# Assignment 4

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### 1 Question 1

This program computes the first derivative of functions

$$f(x) = \cos(x) \tag{1}$$

$$f(x) = exp(x) (2)$$

$$f(x) = \sqrt{(x)} \tag{3}$$

(4)

at points x = 0.1, x = 1, x = 30

#### 1.1 part a

The source code is: assign4/qn1/hw4qn1.f90

The outputs are:

for single precision:

cos01sp.dat,cos1sp.dat,cos30sp.dat for x=0.1,1.0,and30.0 respectively
exp01sp.dat,exp1sp.dat,exp30sp.dat
sq01sp.dat,sq1sp.dat,sq30sp.dat

For double precision:

cos01dp.dat, cos1dp.dat, cos30dp.dat for x=0.1, 1.0, and 30.0 respectively sq01dp.dat, sq1dp.dat, sq30dp.dat

Note: double precision for exp(x) was not required.

I chose h=40.0 for cos(x) and exp(x) and h=0.1 for  $\sqrt{x}$ . The step size was reduced upto the machine precision upto 1e-6 for single precision and 1e-14 for double precision.

#### 1.2 part b

The source code is: hw4qn1.f90

output dat files are: cos01sp.dat,cos01dp.dat etc.

I plotted modulus of logE vs logH for 3 functions for single and double precisions.

The number of decimal places obtained agrees with the estimates in the text.

The graphs looks like this:

#### Plot of absolute error vs h for cos(0.1) single precision 10<sup>1</sup> 10<sup>0</sup> 10<sup>-1</sup> 10<sup>-2</sup> abs error 10<sup>-3</sup> 10<sup>-4</sup> forward central 10<sup>-5</sup> extrapolated fitted forward slope=0.98676 fitted central slope = 1.997110<sup>-6</sup> fitted extrapolated slope = 3.830910<sup>-7</sup> 10<sup>-2</sup> 10<sup>-4</sup> 10<sup>-7</sup> 10<sup>-3</sup> 10<sup>-1</sup> 10<sup>0</sup> 10<sup>-6</sup> 10<sup>-5</sup> $10^1$

Figure 1: Plot of cos(0.1) single precision

#### Plot of absolute error vs h for cos(0.1) double precision 10<sup>-2</sup> 10<sup>-3</sup> 10<sup>-4</sup> 10<sup>-5</sup> 10<sup>-6</sup> 10<sup>-7</sup> absolute error 10<sup>-8</sup> 10<sup>-9</sup> 10<sup>-10</sup> 10<sup>-11</sup> forward central extrapolated 10<sup>-12</sup> $\odot$ fitted forward slope = 0.99699 10<sup>-13</sup> ☐ fitted central slope = 1.9678 10<sup>-14</sup> ◆ fitted extrapolated slope = 3.8972 $10^{-15} \frac{\mathsf{E}}{10^{-9}}$ 10<sup>-2</sup> 10<sup>-3</sup> 10<sup>-5</sup> 10<sup>-4</sup> 10<sup>-1</sup> 10<sup>-7</sup> $10^{-8}$ $10^{-6}$ h

Figure 2: Plot of cos(0.1) double precision

# log-log plot of absolute error vs h

for exp(0.1) single precision first derivative 10<sup>-1</sup> 10<sup>-2</sup> 10<sup>-3</sup> 10<sup>-4</sup> abs error 10<sup>-5</sup> 10<sup>-6</sup> forward central 10<sup>-7</sup> fitted forward slope = 1.0232 extrapolated 10<sup>-8</sup> fitted central slope = 1.9698fitted extrapolated slope = 4.046710<sup>-6</sup> 10<sup>-4</sup> 10<sup>-2</sup> 10<sup>-5</sup> 10<sup>-3</sup> 10<sup>-1</sup> 10<sup>0</sup> 10<sup>1</sup>

h

Figure 3: Plot of  $\exp(0.1)$  single precision

#### 1.3 part c

I plotted the best fit of the graph with log-log fit from xmgrace. Appropriate range for X and Y axes were chosen. From the plots we know that for cosine function in double precision:

slope for forward differentiation is nearly 1. slope for central differentiation is nearly 2. slope for extrapolation differentiation is nearly 4.

The source code is: assign4/qn1/hw4qn1.f90

The output plots are: cos01dp.eps,cos1dp.eps,etc

#### 1.4 part d

I repeated the analysis for  $\cos(x)$  and  $\sqrt{x}$  in double precision and compared to the single precision.

The source code is: assign4/qn1/hw4qn1.f90

The output data files are: cos01dp.dat,cos1dp.dat,etc

The output plots are: cos01dp.eps,cos1dp.eps,etc

#### 1.5 part e

In the above plots I paid special attention to the algorithmic errors. The best fit was plotted for the algorithmic error part. The best fit eps files are inside qn1 folder.

#### 2 Question 2

In this question we studied three-point and five point formula for second order derivative of functions.

#### **2.1** part a

The three-point and five-point formula was derived and the pdf can be found inside: writeup/hw4qn2a.pdf

### Plot of absolute error vs h

for cos(1.0) single precision

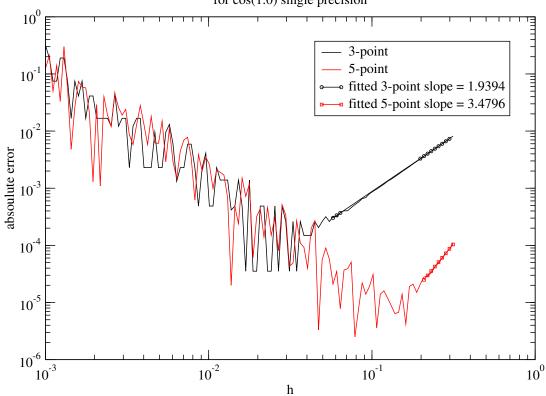


Figure 4: Plot of cos(0.1)

or, qn1/hw4qn2a.pdf

#### 2.2 part b

I wrote the code to calculate 2nd order derivative for cos(x) in single precision for the 3 values of x = 0.1, x = 1, x = 30. I started with h=0.314 and keep going down upto machine precision 1e-6.

While calculating derivative special attention was given in grouping the terms. Similar terms are grouped together.

The grouping can be seen in the source code.

The source code is:

qn2/hw4qn2.f90

and the outpuf files are:

 $hw4qn2\_1.dat, hw4qn2\_01.dat, and, hw4qn2_30.dat. \\$ 

#### 2.3 part c

The derivative and its relative errors were produced. I reduced step size upto 1e-6 for single precision.

### 2.4 part d

The log-log plot of logE versus logH was created. The plots are:  $hw4qn2\_1.eps, hw4qn2\_01.eps, and, hw4qn2_30.eps$ .

The number of decimal places obtained agrees with the estimates in the text.

#### 2.5 part e

We can see truncation error at large h and roundoff error at small h in the graph.

## 3 Question 3: Population Growth Problem

In this problem we solved population growth equation both numerically and analytically.

#### 3.1 part a: when b=0

First we took, b=0 and solved the equation. The source code is: qn3/hw4qn3a.f90

The output is: qn3/hw4qn3a.dat

The plot is: qn3/hw4qn3a.eps

### 3.2 part b: when b=3

```
Here, we took a =10 and b=3. The source code is: qn3/hw4qn3b.f90
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The output is: qn3/hw4qn3b.dat
```

the plot is : qn3/hw4qn3b.eps

Initially the population decreases with time since the  $-bN^2$  term dominates and as the time passes by population growth seems to be constant. We can see this in the plots.