Studies show that listening to music while studying can improve your memory. To demonstrate this, a researcher obtains a sample of 36 college students and gives them a standard memory test while they listen to some background music. Under normal circumstances (without music), the mean score obtained was 25 and standard deviation is 6. The mean score for the sample after the experiment (i.e With music) is 28.

265) What is the null hypothesis in this case?

A) Listening to music while studying will not impact memory.
B) Listening to music while studying may worsen memory.
C) Listening to music while studying may improve memory.
D) Listening to music while studying will not improve memory but can make it worse.

Solution: (D)

The null hypothesis is generally assumed statement, that there is no relationship in the measured phenomena. Here the null hypothesis would be that there is no relationship between listening to music and improvement in memory.

266) What would be the Type I error?

A) Concluding that listening to music while studying improves memory, and it's right.
B) Concluding that listening to music while studying improves memory when it actually doesn't.
C) Concluding that listening to music while studying does not improve memory but it does.

Solution: (B)

Type 1 error means that we reject the null hypothesis when its actually true. Here the null hypothesis is that music does not improve memory. Type 1 error would be that we reject it and say that music does improve memory when it actually doesn't.

267) After performing the Z-test, what can we conclude ____ ?

A) Listening to music does not improve memory.

B)Listening to music significantly improves memory at p

C) The information is insufficient for any conclusion.

D) None of the above

Solution: (B)

Let's perform the Z test on the given case. We know that the null hypothesis is that listening to music does not improve memory.

Alternate hypothesis is that listening to music does improve memory.

In this case the standard error i.e. $\frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{36}} = 1$

The Z score for a sample mean of 28 from this population is

$$Z = \frac{\textit{sample mean} - \textit{population mean}}{\text{standard error}} = \frac{28-25}{1} = 3$$

Z critical value for $\alpha = 0.05$ (one tailed) would be 1.65 as seen from the z table.

Therefore since the Z value observed is greater than the Z critical value, we can reject the null hypothesis and say that listening to music does improve the memory with 95% confidence.

A medical doctor wants to reduce blood sugar level of all his patients by altering their diet. He finds that the mean sugar level of all patients is 180 with a standard deviation of 18. Nine of his patients start dieting and the mean of the sample is observed to 175. Now, he is considering to recommend all his patients to go on a diet.

Note: He calculates 99% confidence interval.

270) What is the standard error of the mean?

A) 9
B) 6
C) 7.5
D) 18

Solution: (B)

The standard error of the mean is the standard deviation by the square root of the number of values. i.e.

Standard error = $18/\sqrt{9}$ = 6

271) What is the probability of getting a mean of 175 or less after all the patients start dieting?

A) 20%
B) 25%
C) 15%
D) 12%

Solution: (A)

This actually wants us to calculate the probability of population mean being 175 after the intervention. We can calculate the Z value for the given mean.

$$Z = \frac{\text{sample mean} - \text{population}}{\text{standard error}} = \frac{175-180}{6}$$

$$Z = -\frac{5}{6} = -0.833$$

If we look at the z table, the corresponding value for z = -0.833 ~ 0.2033.

Therefore there is around 20% probability that if everyone starts dieting, the population mean would be 175.

272) Which of the following statement is correct?

A) The doctor has a valid evidence that dieting reduces blood sugar level.

B) The doctor does not have enough evidence that dieting reduces blood sugar level.

C) If the doctor makes all future patients diet in a similar way, the mean blood pressure will fall below 160.

Solution: (B)

We need to check if we have sufficient evidence to reject the null. The null hypothesis is that dieting has no effect on blood sugar. This is a two tailed test. The z critical value for a 2 tailed test would be ± 2.58.

The z value as we have calculated is -0.833.

Since Z value < Z critical value, we do not have enough evidence that dieting reduces blood sugar.

272) Which of the following statement is correct?

A) The doctor has a valid evidence that dieting reduces blood sugar level.

B) The doctor does not have enough evidence that dieting reduces blood sugar level.

C) If the doctor makes all future patients diet in a similar way, the mean blood pressure will fall below 160.

Solution: (B)

We need to check if we have sufficient evidence to reject the null. The null hypothesis is that dieting has no effect on blood sugar. This is a two tailed test. The z critical value for a 2 tailed test would be ± 2.58.

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Since Z value < Z critical value, we do not have enough evidence that dieting reduces blood sugar.

A researcher is trying to examine the effects of two different teaching methods. He divides 20 students into two groups of 10 each. For group 1, the teaching method is using fun examples. Where as for group 2 the teaching method is using software to help students learn. After a 20 minutes lecture of both groups, a test is conducted for all the students.

We want to calculate if there is a significant difference in the scores of both the groups.

It is given that:

- Alpha=0.05, two tailed.
- Mean test score for group 1 = 10
- Mean test score for group 2 = 7
- Standard error = 0.94

273) What is the value of t-statistic?

A) 3.191
B) 3.395
C) Cannot be determined.
D) None of the above

Solution: (A)

The t statistic of the given group is nothing but the difference between the group means by the standard error.

=(10-7)/0.94 = 3.191

274) Is there a significant difference in the scores of the two groups?

A) Yes
B) No

Solution: (A)

The null hypothesis in this case would be that there is no difference between the groups, while the alternate hypothesis would be that the groups are significantly different.

The t critical value for a 2 tailed test at $\alpha = 0.05$ is ± 2.101. The t statistic obtained is 3.191. Since the t statistic is more than the critical value of t, we can reject the null hypothesis and say that the two groups are significantly different with 95% confidence.

275) What percentage of variability in scores is explained by the method of teaching?

A) 36.13
B) 45.21
C) 40.33
D) 32.97

Solution: (A)

The % variability in scores is given by the R^2 value. The formula for R^2 given by

$$R^2 = \frac{\text{t square}}{\text{t square} + \text{degree of freedom}}$$

The degrees of freedom in this case would be 10+10 -2 since there are two groups with size 10 each. The degree of freedom is 18.

$$R^2 = \frac{3.191 * 3.191}{(3.191 * 3.191) + 18} = 36.13$$

22. How do you calculate needed sample size?

Estimate a population mean:

- General formula is $ME = t \times \frac{S}{\sqrt{n}}$ or $ME = z \times \frac{s}{\sqrt{n}}$
- ME is the desired margin of error
- t is the t score or z score that we need to use to calculate our confidence interval
- s is the standard deviation

Example: we would like to start a study to estimate the average internet usage of households in one week for our business plan. How many households must we randomly select to be 95% sure that the sample mean is within 1minute from the true mean of the population? A previous survey of household usage has shown a standard deviation of 6.95 minutes.

- Z score corresponding to a 95% interval: 1.96 (97.5%, $\frac{\alpha}{2} = 0.025$)
- $s = 6.95$
- $n = (\frac{z \times s}{ME})^2 = (1.96 \times 6.95)^2 = 13.62^2 = 186$

Estimate a proportion:

- Similar: $ME = z \times \sqrt{\frac{p(1-p)}{n}}$

Example: a professor in Harvard wants to determine the proportion of students who support gay marriage. She asks "how large a sample do I need?"

She wants a margin of error of less than 2.5%, she has found a previous survey which indicates a proportion of 30%.

$n = \frac{0.3 \times 0.7}{0.025^2}$

39. In a study of emergency room waiting times, investigators consider a new and the standard triage systems. To test the systems, administrators selected 20 nights and randomly assigned the new triage system to be used on 10 nights and the standard system on the remaining 10 nights. They calculated the nightly median waiting time (MWT) to see a physician. The average MWT for the new system was 3 hours with a variance of 0.60 while the average MWT for the old system was 5 hours with a variance of 0.68. Consider the 95% confidence interval estimate for the differences of the mean MWT associated with the new system. Assume a constant variance. What is the interval? Subtract in this order (New System - Old System).

t confidence interval for the difference of the means assuming equal variances:

$(new - old) \pm t' \times sp \times \sqrt{(\frac{1}{n_1})^2 + (\frac{1}{n_2})^2}$

- t': 97.5% quantile, with 20+10-2=28 degrees of freedom: 2.1
- sp: pooled variance, $\sqrt{\frac{0.6^2 \times 9 + 0.68^2 \times 9}{10+10-2}} = 0.8$
- $\sqrt{1/10 + 1/10} = 0.44$
- We get $[-2.75, -1.25]$

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40. To further test the hospital triage system, administrators selected 200 nights and randomly assigned a new triage system to be used on 100 nights and a standard system on the remaining 100 nights. They calculated the nightly median waiting time (MWT) to see a physician. The average MWT for the new system was 4 hours with a standard deviation of 0.5 hours while the average MWT for the old system was 6 hours with a standard deviation of 2 hours. Consider the hypothesis of a decrease in the mean MWT associated with the new treatment. What does the 95% independent group confidence interval with unequal variances suggest vis a vis this hypothesis? (Because there's so many observations per group, just use the Z quantile instead of the T.)

- Z confidence interval for the differences of the means assuming unequal variances:
  $(new - old) \pm z' \times sp \times \sqrt{(\frac{1}{n_1})^2 + (\frac{1}{n_2})^2}$
- $Z_{97.5}$ quantile
- sp: pooled variance, $\sqrt{\frac{0.5^2 \times 99 + 2^2 \times 99}{(100+100-2)}} = 1.458$
- we get: $[1.6, 2.4]$

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