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"del" operator ∇ has different names, depending on how it is applied.

gradient: ∇f

divergence: $\nabla \cdot f$

curl: $\nabla \times f$

0.1 math 402

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{1}$$

$$f(\vec{r}_0 + \Delta\vec{r}) = e^{\Delta\vec{r}\cdot\vec{\nabla}} f(\vec{r})|_{\vec{r}=\vec{r}_0} = \sum_{m=0}^{\infty} \frac{(\Delta\vec{r}\cdot\vec{\nabla})^m}{m!} f(\vec{r})|_{\vec{r}=\vec{r}_0} \tag{2}$$

Einstein summation notation (relies on covarient versus contravarient notation)

$$\vec{A} \times \vec{B} = \epsilon^{ijk} \hat{x}_i A_j B_k \tag{3}$$

$$\epsilon^{lmn} \epsilon_{ljk} = \delta_{mj}\delta_{nk} - \delta_{mk}\delta_{nj} \tag{4}$$

Dr Hale's favorite formula:

$$df = d\vec{r} \cdot \vec{\nabla} f \tag{5}$$

0.2 partial differential equations

$$\nabla^2 = \frac{\partial}{\partial\rho} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial}{\partial\phi} + \frac{\partial}{\partial z} \tag{6}$$

most are given in 2D, but are applicable to nD

Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{7}$$

more simply for n dimensions

$$\nabla^2 u = 0 \tag{8}$$

Wave equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0 \tag{9}$$

Heat equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = 0 \tag{10}$$

Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = g(x, y) \tag{11}$$

more simply for n dimensions

$$\nabla^2 u = g(x, y) \tag{12}$$