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CIS 352 Homework 5

1

$$g(n) \in \Omega(f(n))$$

$$\text{As } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$$

This is true as  $n^3$  grows much faster than  $n^2$

2

$$f(n) \in \Omega(g(n))$$

$$\text{As } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$$

This is true, as  $g(n)$  is a logarithmic function while  $f(n)$  is a polylogarithmic function which grows faster

3

$$g(n) \in \Omega(f(n))$$

$$\text{As } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$$

This is true because  $4n$  being a linear function, grows faster than the logarithmic function  $g(n)$

4  $f(n) \in O(g(n))$

As  

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$$

This is true because  $6^n$  grows much faster than  $6^{n/2}$

5  $f(n) \in O(g(n))$

As  

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$$

This is true because  $g(n)$  is an exponential which grows faster than  $f(n)$  (a polynomial)

6  $f(n) \in \Theta(g(n))$

As  

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 1$$

If we take the  $\lim_{n \rightarrow \infty}$  of  $\frac{n^{\log_2(n)}}{n^{\log_2(n)}}$  we get 1

$\therefore f(n) \in \Theta(g(n))$

7

For the second equation it is not a problem. In dealing with asymptotic notation we drop the less significant terms and coefficients, so

$$f(n) = (9 + 2 \cos(n))n + 12n^2$$

$n^2$  is all that matters. So we end up with

$$\lim_{n \rightarrow \infty} \frac{n^2}{n^2} = 2 \text{ so } f \in \Theta(n^2)$$

For the first part what cause the issue is  $\cos$  as it does not have a limit when  $n$  goes to infinity

8  $n$ , as there is one loop that runs  $n$  times

9  $n^2$ , as the outer loop will run  $n$  times while the inner loop will run  $n$  times for every  $n$ , so it will run  $n \cdot n$  or  $n^2$

10  $n^2$ , this loop follows the same logic as the previous one with  $j$  being element  $i$  instead of incremented in  $q$

11  $\log_2(n)$  as for any value of  $n$ ,  $m$  will be incremented by  $m \times 2$ , and with  $m$  being compared to  $n$  it will run  $\log_2(n)$