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27$^{\text{th}}$ International Mathematical Olympiad

Warsaw, Poland
Day I
July 9, 1986

1. Let d be any positive integer not equal to 2, 5, or 13. Show that one can find distinct a, b in the set $\{2, 5, 13, d\}$ such that $ab - 1$ is not a perfect square.

2. A triangle $A_1A_2A_3$ and a point P_0 are given in the plane. Define $A_n = A_{n-3}$ for $n \geq 4$. Construct P_n from P_{n-1} by rotation through angle $120°$ clockwise about A_n. Prove that if $P_{1986} = P_0$, then triangle $A_1A_2A_3$ is equilateral.

3. To each vertex of a regular pentagon an integer is assigned such that the sum of all five numbers is positive. If three consecutive vertices are assigned x, y, z respectively and $y < 0$, then replace them by $x+y$, $-y$, $z+y$. Determine whether this procedure necessarily ends after finitely many steps.

27^{th} International Mathematical Olympiad

Warsaw, Poland
Day II
July 10, 1986

4. Let A, B be adjacent vertices of a regular n-gon ($n \geq 5$). A triangle XYZ congruent to OAB moves such that Y, Z trace the boundary of the polygon while X remains inside. Find the locus of X.

5. Find all functions f on non-negative real numbers satisfying:

   (i) $f(xf(y)) = f(x+y)$

   (ii) $f(2) = 0$

   (iii) $f(x) \neq 0$ for $0 \leq x < 2$

6. Given a finite set of lattice points in the plane, is it always possible to color them red and white so that for every line parallel to axes, the difference in counts on the line is at most 1?

28^{th} International Mathematical Olympiad

Havana, Cuba

Day I

July 10, 1987

1. Let $p_n(k)$ be number of permutations of $\{1, \ldots, n\}$ with exactly k fixed points. Prove:
$$\sum_{k=0}^{n} k\, p_n(k) = n!$$

2. In an acute triangle ABC, let internal bisector of $\angle A$ meet BC at L and circumcircle again at N. From L, drop perpendiculars to AB, AC. Prove quadrilateral $AKNM$ and triangle ABC have equal areas.

3. Let $x_1, \ldots, x_n$ satisfy $x_1^2 + \cdots + x_n^2 = 1$. Prove there exist integers $a_1, \ldots, a_n$, not all zero, with
$$|a_i| \leq k - 1$$
and
$$|a_1 x_1 + \cdots + a_n x_n| \leq \frac{(k-1)\sqrt{n}}{k^n - 1}.$$

28^{th} International Mathematical Olympiad

Havana, Cuba
Day II
July 11, 1987

4. Prove there is no function $f : \mathbb{N} \to \mathbb{N}$ such that $f(f(n)) = n + 1987$.

5. Let $n \geq 3$. Prove there exists n points in plane such that all pairwise distances are irrational and every triangle formed has rational area.

6. Let $n \geq 2$. Prove $n^2 + k + n$ is prime for all integers k with $0 \leq k \leq \sqrt{n/3}$ implies $k^2 + k + n$ is prime for $0 \leq k \leq n - 2$.

29^{th} International Mathematical Olympiad

Canberra, Australia

Day I

1. Consider two concentric circles of radii $R > r$. Let P be on smaller circle and B on larger. If perpendicular to BP at P meets smaller circle again at A, find:

   (i) $BO^2 + CA^2 + AB^2$

   (ii) Locus of midpoint of BC

2. Let $A_1, \ldots, A_{2n+1}$ be subsets of a set B satisfying certain intersection properties. For which n can each element of B be assigned 0 or 1 so that each A_i has exactly n zeros?

3. Function f satisfies:

$$f(1) = 1, \quad f(3) = 3$$
$$f(2n) = f(n)$$
$$f(4n+1) = 2f(2n+1) - f(n)$$
$$f(4n+3) = 3f(2n+1) - 2f(n)$$

   Determine number of $n \leq 1988$ with $f(n) = n$.

29^{th} International Mathematical Olympiad

Canberra, Australia

Day II

4. Show solution set of
$$\sum_{k=1}^{70} \frac{k}{x-k} \geq \frac{5}{4}$$
is union of disjoint intervals of total length 1988.

5. In right triangle ABC, prove $S \geq 2T$ where S, T are areas of certain triangles formed by incenters.

6. Let a, b be positive integers such that $ab+1$ divides a^2+b^2. Show
$$\frac{a^2+b^2}{ab+1}$$
is a perfect square.