

Simulation

Bohdan Horak

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Summary

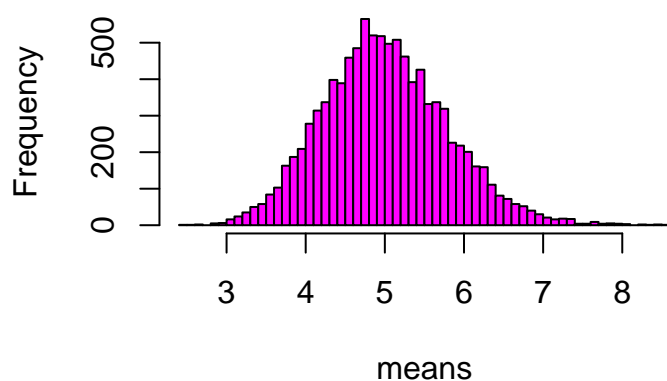
The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. Set $\lambda = 0.2$ for all of the simulations. In this simulation, you will investigate the distribution of averages of 40 exponential(0.2)s. Note that you will need to do a thousand or so simulated averages of 40 exponentials.

Illustrate via simulation and associated explanatory text the properties of the distribution of the mean of 40 exponential(0.2)s. You should ## Question 1

1. Show where the distribution is centered at and compare it to the theoretical center of the distribution.

```
nosim <- 10000
n <- 40
set.seed(1)
means<-apply(matrix( rexp(nosim * n, rate=0.2), nosim), 1, mean)
mean_val<-mean(means)
sds<-sd(means)
theo_sd<-5/sqrt(40)
hist(means,breaks=50,col="magenta")
```

Histogram of means

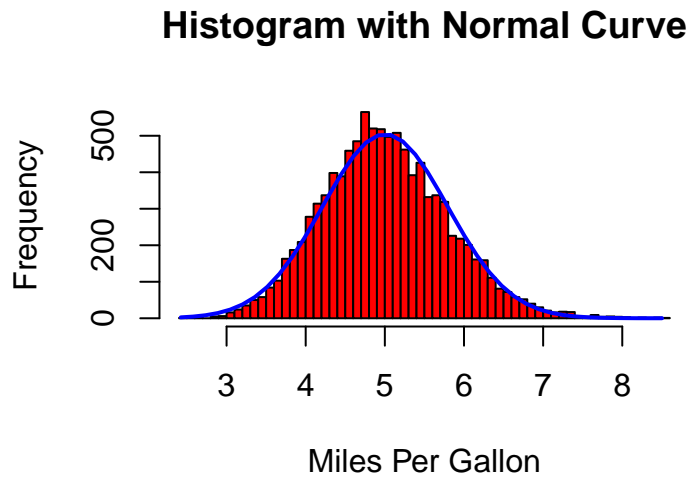


From the resulting simulation the distribution has a mean 5.0029 which is very close to the theoretical mean of $1/\lambda = 1/0.2 = 5$.

2. Show how variable it is and compare it to the theoretical variance of the distribution.

The standard deviation of the simulation is 0.7913 which is also close to the theoretical standard deviation of $SD = s/\sqrt{n} = 5/\sqrt{40} = 0.7906$.

3. Show that the distribution is approximately normal.



This is the same histogram as before but overlaid with a normal curve. Clearly is approximately normal, even with a small bin size.

4. Evaluate the coverage of the confidence interval for $1/\lambda$

```
count<-0
for (i in 1:nosim) {
  temp<-rexp(n,rate=0.2)
  x_bar<-mean(temp)
  sd_bar<-sd(temp)
  ci_l<-x_bar-1.96*sd_bar/sqrt(40)
  ci_u<-x_bar+1.96*sd_bar/sqrt(40)
  if (ci_l<5 & 5<ci_u) {
    count<-count+1
  }
}
coverage<-100*count/nosim
```

Simulated coverage for 104 replications and 40 sample size. The coverage for the confidence interval is about 92.13 %.