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1.7 - The Multinomial Distribution



What if the respondents in the survey had three choices:

1. I feel optimistic,
2. I don't feel optimistic,
3. I am not sure?

What would you consider as an appropriate sampling distribution? This is not a binomial distribution anymore. We have more than two possible outcomes. The multinomial distribution, however, is in many ways an extension of the binomial distribution. Let's look at this new distribution...

Origins

The multinomial distribution arises from an extension of the binomial experiment to situations where each trial has $k \geq 2$ possible outcomes.

Suppose that we have an experiment with

- n independent trials, where
- each trial produces exactly one of the events E_1, E_2, \dots, E_k (i.e. these events are mutually exclusive and collectively exhaustive), and
- on each trial, E_j occurs with probability $\pi_j, j = 1, 2, \dots, k$.

Notice that $\pi_1 + \pi_2 + \dots + \pi_k = 1$. The probabilities, regardless of how many possible outcomes, will always sum to 1.

Let's define the random variables:

X_1 = number of trials in which E_1 occurs,

X_2 = number of trials in which E_2 occurs,

...

X_k = number of trials in which E_k occurs.

Then $X = (X_1, X_2, \dots, X_k)$ is said to have a multinomial distribution with index n and parameter $\pi = (\pi_1, \pi_2, \dots, \pi_k)$. In most problems, n is regarded as fixed and known.

The individual components of a multinomial random vector are binomial and have a binomial distribution,

$X_1 \sim \text{Bin}(n, \pi_1),$

$X_2 \sim \text{Bin}(n, \pi_2),$

...

$X_k \sim \text{Bin}(n, \pi_k).$

The trials or each person's responses are independent, however, the components or the groups of these responses are not independent of each other. The sample sizes are different now and known. The number of responses for one can be determined from the others. In other words, even though the individual X_j 's are random, their sum:

$$X_1 + X_2 + \dots + X_k = n$$

is fixed. Therefore, the X_j 's are negatively correlated.

Notation

If $X = (X_1, X_2, \dots, X_k)$ is multinomially distributed with index n and parameter $\pi = (\pi_1, \pi_2, \dots, \pi_k)$, then we will write:

$$X \sim \text{Mult}(n, \pi)$$

