Assignment – 3

3. If μ= 55, σ_{4a}= 4, $\sigma_{4\beta}$=10, σ_4 = 15, In this which is better.

Given Data:

- Mean (μ) = 55
- Standard Deviation (σ_a) = 4
- Standard Deviation (σ_β) = 10
- Standard Deviation (σc) = 15

The objective of this question is to determine which distribution is better when the mean is constant but the standard deviation differs. To answer this properly in report format, we must first understand the theoretical meaning of mean and standard deviation, then analyze each case carefully using the empirical rule and graphical interpretation.

Introduction to Normal Distribution:

- A normal distribution is a continuous probability distribution that is symmetrical around its mean.
- It is commonly called the bell-shaped curve because its graphical representation resembles a bell.
- In a normal distribution:
  1. The mean, median, and mode are equal.
  2. The curve is perfectly symmetrical about the mean.
  3. The total area under the curve equals 1 (or 100%).
  4. The spread of the curve depends entirely on the standard deviation.

Since the mean (μ = 55) is constant in all three cases, the center of the distribution does not change. However, the spread or dispersion changes according to the value of standard deviation.

Meaning of Standard Deviation:

Standard deviation measures how much the data values deviate from the mean. It tells us how spread out the data points are.

- A small standard deviation indicates that the values are very close to the mean.
- A large standard deviation indicates that the values are widely scattered from the mean.

Mathematically, standard deviation is calculated as:

$\sigma = \sqrt{\Sigma(x - \mu)^2 / N}$

Where:

- x = individual values
- μ = mean
- N = total number of observations

The larger the deviation from the mean, the larger the standard deviation.

Empirical Rule (68–95–99.7 Rule) Application:

The empirical rule helps us understand how data is distributed in a normal curve.

According to this rule:

1. 68% of the data lies within $\pm 1\sigma$
2. 95% of the data lies within $\pm 2\sigma$
3. 99.7% of the data lies within $\pm 3\sigma$

Now let us analyze each case using this rule.

Case Analysis

Case A: $\sigma = 4$

Mean = 55

Within 1 Standard Deviation (68%)

$55 \pm 4 = 51$ to 59

This means 68% of the values lie between 51 and 59, which is a very narrow range of only 8 units.

Within 2 Standard Deviations (95%)

$55 \pm 8 = 47$ to 63

95% of values lie between 47 and 63.

Within 3 Standard Deviations (99.7%)

$55 \pm 12 = 43$ to 67

Almost all values lie within a small interval.

Interpretation

This distribution is highly concentrated around the mean. The curve will be tall and narrow. The variability is very low, and the system is highly consistent.

Case B: $\sigma = 10$

Mean = 55

Within 1 Standard Deviation (68%)

$55 \pm 10 = 45$ to 65

This is a range of 20 units.

Within 2 Standard Deviations (95%)

$55 \pm 20 = 35$ to 75

Within 3 Standard Deviations (99.7%)

$55 \pm 30 = 25$ to 85

Interpretation

This distribution has moderate spread. The curve will be moderately wide. The data is less consistent compared to Case A.

Case C: $\sigma = 15$

Mean = 55

Within 1 Standard Deviation (68%)

$55 \pm 15 = 40$ to 70

This is a wide range of 30 units.

Within 2 Standard Deviations (95%)

$55 \pm 30 = 25$ to 85

Within 3 Standard Deviations (99.7%)

$55 \pm 45 = 10$ to 100

Interpretation

This distribution has very high variability. The curve will be flat and wide. The data points are highly scattered.

Graphical Comparison

If we draw all three normal curves on the same axis:

- $\sigma = 4 \rightarrow$ Tall and narrow curve
- $\sigma = 10 \rightarrow$ Medium height and medium width
- $\sigma = 15 \rightarrow$ Short and very wide curve

Since the mean is the same (55), all three curves are centered at 55. Only the width changes.

Smaller $\sigma \rightarrow$ Higher peak and less spread

Larger $\sigma \rightarrow$ Lower peak and more spread

Which is Better?

The answer depends on context, but generally in statistics, a smaller standard deviation is considered better when we want consistency and stability.

Reasons why $\sigma = 4$ is better:

1. It has the least variability.
2. Data values are closer to the mean.
3. It indicates higher precision.
4. It reduces uncertainty.
5. It is more reliable for prediction.
6. It shows stable performance.

In quality control, finance, education, and manufacturing, lower standard deviation is usually preferred because it means consistent results.

Practical Example:

Suppose 55 represents average marks in an exam.

- If $\sigma = 4 \rightarrow$ Most students scored between 51 and 59 (very similar performance).
- If $\sigma = 10 \rightarrow$ Marks vary moderately.
- If $\sigma = 15 \rightarrow$ Marks vary widely (some very low, some very high).

For a stable academic system, $\sigma = 4$ would be considered better because performance is consistent.

Final Conclusion:

- Since all three distributions have the same mean ($\mu = 55$), the deciding factor is the standard deviation. The distribution with $\sigma = 4$ has the smallest spread, highest concentration around the mean, lowest variability, and highest consistency.
- Therefore, among $\sigma = 4$, $\sigma = 10$, and $\sigma = 15$, the best and most stable distribution is $\sigma = 4$, because it represents minimal dispersion and maximum reliability.