Exercise 9

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Exercise 9

10.2

b. *Prove with Boolean algebra the absorption property,* $x \cdot x + y = x$*. Give a reason for each step in your proof.*

Property:	$x \cdot x + y = x$
distributive:	$(x \cdot y) + (x \cdot x)$
commutative:	$x \cdot y + x$
identity:	$x \cdot y + x \cdot 1$
distributive:	$x(y + 1)$
zero-theorem:	$x \cdot 1$
identity:	x

10.3

b. *Prove with Boolean algebra the consensus theorem. Give a reason for each step in your proof.* $(x + y) \cdot (x' + z) \cdot (y + z) = (x + y) \cdot (x' + z)$

Property:	$(x + y) \cdot (x' + z) \cdot (y + z) = (x + y) \cdot (x' + z)$
duality:	$(x \cdot y) + (x' \cdot z) + (y \cdot z)$
complement:	$(x \cdot y) + (x' \cdot z) + (x + x') \cdot (y \cdot x)$
distributive:	$(x \cdot y) + (x' \cdot z) + (x \cdot y \cdot z) \cdot (x' \cdot y \cdot z)$
commutative:	$(x \cdot y + x \cdot y \cdot z) + (x' \cdot z + x' \cdot y \cdot z)$
distributive:	$x \cdot y \cdot (1 + z) + x' \cdot z \cdot (1 + y)$
zero-theorem:	$x \cdot y \cdot 0 + x' \cdot z \cdot 0$
identity:	$x \cdot y + x' \cdot z$
duality:	$(x + y) \cdot (x' + z)$

10.4

Prove DeMorgan's Law $(a+b)' = a' \cdot b'$ by giving the dual of the proof in the text. Give a reason for each step in the proof.

First Part:

Property:	$(a+b)' + (a' \cdot b') = 1$
distributive:	$(a+b) + a' \cdot (a+b) + b'$
commutative:	$a + a' + b \cdot a + b + b'$
complementary:	$1 + b \cdot b + 1$
zero-theorem:	$1 \cdot 1$
identity:	1

Second Part:

Property:	$(a+b) \cdot (a' \cdot b') = 0$
distributive:	$a' \cdot b' \cdot a + a' \cdot b' \cdot b$
complementary:	$0 \cdot b' + a' \cdot 0$
zero-theorem:	$0 + 0$
identity:	0

10.6

b. *Prove with Boolean algebra that $(x \cdot y) + (x' \cdot y) = y$. Give a reason for each step in your proof.*

Property:	$(x \cdot y) + (x' \cdot y) = y$
distributive:	$y \cdot (x + x')$
complementary:	$y \cdot 1$
identity:	y

10.7

b. *Prove with Boolean algebra that* $(x \cdot y) \cdot (y + x') = x \cdot y$. *Give a reason for each step in your proof.*

Property	$(x \cdot y) \cdot (y + x') = x \cdot y$
distributive:	$y(x \cdot y) + x'(x \cdot y)$
commutative:	$x \cdot y \cdot y + y$
distributive:	$y(x \cdot y + 1)$
zero-theorem:	$y(x \cdot 1)$
identity:	$y(x)$
commutative:	$x \cdot y$

References

Warford, J. (2009). *Computer systems* (4th ed.). Jones and Bartlett.