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1 coordinate systems

(from cover of [1].) Most can be found in Schaum's. the following is in Schaum's, pg 120

1.1 cartesian

$$dV = dx \cdot dy \cdot dz \tag{1}$$

gradient:

$$\vec{\nabla} t = \widehat{x}\frac{\partial t}{\partial x} + \widehat{y}\frac{\partial t}{\partial y} + \widehat{z}\frac{\partial t}{\partial z} \tag{2}$$

divergence (equ 1.40, page 17 [1])

$$\vec{\nabla} \cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \tag{3}$$

laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{4}$$

1.2 spherical

$$dV = r^2\ \sin\theta\ dr\ d\theta\ d\phi \tag{5}$$

page 40, [1]

$$d\vec{a} = \hat{r}\ r^2 \sin\theta\ d\theta\ d\phi \tag{6}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \tag{7}$$

Schaum's, pg 126 laplacian. See planetMath laplacian rectangualr to cartesian and planetMath laplacian (equ 3.53, page 137 [1])

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} \tag{8}$$

1.3 cylindrical

$$dV = s\ ds\ d\phi\ d\theta \tag{9}$$

$$\vec{\nabla} = \hat{s}\frac{\partial}{\partial s} + \hat{\phi}\frac{1}{s}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial a} \tag{10}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2} \tag{11}$$

2 formulas

material in this section can NOT be found in Schaum's mathematical handbook of formulas, which is allowed on the day of the exam. Schaum's, 123
Stoke's thm (equ 1.57, page 34 [1])

$$\int_{surface} (\vec{\nabla} \cdot \vec{v}) \cdot d\vec{a} = \oint_{volume} \vec{v} \cdot d\vec{l} \tag{12}$$

Greene's thm (equ 1.56, page 31 [1])

$$\int_{volume} (\vec{\nabla} \cdot \vec{v}) d\tau = \oint_{surface} \vec{v} \cdot d\vec{a} \tag{13}$$

Taylor series expansion is in Schaum's, but the easy approximation is not. (assuming small x)

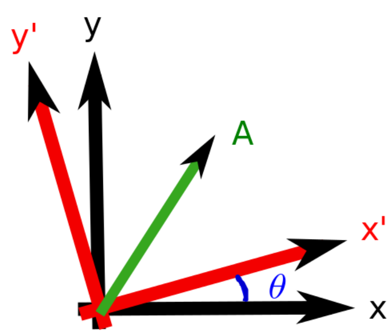
$$(1+x)^n \approx 1 + nx \tag{14}$$

in Schuam's
Taylor series expansion

$$f(x) = f(a) + f'(x)|_a (x-a) + \frac{1}{2!} f''(x)|_a (x-a)^2 + \cdots \tag{15}$$

3 Math

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} A_x \\ A_y \end{bmatrix} = \begin{bmatrix} A_{x'} \\ A_{y'} \end{bmatrix} \tag{16}$$



Variance

$$(\Delta A)^2 = \langle A^2 \rangle - \langle A \rangle^2 \tag{17}$$

Quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{18}$$

4 Electromagnetics

4.1 Maxwell equations

See also page 22, [4]; page 2, [2]
Faraday's law (equation 7.16, page 302, [1])

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{19}$$

Ampere's law (equation 5.54, page 225 and equation 5.44, page 222 [1])

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \tag{20}$$

where

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{21}$$

Gauss's law, electricity (equation 2.14, page 69 [1])

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \tag{22}$$

Guass's law, magnetism (equation 5.48, page 223 [1])

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{23}$$

Integral forms:

flux of electric field (equation 2.11, page 67 [1])

$$\Phi_E = \int_{any\ surface} \vec{E} \cdot d\vec{a} \tag{24}$$

Gauss's law, electricity. (equation 2.13, page 68 [1])

$$\oint_{closed\ surface} \vec{E} \cdot d\vec{a} = \frac{Q_{enclosed}}{\epsilon_0} \tag{25}$$

Faraday's Law (equation 7.18, page 306 [1])

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \tag{26}$$

Ampere's law (equation 5.55, page 225 and 5.42, page 222 [1])

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} \tag{27}$$

Ampere's law (page 306, [1])

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0 \epsilon_0 \int \left(\frac{d}{dt} \vec{E} \right) \cdot d\vec{a} \tag{28}$$

Lorentz force law (equation 5.2, page 204 [1]). Also equation 1-61, page 22 [3]

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{29}$$

4.2 general EM

equation 1-62, page 22 [3]

$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{30}$$

Coulomb's Law

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{31}$$

where q_2 is the test charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho}{r^2} d\tau \hat{r} \tag{32}$$

electric displacement (see also eq 43)

$$\vec{D} = \epsilon_0 \vec{E} \tag{33}$$

Electric field due to point charge (equation 2.4, page 60 [1])

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i \tag{34}$$

electrostatic potential (equ 2.21, page 78 and equ 7.10, page 293[1])

$$V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} \tag{35}$$

electric field and electrostatic potential (equation 2.23, page 78, [1])

$$\vec{E} = -\vec{\nabla} V \tag{36}$$

electrostatic potential (equation 2.27, page 84 [1])

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \tag{37}$$

electrostatic potential (equation 2.29, page 84 [1])

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r} d\tau \tag{38}$$

work (equation 2.42, page 92 [1])

$$W = \frac{1}{2}\sum q_i V(r_i) \tag{39}$$

work and electrostatic potential (equ 2.45, page 94 [1])

$$W = \frac{\epsilon_0}{2}\int_{allspace} E^2 d\tau \tag{40}$$

surface charge density and electrostatic potential (where n is the normal direction (equation 2.49, page 102 [1])

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n} \tag{41}$$

equation 2.48, page 102 [1]

$$\vec{E} = \frac{\sigma}{\epsilon}\hat{n} \tag{42}$$

electric displacement, including polarization (equ 4.21, page 175 [1]) (see also eq 33)

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \tag{43}$$

work and electrostatic potential (equ 2.55, page 106, [1])

$$W = \frac{1}{2} C V^2 \tag{44}$$

electrostatic potential due to a monopole (equation 3.97, page 149 [1])

$$V_{monopole} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \tag{45}$$

electrostatic potential due to a dipole (equation 3.99, page 149 [1])

$$V_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \tag{46}$$

where dipole moment (which points from the (-) charge to the (+) is
(equation 3.98, page 149 [1]) (equation 3.100, page 150 [1])

$$\vec{p} = \int \vec{r}\,'\rho(\vec{r}\,')d\tau\,' = \sum_i q_i r_i \tag{47}$$

Note: E_{dipole}, equation 3.104, page 155; is given on 2002 B10, but as a derivation (see also problem 3.33).

Torque (N) is on a dipole ($\vec{p} = q\vec{d}$) due to E field (equation 4.4, page 164, [1])

$$\vec{N} = \vec{p} \times \vec{E} \tag{48}$$

potential energy of a dipole (problem 4.7) (equation 4.6, page 165, [1])

$$U = -\vec{p}\cdot\vec{E} \tag{49}$$

Similarly for magnetic moment ($\vec{\mu} = (\vec{I}\cdot\vec{A})\hat{n}$)
(equation 6.1, page 257, [1])

$$\vec{N} = \vec{\mu} \times \vec{B} \tag{50}$$

potential energy (equation 6.34, page 281, [1])

$$U = -\vec{\mu}\cdot\vec{B} \tag{51}$$

force due to magnetic field (equ 5.16, page 205 [1])

$$F_{magnetic} = I\int d\vec{l} \times \vec{B} \tag{52}$$

(equ 6.31, page 275 [1])

$$\vec{B} = \mu\vec{H} \tag{53}$$

(equ 6.32, page 275 [1])

$$\mu = \mu_0(1+\chi_m) \tag{54}$$

(equ 7.3, page 285 [1])

$$\vec{J} = \sigma\vec{E} \tag{55}$$

flux (equ 7.12, page 295 [1])

$$\Phi = \int \vec{B}\cdot d\vec{a} \tag{56}$$

work (equ 7.29, page 317 [1]). Used in problems 7.26, 7.27.

$$W = \frac{1}{2}LI^2 \tag{57}$$

work (equ 7.34, page 317 [1])

$$W = \frac{1}{2\mu_0}\int_{allspace} B^2 d\tau \tag{58}$$

flux, inductance (equ 7.25, page 313 [1])

$$\Phi = LI \tag{59}$$

Power (equ 8.11, page 347 [1]), positive on page 349

$$Power = \int \vec{S} \cdot d\vec{a} \tag{60}$$

total potential energy in a field (equation 8.5, page 346 [1]) (combines 58 and 40)

$$U_{em} = \frac{1}{2} \int (\epsilon_0 E^2 + \frac{1}{\mu_0} B^2) d\tau \tag{61}$$

Poynting vector (equation 8.10, page 347 [1])

$$\vec{S} = \frac{1}{\mu} \vec{E} \times \vec{B} \tag{62}$$

Power and Poynting vector (equation 8.11, page 347 [1])

$$P = \frac{dW}{dt} = -\frac{dU_{em}}{dt} - \oint \vec{S} \cdot d\vec{a} \tag{63}$$

4.3 circuits

capacitance (equ 2.53, page 104, [1])

$$C \equiv \frac{Q}{V} \tag{64}$$

$$P = I^2 R \tag{65}$$

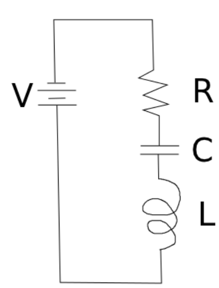
$$V = IR \tag{66}$$

Power and electrostatic potential ($P = I^2 R$ can be derived from $V = IR$) (equation 7.7, page 290, [1])

$$Power = IV \tag{67}$$

circuit sum (equation 7.4, page 290, [1]) [resistor, capacitor, inductor]

$$V = IR + \frac{Q}{C} + L\frac{dI}{dt} \tag{68}$$



$$I = \frac{dQ}{dt} \tag{69}$$

capacitance, A is the area, d is the seperation (equation 2.54, page 105 [1])

$$C = \frac{A\epsilon_0}{d} \tag{70}$$

4.4 relativity

Lorentz factor, [$\gamma > 1$](equation 12.6, page 486 [1])

$$\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}} \tag{71}$$

total relativistic energy for free particle (equation 12.55, page 551 [1])

$$E = \sqrt{m^2c^4 + p^2c^2} \tag{72}$$

$$E = \gamma mc^2 \tag{73}$$

relativistic momentum (equation 2-7, page 71 [9])

$$p = \gamma mv \tag{74}$$

4.5 EM concepts

- hollow charged conductors have no interior electric field (page 97, [1])
- solid evenly-charged spheres have linearly increasing electric field inside; outside E field falls of as $\frac{1}{r^2}$
- potential of a charge falls off as $\frac{1}{r}$
- length contraction, time dilations (chapter 12, [1])
- images above an ∞ grounded conducting plane $\rightarrow$ use method of images (section 3.2, see also problem 4.6 in [1])
- changing electric field induces a magnetic field (page 323, [1])
- magnetic forces do no work (page 207, [1])
- $E_{tangential}$ is continuous, E_{normal} is discontinuous
- to solve Laplace's equation ($\nabla^2 V = 0$) with V specified on the boundaries, use seperation of variables (Chapter 3, [1])
- magnetic forces do no work (page 207, [1])

5 Thermodynamics

most generally, specific heat is

$$C_x = \frac{dQ}{dT}|_x \tag{75}$$

where x is the variable that is held constant.

$$\frac{1}{T} = \frac{\partial S}{\partial E}|_v \tag{76}$$

Then from equation $5 \cdot 6 \cdot 1$ page 164; page 153; [4]

$$T \cdot dS = dE + p \cdot dV \tag{77}$$

solve for dE, which is dQ, and hold V constant to get
specific heat, constant volume;

$$C_V = \frac{\partial E}{\partial T}|_v \tag{78}$$

using the definition of enthalpy, add $d(pV)$ to Eq. 77 to get

$$dH = T \cdot dS + V \cdot dp \tag{79}$$

and hold pressure p constant to get
specific heat, constant pressure

$$C_p = \frac{\partial H}{\partial T}|_p \tag{80}$$

$$C_P = C_V + NK_{Boltzmann} = C_V + \nu R \tag{81}$$

$C_P > C_V$ always

$$pV = \nu RT \tag{82}$$

where ν is the number of moles, $\frac{N}{N_{Av}}$ and $R = N_{Av}K_{Boltzmann}$.

equations $3 \cdot 3 \cdot 12$; $3 \cdot 11 \cdot 6$, page 123; [4])

$$S = k \ln \Omega \tag{83}$$

ideal gas; relating average pressure, Volume, Temperature (equations $3 \cdot 12 \cdot 8$ page 125; $7 \cdot 2 \cdot 8$ page 241; [4])

$$\overline{p}V = Nk_{Boltzmann}T \tag{84}$$

$pV = nRT$
ideal gas; relating to average Kinetic energy

$$\overline{p}V = \frac{2}{3}N\overline{K} \tag{85}$$

where number density $n = \frac{N}{V}$ is number of molecules per volume.
mean kinetic energy per particle; applies to fluids in closed systems

$$\overline{K} = \frac{3}{2}k_{boltzmann}T \tag{86}$$

partition function for a single particle (equation $6 \cdot 5 \cdot 3$ page 213; [4])

$$z \equiv \sum_{r}^{\infty} e^{-\beta E_r} \tag{87}$$

and total partition function $Z = z^{3N}$
partition function and energy (equation $6 \cdot 5 \cdot 4$ page 213; [4])

$$E = -\frac{\partial(\ln Z)}{\partial \beta}|_V \tag{88}$$

Fermi-Dirac distribution (equation $9 \cdot 3 \cdot 14$ page 341; [4])
mean occupation number; number of particles with energy ϵ_s

$$n = \frac{1}{e^{\alpha+\beta\epsilon_s} + 1} \tag{89}$$

Bose-Einstein distribution (equation $9 \cdot 3 \cdot 22$ page 342; [4])

$$n = \frac{1}{e^{\alpha+\beta\epsilon_s} - 1} \tag{90}$$

since when the exponent is 0, then $n \to \infty$, so all the particles form a BEC.
Maxwell-Boltzmann distribution (equation $9 \cdot 4 \cdot 7$ page 345; [4]) (see also page 133, [9])

$$n = N\frac{e^{-\beta\epsilon_s}}{\sum_r e^{-\beta\epsilon_r}} \tag{91}$$

5.1 Thermodynamics concepts

- good review of basics on page 122 of [4], including the laws
- second law: $\Delta S \geq 0$
- classical equipartition theorm: each squared term of the Hamiltonian has $E = \frac{1}{2}k_b T$ (equation $7 \cdot 5 \cdot 7$ page 249; [4])
- entropy is maximum for a closed, isolated system at equilibrium
- all states are equally probable. Equilibrium is the most probable configuration – the largest number of states sharing the same configuration

6 Classical Mechanics

the "basics:"
Force and potential energy, (page 20, [3])

$$\vec{F} = -\vec{\nabla} V \tag{92}$$

$$T = \frac{1}{2} I \omega^2 \tag{93}$$

for circular motion, tangential velocity is

$$v = \omega r \tag{94}$$

and centripital acceleration (also circular motion)

$$\frac{v^2}{R} \tag{95}$$

Angular momentum $\vec{L}$

$$\vec{L} = \vec{r} \times \vec{p} \tag{96}$$

Torque $\vec{N}$

$$\vec{N} = \vec{r} \times \vec{F} = \frac{\partial \vec{L}}{\partial t} \tag{97}$$

Force F, potential V, momentum p

$$\vec{F} = -\vec{\nabla} V = \frac{\partial p}{\partial t} \tag{98}$$

$$\begin{aligned} F_{spring} &= -k\ x \\ V_{spring} &= \frac{1}{2} k\ x^2 \end{aligned} \tag{99}$$

6.1 Lagrange's Equations of Motion

kinetic energy (cartesian, cylindrical)

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2) \tag{100}$$

Lagrangian $L = L(q_j, \dot{q}_j, t) = T - U$ (equation 1-56, page 20 [3]).
(equation 1-57, page 21 [3])

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0 \tag{101}$$

6.2 Hamilton's Equations of Motion

$H = H(q_j, p_j, t)$
usually $H = T + V$
(equation 8-8, page 341 [3])

$$H = \sum_j p_j \dot{q}_j - L \tag{102}$$

Once Lagrangian is found, get canonical momentum

$$p_j \equiv \frac{\partial L}{\partial \dot{q}_j} \tag{103}$$

and solve for $\dot{q}_j$ for later

$$h = \left(\sum \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) - L \tag{104}$$

then substitue in $\dot{q}$ to get H.
Finally, get Hamilton Equations of motion by
(equation 8-12, page 342 [3])

$$\begin{aligned} \dot{q}_j &= \frac{\partial H}{\partial p_j} \\ \dot{p}_j &= -\left(\frac{\partial H}{\partial q_j} \right) \end{aligned} \tag{105}$$

then integrate to get q and p

6.3 2-body central force

standard orbit is Counter Clock-Wise (CCW).
Given the Lagrangian in polar coordinates, one derives the canonical angular momentum
(equation 3-8, page 73 [3]) (see eq 103)

$$l = \mu r^2 \dot{\theta} \tag{106}$$

for perturbed orbits, $\tau > \tau_c$ implies CCW (positive $\Delta\theta$) motion (forward orbital motion)

$$\Delta\theta = \frac{l}{mr^2}(\tau - \tau_c) \tag{107}$$

circular orbit condition (equation 3-12, page 74 [3])

$$f^{effective}(r_c) = 0 = f(r_c) + \frac{l^2}{\mu r_c^3} \tag{108}$$

where for the kepler problem $f(r) = -\frac{k}{r^2}$
small oscillations of orbit

$$r = r_c + \epsilon \tag{109}$$

not included: small oscillation (chapter 6 of [3]).

7 Quantum

expectation value, average, mean

$$\langle A\rangle = \langle\psi|A|\psi\rangle = \int_{-\infty}^{\infty} \psi^* A\psi \, dx \tag{110}$$

probability amplitude ([7] page 118 calls this the add-mixture coefficient $|C_n|^2$)

$$|\psi|^2 = \langle\psi|\psi\rangle = \int \langle\psi|x\rangle\langle x|\psi\rangle dx \tag{111}$$

Ehrenfest Theorem

$$\frac{d}{dt}\langle a\rangle = \left\langle \frac{\partial a}{\partial t}\right\rangle + i\hbar \left\langle [a, H]\right\rangle \tag{112}$$

and for sudden change, from the ground state (1) to the first excited state (2), probability of transition is

$$|\langle\phi_2|\psi_1\rangle|^2 \tag{113}$$

uncertianty

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} \tag{114}$$

commutator (equation 7.24, page 194 [7])

$$[x, p] = i\hbar \tag{115}$$

frequency

$$\omega = \sqrt{\frac{k}{m}} \tag{116}$$

time evolution (equ 2.490, page 98, [6])

$$|\Psi(t)\rangle = e^{-i\frac{H}{\hbar}t}|\Psi_0\rangle \tag{117}$$

$$|\Psi(t)\rangle = \sum_n e^{-i\frac{E_n}{\hbar}t}\Psi_n(0)|n\rangle \tag{118}$$

total spin number

$$m_{total} = m_1 + m_2 \tag{119}$$

where $m = -j, \ldots, j$
total momentum

$$s_{total} = (s_1 + s_2), (s_1 + s_2 - 1), \ldots, |s_1 - s_2| \tag{120}$$

where $s_{total} \geq 0$

7.1 energy eigenvalue equation

In position representation, derive the energy eigenvalue equation from

$$\vec{K} = -i\vec{\nabla} \tag{121}$$

in position representation, (equation 130)

$$\vec{P} = \hbar\vec{K} \tag{122}$$

thus momentum (page 14, [6]) (equation 3.2, page 69 [7]

$$\vec{P} = -i\hbar\vec{\nabla} \tag{123}$$

From eq 123, knowing the Hamiltonian is $H = T + V$

$$\hat{H} = \frac{p^2}{2m} + V \tag{124}$$

Then plug 123 into 124 and use

$$H\psi = E\psi \tag{125}$$

to get the energy eigenvalue equation,

$$\hat{H} = \frac{-\hbar^2\nabla^2}{2m}\Psi + V\Psi = E\Psi \tag{126}$$

Schrodinger Equation

$$i\hbar\frac{d}{dt}|\Psi\rangle = H|\Psi\rangle \tag{127}$$

equation 9.19, page 358 [7]

$$\begin{aligned} J_+ &= J_x + iJ_y \\ J_- &= J_x - iJ_y \end{aligned} \tag{128}$$

7.2 harmonic oscillator

SHO energy levels

$$E_n = (n + \frac{1}{2})\hbar\omega \tag{129}$$

harmonic oscillator Hamiltonian (equation 3.2, page 103 [6])

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 \tag{130}$$

factor out $\frac{\hbar\omega}{2}$ to derive harmonic oscillator Hamiltonian in dimensionless operators
Note: $\hat{P}$ and $\hat{Q}$ are dimensionless, whereas $\hat{p}$ and $\hat{q}$ are dimensional (equation 3.18, page 105 [6])

$$H = \frac{\hbar\omega}{2}(\hat{P}^2 + \hat{Q}^2) \tag{131}$$

by comparing 131 and 130 the relationship between dimensionless q,p and dimensional x,p can be found.
in terms of energy operators, harmonic oscillator Hamiltonian (equation 3.34, page 107 [6])

$$H = \hbar\omega(N + \frac{1}{2}) \tag{132}$$

where $N = a^+a$ (equation 3.33)
Lowering, raising operators

$$\begin{aligned} a|n\rangle &= \sqrt{n}|n-1\rangle \\ a^+|n\rangle &= \sqrt{n+1}|n+1\rangle \end{aligned} \tag{133}$$

dimensionless position, momentum (equation 3.29, page 106 [6])

$$\begin{aligned} \hat{q} &= \frac{1}{\sqrt{2}}(\hat{a}^+ + \hat{a}) \\ \hat{p} &= \frac{i}{\sqrt{2}}(\hat{a}^+ - \hat{a}) \end{aligned} \tag{134}$$

7.3 time independent non-degenerate perturbation theory

perturbation theory says the exact solution can be approximated as

$$E \cong E^{(0)} + E^{(1)} + E^{(2)} + \dots \tag{135}$$

(eq. 136 through 139 first order correction to energy (equ 5.41, page 138, [6]), (page 685 [7])

$$E_n^{(1)} = \langle n^{(0)}|V|n^{(0)}\rangle \tag{136}$$

first order correction to state (equ 5.42, page 138, [6]), (equation 13.8, page 685 [7])

$$|n^{(1)}\rangle = \sum_{m\neq n} |m^{(0)}\rangle\langle m^{(0)}|n^{(1)}\rangle \tag{137}$$

where (equation 5.43, page 138, [6])

$$\langle m^{(0)}|n^{(1)}\rangle = -\frac{\langle m^{(0)}|V|n^{(0)}\rangle}{E_m^{(0)} - E_n^{(0)}} \tag{138}$$

where $m \neq n$

second order correction to energy

$$E_n^{(2)} = \sum_{m\neq n} \frac{|\langle m^{(0)}|V|n^{(0)}\rangle|^2}{E_m^{(0)} - E_n^{(0)}} \tag{139}$$

7.4 time-independent degenerate perturbation theory

characteristic equation

$$det(H - \lambda I) = 0 \tag{140}$$

where λ are eigenvalues

7.5 particle in a box

infinite square well of size L, $0 \leq x \leq L$ (equation 4.15, page 93 [7])

$$\psi_n(x) = \sqrt{\frac{2}{L}} sin\left(\frac{n\pi x}{L}\right) \tag{141}$$

where $n = 1, 2, 3, \dots$ (equation 4.14, page 93 [7])

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \tag{142}$$

Whereas if $\frac{-L}{2} \leq x \leq \frac{L}{2}$

$$\begin{aligned} \psi_m(x) &= \sqrt{\frac{2}{L}} cos\left(\frac{m\pi x}{L}\right) \\ \psi_n(x) &= \sqrt{\frac{2}{L}} sin\left(\frac{n\pi x}{L}\right) \end{aligned} \tag{143}$$

where $m = 1, 3, \dots$ and $n = 2, 4, \dots$
this can also be derived by shifting eq. 141 by $\frac{L}{2}$.

7.6 free particle

for a free particle, assume $V = 0$, then $H = \frac{p^2}{2m}$. The wave function is

$$\phi_k(r) = \frac{1}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{r}} \tag{144}$$

and the energy is (equation 3.19, page 71 [7])

$$E_k = \frac{\hbar^2 k^2}{2m} \tag{145}$$

and since $E = \hbar\omega$, then

$$\omega_k = \frac{\hbar k^2}{2m} \tag{146}$$

time dependent wave function

$$\psi(\vec{r}, t) = \langle \vec{r}|\psi(t)\rangle = \langle \vec{r}| \int d^3k \; e^{-i\omega_k t}\psi(\vec{k}, 0)|\vec{k}\rangle \tag{147}$$

7.7 quantum concepts

- the same spectrum (eigenvalues) occur in any basis
- Q: when is perturbation valid? A: when the perturbation term is much smaller than the unperturbed term
- a $\delta(x)$ perturbation in the middle of an ∞ square well has no effect on even states

Bosons	Fermions
zero or integer spin	half-integer spin
symmetric wave functions	anti-symmetric wave functions
Example: photons	Example: electrons, proton, neutron
more than one particle may occupy the same quantum state	Pauli exclusion principle: one particle per state
Bose-Einstien statistics	Fermi-Dirac statistics

8 modern

$$\frac{h}{2\pi} = \hbar \tag{148}$$

period and frequency

$$T = \frac{1}{f} \tag{149}$$

Bragg condition (equation 3-38, page 144 [9]; equ 1 page 25 KittelSS)

$$2d \; sin\theta = m\lambda \tag{150}$$

where m is an integer.

angular frequency (equation 5-13a, page 211 [9])

$$\omega = 2\pi f \tag{151}$$

de Broglie relations (equation 5-1, page 198 [9])

$$E = hf \tag{152}$$

de Broglie wavelength (equation 5-2, page 198 [9]) (equation 3.10, page 70 [7])

$$\lambda = \frac{h}{p} \tag{153}$$

frequency

$$f = \frac{c}{\lambda} \tag{154}$$

(page 199, [9])

$$E = pc = hf = \frac{hc}{\lambda} \tag{155}$$

wavenumber (equation 5-13b, page 211 [9]) (equation 3.9, page 70 [7])

$$k = \frac{2\pi}{\lambda} \tag{156}$$

phase velocity (equation 5-14, page 211 [9])

$$v_{phase} = f\lambda \tag{157}$$

group velocity (equation 5-22, page 211 [9]) (equation 3.62, page 83 [7])

$$v_{group} = \frac{d\omega}{dk} \tag{158}$$

energy and frequency (equation 5-24, page 218 [9])

$$E = \hbar\omega \tag{159}$$

momentum (equation 5-25, page 218 [9])

$$p = \hbar k \tag{160}$$

uncertainty (equation 5-29, page 223 [9]) (problem 5.28, page 143 [7])

$$\begin{aligned} \Delta x \Delta p &\geq \frac{\hbar}{2} \\ \Delta E \Delta t &\geq \frac{\hbar}{2} \end{aligned} \tag{161}$$

work function: energy need to remove an electron (equation 3-37, page 139 [9])

$$\phi = hf_{threshold} = \frac{hc}{\lambda_{threshold}} \tag{162}$$

8.1 Modern concepts

- The mass of a bound system is greater than that of the individual constituent particles due to contribution of binding energy. For instance, $E_{hydrogen} = 13.6eV$ is the binding energy for hydrogen. (see page 86, [9])
- 4 forces in order of decreasing strength and [force carrier particle]
  1. strong [gluons]
  2. electromagnetic [photon]
  3. weak [W and Z bosons]
  4. gravity [graviton]

8.2 constants

- radius of a simple atom is $\approx .5\dot{A}$ (half an angstrom). (Bohr radius, page 173 [9])
- $hc = 1240\ eV\ nm$

9 optics

snell's law

$$n_1 sin(\theta_1) = n_2 sin(\theta_2) \tag{163}$$

10 Symbol notations

See page 631-642 of [4]

$C_v \equiv$ specific heat for constant volume
$k_b \equiv$ Boltzmann constant
$\vec{E} \equiv$ electric field
$\vec{B} \equiv$ magnetic field
$\vec{D} \equiv$ electric displacement, equ 4.21, page 175 [1]
$\vec{N} \equiv$ torque
$W \equiv$ work
$\vec{v} \equiv$ velocity
$V \equiv$ electrostatic potential [Volts]
$V \equiv$ volume
$U \equiv$ potential energy [Volts]
$\vec{P} \equiv$ polarization, page 166 [1]
$I \equiv$ current [Amps]
$\vec{F} \equiv$ force [Newtons, $\frac{kg \cdot m}{s^2}$]
$E \equiv$ energy
$T \equiv$ kinetic energy
$T \equiv$ temperature
$L \equiv$ lagrangian
$L \equiv$ capacitance
$\vec{L} \equiv$ classical (orbital) angular momentum
$l \equiv$ quantum (orbital) angular momentum
$\vec{p} \equiv$ linear momentum
$\omega \equiv$ angular frequency
$z \equiv$ single particle partition function
$Z \equiv$ total partition function

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