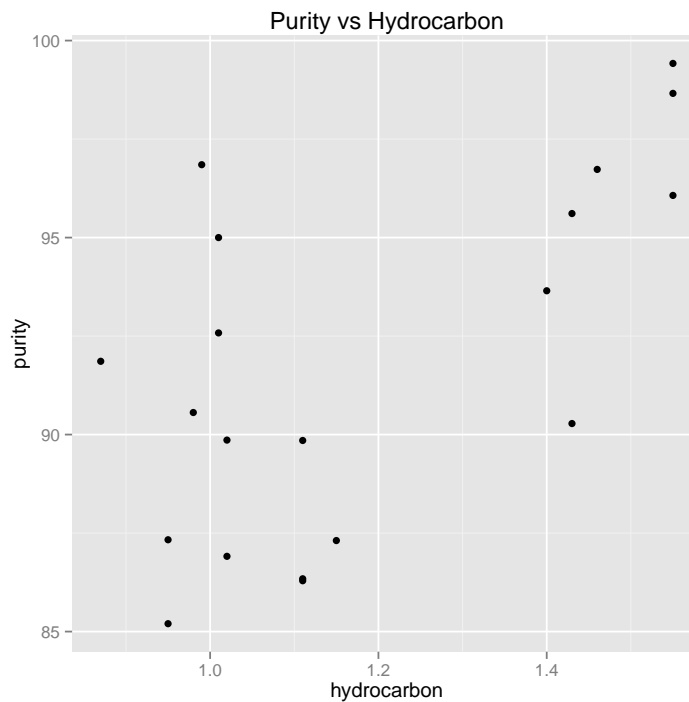


Question 2

A: Plot Purity vs Hydrocarbon. Discuss what you see in relation to the SLR assumptions.

Assumption 1: There does appear to be a linear relationship between purity and hydrocarbons. The expectation of residuals does appear to be approximately zero. **Assumption 2:** The error or residual values do not have constant variances. As seen from the plot the hypothetical error variables are considerably more variable for higher hydrocarbon values than for lower ones. The homoscedasticity assumption might be violated in this case. **Assumption 3:** The error/residuals do seem to be independent from one another. **Assumption 4:** Even though their variance isn't constant, the error values do seem to be normally distributed.

```
> ## loading useful packages
> library(knitr);library(xtable);library(ggplot2)
> ## read in data
> data<-read.table("A1_data.txt",sep=" ",header=T)
> ## rename the variables
> names(data)<-c("purity","hydrocarbon")
> ## Plot graph
> plot_1<-qplot(hydrocarbon,purity,data=data,main="Purity vs Hydrocarbon")
> print(plot_1)
```



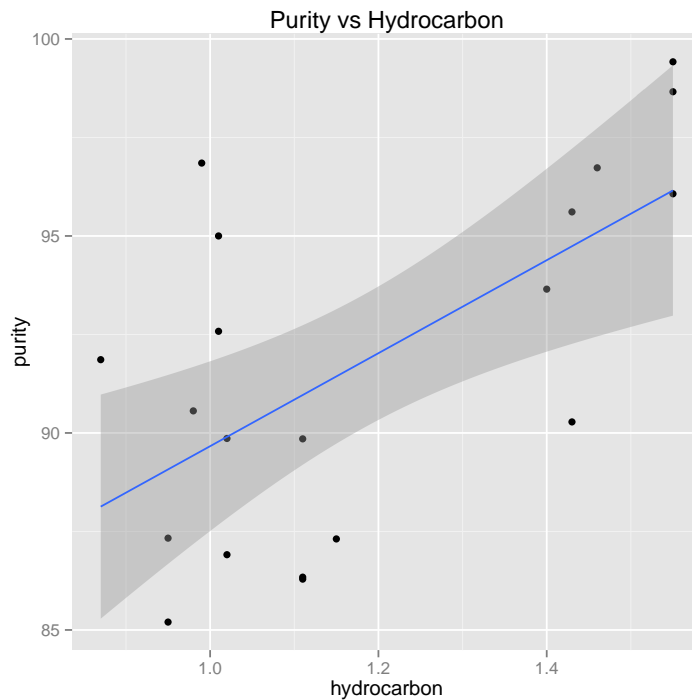
B: Fit a linear model and plot the fitted line to (A). Explain the model

```
> fit1<-lm(purity~hydrocarbon,data=data)
> print(xtable(summary(fit1)))
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	77.8633	4.1989	18.54	0.0000
hydrocarbon	11.8010	3.4851	3.39	0.0033

The value of slope or the B1 estimator, 11.8010281931501 is interpreted as the expected change in purity given a 1 unit change of hydrocarbons. Similarly, the intercept value of 77.8632841616 or the B0 estimator represents the expected value of purity at 0 hydrocarbons, though this might not make any contextual sense in some cases.

```
> plot_1<-plot_1+geom_smooth(method = "lm")
> print(plot_1)
```



C: Fit a 95 percent prediction and confidence interval for the purity level when the hydrocarbon percentage is equal to 1.0. Explain the PI and CI.

```
> confidence_interval<-predict(fit1,newdata=data.frame(hydrocarbon=1),
+                             interval="confidence",level=0.95)
> prediction_interval<-predict(fit1,newdata=data.frame(hydrocarbon=1),
+                              interval="prediction",level=0.95)
> print(confidence_interval)

      fit      lwr      upr
1 89.66431 87.51017 91.81845

> print(prediction_interval)

      fit      lwr      upr
1 89.66431 81.80716 97.52146
```

A confidence interval expresses uncertainty about the expected value of y-values at a given x. A prediction interval expresses uncertainty surrounding the predicted y-value of a single sampled point with that value of x. In regards, to our experiment we say that the confidence interval is the interval in which we say that 95 percent of the sample means we observe will fall in this interval, given that the hydrocarbon value is equal to 1.0. The prediction interval is the interval in which we are 95 percent certain the values of the next observation will fall within this interval, again given that the hydrocarbon value is zero. As such, the prediction interval is wider than the confidence interval.

D: Do the hypothesis test for $B_1=0$ at 0.05 level of significance. Explain.

```
> print(xtable(summary(fit1)))
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	77.8633	4.1989	18.54	0.0000
hydrocarbon	11.8010	3.4851	3.39	0.0033

As we can see the table summary of coefficients automatically calculates the hypothesis test for us. At a significance level of 5 percent the slope coefficient is statistically significant.

In more detail, we do a double sided t-test with a null hypothesis that the slope estimator is 0 i.e. there is no relation between purity and hydrocarbons. The alternative hypothesis is that this value is different than 0. In order to evaluate this we use a student's t-test with 2 degrees of freedom. The resulting value of this t-test ends up being 3.3861194436304 and the corresponding p-value of 0.00329112239545355. As we can see this p-value is close to zero and is less than our significance figure of 0.05, so we can say that the slope estimator is indeed statistically significant.