HW4

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QN 2a For 3 point formula $(x-h, x, x+h)$

we know that, From Taylor expansion:

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^{iv}(x) + \cdots \quad \text{—— (1)}$$

also,

$$f(x-h) = f(x) - h f'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(x) + \frac{h^4}{24} f^{iv}(x) + \cdots \quad \text{—— (2)}$$

applying (1) + (2):

$$f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + \underbrace{\frac{h^4}{12} f^{iv}(x) + \cdots}_{\text{neglect}}$$

solve,

$$\boxed{f''(x) = \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} - O(h^2)} \quad \text{—— (3)}$$

proved!

Five point formula ($x-2h,\ x-h,\ x,\ x+h,\ x+2h$):

From Taylor expansion;

$$f(x+2h) = f(x) + 2hf'(x) + \frac{4h^2}{2} f''(x) + \frac{8h^3}{6} f'''(x) + \frac{16h^4}{24} f^{iv}(x) + \frac{32h^5}{120} f^{v}(x) + \frac{64h^6}{720} f^{vi}(x) + \cdots \quad \text{--- (4)}$$

also,

$$f(x-2h) = f(x) - 2hf'(x) + \frac{4h^2}{2} f''(x) - \frac{8h^3}{6} f'''(x) + \frac{16h^4}{24} f^{iv}(x) - \frac{32h^5}{120} f^{v}(x) + \frac{64h^6}{720} f^{vi}(x) - \cdots \quad \text{--- (5)}$$

Adding (4) + (5)

LOOK !

$$f(x+2h) + f(x-2h) = 2f(x) + 4h^2 f''(x) + \frac{16h^4}{12} f^{iv}(x) + \underbrace{\frac{64h^6}{360}}_{\text{neglect}} f^{vi}(x) + \cdots$$

$$f''(x) = \frac{f(x+2h) + f(x-2h) - 2f(x)}{4h^2} - \frac{\cancel{4}^{1}}{\cancel{12}_{3}} h^4 f^{iv}(x)$$

$$- O(h^4)$$

--- (6)

Again, 4 × ③ yields,

$$4f''(x) = \frac{4f(x+h)+4f(x-h)-8f(x)}{h^2} - \frac{h^4}{3} f^{iv}(x) - O(h^4)$$

$$f''(x) = \frac{f(x+2h)+f(x-2h)-2f(x)}{4h^2} - \frac{h^4}{3} f^{iv}(x) - O(h^4) \qquad (\text{eqn } ⑥)$$

subtracting,

$$3f''(x) = \frac{4f(x+h)+4f(x-h)-8f(x)}{h^2} - \frac{f(x+2h)+f(x-2h)-2f(x)}{4h^2} - O(h^4)$$

$$\text{or, } f''(x) = \frac{4f(x+h)+4f(x-h)-8f(x)}{3h^2} - \frac{f(x+2h)+f(x-2h)-2f(x)}{12h^2} - O(h^4)$$

$$\boxed{f''(x) = \frac{-f(x-2h)+16f(x-h)-30f(x)+16f(x+h)-f(x)-f(x+2h)}{12h^2} - O(h^4)}$$

proved!