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Forces review

Simple problems

1. A cow is pushed simultaneously by two strong, independent persons of equal strength on opposing sides, at equal force. What is the net force on the cow?

   (a) Add the forces.

$$\text{let F be the force of one person.} \tag{1}$$
$$F_{\text{net}} = F + (-F) \qquad \text{Substitute.} \tag{2}$$
$$F_{\text{net}} = 0\,\text{N} \qquad \text{Evaluate.} \tag{3}$$

   The net force exerted on the truck is 0 N.

2. A vector of magnitude 5 and direction of 0° is added to a vector of magnitude 10 and direction of 0°. What is the resulting vector?

   (a) Add the vectors.

$$\vec{V} = 5 + 10 \qquad \text{Add the magnitudes.} \tag{4}$$

   The net vector is 15, 0°.

3. A vector of magnitude 3 and direction of 90° is added to a vector of magnitude 4 and direction of 0°. What is the resulting vector?

   (a) Find the magnitude of the resulting vector.

$$\vec{V}^2 = 3^2 + 4^2 \qquad \text{Use the Pythagorean theorem and substitute.} \tag{5}$$
$$\vec{V}^2 = 5 \qquad \text{Solve for } \vec{V}^2. \tag{6}$$

   The net vector is 15, 0°.

4. A gorilla pulls straight downwards with a force of 1000.0 N on a 100.0 g apple. What is the net force on the apple?

   (a) Evaluate for the force of gravity.

$$F = mg \qquad \text{From F=ma, substituting g for a.} \tag{7}$$
$$F = 0.98\,\text{N} \qquad \text{Substitute and evaluate} \tag{8}$$

(b) Add the magnitudes of the forces.

$$F_{\text{net}} = 0.98\,\text{N} + 1000.0\,\text{N} \quad \text{Add the forces.} \tag{9}$$
$$F_{\text{net}} = 1001.0\,\text{N} \quad \text{Evaluate.} \tag{10}$$

The net force exerted on the apple is 1001.0 N.

5. A 2000.0 kg car accelerates at $11.5\,\text{m}\,\text{s}^{-2}$. If the coefficient of friction acting on the car is roughly 0.1, how much force does the engine exert?

(a) Solve for the normal force.

$$F_N = F_g \tag{11}$$
$$F_N = mg \quad \text{Substitute.} \tag{12}$$
$$F_N = 2000.0\,\text{kg} \cdot g \quad \text{Substitute.} \tag{13}$$
$$F_N = 19\,613\,\text{N} \quad \text{Evaluate.} \tag{14}$$

(b) Solve for the force of friction.

$$F_f = \mu F_N \tag{15}$$
$$F_f = 0.1 \cdot 19\,613\,\text{N} \quad \text{Substitute.} \tag{16}$$
$$F_f = 1961.3\,\text{N} \quad \text{Evaluate.} \tag{17}$$

(c) Solve for the applied force.

$$F_{\text{net}} = F_a - F_f = ma \tag{18}$$
$$F_a - 1961.3\,\text{N} = 2000.0\,\text{kg} \cdot 11.5\,\frac{\text{m}}{\text{s}^2} \quad \text{Substitute.} \tag{19}$$
$$F_a = 24\,900\,\text{N} \quad \text{Solve for } F_a. \tag{20}$$

The net force exerted by the engine is 24 900 N.

Standard problems

1. How much force does a railgun exert on a 25.0 kg projectile as it accelerates from rest to $3500.0\,\text{m}\,\text{s}^{-1}$ in the 10.0 m long rails?

(a) Solve for acceleration.

$$v^2 = v_0^2 + 2ad \quad \text{From kinematics.} \tag{21}$$
$$(3500\,\frac{\text{m}}{\text{s}})^2 = (0\,\frac{\text{m}}{\text{s}})^2 + 2 \cdot a \cdot 10.0\,\text{m} \quad \text{Substitute.} \tag{22}$$
$$a = 6.12 \times 10^5\,\frac{\text{m}}{\text{s}^2} \quad \text{Solve for acceleration.} \tag{23}$$

(b) Substitute into $F = ma$ for force.

$$F = ma \quad \text{From Newton's second law.} \tag{24}$$
$$F = 25.0\,\text{kg} \cdot 6.12 \times 10^5\,\frac{\text{m}}{\text{s}^2} \quad \text{Substitute.} \tag{25}$$
$$F = 1.53 \times 10^7\,\text{N} \quad \text{Evaluate.} \tag{26}$$

The force exerted on the projectile is 1.53×10^7 N.

2. A 7.29 kg bowling ball is kicked simultaneously by two bowlers. When looking at the situation from top-down, the first bowler kicks it with 100.0 N of force north, and the second bowler kicks it with 78.56 N of force west.

(a) What is the force on the ball?

i. Solve for the magnitude of the force.

$$c^2 = a^2 + b^2 \quad \text{Use the Pythagorean theorem.} \tag{27}$$
$$c^2 = (78.56\,\text{N})^2 + (100.0\,\text{N})^2 \quad \text{Substitute.} \tag{28}$$
$$c = 127.2\,\text{N} \quad \text{Solve for c.} \tag{29}$$

ii. Solve for the direction of the force.

$$\theta = \arctan(\frac{78.56}{100.0}) \quad \text{Use the inverse tangent function.} \tag{30}$$
$$\theta = 38.15^\circ \quad \text{Evaluate for } \theta. \tag{31}$$

The force on the ball is 127.2 N 38.15° west of north.

(b) What is the ball's acceleration?

i. Substitute into $F = ma$ for acceleration.

$$F = ma \quad \text{From Newton's second law.} \tag{32}$$
$$127.2\,\text{N} = 7.29\,\text{kg} \cdot a \quad \text{Substitute.} \tag{33}$$
$$a = 17.44\,\frac{\text{m}}{\text{s}^2} \quad \text{Solve for acceleration.} \tag{34}$$

The acceleration of the ball is $17.44\,\text{m}\,\text{s}^{-2}$ 38.15° west of north.

3. A 34.8 kg medium-sized furry animal is sliding down a friction-less 1° inclined plane. What is its acceleration?

(a) Solve for the force of the animal due to gravity.

$$F = mg \quad \text{From F=ma, substituting g for a.} \tag{35}$$
$$F = 34.8\,\text{kg} \cdot g \quad \text{Substitute.} \tag{36}$$
$$F = 341\,\text{N} \quad \text{Evaluate.} \tag{37}$$

(b) Solve for the force in the x direction..

$$F = 341\,\mathrm{N} \cdot \sin(1^\circ) \qquad \text{Substitute.} \tag{38}$$
$$F = 5.95\,\mathrm{N} \qquad \text{Evaluate.} \tag{39}$$

(c) Solve for the acceleration.

$$F = ma \tag{40}$$
$$5.95\,\mathrm{N} = 34.8\,\mathrm{kg} \cdot a \qquad \text{Substitute.} \tag{41}$$
$$a = 0.171\,\frac{\mathrm{N}}{\mathrm{kg}} \qquad \text{Evaluate.} \tag{42}$$

The acceleration of the furry animal is $0.171\,\mathrm{N\,kg^{-1}}$ in the x direction.

4. A 2.00 kg small furry animal is on a friction-less table. Attached to it on a pulley and dangling off the table is a 999.99 kg anvil. What is the acceleration of the animal?

(a) Solve for the force of the anvil due to gravity.

$$F_g = mg \qquad \text{From F=ma, substituting g for a.} \tag{43}$$
$$F = 999.99\,\mathrm{kg} \cdot g \qquad \text{Substitute.} \tag{44}$$
$$F = 9806.6\,\mathrm{N} \qquad \text{Evaluate.} \tag{45}$$

(b) Solve for the force applied to the 2.00 kg small furry animal.

$$9806.6\,\mathrm{N} = (2.00\,\mathrm{kg} + 999.99\,\mathrm{kg})\cdot \qquad \text{Substitute from F=ma.} \tag{46}$$
$$F = 5.95\,\mathrm{N} \qquad \text{Evaluate.} \tag{47}$$

(c) Solve for the acceleration.

$$F = ma \tag{48}$$
$$5.95\,\mathrm{N} = 34.8\,\mathrm{kg} \cdot a \qquad \text{Substitute.} \tag{49}$$
$$a = 0.171\,\frac{\mathrm{N}}{\mathrm{kg}} \qquad \text{Evaluate.} \tag{50}$$

The acceleration of the furry animal is $0.171\,\mathrm{N\,kg^{-1}}$ in the x direction.

5. Two rabbits are tied together with a string. There is a friction-less right triangular ramp. One rabbit is dangling from a pulley, the other situated on the triangular ramp. The pulley is attached to the top vertex of the triangle.
The triangle's angles are as follows: 90°, 30°, which are situated on the ground, and 60°. The rabbits both weigh 1500.0 kg. What is the acceleration of the dangling rabbit?

(a) Find the force due to gravity of the dangling rabbit.

$$F = mg \tag{51}$$
$$F = 1500.0\,\mathrm{kg} \cdot g \tag{52}$$
$$F = 14\,709\,\mathrm{N} \tag{53}$$

(b) Find the force due to gravity in the x direction of the rabbit on the ramp.

$$
\begin{aligned}
F &= mg \cdot \sin(30°) && (54)\\
F &= 1500.0\,\mathrm{kg} \cdot g \cdot \sin(30°) && (55)\\
F &= 7355.0\,\mathrm{N} && (56)
\end{aligned}
$$

(c) Find the acceleration of the system of rabbits.

$$
\begin{aligned}
F &= ma && (57)\\
7355.0\,\mathrm{N} &= 3000.0\,\mathrm{kg} \cdot a && (58)\\
a &= 2.452\,\frac{\mathrm{N}}{\mathrm{kg}} && (59)
\end{aligned}
$$

The acceleration of the dangling rabbit is $2.452\,\mathrm{N\,kg^{-1}}$.

"Challenging" questions

1. When a 500.0 kg hamster is sliding down a friction-less ramp, it takes 400.0 N of force directed up the ramp to keep its velocity constant. Calculate the angle of the incline.

(a) Write an expression equating the forces.

$$
\begin{aligned}
F_{gx} &= F_A && && (60)\\
\sin(\theta) \cdot 500.0\,\mathrm{kg} \cdot g &= 400.0\,\mathrm{N} && \text{Substitute.} && (61)\\
\theta &= 4.68° && \text{Solve for theta.} && (62)
\end{aligned}
$$

The angle is 4.68°.

2. When that same hamster is being sliding down a new, frictioned ramp, it takes 600.0 N to keep its velocity constant. When it is pushed up, it takes 1819.73 N to keep the velocity constant. What is the new angle and coefficient of friction?

(a) Write an expression equating the forces when sliding down.

$$
\begin{aligned}
F_{gx} &= F_A + F_f && && (63)\\
\sin(\theta) \cdot 500.0\,\mathrm{kg} \cdot g &= 600.0\,\mathrm{N} + F_f && \text{Substitute.} && (64)\\
F_f &= F_{gx} - 600.0\,\mathrm{N} && \text{Solve for } F_f. && (65)
\end{aligned}
$$

(b) Write an expression equating the forces when pushed up.

$$
\begin{aligned}
1819.73\,\mathrm{N} &= F_f + F_{gx} && && (66)\\
1819.73\,\mathrm{N} &= 2F_{gx} + 600.0\,\mathrm{N} && \text{Substitute.} && (67)\\
F_{gx} &= 1210.0\,\mathrm{N} && \text{Solve for } F_{gx}. && (68)
\end{aligned}
$$

(c) Solve for the angle.

$$F_{gx} = \sin(\theta) \cdot 500.0\,\mathrm{kg} \cdot g \tag{69}$$
$$1210.0\,\mathrm{N} = \sin(\theta) \cdot 500.0\,\mathrm{kg} \cdot g \qquad \text{Substitute.} \tag{70}$$
$$\theta = 14.30^\circ \qquad \text{Solve for } \theta. \tag{71}$$

(d) Solve for the coefficient of friction.

$$F_f = \mu F_N \tag{72}$$
$$F_{gx} - 600.0\,\mathrm{N} = \mu \cdot \cos(\theta) \cdot 500.0\,\mathrm{kg} \cdot g \qquad \text{Substitute.} \tag{73}$$
$$609.8\,\mathrm{N} = \mu \cdot \cos(14.30^\circ) \cdot 500.0\,\mathrm{kg} \cdot g \qquad \text{Further substitute.} \tag{74}$$
$$\mu = 0.128 \qquad \text{Solve for } \mu. \tag{75}$$

The new angle is 14.30° and the coefficient of friction is 0.128.

Conceptual questions

1. What is the $\int_a^b F(x)dx$?
   Work.

2. Is the acceleration of a free falling object proportional to its mass?
   No. In a gravitational field with a large enough difference between m_1 and m_2, the acceleration of a free falling object is the same regardless of the mass of the falling object.

3. Does a constant net force result in a constant velocity?
   No. At not-close-to-light speeds, constant net force results in a constant acceleration, as acceleration is proportional to force.

4. Would a graph of acceleration with respect to mass be concave up, down, or $\frac{d^2a}{dm^2} = 0$?
   As a is inversely proportional to mass ceteris paribus, $\frac{d}{dm}\frac{F}{m} = \frac{-F}{m^2}$. $\frac{d^2}{dm^2}\frac{F}{m} = \frac{2F}{m^3}$. Therefore, if force is positive and mass is positive, such a graph must be concave up.