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Multinomial Distribution

Let a set of random variates X_1, X_2, \dots, X_n have a probability function

$$P(X_1 = x_1, \dots, X_n = x_n) = \frac{N!}{\prod_{i=1}^n x_i!} \prod_{i=1}^n \theta_i^{x_i} \tag{1}$$

where x_i are [nonnegative integers](#) such that

$$\sum_{i=1}^n x_i = N, \tag{2}$$

and θ_i are constants with $\theta_i > 0$ and

$$\sum_{i=1}^n \theta_i = 1. \tag{3}$$

Then the joint distribution of X_1, \dots, X_n is a multinomial distribution and $P(X_1 = x_1, \dots, X_n = x_n)$ is given by the corresponding coefficient of the [multinomial series](#)

$$(\theta_1 + \theta_2 + \dots + \theta_n)^N. \tag{4}$$

In the words, if X_1, X_2, \dots, X_n are mutually exclusive events with $P(X_1 = x_1) = \theta_1, \dots, P(X_n = x_n) = \theta_n$. Then the probability that X_1 occurs x_1 times, ..., X_n occurs x_n times is given by

$$P_N(x_1, x_2, \dots, x_n) = \frac{N!}{x_1! \cdots x_n!} \theta_1^{x_1} \cdots \theta_n^{x_n}. \tag{5}$$

(Papoulis 1984, p. 75).

The [mean](#) and [variance](#) of X_i are

$$\begin{aligned} \mu_i &= N \theta_i \\ \sigma_i^2 &= N \theta_i (1 - \theta_i). \end{aligned} \tag{6}$$

(7)

The [covariance](#) of X_i and X_j is

$$\sigma_{ij}^2 = -N \theta_i \theta_j. \tag{8}$$

SEE ALSO:
[Binomial Distribution](#), [Multinomial Coefficient](#)

REFERENCES:
Beyer, W. H. *CRC Standard Mathematical Tables, 28th ed.* Boca Raton, FL: CRC Press, p. 532, 1987.
Papoulis, A. *Probability, Random Variables, and Stochastic Processes, 2nd ed.* New York: McGraw-Hill, 1984.

Referenced on Wolfram|Alpha: [Multinomial Distribution](#)

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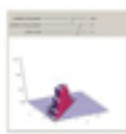
Bernoulli distribution



THINGS TO TRY:

- = Bernoulli distribution
- = discrete distributions
- = beta binomial distribution

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The Trinomial
Distribution

Step-by-Step
Math, Algebra &
Calculus Solver

STEP 2

For the integrand $\sec^{-1}(\sqrt{t})$, sub-
and $du = \frac{1}{2\sqrt{t}} dt$:
 $= 2 \int u \sec^{-1}(u) du$

STEP 3

Multiple intermediate steps

For the integrand $u \sec^{-1}(u)$, integrate by parts,
 $\int f dg = fg - \int g df$, where $f = \sec^{-1}(u)$, dg
 $df = \frac{1}{u\sqrt{u^2-1}} du$, $g = \frac{u^2}{2}$:
 $= u^2 \sec^{-1}(u) - \int \frac{u}{\sqrt{u^2-1}} du$

Next step

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