Effect Sizes

- Interpreting Cohen's d Effect Size

Difference family: Effect sizes based on differences between means [edit]

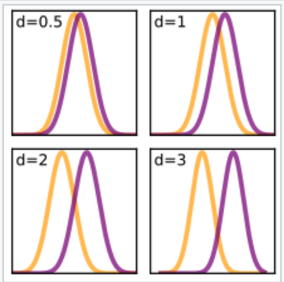
A (population) effect size θ based on means usually considers the standardized mean difference between two populations[20]:78

$$\theta = \frac{\mu_1 - \mu_2}{\sigma},$$

where μ_1 is the mean for one population, μ_2 is the mean for the other population, and σ is a standard deviation based on either or both populations.

In the practical setting the population values are typically not known and must be estimated from sample statistics. The several versions of effect sizes based on means differ with respect to which statistics are used.

This form for the effect size resembles the computation for a *t*-test statistic, with the critical difference that the *t*-test statistic includes a factor of $\sqrt{n}$. This means that for a given effect size, the significance level increases with the sample size. Unlike the *t*-test statistic, the effect size aims to estimate a population parameter and is not affected by the sample size.



Plots of Gaussian densities illustrating various values of Cohen's d.

Cohen's *d* [edit]

Cohen's *d* is defined as the difference between two means divided by a standard deviation for the data, *i.e.*

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s} = \frac{\mu_1 - \mu_2}{s}.$$

Jacob Cohen defined *s*, the pooled standard deviation, as (for two independent samples):[8]:67

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

where the variance for one of the groups is defined as

$$s_1^2 = \frac{1}{n_1 - 1}\sum_{i=1}^{n_1}(x_{1,i} - \bar{x}_1)^2,$$

and similarly for the other group.

The table below contains descriptors for magnitudes of $d = 0.01$ to 2.0, as initially suggested by Cohen and expanded by Sawilowsky.[9]

Effect size	*d*	Reference
Very small	0.01	[9]
Small	0.20	[8]
Medium	0.50	[8]
Large	0.80	[8]
Very large	1.20	[9]
Huge	2.0	[9]

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s_{pooled}}$$

where

$$s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

COHEN'S D

- Now compare to the one-sample t-statistic
- So $t = \frac{\bar{X} - \mu_X}{\frac{s}{\sqrt{N}}}$ $\quad t = d\sqrt{N} \quad and \quad d = \frac{t}{\sqrt{N}}$
- This shows how the test statistic (and its observed p-value) is in part determined by the effect size, but is confounded with sample size
- This means small effects may be statistically significant in many studies (esp. social sciences)

$$\text{Cohen's } d = \frac{M_1 - M_2}{SD_{pooled}}$$

$$\text{Glass's } \Delta = \frac{M_1 - M_2}{SD_{control}}$$

$$\text{Hedges' } g = \frac{M_1 - M_2}{SD^*_{pooled}}$$

Cohens d

- datacamp example

```
import numpy as np

def cohen_d(x,y):
    n1 = len(x)
    n2 = len(y)
    dof = n1 + n2 - 2
    diff = np.mean(x) - np.mean(y)
    #s_pooled = np.sqrt(((n1-1)*np.std(x, ddof=1) ** 2 + (n2-1)*np.std(y,
ddof=1) ** 2) / dof)
    s_pooled = np.sqrt(((n1-1)*np.var(x, ddof=1) + (n2-1)*np.var(y,
ddof=1)) / dof)
    cohens_d = diff / s_pooled
    return cohens_d

#dummy data
x = [2,4,7,3,7,35,8,9]
y = [i*2 for i in x]
# extra element so that two group sizes are not equal.
x.append(10)

# calculate d
d = cohen_d(x,y)
print(d) # -0.5720156046660209
```