

coin toss

$$H_0: p=0.5 \quad H_1: p<0.5$$

$$\hat{p} = 0.38, \quad p_o = 0.50$$

if H_0 is true, if the coin is fair, then

$$SD(p_o) = \sqrt{\frac{p_o q_o}{n}} = \sqrt{\frac{0.5 \times 0.5}{100}} = 0.05$$

$$z = \frac{(\hat{p} - p_o)}{SD(\hat{p})} = \frac{(.38 - .50)}{0.05} = -2.4$$

code

```
import numpy as np
import pandas as pd
from scipy import stats

p = 0.5
q = 1-p
h = 38
n = 100
p_hat = float(h)/n

std = np.sqrt(p*q/n)
z_score = (p_hat-p)/std

print(f'z_score = {z_score}') # negative means left side of curve.
# z_score = -2.4

p_value = stats.norm.cdf(z_score) # one-sided
print(f"p_value = {p_value:.4f}") # p_value = 0.0082

alpha = 0.01 # 1% significance level alpha (or, 99% confidence level
gamma)

if p_value < alpha:
    print('We reject Null hypothesis that the coin is unbiased.')
    print('The coin is biased towards the tail.')
```

```
else:
    print('We fail to reject Null Hypothesis')
    print('We do not have sufficient evidence to say that coin is biased
towards tail.')

# We reject Null hypothesis that the coin is unbiased.
# The coin is biased towards the tail.
```

Questions


A coin is tossed 6 times. What is the probability of getting at least one heads? What about all heads?

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 21 Answers



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Answered 3 years ago · Upvoted by Richard Enison, M.A. Mathematics & Mathematical Logic and Foundations, University of California, Berkeley (1974) · Author has **508** answers and **795.7K** answer views

Originally Answered: A coin is tossed 6 times. What is the probability of getting at least one head? All heads?

Let X represent the number of times a coin comes up heads. Assuming that the probability that heads and tails are equal, then X follows a binomial distribution with $n = 6$ and $p = 0.5$.

The probability mass function for a binomial distribution is

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

where n is the number of trials, x is the number of successes (for this problem we consider getting a head as a success), p is the probability of success and q is the probability of failure (where $q = 1 - p$).

To find the probability of getting at least one head, you can do the following:

(The first equality below is due to the complement rule $P(A) = 1 - P(A')$ where A' is the complement of the event A .)

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - \binom{6}{0} (0.5)^0 (0.5)^6$$

$$= 0.984375$$

and you get the same answer of 0.984375.

The probability of getting all heads is

$$P(X = 6) = \binom{6}{6} (0.5)^6 (0.5)^0$$

$$= 0.015625$$

The line of code below shows how you can do this in R

```
1 dbinom(x=6,size = 6,prob = 0.5)
```

Again, you get the answer for $P(X = 6)$ to be 0.015625.

If you think of the tosses as a sequence of 6 flips, there are 2^6 total possibilities.

There is only 1 way to get all heads, so the probability of getting all heads is $\frac{1}{2^6} = \frac{1}{64}$.

To get the probability of getting at least one head, this is the opposite of the probability of getting no heads - i.e. all tails.

The probability of getting all tails is $\frac{1}{2^6} = \frac{1}{64}$.

To get the probability of getting at least one head, we subtract this from 1 to get:

$$1 - \frac{1}{2^6} = 1 - \frac{1}{64} = \frac{63}{64}.$$

Examples

Draw samples from the distribution:

```
>>> n, p = 10, .5 # number of trials, probability of each trial
>>> s = np.random.binomial(n, p, 1000)
# result of flipping a coin 10 times, tested 1000 times.
```

A real world example. A company drills 9 wild-cat oil exploration wells, each with an estimated probability of success of 0.1. All nine wells fail. What is the probability of that happening?

Let's do 20,000 trials of the model, and count the number that generate zero positive results.

```
>>> sum(np.random.binomial(9, 0.1, 20000) == 0)/20000.
# answer = 0.38885, or 38%.
```

Question 1: Nathan makes 60% of his free-throw attempts. If he shoots 12 free throws, what is the probability that he makes exactly 10?

```
from scipy.stats import binom

#calculate binomial probability
binom.pmf(k=10, n=12, p=0.6)

0.0639
```

The probability that Nathan makes exactly 10 free throws is **0.0639**.

Question 2: Marty flips a fair coin 5 times. What is the probability that the coin lands on heads 2 times or fewer?

```
from scipy.stats import binom

#calculate binomial probability
binom.cdf(k=2, n=5, p=0.5)

0.5
```

The probability that the coin lands on heads 2 times or fewer is **0.5**.

Question 3: It is known that 70% of individuals support a certain law. If 10 individuals are randomly selected, what is the probability that between 4 and 6 of them support the law?

```
from scipy.stats import binom

#calculate binomial probability
binom.cdf(k=6, n=10, p=0.7) - binom.cdf(k=3, n=10, p=0.7)

0.3398
```

The probability that between 4 and 6 of the randomly selected individuals support the law is **0.3398**.