Find an anti-derivative (or integral) of the following functions by the method of inspection.

1. $\sin 2x$

3. e^{2x}

5. $\sin 2x - 4e^{3x}$

2. $\cos 3x$

4. $(ax + b)^2$

Find the following integrals in Exercises 6 to 20:

6. $\int (4e^{3x} + 1)\, dx$

9. $\int (2x^2 + e^x)\, dx$

12. $\int \dfrac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

7. $\int x^2 \left(1 - \frac{1}{x^2}\right) dx$

10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

13. $\int \dfrac{x^3 - x^2 + x - 1}{x - 1}\, dx$

8. $\int (ax^2 + bx + c)\, dx$

11. $\int \dfrac{x^3 + 5x^2 - 4}{x^2}\, dx$

14. $\int (1 - x)\sqrt{x}\, dx$

15. $\int \sqrt{x}(3x^2 + 2x + 3)\, dx$

19. $\int \dfrac{\sec^2 x}{\csc^2 x}\, dx$

16. $\int (2x - 3\cos x + e^x)\, dx$

17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

18. $\int \sec x(\sec x + \tan x)\, dx$

20. $\int \dfrac{2 - 3\sin x}{\cos^2 x}\, dx$

Choose the correct answer in Exercises 21 and 22.

21. The anti-derivative of $\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{3/2} + 2x^{1/2} + C$ (B) $\frac{2}{3}x^{3/2} + \frac{1}{2}x^{1/2} + C$

(C) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$ (D) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

22. If $\dfrac{d}{dx} f(x) = 4x^3 - \dfrac{3}{x^4}$ such that $f(2) = 0$, then $f(x)$ is

(A) $x^4 + \dfrac{1}{x^3} - \dfrac{129}{8}$ (B) $x^3 + \dfrac{1}{x^4} + \dfrac{129}{8}$

(C) $x^4 + \dfrac{1}{x^3} + \dfrac{129}{8}$ (D) $x^3 + \dfrac{1}{x^4} - \dfrac{129}{8}$

Find the integrals of the functions in Exercises 1 to 22:

1. $\sin^2(2x + 5)$

7. $\sin 4x \sin 8x$

12. $\dfrac{\sin^2 x}{1 + \cos x}$

2. $\sin 3x \cos 4x$

8. $\dfrac{1 - \cos x}{1 + \cos x}$

3. $\cos 2x \cos 4x \cos 6x$

13. $\dfrac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$

9. $\dfrac{\cos x}{1 + \cos x}$

4. $\sin^3(2x + 1)$

14. $\dfrac{\cos x - \sin x}{1 + \sin 2x}$

5. $\sin^3 x \cos^3 x$

10. $\sin^4 x$

6. $\sin x \sin 2x \sin 3x$

11. $\cos^4 2x$

15. $\tan^3 2x \sec 2x$

1

16. $\tan^4 x$

17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

18. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

19. $\dfrac{1}{\sin x \cos^3 x}$

20. $\dfrac{\cos 2x}{(\cos x + \sin x)^2}$

21. $\sin^{-1}(\cos x)$

22. $\dfrac{1}{\cos(x-a)\cos(x-b)}$

Choose the correct answer in Exercises 23 and 24.

23. $\displaystyle\int \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$ is equal to

(A) $\tan x + \cot x + C$ (B) $\tan x + \csc x + C$ (C) $-\tan x + \cot x + C$ (D) $\tan x + \sec x + C$

24. $\displaystyle\int \dfrac{e^x(1+x)}{\cos^2(e^x)}\, dx$ equals

(A) $-\cot(e^x) + C$ (B) $\tan(xe^x) + C$ (C) $\tan(e^x) + C$ (D) $\cot(e^x) + C$

$$\boxed{\textbf{EXERCISE 7.4}}$$

Integrate the functions in Exercises 1 to 23:

1. $\dfrac{3x^2}{x^6 + 1}$

2. $\dfrac{1}{\sqrt{1 + 4x^2}}$

3. $\dfrac{1}{\sqrt{(2 - x)^2 + 1}}$

4. $\dfrac{1}{\sqrt{9 - 25x^2}}$

5. $\dfrac{3x}{1 + 2x^4}$

6. $\dfrac{x^2}{1 - x^6}$

7. $\dfrac{x - 1}{\sqrt{x^2 - 1}}$

8. $\dfrac{x^2}{\sqrt{x^6 + a^6}}$

9. $\dfrac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

10. $\dfrac{1}{\sqrt{x^2 + 2x + 2}}$

11. $\dfrac{1}{9x^2 + 6x + 5}$

12. $\dfrac{1}{\sqrt{7 - 6x - x^2}}$

13. $\dfrac{1}{\sqrt{(x - 1)(x - 2)}}$

14. $\dfrac{1}{\sqrt{8 + 3x - x^2}}$

15. $\dfrac{1}{\sqrt{(x - a)(x - b)}}$

16. $\dfrac{4x + 1}{\sqrt{2x^2 + x - 3}}$

17. $\dfrac{x + 2}{\sqrt{x^2 - 1}}$

18. $\dfrac{5x - 2}{1 + 2x + 3x^2}$

19. $\dfrac{6x + 7}{\sqrt{(x - 5)(x - 4)}}$

20. $\dfrac{x + 2}{\sqrt{4x - x^2}}$

21. $\dfrac{x + 2}{\sqrt{x^2 + 2x + 3}}$

22. $\dfrac{x + 3}{x^2 - 2x - 5}$

23. $\dfrac{5x + 3}{\sqrt{x^2 + 4x + 10}}$

Choose the correct answer in Exercises 24 and 25.

24. $\displaystyle\int \dfrac{dx}{x^2 + 2x + 2}$ equals

(A) $x\tan^{-1}(x + 1) + C$

(B) $\tan^{-1}(x + 1) + C$

(C) $(x + 1)\tan^{-1} x + C$

(D) $\tan^{-1} x + C$

25. $\int \dfrac{dx}{\sqrt{9x - 4x^2}}$ equals

(A) $\dfrac{1}{9} \sin^{-1}\left(\dfrac{9x - 8}{8}\right) + C$

(B) $\dfrac{1}{2} \sin^{-1}\left(\dfrac{8x - 9}{9}\right) + C$

(C) $\dfrac{1}{3} \sin^{-1}\left(\dfrac{9x - 8}{8}\right) + C$

(D) $\dfrac{1}{2} \sin^{-1}\left(\dfrac{9x - 8}{9}\right) + C$

EXERCISE 7.5

Integrate the rational functions in Exercises 1 to 21:

1. $\dfrac{x}{(x + 1)(x + 2)}$

2. $\dfrac{1}{x^2 - 9}$

3. $\dfrac{3x - 1}{(x - 1)(x - 2)(x - 3)}$

4. $\dfrac{x}{(x - 1)(x - 2)(x - 3)}$

5. $\dfrac{2x}{x^2 + 3x + 2}$

6. $\dfrac{1 - x^2}{x(1 - 2x)}$

7. $\dfrac{x}{(x^2 + 1)(x - 1)}$

8. $\dfrac{x}{(x - 1)^2(x + 2)}$

9. $\dfrac{3x + 5}{x^3 - x^2 - x + 1}$

10. $\dfrac{2x - 3}{(x^2 - 1)(2x + 3)}$

11. $\dfrac{5x}{(x + 1)(x^2 - 4)}$

12. $\dfrac{x^3 + x + 1}{x^2 - 1}$

13. $\dfrac{2}{(1 - x)(1 + x^2)}$

14. $\dfrac{3x - 1}{(x + 2)^2}$

15. $\dfrac{1}{x^4 - 1}$

16. $\dfrac{1}{x(x^n + 1)}$

17. $\dfrac{\cos x}{(1 - \sin x)(2 - \sin x)}$

18. $\dfrac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)}$

19. $\dfrac{2x}{(x^2 + 1)(x^2 + 3)}$

20. $\dfrac{1}{x(x^4 - 1)}$

21. $\dfrac{1}{e^x - 1}$

Choose the correct answer in each of the Exercises 22 and 23.

22. $\int \dfrac{x\, dx}{(x - 1)(x - 2)}$ equals

(A) $\log\left|\dfrac{(x - 1)^2}{x - 2}\right| + C$

(B) $\log\left|\dfrac{(x - 2)^2}{x - 1}\right| + C$

(C) $\log\left|\dfrac{x - 1}{x - 2}\right|^2 + C$

(D) $\log |(x-1)(x-2)| + C$

23. $\int \dfrac{dx}{x(x^2+1)}$ equals

(A) $\log |x| - \dfrac{1}{2} \log(x^2+1) + C$

(B) $\log |x| + \dfrac{1}{2} \log(x^2+1) + C$

(C) $-\log |x| + \dfrac{1}{2} \log(x^2+1) + C$

(D) $\dfrac{1}{2} \log |x| + \log(x^2+1) + C$