

Panel 1

# Purely Functional Data Structures

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Panel 2

# Why?

- **Functional...**
- **Immutable ("values"):**
  - **Copy by sharing**
  - **Thread-safe**
  - **Persistent?**

Panel 3

**Persistent (adj.): not ephemeral.**

**Fully persistent: can access and modify all versions**

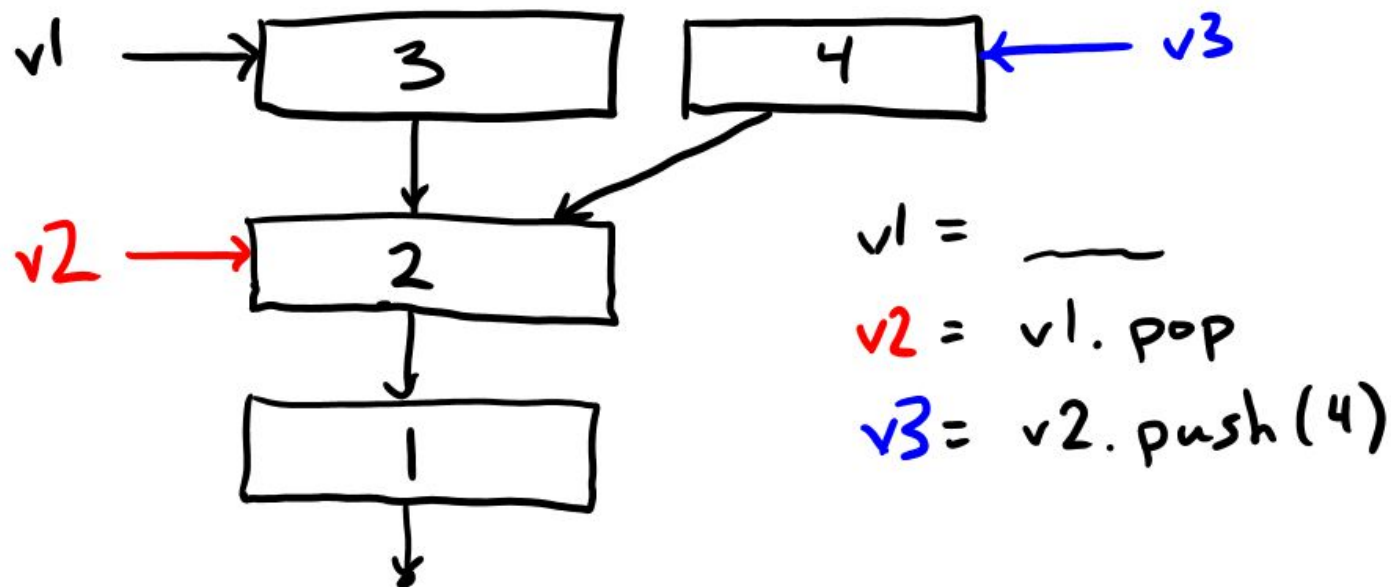
***Partially persistent: can access all versions and modify current version***

***Confluently persistent: can merge previous versions into new version***

Panel 4

# Linked List, used as a Stack

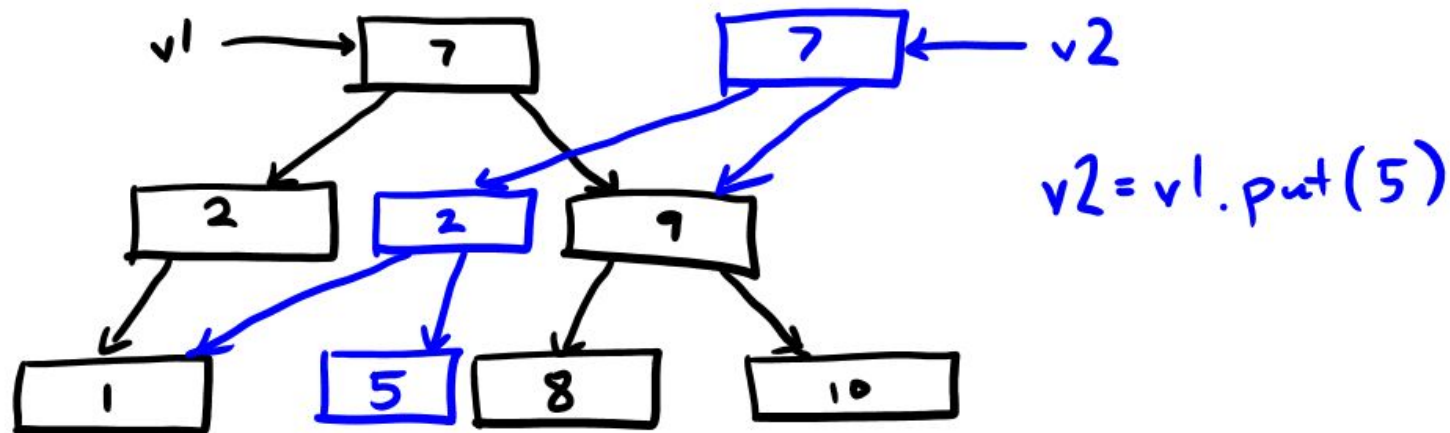
## Fully Persistent, $O(1)$ push/pop/top



Panel 5

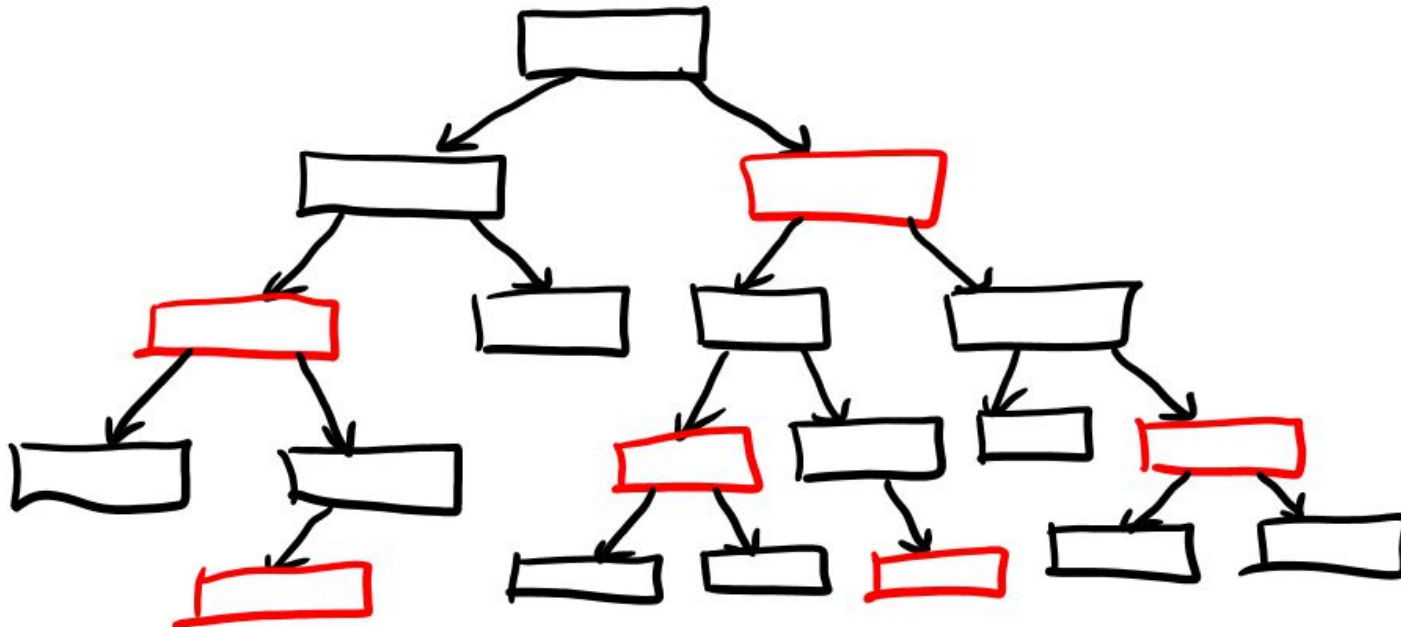
# Binary Search Tree (Set or Map)

Fully Persistent, Average  $O(\log N)$   
get/put



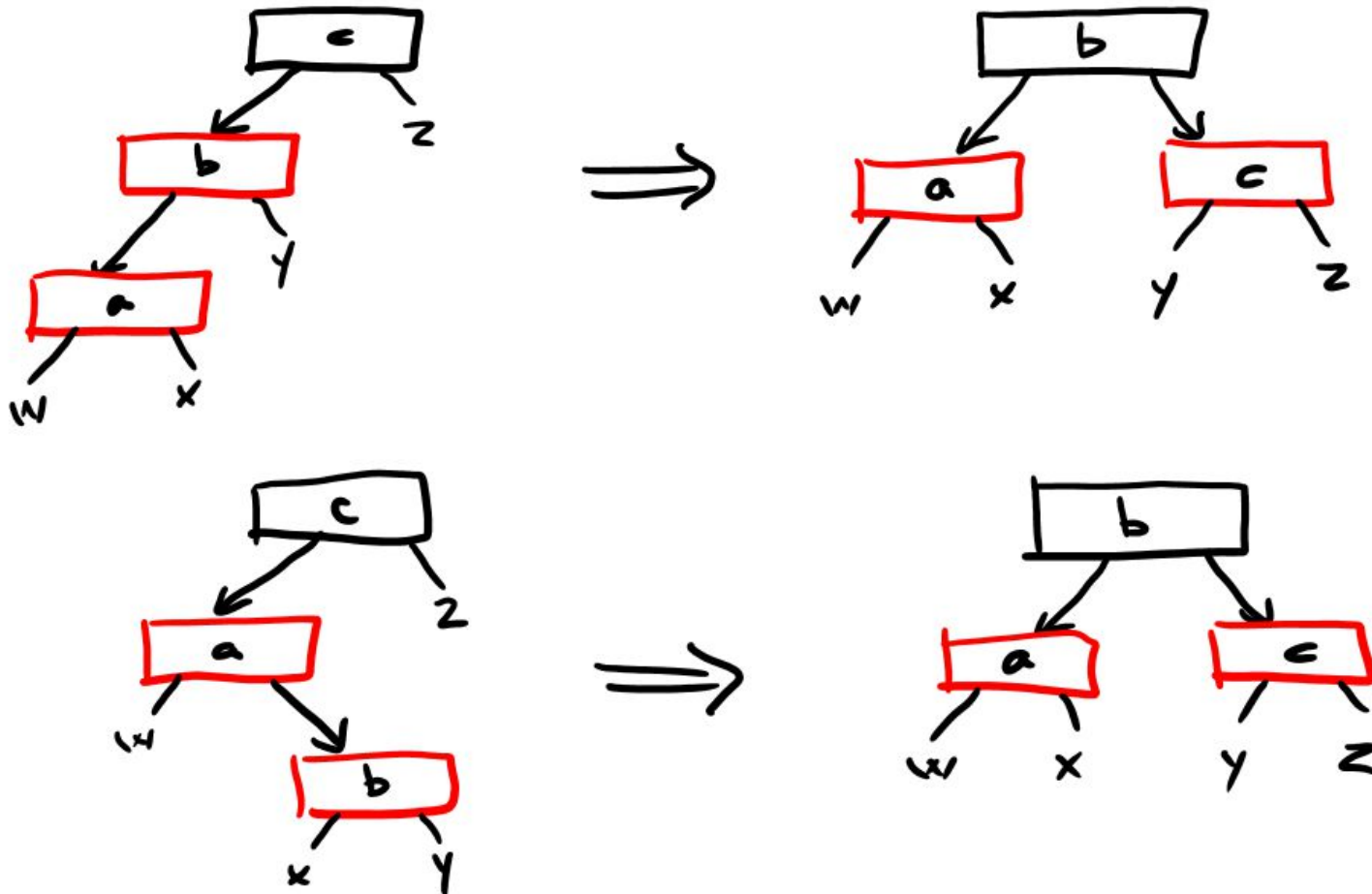
Panel 6

**Red-Black Tree: add balance condition**  
**(same # black nodes per path to leaf;**  
**no adjacent red nodes on any path)**  
**Worst-case  $O(\log N)$**



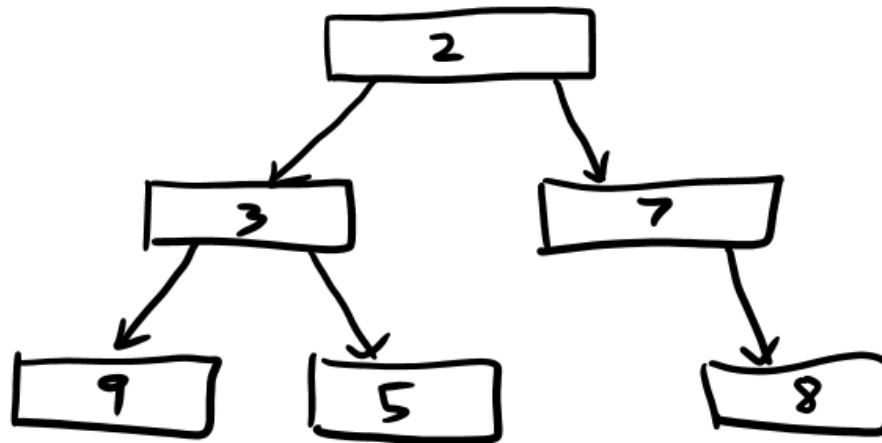
Panel 7

# Rotations



Panel 8

## Priority Queue: tree with "heap order"



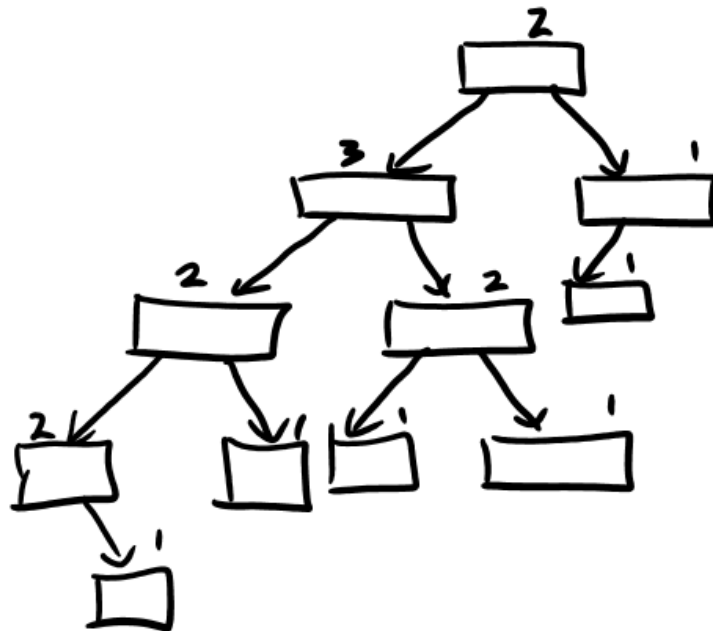
**head:  $O(1)$**

**enqueue/dequeue: average  $O(\log N)$**



Panel 9

**Leftist Heap: add balance condition  
(left children have  $\geq$  "rank" --  
length of right "spine")**

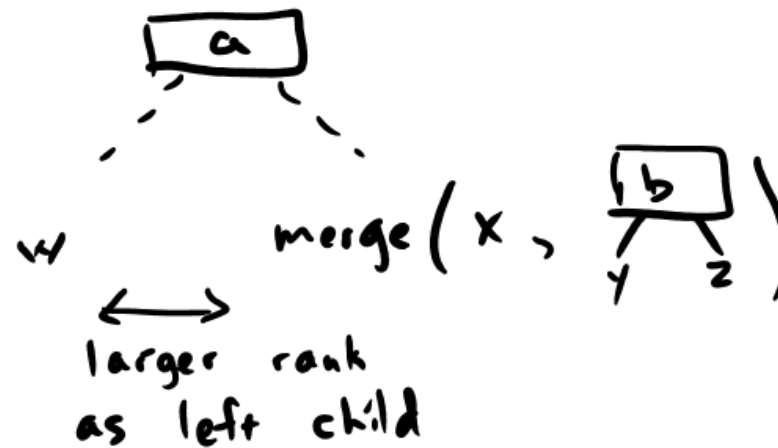


Panel 10

# Merge operation preserves balance:

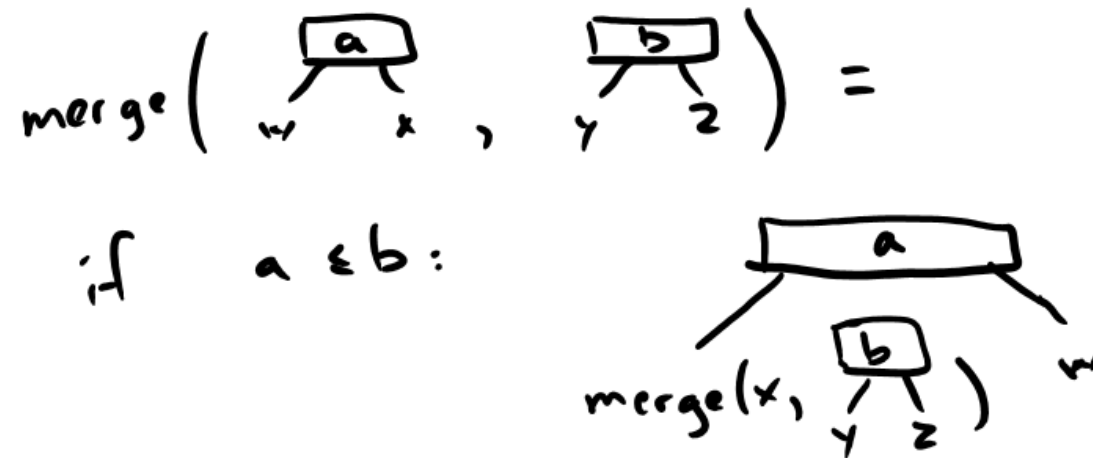


if  $a \leq b$ :



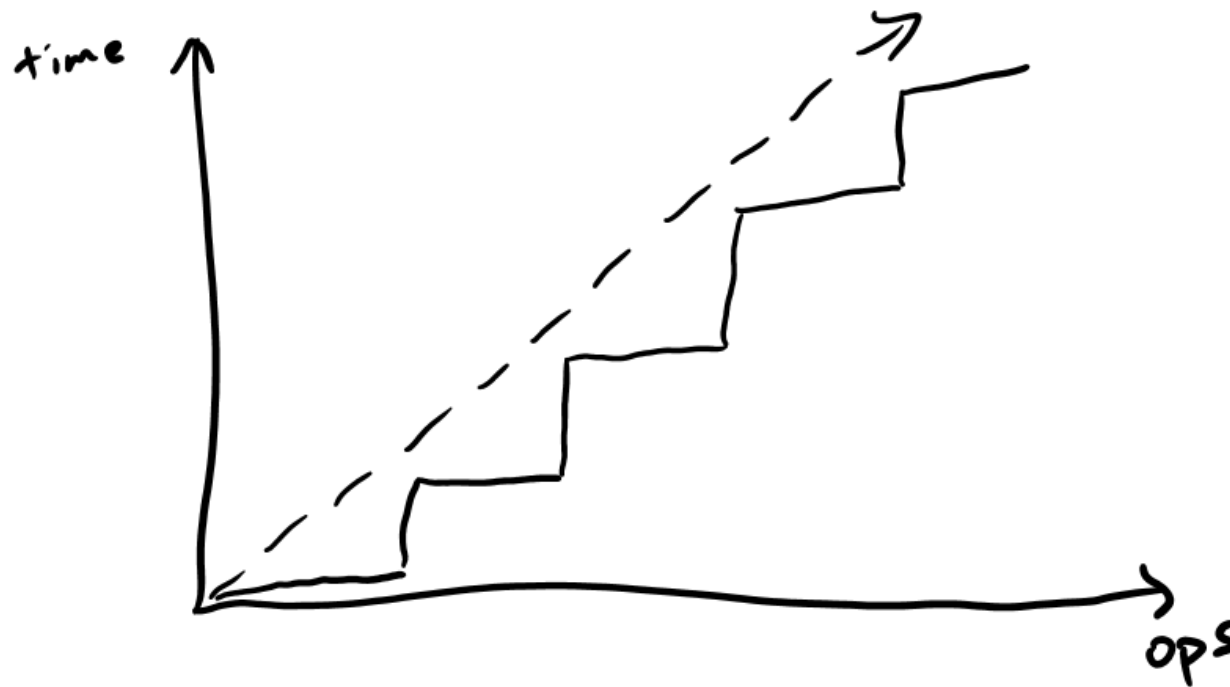
Panel 11

**Skew Heap: *always* swap children  
when merging**  
**Simpler, but only "amortized"  $O(\log N)$**



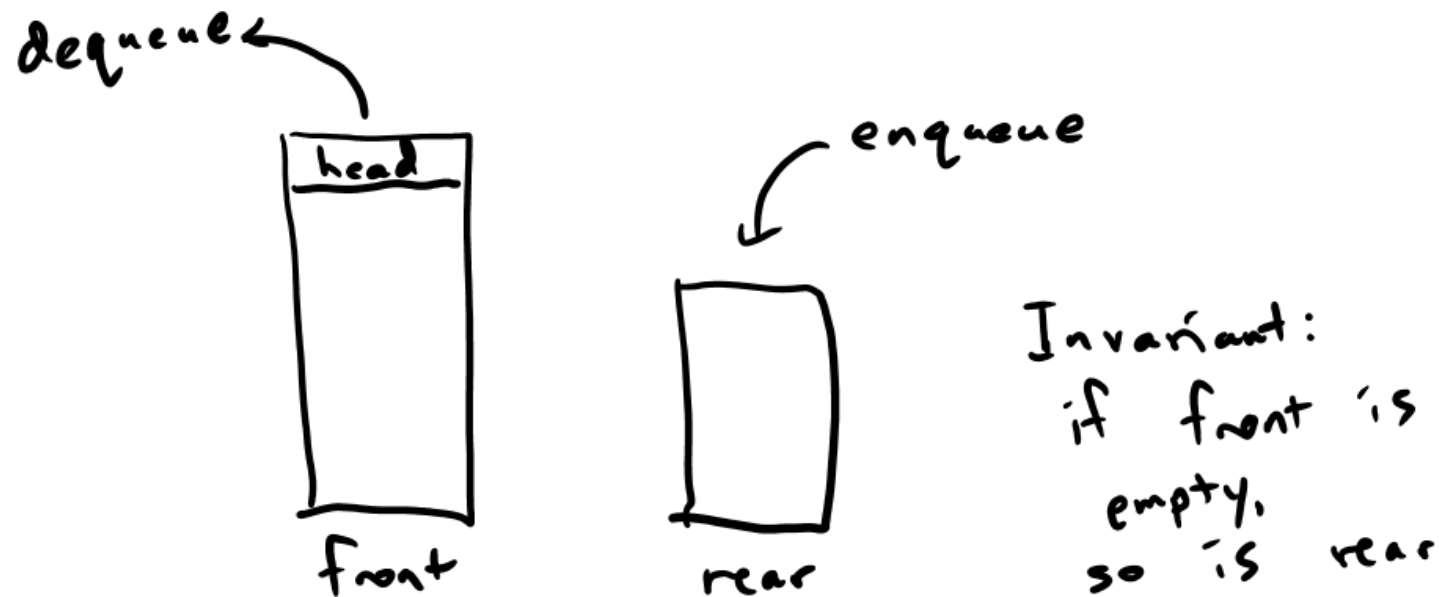
Panel 12

**Amortization: expensive operations have been "pre-paid", so the overall average is still cheap**



Panel 13

**Batched Queue: pair of stacks**  
**worst-case  $O(1)$  head/enqueue**  
**amortized  $O(1)$  dequeue (*not persistent!*)**



Panel 14

**Real-Time Queue (Okasaki):  
use "laziness" to "schedule" an  
incremental reversal of rear to front**

**Fully-persistent, worst-case  $O(1)$   
(but not technically immutable!)**