Probability mass functions

EXPLORATORY DATA ANALYSIS IN PYTHON

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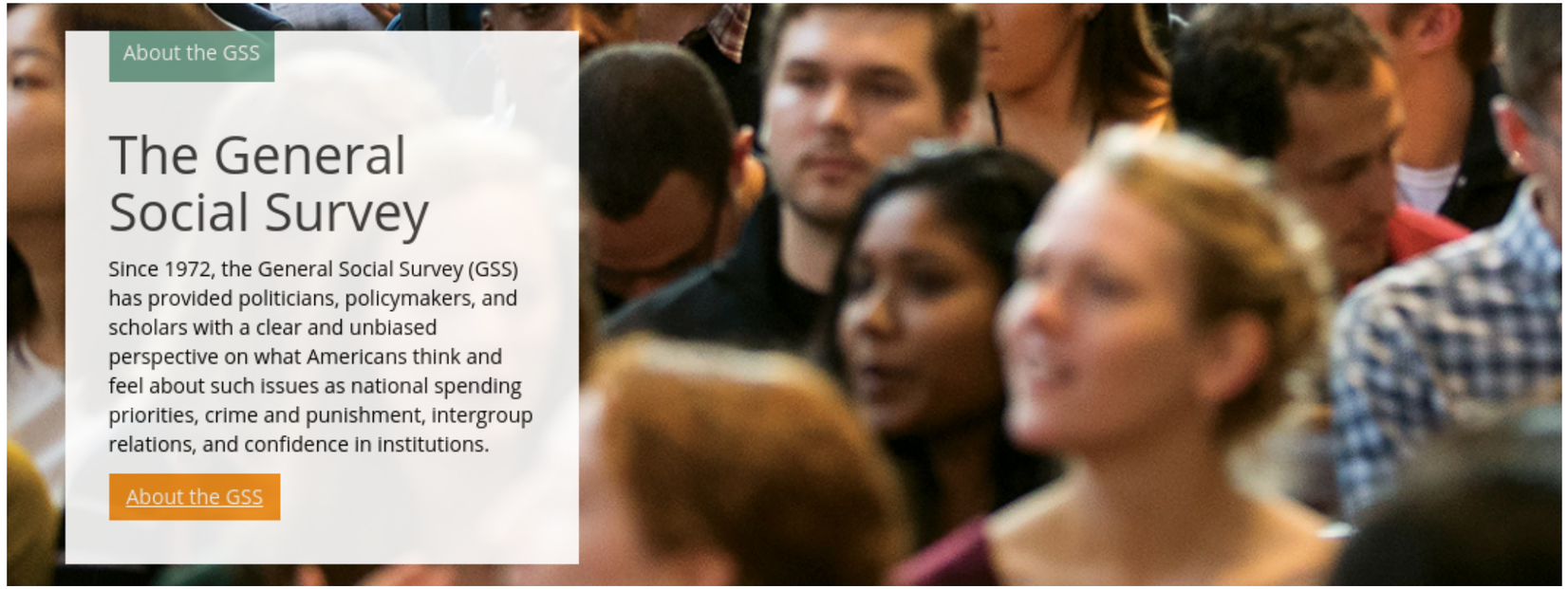
GSS

- Annual sample of U.S. population.
- Asks about demographics, social and political beliefs.
- Widely used by policy makers and researchers.

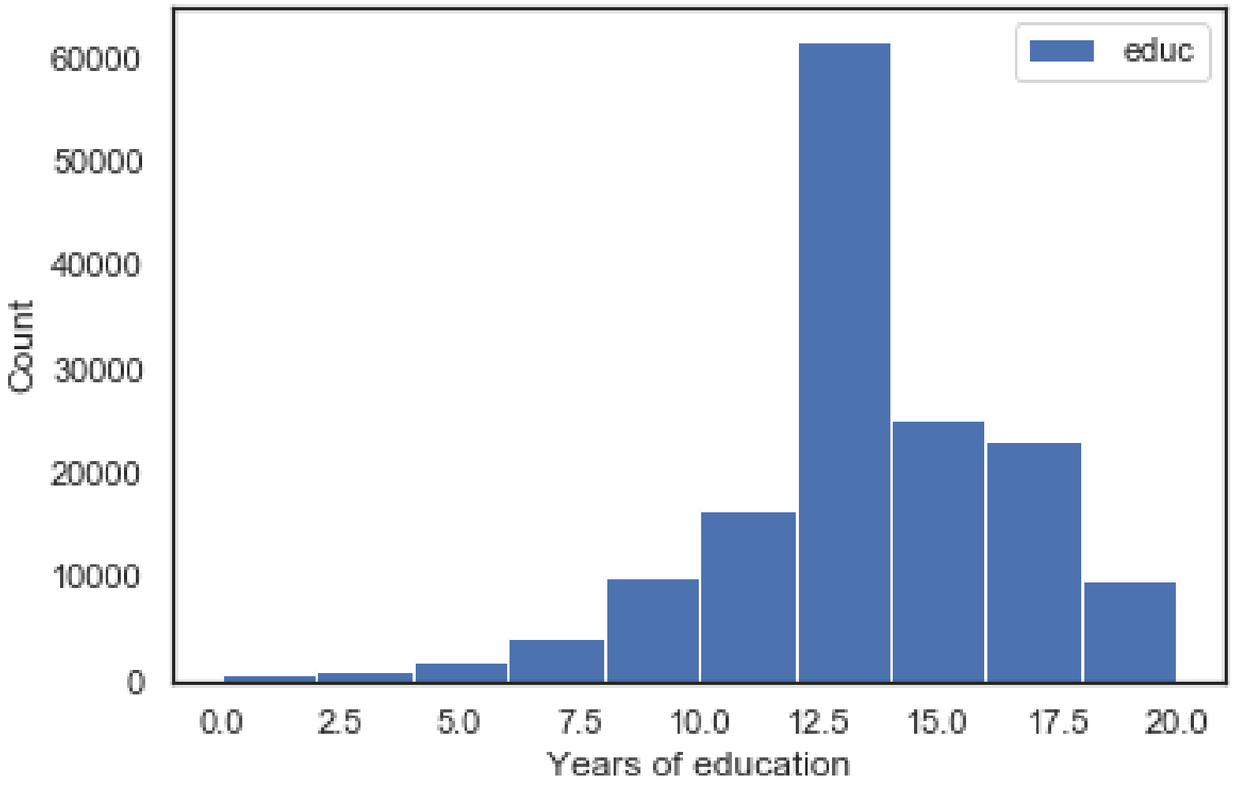


Read the data

```
gss = pd.read_hdf('gss.hdf5', 'gss')

gss.head()
```

	year	sex	age	cohort	race	educ	realinc	wtssall
0	1972	1	26.0	1946.0	1	18.0	13537.0	0.8893
1	1972	2	38.0	1934.0	1	12.0	18951.0	0.4446
2	1972	1	57.0	1915.0	1	12.0	30458.0	1.3339
3	1972	2	61.0	1911.0	1	14.0	37226.0	0.8893
4	1972	1	59.0	1913.0	1	12.0	30458.0	0.8893

```
educ = gss['educ']

plt.hist(educ.dropna(), label='educ')
plt.show()
```



PMF

```
pmf_educ = Pmf(educ, normalize=False)

pmf_educ.head()
```

```
0.0    566
1.0    118
2.0    292
3.0    686
4.0    746
Name: educ, dtype: int64
```

PMF

```
pmf_educ[12]
```

```
47689
```

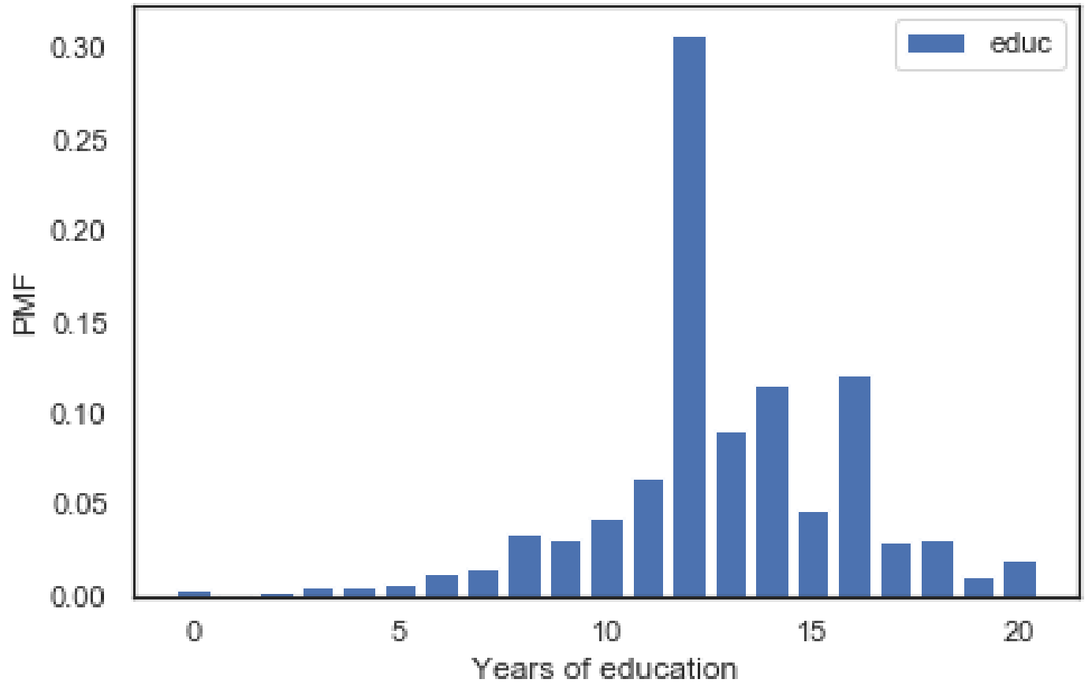
```
pmf_educ = Pmf(educ, normalize=True)

pmf_educ.head()
```

```
0.0    0.003663
1.0    0.000764
2.0    0.001890
3.0    0.004440
4.0    0.004828
Name: educ, dtype: int64
```

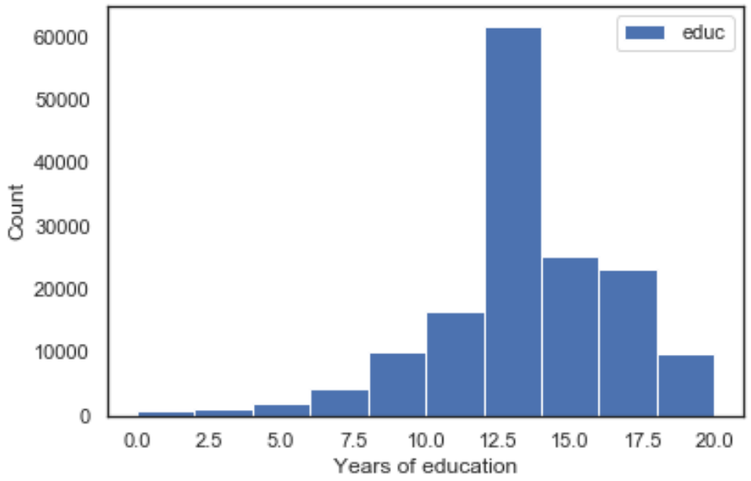
```
pmf_educ[12]
```

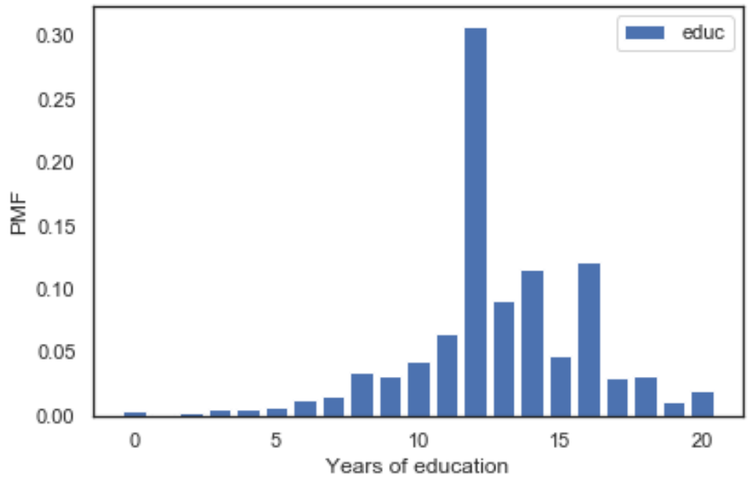
```
0.3086386994058790 7
```

```
pmf_educ.bar(label='educ')
plt.xlabel('Years of education')
plt.ylabel('PMF')
plt.show()
```



Histogram vs. PMF





Let's make some PMFs!

EXPLORATORY DATA ANALYSIS IN PYTHON

Cumulative distribution functions

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From PMF to CDF

If you draw a random element from a distribution:

- PMF (Probability Mass Function) is the probability that you get exactly x
- CDF (Cumulative Distribution Function) is the probability that you get a value <= x

for a given value of x.

Example

PMF of {1, 2, 2, 3, 5}

PMF(1) = 1/5

PMF(2) = 2/5

PMF(3) = 1/5

PMF(5) = 1/5

CDF is the cumulative sum of the PMF.

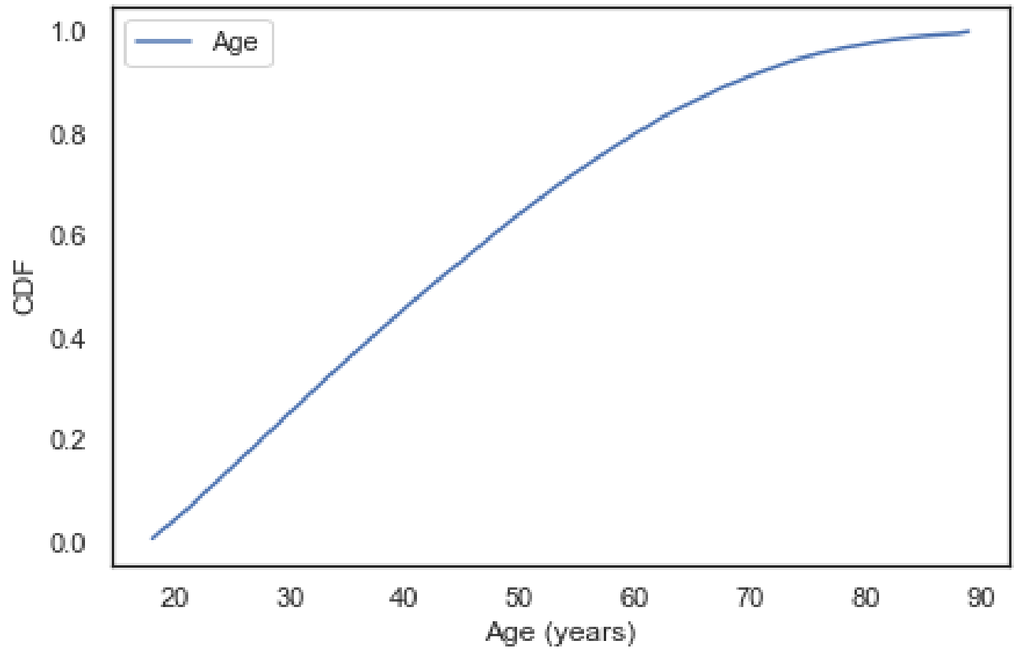
CDF(1) = 1/5

CDF(2) = 3/5

CDF(3) = 4/5

CDF(5) = 1

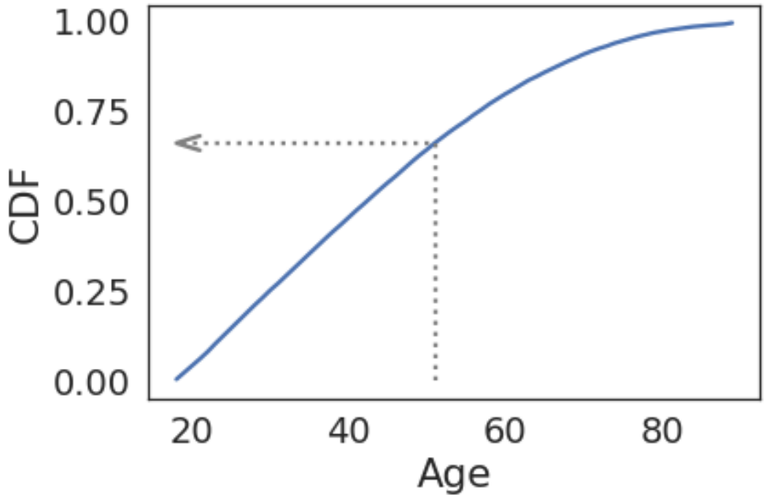
```
cdf = Cdf(gss['age'])
cdf.plot()
plt.xlabel('Age')
plt.ylabel('CDF')
plt.show()
```



Evaluating the CDF

```
q = 51
p = cdf(q)
print(p)
```

```
0.66
```



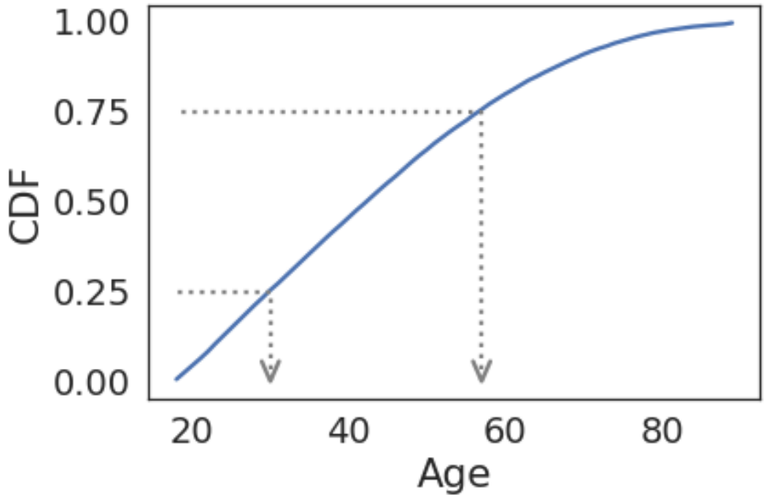
Evaluating the inverse CDF

```
p = 0.25
q = cdf.inverse(p)
print(q)
```

```
30
```

```
p = 0.75
q = cdf.inverse(p)
print(q)
```

```
57
```



Let's practice!

EXPLORATORY DATA ANALYSIS IN PYTHON

Comparing distributions

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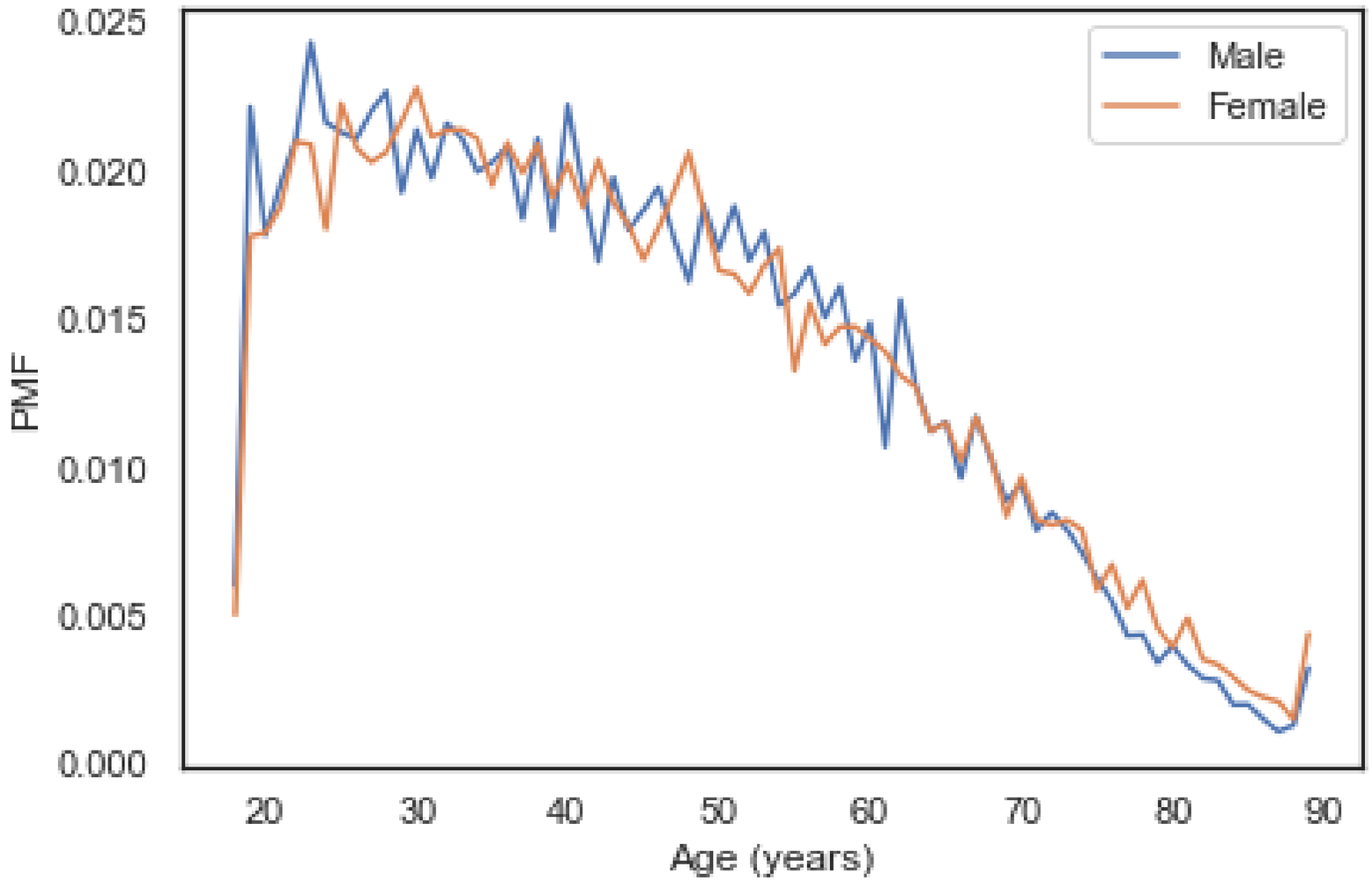
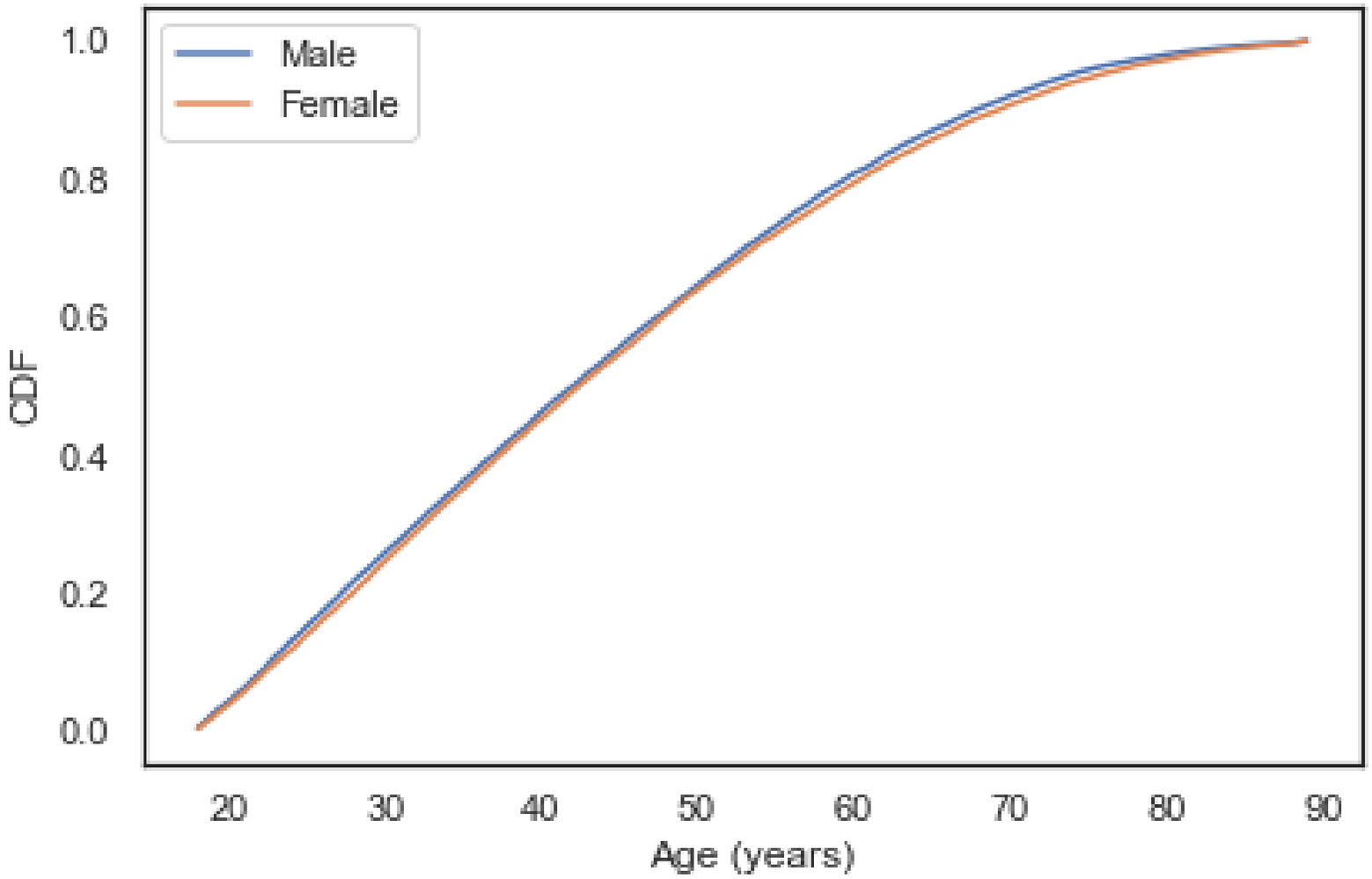
Multiple PMFs

```
male = gss['sex'] == 1

age = gss['age']

male_age = age[male]
female_age = age[~male]

Pmf(male_age).plot(label='Male')
Pmf(female_age).plot(label='Female')

plt.xlabel('Age (years)')
plt.ylabel('Count')
plt.show()
```


0.025
0.020
0.015
0.010
0.005
0.000
PMF
20
30
40
50
60
70
80
90
Age (years)
Male
Female

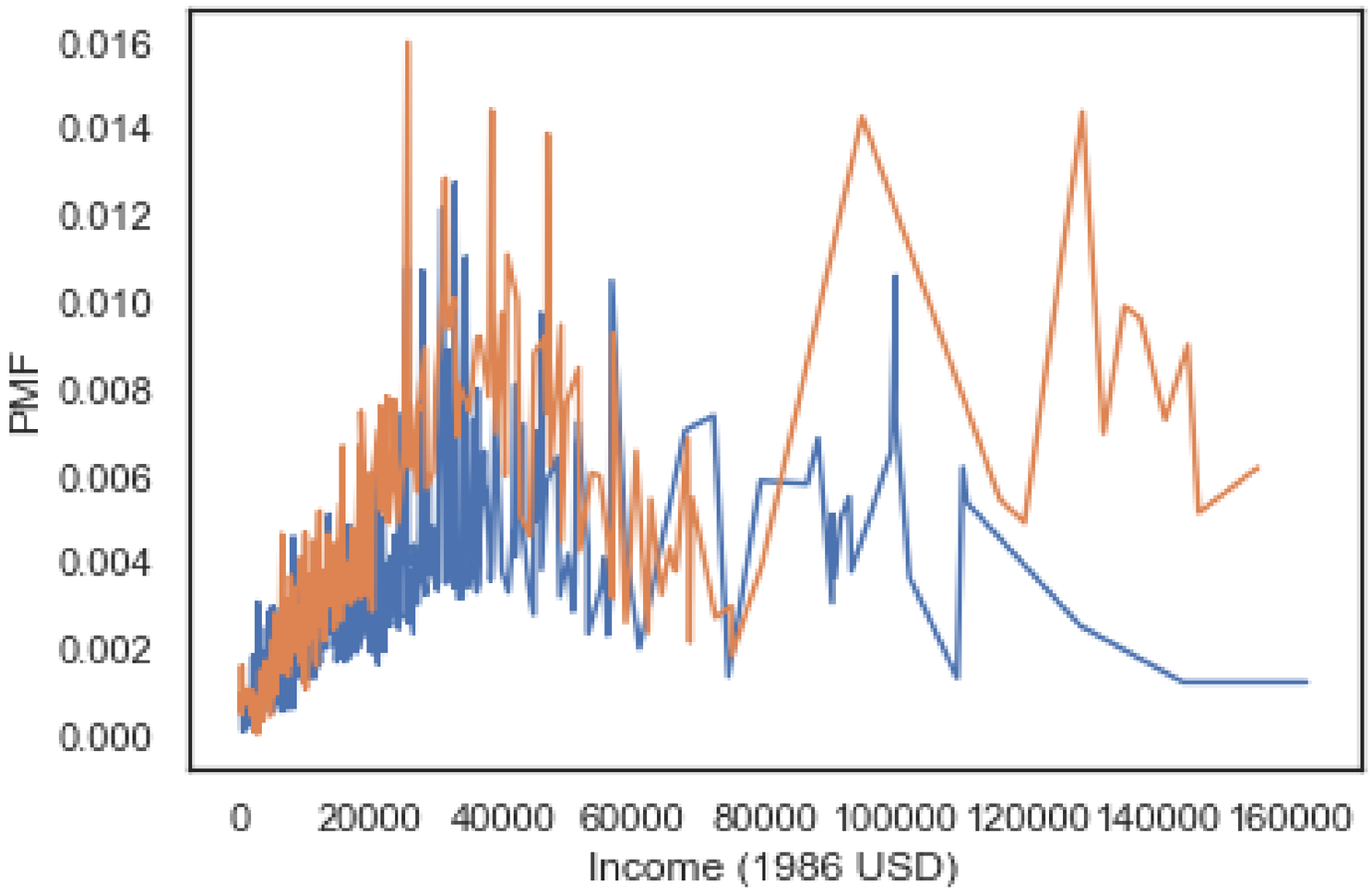
Multiple CDFs

```
Cdf(male_age).plot(label='Male')
Cdf(female_age).plot(label='Female')

plt.xlabel('Age (years)')
plt.ylabel('Count')
plt.show()
```


Male
Female
CDF
1.0
0.8
0.6
0.4
0.2
0.0
20
30
40
50
60
70
80
90
Age (years)

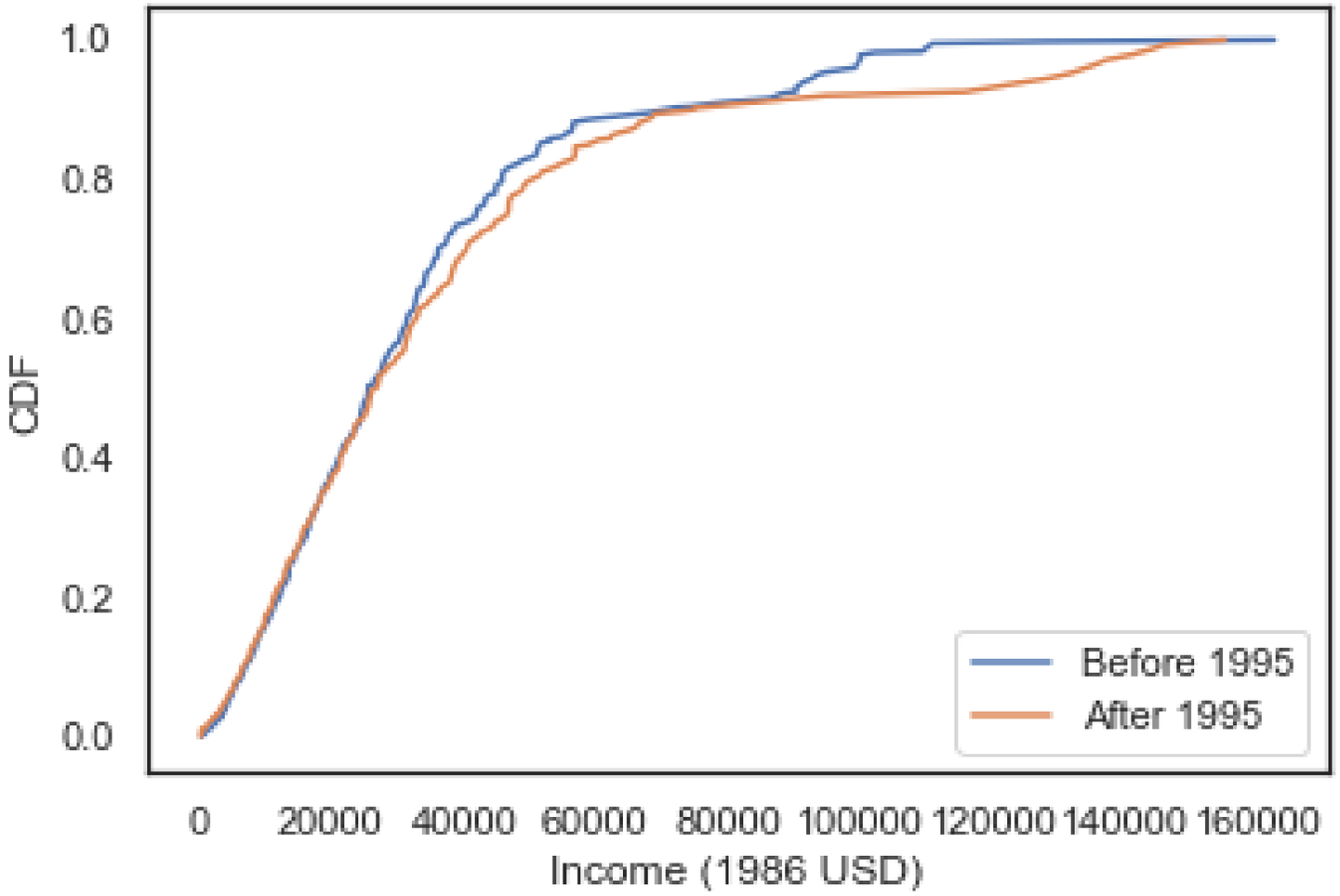
Income distribution

```
income = gss['realinc']

pre95 = gss['year'] < 1995

Pmf(income[pre95]).plot(label='Before 1995')
Pmf(income[~pre95]).plot(label='After 1995')

plt.xlabel('Income (1986 USD)')
plt.ylabel('PMF')
plt.show()
```


0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
PMF
0
20000
40000
60000
80000
100000
120000
140000
160000
Income (1986 USD)

Income CDFs

```
Cdf(income[pre95]).plot(label='Before 1995')
Cdf(income[~pre95]).plot(label='After 1995')
```


1.0
0.8
0.6
0.4
0.2
0.0
CDF
0
20000
40000
60000
80000
100000
120000
140000
160000
Income (1986 USD)
Before 1995
After 1995

Let's practice!

EXPLORATORY DATA ANALYSIS IN PYTHON

Modeling distributions

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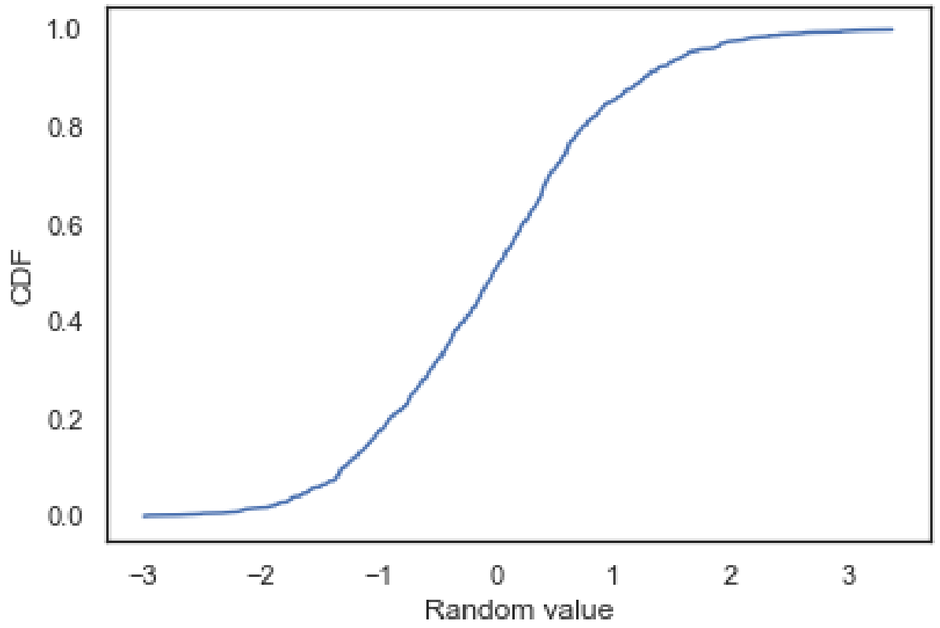
The normal distribution

```
sample = np.random.normal(size=1000)
Cdf(sample).plot()
```



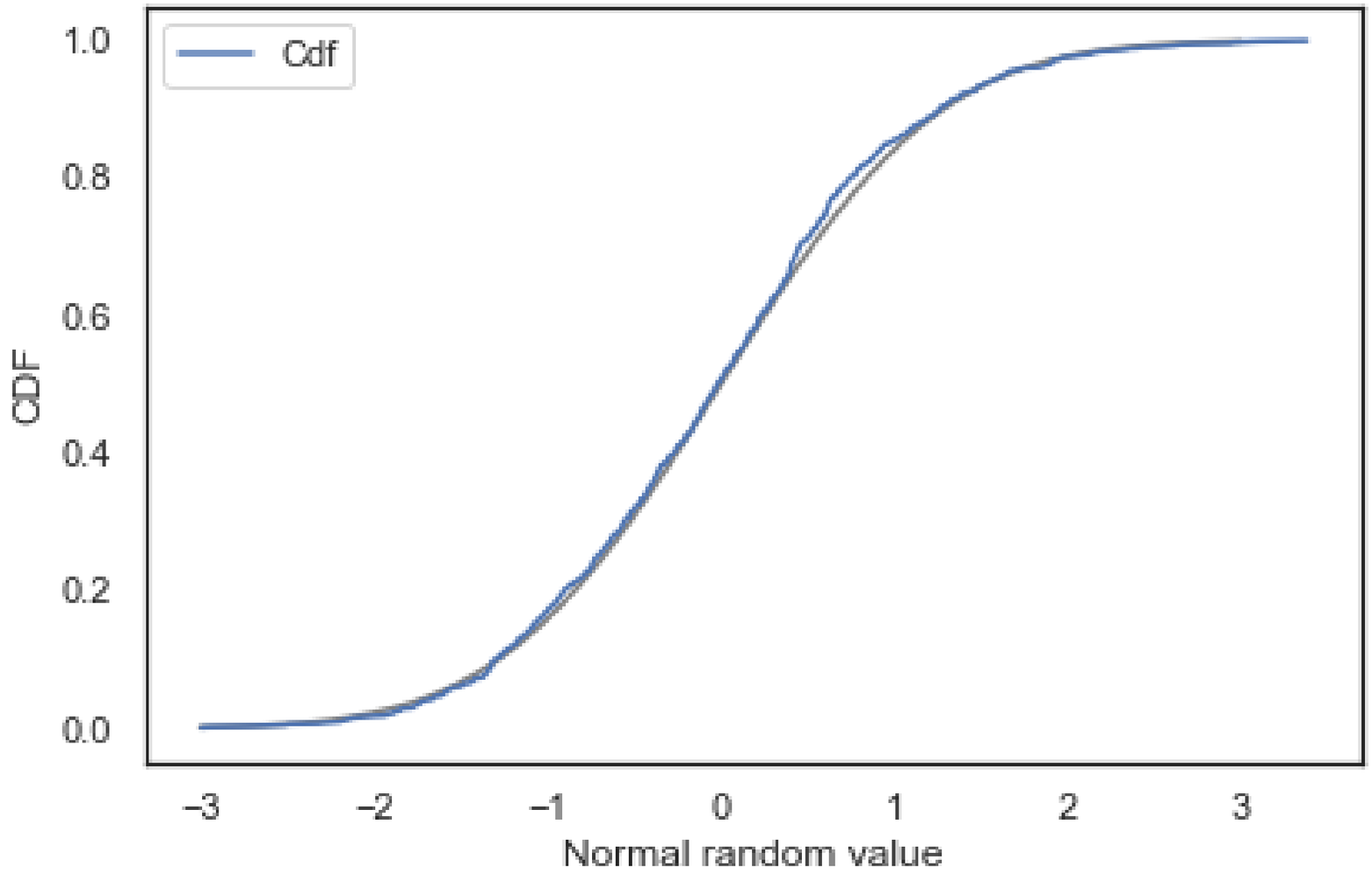
The normal CDF

```
from scipy.stats import norm
```

```
xs = np.linspace(-3, 3)
ys = norm(0, 1).cdf(xs)
```

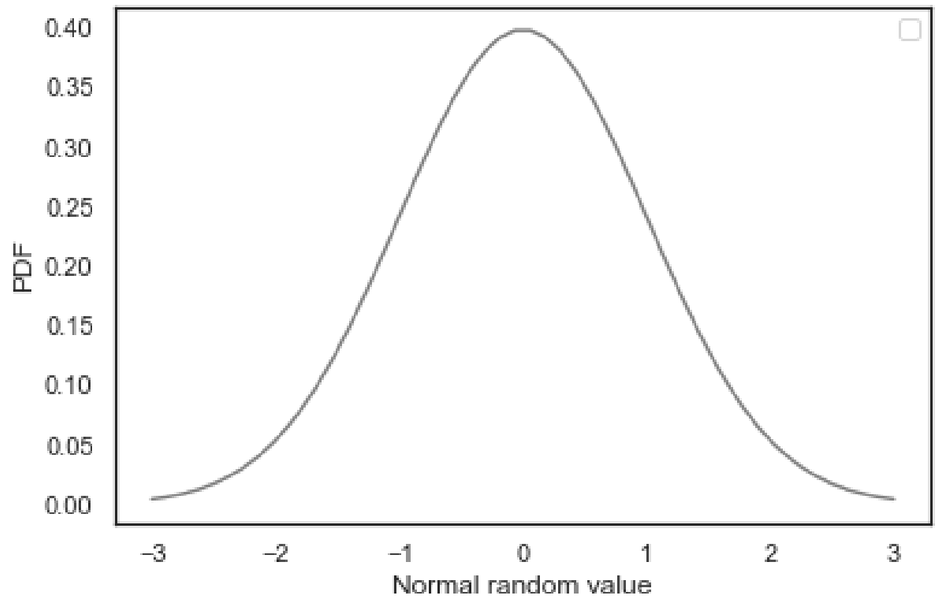
```
plt.plot(xs, ys, color='gray')
```

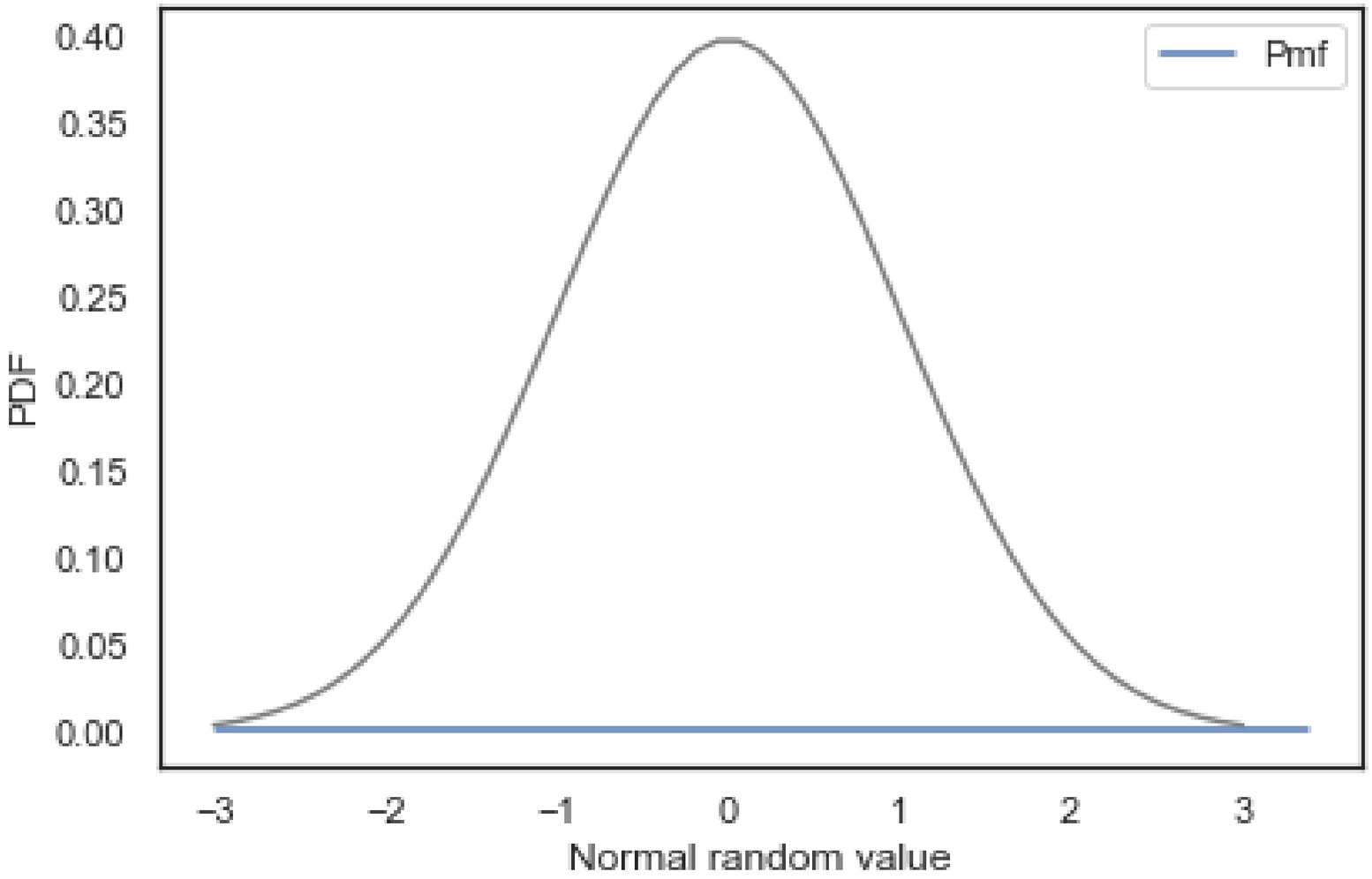
```
Cdf(sample).plot()
```


Cdf
1.0
0.8
0.6
0.4
0.2
0.0
CDF
−3
−2
−1
0
1
2
3
Normal random value

The bell curve

```
xs = np.linspace(-3, 3)
ys = norm(0,1).pdf(xs)
plt.plot(xs, ys, color='gray')
```

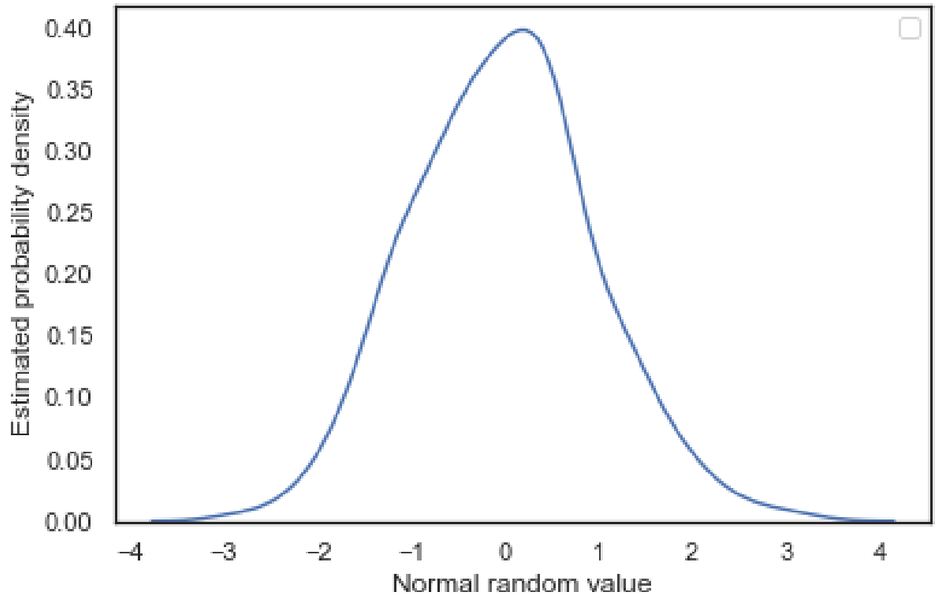



Pmf
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
PDF
−3
−2
−1
0
1
2
3
Normal random value

KDE plot

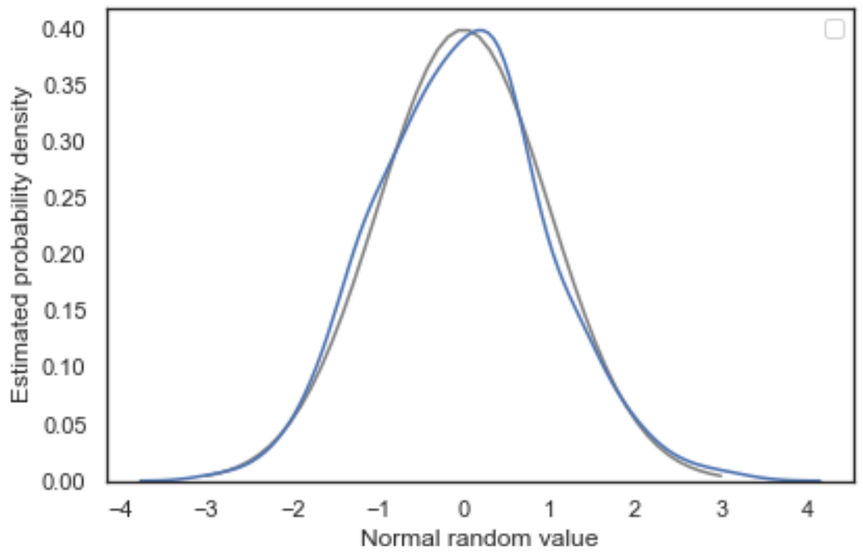
```
import seaborn as sns

sns.kdeplot(sample)
```



KDE and PDF

```
xs = np.linspace(-3, 3)
ys = norm.pdf(xs)
plt.plot(xs, ys, color='gray')
sns.kdeplot(sample)
```



PMF, CDF, KDE

- Use CDFs for exploration.
- Use PMFs if there are a small number of unique values.
- Use KDE if there are a lot of values.

Let's practice!

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