Maximum Likelihood Estimates
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1 Learning Goals

1. Be able to define the likelihood function for a parametric model given data.
2. Be able to compute the maximum likelihood estimate of unknown parameter(s).

2 Introduction

Suppose we know we have data consisting of values $x_1, \ldots, x_n$ drawn from an exponential distribution. The question remains: which exponential distribution?!

We have casually referred to *the* exponential distribution or *the* binomial distribution or *the* normal distribution. In fact the exponential distribution $\exp(\lambda)$ is not a single distribution but rather a one-parameter family of distributions. Each value of λ defines a different distribution in the family, with pdf $f_\lambda(x) = \lambda e^{-\lambda x}$ on $[0, \infty)$. Similarly, a binomial distribution $\text{bin}(n, p)$ is determined by the two parameters n and p, and a normal distribution $N(\mu, \sigma^2)$ is determined by the two parameters μ and σ^2 (or equivalently, μ and σ). Parameterized families of distributions are often called parametric distributions or parametric models.

We are often faced with the situation of having random data which we know (or believe) is drawn from a parametric model, whose parameters we do not know. For example, in an election between two candidates, polling data constitutes draws from a Bernoulli(p) distribution with unknown parameter p. In this case we would like to use the data to estimate the value of the parameter p, as the latter predicts the result of the election. Similarly, assuming gestational length follows a normal distribution, we would like to use the data of the gestational lengths from a random sample of pregnancies to draw inferences about the values of the parameters μ and σ^2.

Our focus so far has been on computing the probability of data arising from a parametric model with known parameters. Statistical inference flips this on its head: we will estimate