# Multinomial Distribution

A **multinomial distribution** is the probability distribution of the outcomes from a multinomial experiment.

## Multinomial Experiment

A **multinomial experiment** is a statistical experiment that has the following properties:

- The experiment consists of *n* repeated trials.
- Each trial has a discrete number of possible outcomes.
- On any given trial, the probability that a particular outcome will occur is constant.
- The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.

Consider the following statistical experiment. You toss two dice three times, and record the outcome on each toss. This is a multinomial experiment because:

- The experiment consists of repeated trials. We toss the dice three times.
- Each trial can result in a discrete number of outcomes - 2 through 12.
- The probability of any outcome is constant; it does not change from one toss to the next.
- The trials are independent; that is, getting a particular outcome on one trial does not affect the outcome on other trials.

**Note:** A binomial experiment is a special case of a multinomial experiment. Here is the main difference. With a binomial experiment, each trial can result in two - and only two - possible outcomes. With a multinomial experiment, each trial can have two *or more* possible outcomes.

## Multinomial Distribution

A **multinomial distribution** is the probability distribution of the outcomes from a multinomial experiment. The multinomial formula defines the probability of any outcome from a multinomial experiment.

**Multinomial Formula**. Suppose a multinomial experiment consists of *n* trials, and each trial can result in any of *k* possible outcomes: $E_1, E_2, \ldots, E_k$. Suppose, further, that each possible outcome can occur with probabilities $p_1, p_2, \ldots, p_k$. Then, the probability (P) that $E_1$ occurs $n_1$ times, $E_2$ occurs $n_2$ times, . . . , and $E_k$ occurs $n_k$ times is:

$$P = [ n! / ( n_1! * n_2! * ... n_k! ) ] * ( p_1^{n_1} * p_2^{n_2} * . . . * p_k^{n_k} )$$

where $n = n_1 + n_2 + . . . + n_k$.

The examples below illustrate how to use the multinomial formula to compute the probability of an outcome from a multinomial experiment.

## Test Your Understanding

**Problem 1**

Suppose a card is drawn randomly from an ordinary deck of playing cards, and then put back in the deck. This exercise is repeated five times. What is the probability of drawing 1 spade, 1 heart, 1 diamond, and 2 clubs?

*Solution:* To solve this problem, we apply the multinomial formula. We know the following:

- The experiment consists of 5 trials, so n = 5.
- The 5 trials produce 1 spade, 1 heart, 1 diamond, and 2 clubs; so $n_1 = 1$, $n_2 = 1$, $n_3 = 1$, and $n_4 = 2$.
- On any particular trial, the probability of drawing a spade, heart, diamond, or club is 0.25, 0.25, 0.25, and 0.25, respectively. Thus, $p_1 = 0.25$, $p_2 = 0.25$, $p_3 = 0.25$, and $p_4 = 0.25$.

We plug these inputs into the multinomial formula, as shown below:

$$P = [ n! / ( n_1! * n_2! * ... n_k! ) ] * ( p_1^{n_1} * p_2^{n_2} * . . . * p_k^{n_k} )$$

$$P = [ 5! / ( 1! * 1! * 1! * 2! ) ] * [ (0.25)^1 * (0.25)^1 * (0.25)^1 * (0.25)^2 ]$$

$$P = 0.05859$$

Thus, if we draw five cards with replacement from an ordinary deck of playing cards, the probability of drawing 1 spade, 1 heart, 1 diamond, and 2 clubs is 0.05859.

### Multinomial Calculator

The multinomial formula requires many time-consuming computations. The Multinomial Calculator can do this work for you - quickly, easily, and error-free. Use the Multinomial Calculator to compute the probability of outcomes from multinomial experiments. The calculator is free. It can found in the Stat Trek main menu under the Stat Tools tab. Or you can tap the button below.

Multinomial Calculator

**Problem 2**

Suppose we have a bowl with 10 marbles - 2 red marbles, 3 green marbles, and 5 blue marbles. We randomly select 4 marbles from the bowl, with replacement. What is the probability of selecting 2 green marbles and 2 blue marbles?

*Solution:* To solve this problem, we apply the multinomial formula. We know the following:

- The experiment consists of 4 trials, so n = 4.
- The 4 trials produce 0 red marbles, 2 green marbles, and 2 blue marbles; so $n_{red} = 0$, $n_{green} = 2$, and $n_{blue} = 2$.
- On any particular trial, the probability of drawing a red, green, or blue marble is 0.2, 0.3, and 0.5, respectively. Thus, $p_{red} = 0.2$, $p_{green} = 0.3$, and $p_{blue} = 0.5$

We plug these inputs into the multinomial formula, as shown below:

$$P = [ n! / ( n_1! * n_2! * ... n_k! ) ] * ( p_1^{n_1} * p_2^{n_2} * . . . * p_k^{n_k} )$$

$$P = [ 4! / ( 0! * 2! * 2! ) ] * [ (0.2)^0 * (0.3)^2 * (0.5)^2 ]$$

$$P = 0.135$$

Thus, if we draw 4 marbles with replacement from the bowl, the probability of drawing 0 red marbles, 2 green marbles, and 2 blue marbles is 0.135.

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