Multinomial Distribution

Let a set of random variates X_1, X_2, ..., X_n have a probability function

$$P(X_1 = x_1, \ldots, X_n = x_n) = \frac{N!}{\prod_{i=1}^n x_i!} \prod_{i=1}^n \theta_i^{x_i} \qquad (1)$$

where x_i are nonnegative integers such that

$$\sum_{i=1}^n x_i = N, \qquad (2)$$

and θ_i are constants with $\theta_i > 0$ and

$$\sum_{i=1}^n \theta_i = 1. \qquad (3)$$

Then the joint distribution of X_1, ..., X_n is a multinomial distribution and $P(X_1 = x_1, \ldots, X_n = x_n)$ is given by the corresponding coefficient of the multinomial series

$$(\theta_1 + \theta_2 + \ldots + \theta_n)^N. \qquad (4)$$

In the words, if X_1, X_2, ..., X_n are mutually exclusive events with $P(X_1 = x_1) = \theta_1$, ..., $P(X_n = x_n) = \theta_n$. Then the probability that X_1 occurs x_1 times, ..., X_n occurs x_n times is given by

$$P_N(x_1, x_2, \ldots, x_n) = \frac{N!}{x_1! \cdots x_n!} \theta_1^{x_1} \cdots \theta_n^{x_n}. \qquad (5)$$

(Papoulis 1984, p. 75).

The mean and variance of X_i are

$$\mu_i = N\,\theta_i \qquad (6)$$
$$\sigma_i^2 = N\,\theta_i\,(1 - \theta_i). \qquad (7)$$

The covariance of X_i and X_j is

$$\sigma_{ij}^2 = -N\,\theta_i\,\theta_j. \qquad (8)$$

SEE ALSO:
Binomial Distribution, Multinomial Coefficient

REFERENCES:
Beyer, W. H. *CRC Standard Mathematical Tables, 28th ed.* Boca Raton, FL: CRC Press, p. 532, 1987.
Papoulis, A. *Probability, Random Variables, and Stochastic Processes, 2nd ed.* New York: McGraw-Hill, 1984.

Referenced on Wolfram|Alpha: Multinomial Distribution

CITE THIS AS:
Weisstein, Eric W. "Multinomial Distribution." From *MathWorld*--A Wolfram Web Resource. https://mathworld.wolfram.com/MultinomialDistribution.html