A I plotted the graph, which is shown in Qn 6.

9 Qn 9: Expectation value of kinetic energy operator

In this part I calculated the expectation value of kinetic energy of the deuteron. For $V_A = 1085.3073 Mevfm$ I found $H_0 = 0.001 Mev$.
For $V_A = 985.3073 Mevfm$ I found $H_0 = 6.9E-3 Mev$.
The total mass of free neutron and proton is different than that of deuteron. The mass difference is called mass deficit. This mass deficit in terms of energy is called binding energy of the deuteron. Its value is about 2.23 Mev. This means if we combine one proton and one neutron to form a deuteron, the gamma ray of energy 2.23 Mev will be emitted. Now, when we supply the gamma ray of at least 2.23 Mev to a bound state deuteron, they will separate apart and will gain kinetic energy. At any time,
Total energy = Kinetic energy + Potential Energy
is valid.

10 Appendix

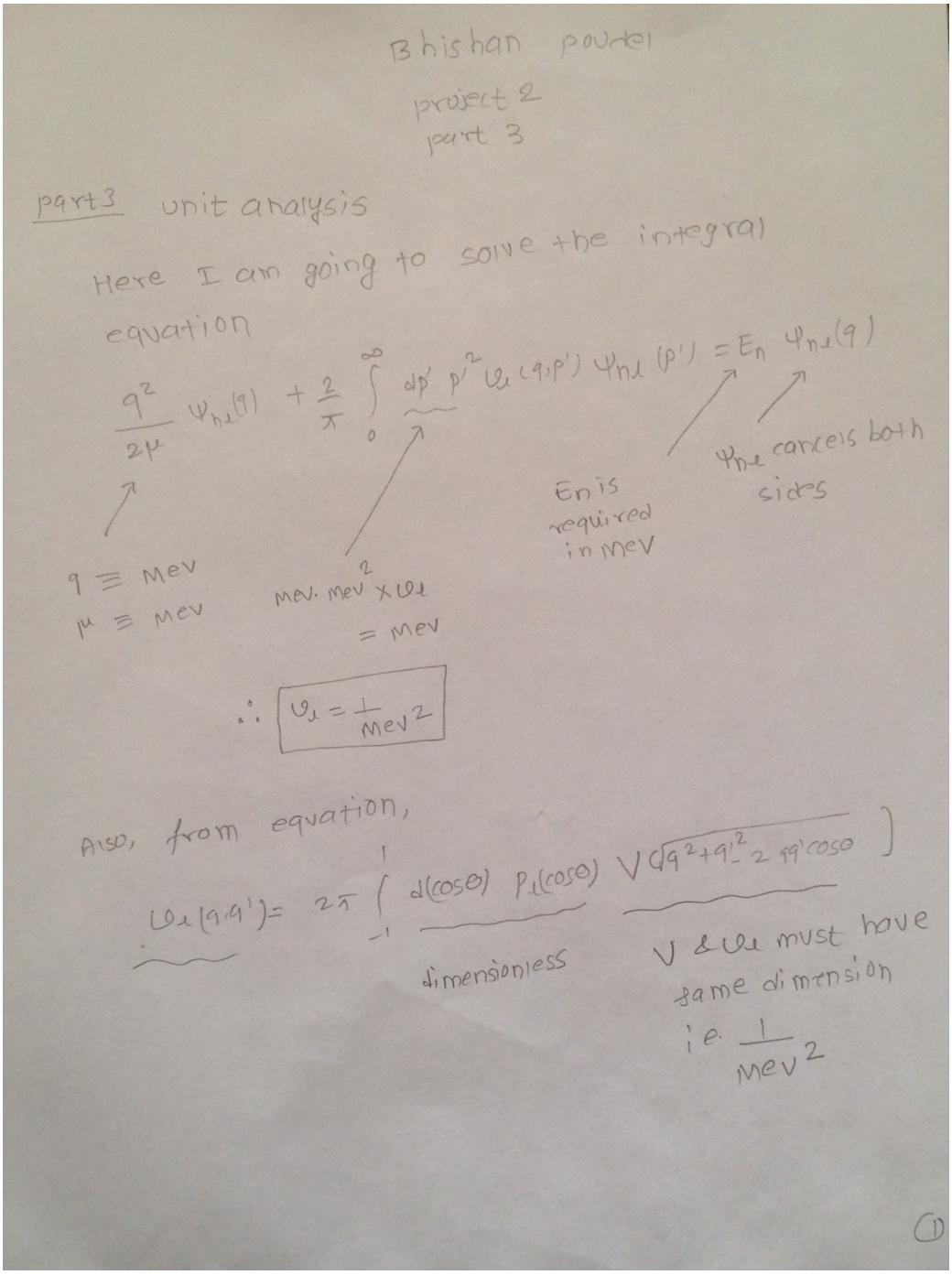


Figure 4: Unit Analysis page 1

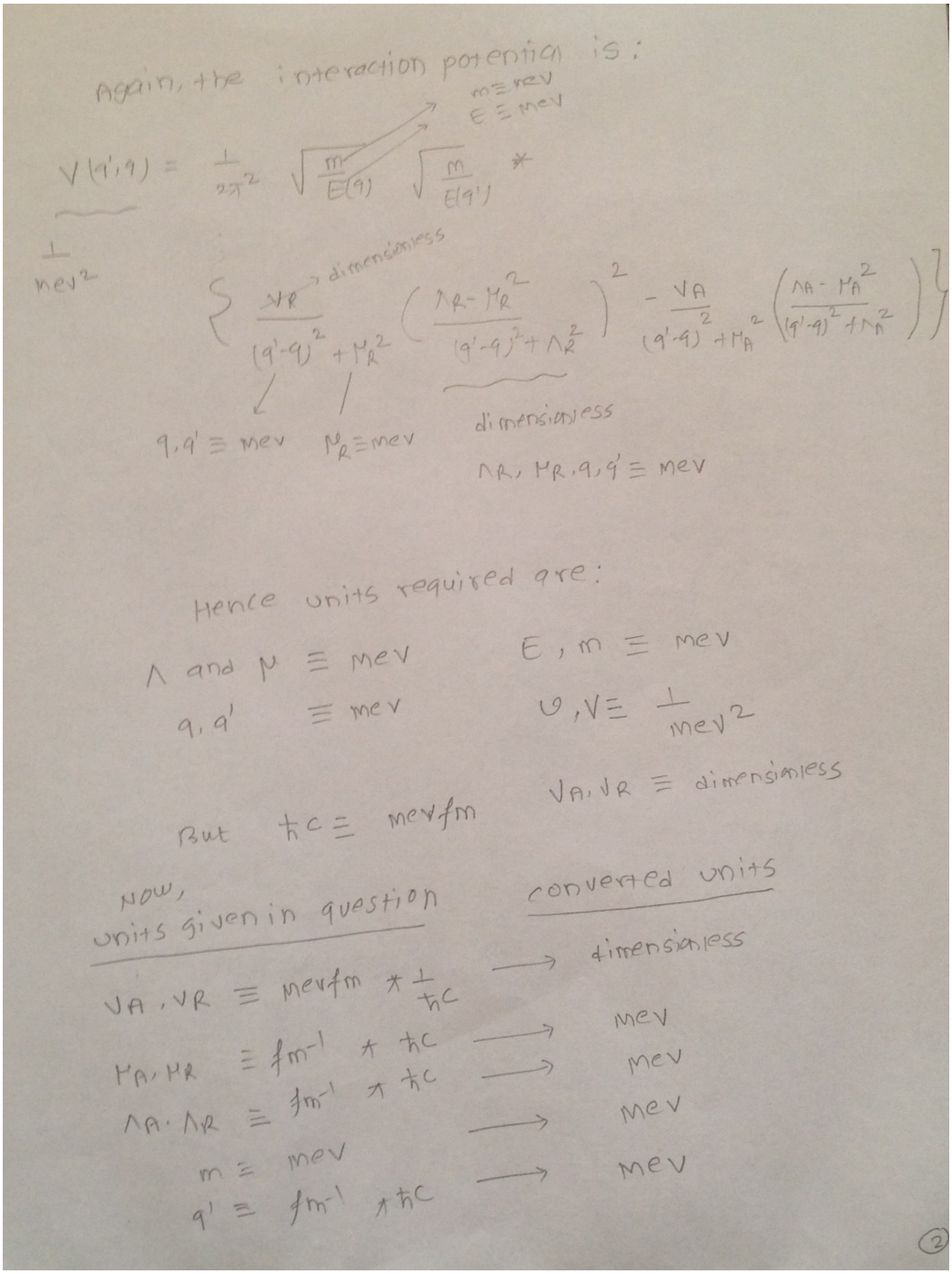


Figure 5: Unit Analysis page 2