Homework 6: Linear Regression, Features and Regularization

Due Date: Tuesday 12/10, 11:59 PM

Following our exploration of housing prices in Boston, Massachusetts, we want to study housing prices in Ames, Iowa. We will use linear regression to predict sale prices from features such as size, number of bathrooms and neighborhood. While we want to add many features into the model, we need to avoid overfitting the training set. So along the way, we will get experience with regularization. We will use Ridge Regression to shrink the weights. Since we need an extra parameter to control the amount of shrinking, we will use validation to guess-and-check different values for the extra parameter. By completing Homework 6, you should get...

- Practice reasoning about features for linear regression particularly polynomial features
- Intuition about regularization particularly Ridge Regression
- Understanding of feature normalization to prevent against differ scales between features

We will guide you through some exploratory data analysis, laying out an approach to selecting features for the model. After incorporating the features, we will fit a Ridge Regression model to predict housing prices. Finally we will analyze the error of the model. Along the way, we will try to pull together reusable code for each step. Using packages such as pandas and scikit-learn allows us to build a pipeline rather than rewrite different code for different approaches.

We encourage you to think about ways to improve the model's performance with your classmates. If you are interested in try out your ideas, then you could take part in a related modeling competition (https://www.kaggle.com/c/house-prices-advanced-regression-techniques)

Submission Instructions

You are required to **submit a copy of the notebook to Gradescope**. Follow these steps

1. Download as HTML (`File->Download As->HTML(.html)`).
2. Open the HTML in the browser. Print to .pdf
3. Upload to Gradescope.
4. Map your answers to our questions. Otherwise you may lose points. Please see the rubric below.

Consult the instructional video (https://www.gradescope.com/get_started#student-submission) for more information. Note that

- You should break long lines of code into multiple lines. Otherwise your code will extend out of view from the cell. Consider using `\` followed by a new line.
- For each textual response, please include relevant code that informed your response. For each plotting question, please include the code used to generate the plot. If your plot does not appear in the HTML / pdf output, then use `Image('name_of_file')` to embed it.
- You should not display large output cells such as all rows of a table. Instead convert the input cell from Code to Markdown back to Code to remove the output cell.

You are encouraged to **submit the notebook on Jupyter Hub**. Please navigate to the Assignments tab to submit fetch, modify and submit your notebook. Consult the instructional video (https://nbgrader.readthedocs.io/en/stable/user_guide/highlights.html#student-assignment-list-extension-for-jupyter-notebooks) for more information.

Collaboration Policy

Data science is a collaborative activity. While you may talk with others about the homework, we ask that you **write your solutions individually**. If you do discuss the assignments with others please **include their names** at the top of your solution.

Collaborators: *list names here*

Rubric

Question	Points
Gradescope	2
Question 1	1
Question 2	3
Question 3	3
Question 4a	3
Question 4b	2
Question 5	1
Question 6	3
Question 7	2
Question 8a	2
Question 8b	1
Question 8c (optional)	2
Total	23

Getting Started

In [1]:
```
from IPython.display import Image, Markdown, display

# Import standard packages
import pandas as pd
import numpy as np
import csv
import re

# Set some parameters
np.random.seed(47)
np.set_printoptions(4)
pd.options.display.max_rows = 20
pd.options.display.max_columns = 15
pd.set_option('precision', 2)

# Import standard plotting packages
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns

# Set some parameters
plt.rcParams['figure.figsize'] = (12, 9)
plt.rcParams['font.size'] = 12

from sklearn.linear_model import Ridge
```

Fetching the Data

The Ames dataset (http://jse.amstat.org/v19n3/decock.pdf) consists of 2928 records taken from the Ames, Iowa, Assessor's Office describing houses sold in Ames from 2006 to 2010. The data set has 23 nominal, 23 ordinal, 14 discrete, and 20 continuous variables (and 2 additional observation identifiers) --- 82 features in total. An explanation of each variable can be found in the included `codebook.txt` file. The information was used in computing assessed values for individual residential properties sold in Ames, Iowa from 2006 to 2010.

The data are split into training and test sets with 1998 and 930 observations, respectively.

In [2]:
```
training_data = pd.read_csv("training.csv")
testing_data = pd.read_csv("testing.csv")
```

We shold verify that the data shape matches the description.

In [3]:
```
# TEST

# 2000 observations and 82 features in training data
assert training_data.shape == (1998, 82)

# 930 observations and 81 features in test data
assert testing_data.shape == (930, 82)

# training and testing should have the same columns
assert len(np.intersect1d(testing_data.columns.values,
                          training_data.columns.values)) == 82
```

The Ames data set contains information that typical homebuyers would want to know. A more detailed description of each variable is included in `codebook.txt`. You should take some time to familiarize yourself with the codebook before moving forward.

In [4]:
```
# RUN

training_data.columns.values
```

Out[4]:
```
array(['Order', 'PID', 'MS_SubClass', 'MS_Zoning', 'Lot_Frontage',
       'Lot_Area', 'Street', 'Alley', 'Lot_Shape', 'Land_Contour',
       'Utilities', 'Lot_Config', 'Land_Slope', 'Neighborhood',
       'Condition_1', 'Condition_2', 'Bldg_Type', 'House_Style',
       'Overall_Qual', 'Overall_Cond', 'Year_Built', 'Year_Remod/Add',
       'Roof_Style', 'Roof_Matl', 'Exterior_1st', 'Exterior_2nd',
       'Mas_Vnr_Type', 'Mas_Vnr_Area', 'Exter_Qual', 'Exter_Cond',
       'Foundation', 'Bsmt_Qual', 'Bsmt_Cond', 'Bsmt_Exposure',
       'BsmtFin_Type_1', 'BsmtFin_SF_1', 'BsmtFin_Type_2', 'BsmtFin_SF_2',
       'Bsmt_Unf_SF', 'Total_Bsmt_SF', 'Heating', 'Heating_QC',
       'Central_Air', 'Electrical', '1st_Flr_SF', '2nd_Flr_SF',
       'Low_Qual_Fin_SF', 'Gr_Liv_Area', 'Bsmt_Full_Bath',
       'Bsmt_Half_Bath', 'Full_Bath', 'Half_Bath', 'Bedroom_AbvGr',
       'Kitchen_AbvGr', 'Kitchen_Qual', 'TotRms_AbvGrd', 'Functional',
       'Fireplaces', 'Fireplace_Qu', 'Garage_Type', 'Garage_Yr_Blt',
       'Garage_Finish', 'Garage_Cars', 'Garage_Area', 'Garage_Qual',
       'Garage_Cond', 'Paved_Drive', 'Wood_Deck_SF', 'Open_Porch_SF',
       'Enclosed_Porch', '3Ssn_Porch', 'Screen_Porch', 'Pool_Area',
       'Pool_QC', 'Fence', 'Misc_Feature', 'Misc_Val', 'Mo_Sold',
       'Yr_Sold', 'Sale_Type', 'Sale_Condition', 'SalePrice'],
      dtype=object)
```

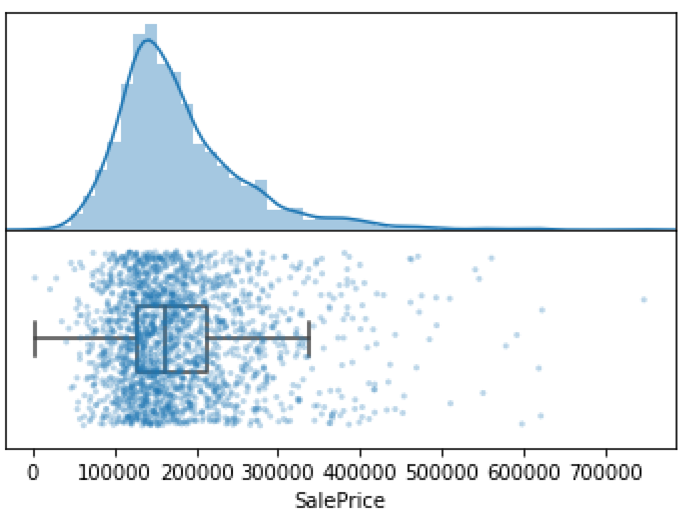
Exploratory Data Analysis

We will generate a couple visualizations to understand the relationship between `SalePrice` and other features. Note that we will examine the training data so that information from the testing data does not influence our modeling decisions. Looking at the testing data introduces bias (https://en.wikipedia.org/wiki/Data_dredging) into the model.

Question 1

We begin by examining a raincloud plot (https://micahallen.org/2018/03/15/introducing-raincloud-plots/amp/?__twitter_impression=true) (a combination a histogram with density, a strip plot, and a box plot) of our target variable `SalePrice` . At the same time, we also take a look at some descriptive statistics of this variable.

In [5]:
```
fig, axs = plt.subplots(nrows=2)

sns.distplot(
    training_data['SalePrice'],
    ax=axs[0]
)
sns.stripplot(
    training_data['SalePrice'],
    jitter=0.4,
    size=3,
    ax=axs[1],
    alpha=0.3
)
sns.boxplot(
    training_data['SalePrice'],
    width=0.3,
    ax=axs[1],
    showfliers=False,
)

# Align axes
spacer = np.max(training_data['SalePrice']) * 0.05
xmin = np.min(training_data['SalePrice']) - spacer
xmax = np.max(training_data['SalePrice']) + spacer
axs[0].set_xlim((xmin, xmax))
axs[1].set_xlim((xmin, xmax))

# Remove some axis text
axs[0].xaxis.set_visible(False)
axs[0].yaxis.set_visible(False)
axs[1].yaxis.set_visible(False)

# Put the two plots together
plt.subplots_adjust(hspace=0)

# Adjust boxplot fill to be white
axs[1].artists[0].set_facecolor('white')
```



In [6]:
```
training_data['SalePrice'].describe()
```

Out[6]:
```
count      1998.00
mean     180785.11
std       81619.69
min        2489.00
25%      128600.00
50%      162000.00
75%      213175.00
max      747800.00
Name: SalePrice, dtype: float64
```

To check your understanding of the graph and summary statistics above, answer the following `True` or `False` questions:

1. The distribution of `SalePrice` in the training set is left-skew.
2. The mean of `SalePrice` in the training set is greater than the median.
3. At least 25% of the houses in the training set sold for more than $200,000.00.

In [7]:
```
q1statement1 = False
q1statement2 = True
q1statement3 = True
#raise NotImplementedError()
```

In [8]:
```
# TEST
set([q1statement1, q1statement2, q1statement3]).issubset({False, True})
```

Out[8]:
```
True
```

In [ ]:

Question 2

We know that Total Bathrooms can be calculated as:

$$\text{TotalBathrooms} = (\text{BsmtFullBath} + \text{FullBath}) + \frac{1}{2}(\text{BsmtHalfBath} + \text{HalfBath})$$

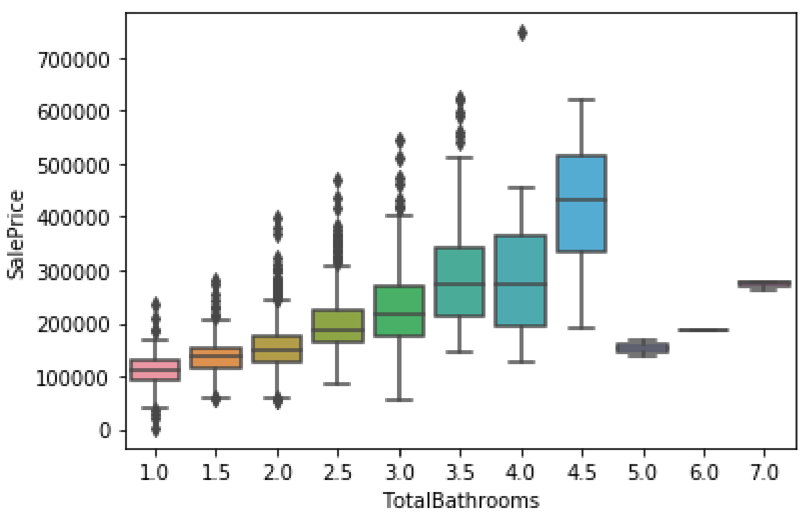
Write a function `add_total_bathrooms(data)` that returns a copy of `data` with an additional column called `TotalBathrooms` computed by the formula above. Treat missing values as zeros. Remember that you can make use of vectorized code here; you shouldn't need any `for` statements.

In [9]:
```
def add_total_bathrooms(data):
    """
    Input:
      data (data frame): a data frame containing at least 4 numeric columns
            Bsmt_Full_Bath, Full_Bath, Bsmt_Half_Bath, and Half_Bath
    """
    with_bathrooms = data.copy()
    bath_vars = ['Bsmt_Full_Bath', 'Full_Bath', 'Bsmt_Half_Bath', 'Half_Bath']
    weights = pd.Series([1, 1, 0.5, 0.5], index=bath_vars)
    # YOUR CODE HERE

    with_bathrooms = with_bathrooms.fillna({null: 0 for null in bath_vars})

    with_bathrooms['TotalBathrooms'] = with_bathrooms[bath_vars].dot(weights)
    #raise NotImplementedError()
    return with_bathrooms

training_data = add_total_bathrooms(training_data)
```

In [10]:
```
# TEST
assert not training_data['TotalBathrooms'].isnull().any() # Check that missing values are dealt with
assert training_data['TotalBathrooms'].sum() == 4421.5 # Check that the values are as expected
```

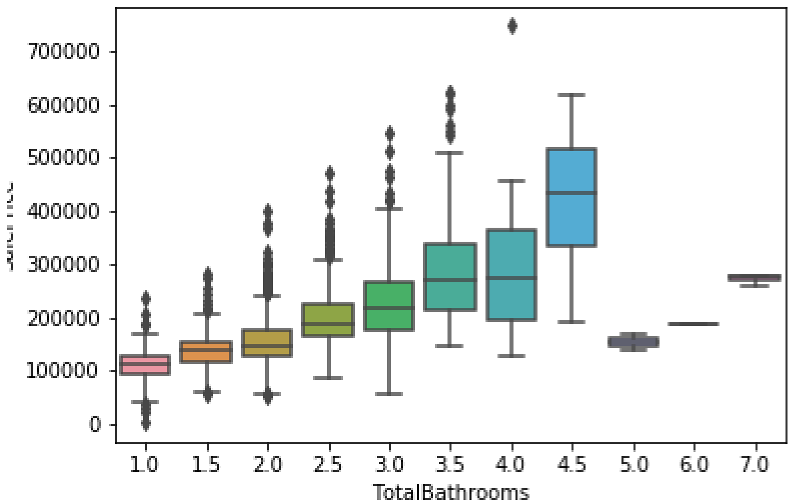
Using a boxplot, shows that `TotalBathrooms` is associated with `SalePrice`. Save your vizualization as `boxplot.png`

In [11]:
```
# YOUR CODE HERE
#raise NotImplementedError()
plot = sns.boxplot(x='TotalBathrooms', y='SalePrice', data=training_data)
plt.savefig('boxplot.png')
```



In [12]:
```
# RUN

Image('boxplot.png')
```

Out[12]:



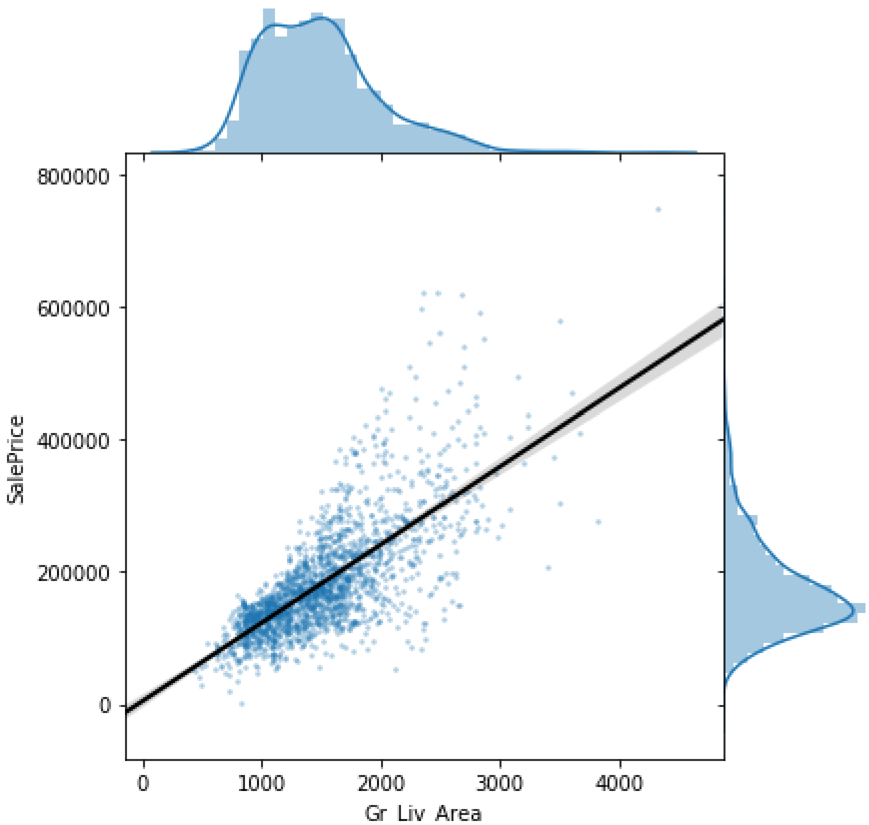
Encoding Features

We will create new features out of old features through some data transformations.

Question 3

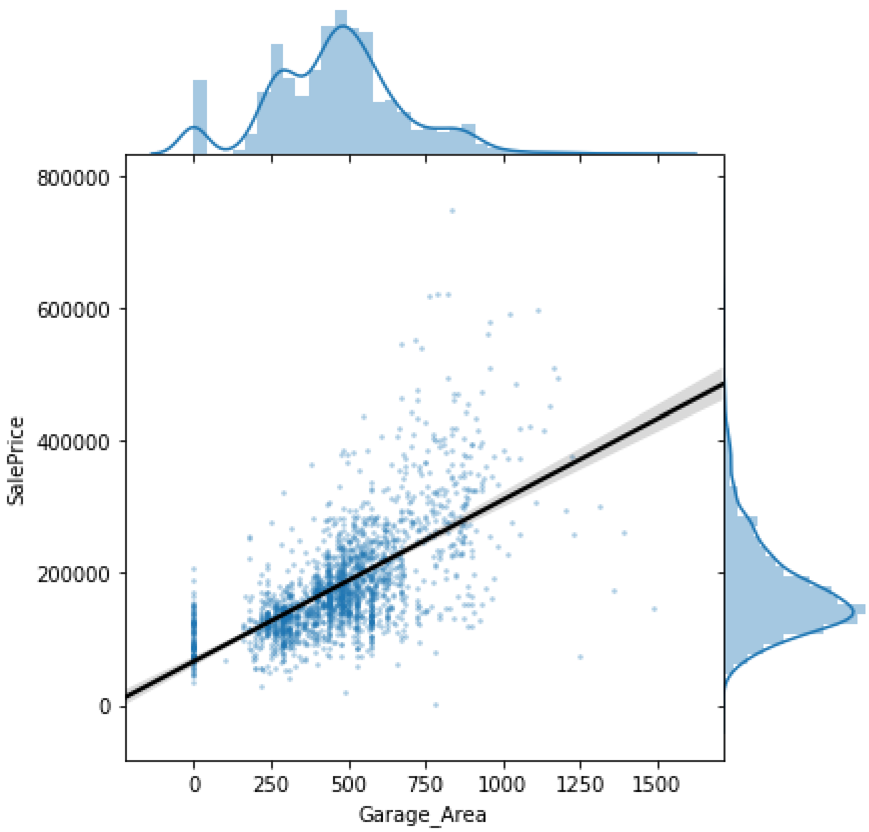
We can visualize the association between `SalePrice` and `Gr_Liv_Area`. The `codebook.txt` file tells us that `Gr_Liv_Area` measures "above grade (ground) living area square feet." This variable represents the square footage of the house excluding anything underground.

In [13]:
```
sns.jointplot(
    x='Gr_Liv_Area',
    y='SalePrice',
    data=training_data,
    stat_func=None,
    kind="reg",
    ratio=4,
    space=0,
    scatter_kws={
        's': 3,
        'alpha': 0.25
    },
    line_kws={
        'color': 'black'
    }
);
```



Since `Gr_Liv_Area` excludes the garage space, we visualize the association between `SalePrice` and `Garage_Area`. The `codebook.txt` file tells us that `Gr_Liv_Area` measures "Size of garage in square feet."

In [14]:
```
sns.jointplot(
    x='Garage_Area',
    y='SalePrice',
    data=training_data,
    stat_func=None,
    kind="reg",
    ratio=4,
    space=0,
    scatter_kws={
        's': 3,
        'alpha': 0.25
    },
    line_kws={
        'color': 'black'
    }
);
```



Write a function called `add_power` that inputs

- a table `data`
- a column name `column_name` of the table
- positive integer `degree`

and outputs

- a copy of `data` with an additional column called `column_name2` containing all entries of `column_name` raised to power `degree`.

In [15]:
```
def add_power(data, column_name, degree):
    """
    Input:
      data (data frame): a data frame containing column called column_name
      column_name (string): a column in data
      degree: positive integer

    Output:
      copy of data containing a column called column_name2 with entries of col
umn_name to power degree
    """
    with_power = data.copy()
    # YOUR CODE HERE
    column_name2 = column_name + '2'
    with_power[column_name2] = with_power[column_name] ** degree

    #raise NotImplementedEq1statement1rror()
    return with_power

training_data = add_power(training_data, "Garage_Area", 2)
training_data = add_power(training_data, "Gr_Liv_Area", 2)
training_data.head()
```

Out[15]:

	Order	PID	MS_SubClass	MS_Zoning	Lot_Frontage	Lot_Area	Street	...	Yr_Sold	Sa
0	1	526301100	20	RL	141.0	31770	Pave	...	2010	
1	2	526350040	20	RH	80.0	11622	Pave	...	2010	
2	3	526351010	20	RL	81.0	14267	Pave	...	2010	
3	4	526353030	20	RL	93.0	11160	Pave	...	2010	
4	5	527105010	60	RL	74.0	13830	Pave	...	2010	

5 rows × 85 columns

Among `Gr_Liv_Area` , `Gr_Liv_Area2` , `Garage_Area` , `Garage_Area2` which has the largest correlation with `SalePrice` ? Note that pd.DataFrame.corr would be helpful.

In [16]:
```
tester = training_data[['Gr_Liv_Area', 'Gr_Liv_Area2', 'Garage_Area', 'Garage_
Area2', 'SalePrice']]
tester.corr(method='pearson')
highest_variable = 'Gr_Liv_Area'
# YOUR CODE HERE
#raise NotImplementedError()
```

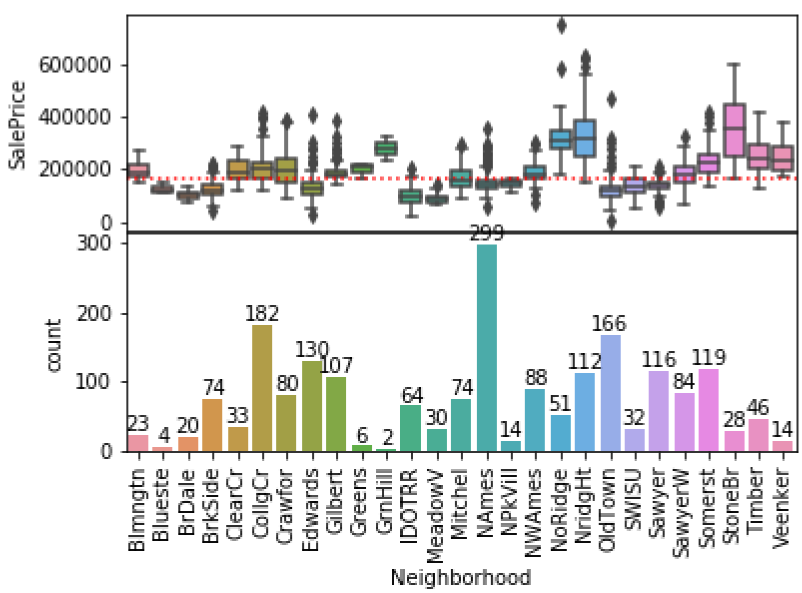
In [17]:
```
# TEST
assert highest_variable in ['Gr_Liv_Area', 'Gr_Liv_Area2', 'Garage_Area', 'Gar
age_Area2']
```

In [ ]:
```

```

Question 4

Let's take a look at the relationship between neighborhood and sale prices of the houses in our data set.

In [18]:
```
fig, axs = plt.subplots(nrows=2)

sns.boxplot(
    x='Neighborhood',
    y='SalePrice',
    data=training_data.sort_values('Neighborhood'),
    ax=axs[0]
)

sns.countplot(
    x='Neighborhood',
    data=training_data.sort_values('Neighborhood'),
    ax=axs[1]
)

# Draw median price
axs[0].axhline(
    y=training_data['SalePrice'].median(),
    color='red',
    linestyle='dotted'
)

# Label the bars with counts
for patch in axs[1].patches:
    x = patch.get_bbox().get_points()[:, 0]
    y = patch.get_bbox().get_points()[1, 1]
    axs[1].annotate(f'{int(y)}', (x.mean(), y), ha='center', va='bottom')

# Format x-axes
axs[1].set_xticklabels(axs[1].xaxis.get_majorticklabels(), rotation=90)
axs[0].xaxis.set_visible(False)

# Narrow the gap between the plots
plt.subplots_adjust(hspace=0.01)
```



We find a lot of variation in prices across neighborhoods. Moreover, the amount of data available is not uniformly distributed among neighborhoods. North Ames, for example, comprises almost 15% of the training data while Green Hill has only 2 observations in this data set.

One way we can deal with the lack of data from some neighborhoods is to create a new feature that bins neighborhoods together. Let's categorize our neighborhoods in a crude way: we'll take the top 3 neighborhoods measured by median `SalePrice` and identify them as "rich neighborhoods"; the other neighborhoods are not marked.

Question 4a

Write a function that returns a list of the top `n` neighborhoods as measured by our choice of aggregating function. For example, in the setup above, we would want to call `find_rich_neighborhoods(training_data, 3, np.median)` to find the top 3 neighborhoods measured by median `SalePrice`.

Hint Try using the operations `groupby`, `apply` and `sort_values`.

In [19]:
```
training_data.head()
```

Out[19]:

	Order	PID	MS_SubClass	MS_Zoning	Lot_Frontage	Lot_Area	Street	...	Yr_Sold	Sa
0	1	526301100	20	RL	141.0	31770	Pave	...	2010	
1	2	526350040	20	RH	80.0	11622	Pave	...	2010	
2	3	526351010	20	RL	81.0	14267	Pave	...	2010	
3	4	526353030	20	RL	93.0	11160	Pave	...	2010	
4	5	527105010	60	RL	74.0	13830	Pave	...	2010	

5 rows × 85 columns

In [20]:
```
def find_rich_neighborhoods(data, n=3, metric=np.median):
    """
    Input:
      data (data frame): should contain at least a string-valued Neighborhood
        and a numeric SalePrice column
      n (int): the number of top values desired
      metric (function): function used for aggregating the data in each neighb
orhood.
        for example, np.median for median prices

    Output:
      a list of the top n richest neighborhoods as measured by the metric func
tion
    """
    neighborhoods = data.groupby('Neighborhood').agg(np.median).sort_values(by
='SalePrice', ascending=False)
    neighborhoods = neighborhoods.iloc[:n]
    neighborhoods = list(neighborhoods.index.values)
    # YOUR CODE HERE
    #raise NotImplementedError()
    return neighborhoods

rich_neighborhoods = find_rich_neighborhoods(training_data, 3, np.median)
rich_neighborhoods
```

Out[20]: `['StoneBr', 'NridgHt', 'NoRidge']`

In [21]:
```
# TEST
assert len(find_rich_neighborhoods(training_data, 5, np.median))
assert isinstance(rich_neighborhoods, list)
assert all([isinstance(neighborhood, str) for neighborhood in rich_neighborhoo
ds])
```

In [ ]:

Question 4b

We now have a list of neighborhoods we've deemed as richer than others. Let's use that information to make a new variable `in_rich_neighborhood` . Write a function `add_rich_neighborhood` that adds an indicator variable which takes on the value 1 if the house is part of `rich_neighborhoods` and the value 0 otherwise.

Note that `pd.Series.astype` (https://pandas.pydata.org/pandas-docs/version/0.23.4/generated/pandas.Series.astype.html) may be useful for converting True/False values to integers.

In [22]:
```
def add_in_rich_neighborhood(data, neighborhoods):
    """
    Input:
      data (data frame): a data frame containing a 'Neighborhood' column with
 values
        found in the codebook
      neighborhoods (list of strings): strings should be the names of neighbor
hoods
    Output:
      data frame identical to the input with the addition of a binary
      in_rich_neighborhood column
    """
    temp = data['Neighborhood'].isin(neighborhoods)
    data['in_rich_neighborhood'] = temp
    data['in_rich_neighborhood'] = data['in_rich_neighborhood'].astype('int32'
)
    # YOUR CODE HERE
    #raise NotImplementedError()
    return data

rich_neighborhoods = find_rich_neighborhoods(training_data, 3, np.median)
training_data = add_in_rich_neighborhood(training_data, rich_neighborhoods)
```

In [23]:
```
training_data.loc[training_data['Neighborhood'] == 'StoneBr']
```

Out[23]:

	Order	PID	MS_SubClass	MS_Zoning	Lot_Frontage	Lot_Area	Street	...	Sale_Typ
6	8	527145080	120	RL	43.0	5005	Pave	...	W
7	9	527146030	120	RL	39.0	5389	Pave	...	W
12	18	527258010	20	RL	88.0	11394	Pave	...	Ne
248	350	527127100	120	RL	28.0	7296	Pave	...	W
249	352	527132090	120	RL	61.0	7380	Pave	...	W
261	366	527182110	120	RL	NaN	5814	Pave	...	CC
262	367	527214050	20	RL	63.0	17423	Pave	...	Ne
263	368	527254020	20	RL	80.0	11844	Pave	...	Ne
264	369	527258020	20	RL	124.0	16158	Pave	...	W
673	1001	527131110	120	RL	45.0	6264	Pave	...	W
...	...	...	...	...	...	...	...	...	
1109	1642	527256030	20	RL	85.0	14082	Pave	...	W
1110	1643	527256040	20	RL	81.0	13870	Pave	...	Ne
1572	2323	527146135	160	RL	68.0	13108	Pave	...	W
1575	2328	527190050	160	RL	44.0	5306	Pave	...	W
1576	2330	527210030	60	RL	59.0	16023	Pave	...	Ne
1577	2333	527212030	60	RL	85.0	16056	Pave	...	Ne
1578	2334	527212040	60	RL	82.0	12438	Pave	...	Ne
1579	2335	527214060	60	RL	82.0	16052	Pave	...	Ne
1580	2336	527216010	60	RL	92.0	15922	Pave	...	Ne
1583	2342	527256120	20	RL	90.0	18261	Pave	...	W

28 rows × 86 columns

In [24]:
```
# TEST
assert sum(training_data.loc[:, 'in_rich_neighborhood']) == 191
assert sum(training_data.loc[:, 'in_rich_neighborhood'].isnull()) == 0
```

In [ ]:

Modeling

We can use the features from Question 2, Question 3, and Question 4 to determine a model.

Question 5

Remember that we need to normalize features for regularization. If the features have different scales, then regularization will unduly shrink the weights for features with smaller scales.

Write a function called `standardize` that inputs either a 1 dimensional array or a 2 dimensional array `Z` of numbers and outputs a copy of `Z` where the columns have been transformed to have mean 0 and standard deviation 1.

To avoid dividing by a small number, you should add 0.00001 to the standard deviation in the denominator.

In [25]:
```
from sklearn import preprocessing

def standardize(Z):
    """
    Input:
       Z: 1 dimensional or 2 dimensional array
    Outuput
       copy of Z with columns having mean 0 and variance 1
    """
    Z = (Z - np.mean(Z, axis=0)) / np.std(Z, axis=0)

    return Z
    # YOUR CODE HERE
    raise NotImplementedError()
```

In [26]:
```
Z = training_data[['Garage_Area','Gr_Liv_Area']].values
Z
```

Out[26]:
```
array([[ 528, 1656],
       [ 730,  896],
       [ 312, 1329],
       ...,
       [ 484,  902],
       [   0,  970],
       [ 650, 2000]])
```

In [27]:
```
# TEST

Z = training_data[['Garage_Area','Gr_Liv_Area']].values

assert np.all(np.isclose(standardize(Z).sum(axis = 0), [0,0]))
```

Question 6

Let's split the training set into a training set and a validation set. We will use the training set to fit our model's parameters. We will use the validation set to estimate how well our model will perform on unseen data. If we used all the data to fit our model, we would not have a way to estimate model performance on unseen data.

In [28]:
```
# RUN

training_data_copy = pd.read_csv('training.csv')
```

Split the data in `training_data_copy` into two DataFrames named `training_data` and `validating_data`. Let `training_data` contain 80% of the data, and let `validating_data` contain the remaining 20% of the data.

To do this, first create two NumPy arrays named `train_indices` and `validate_indices`. `train_indices` should contain a *random* 80% of the indices, and `validate_indices` should contain the remaining 20% of the indices. Then, use these arrays as indices to break `training_data_copy` into two pieces.

In [29]:
```
# This makes the train-test split in this section reproducible across differen
t runs
# of the notebook. You do not need this line to run train_test_split in genera
l
np.random.seed(47)

training_data_len = len(training_data_copy)
indices = np.arange(training_data_len)
shuffled_indices = np.random.permutation(indices)
```

In [30]:
```
# Set train_indices to the first 80% of shuffled_indices and validate_indices
 to the rest.

# YOUR CODE HERE

train_indices = shuffled_indices[:int(training_data_len * 0.8)]
validate_indices = shuffled_indices[int(training_data_len * 0.8):]
#raise NotImplementedError()
```

In [31]:
```
# Create training_data and validating_data by indexing training_data_copy with
# `train_indices` and `validate_indices`

# YOUR CODE HERE

training_data = training_data_copy.loc[train_indices]
validating_data = training_data_copy.loc[validate_indices]
#raise NotImplementedError()
```

In [32]:
```
# TEST
assert training_data.shape == (1598, 82) # train should contain 80% of the dat
a
assert validating_data.shape == (400, 82) # test should contain 20% of the dat
a
```

In [33]:
```
# TEST
assert np.intersect1d(train_indices, validate_indices).size == 0 # make sure t
rain_indices and test_indices have no overlap
assert np.intersect1d(training_data['PID'], validating_data['PID']).size == 0
# make sure train and test have no overlapping houses
```

Reusable Pipeline

We want to try a couple different models. For each model, we will have to apply transformations to the data. By bundling the transformations together, we can apply can efficiently pass data to the different models.

We use a single function called `process_data` . We select a handful of features to use from the many that are available.

In [34]:
```
# RUN

def select_columns(data, columns):
    """Select only columns passed as arguments."""
    return data.loc[:, columns]

def process_data(data):
    """Process the data for a guided model."""

    # Transform Data, Select Features
    nghds = find_rich_neighborhoods(data, 3, metric=np.median)

    data = ( data.pipe(add_total_bathrooms)
                 .pipe(add_power,'Gr_Liv_Area', 2)
                 .pipe(add_power,'Garage_Area', 2)
                 .pipe(add_in_rich_neighborhood, nghds)
                 .pipe(select_columns, ['SalePrice',
                                         'Gr_Liv_Area',
                                         'Garage_Area',
                                         'Gr_Liv_Area2',
                                         'Garage_Area2',
                                         'TotalBathrooms',
                                         'in_rich_neighborhood']) )

    # Return predictors and response variables separately
    data.dropna(inplace = True)
    X = data.drop(['SalePrice'], axis = 1)
    X = standardize(X)
    y = data.loc[:, 'SalePrice']
    y = standardize(y)

    return X, y
```

Note that we split our data into `X` , a matrix of features, and `y` , a vector of sale prices.

Run the cell below to feed our training, validating and testing data through the pipeline, generating `X_train` , `y_train` , `X_test` , and `y_test` .

In [35]:
```
# RUN
# Pre-process our training and test data in exactly the same way

X_train, y_train = process_data(training_data)
X_validate, y_validate = process_data(validating_data)
X_test, y_test = process_data(testing_data)
```

Fitting the Model

We are ready to fit a model. The model we will fit can be written as follows:

$$\begin{aligned} \text{SalePrice} = \theta_0 + \theta_1 \cdot \text{Gr\_Liv\_Area} + \theta_2 \cdot \text{Gr\_Liv\_Area2} \\ + \theta_3 \cdot \text{Garage\_Area} + \theta_4 \cdot \text{Garage\_Area2} \\ + \theta_5 \cdot \text{is\_in\_rich\_neighborhood} \\ + \theta_6 \cdot \text{TotalBathrooms} \end{aligned}$$

Here `Gr_Liv_Area` , `Gr_Liv_Area2` , `Garage_Area` , and `Garage_Area2` are continuous variables and `is_in_rich_neighborhood` and `TotalBathrooms` are discrete variables. While `is_in_rich_neighborhood` is a one-hot encoding of categories, `TotalBathrooms` can be understsood as a number.

Question 7

We will use a [`sklearn.linear_model.Ridge` (https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Ridge.html)](https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Ridge.html) to implement Ridge Regression. We must specify three inputs.

- `normalize` : Having applied the function `standardize` to the data, we should set `normalize` to False
- `fit_intercept` : Having applied the function `standardize` to the data, we should set `fit_intercept` to False. The intercept of our model corresponds to θ_0. Since the mean of the columns is 0, we know that $\widehat{\theta_0} = 0$
- `alpha` : We need an extra parameter to specify the emphasis on regularization. Large values of `alpha` mean greater emphasis on regularization. We will try a range of values.

For each value of alpha, generate a `Ridge` model. Store in the dictionary `models` .

In [36]:
```
models = dict()
alphas = np.logspace(-4,4,10)

for alpha in alphas:
    ridge_regression_model = Ridge(alpha, normalize=False, fit_intercept=False
)
    # YOUR CODE HERE
   # raise NotImplementedError()
    models[alpha] = ridge_regression_model
```

In [ ]:

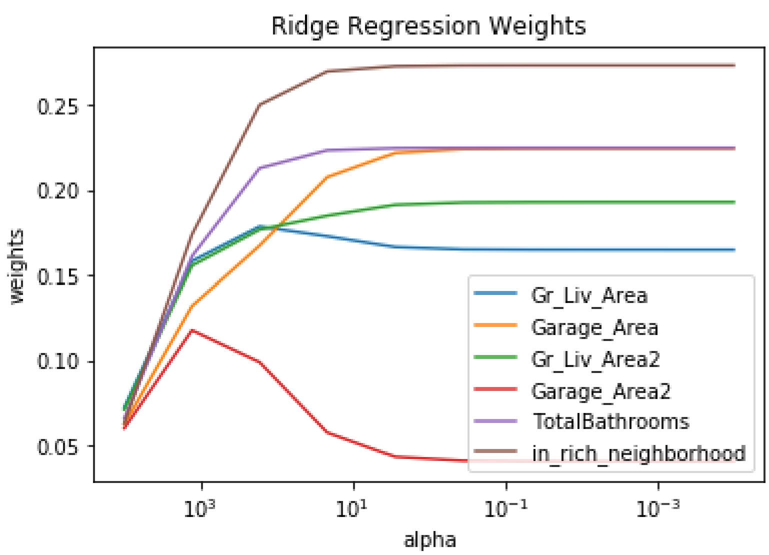
For each `alpha` , fit the corresponding model in `models` with `X_train` , `y_train` .

In [37]:
```
for alpha, model in models.items():
    models[alpha].fit(X_train, y_train)

    # YOUR CODE HERE
    #raise NotImplementedError()
```

In [ ]:

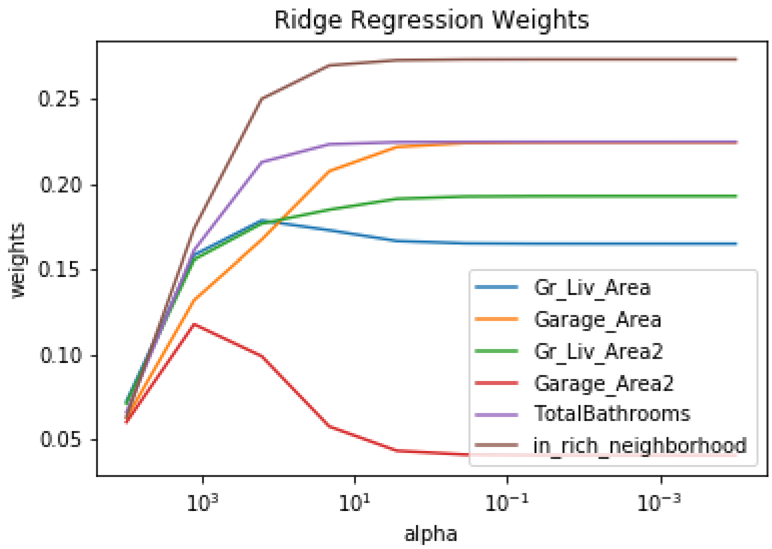
Plot the weights for each value of `alpha` .

In [38]:
```
# RUN

labels =   ['Gr_Liv_Area',
            'Garage_Area',
            'Gr_Liv_Area2',
            'Garage_Area2',
            'TotalBathrooms',
            'in_rich_neighborhood']

coefs = []
for alpha, model in models.items():
    coefs.append(model.coef_)

coefs = zip(*coefs)

fig, ax = plt.subplots(ncols=1, nrows=1)

for coef, label in zip(coefs, labels):
    plt.plot(alphas, coef, label = label)

ax.set_xscale('log')
ax.set_xlim(ax.get_xlim()[::-1])  # reverse axis
plt.xlabel('alpha')
plt.ylabel('weights')
plt.title('Ridge Regression Weights')
plt.legend()
plt.savefig('reg_path.png')
```



In [39]:
```
# RUN

Image('reg_path.png')
```

Out[39]:



Evaluating the Model

Is our linear model any good at predicting house prices? Let's measure the quality of our model by calculating the Root-Mean-Square Error (RMSE) between our predicted house prices and the true prices stored in `SalePrice`.

$$\text{RMSE} = \sqrt{\frac{\sum_{\text{houses in test set}}(\text{actual price of house} - \text{predicted price of house})^2}{\text{\# of houses in data set}}}$$

Here we have a function called `rmse` that calculates the RMSE of a model.

In [40]:
```
# RUN

def rmse(actual, predicted):
    """
    Calculates RMSE from actual and predicted values
    Input:
      actual (1D array): vector of actual values
      predicted (1D array): vector of predicted/fitted values
    Output:
      a float, the root-mean square error
    """
    return np.sqrt(np.mean((actual - predicted)**2)) # SOLUTION
```

Question 8a

For each `alpha`, use `rmse` to calculate the training error and validating error.

In [41]:
```
rmse_training = dict()
rmse_validating = dict()

for alpha, model in models.items():

    rmse_training[alpha] = rmse(X_train[alpha], models[alpha])
    rmse_validating[alpha] = rmse(X_validate[alpha], models[alpha])


rmse_training = sorted(rmse_training.items(), key = lambda k: k[1])
rmse_validating = sorted(rmse_validating.items(), key = lambda k: k[1])
```

```
---------------------------------------------------------------------------
KeyError                                  Traceback (most recent call last)
/share/apps/jupyterhub/2019-FA-DS-UA-112/lib/python3.7/site-packages/pandas/c
ore/indexes/base.py in get_loc(self, key, method, tolerance)
   2656             try:
-> 2657                 return self._engine.get_loc(key)
   2658             except KeyError:

pandas/_libs/index.pyx in pandas._libs.index.IndexEngine.get_loc()

pandas/_libs/index.pyx in pandas._libs.index.IndexEngine.get_loc()

pandas/_libs/hashtable_class_helper.pxi in pandas._libs.hashtable.PyObjectHas
hTable.get_item()

pandas/_libs/hashtable_class_helper.pxi in pandas._libs.hashtable.PyObjectHas
hTable.get_item()

KeyError: 0.0001

During handling of the above exception, another exception occurred:

KeyError                                  Traceback (most recent call last)
<ipython-input-41-dbbd5d7afb92> in <module>
      4 for alpha, model in models.items():
      5 
----> 6     rmse_training[alpha] = rmse(X_train[alpha], models[alpha])
      7     rmse_validating[alpha] = rmse(X_validate[alpha], models[alpha])
      8 

/share/apps/jupyterhub/2019-FA-DS-UA-112/lib/python3.7/site-packages/pandas/c
ore/frame.py in __getitem__(self, key)
   2925             if self.columns.nlevels > 1:
   2926                 return self._getitem_multilevel(key)
-> 2927             indexer = self.columns.get_loc(key)
   2928             if is_integer(indexer):
   2929                 indexer = [indexer]

/share/apps/jupyterhub/2019-FA-DS-UA-112/lib/python3.7/site-packages/pandas/c
ore/indexes/base.py in get_loc(self, key, method, tolerance)
   2657                 return self._engine.get_loc(key)
   2658             except KeyError:
-> 2659                 return self._engine.get_loc(self._maybe_cast_indexer(
key))
   2660         indexer = self.get_indexer([key], method=method, tolerance=to
lerance)
   2661         if indexer.ndim > 1 or indexer.size > 1:

pandas/_libs/index.pyx in pandas._libs.index.IndexEngine.get_loc()

pandas/_libs/index.pyx in pandas._libs.index.IndexEngine.get_loc()

pandas/_libs/hashtable_class_helper.pxi in pandas._libs.hashtable.PyObjectHas
hTable.get_item()

pandas/_libs/hashtable_class_helper.pxi in pandas._libs.hashtable.PyObjectHas
hTable.get_item()
```

```
KeyError: 0.0001
```

Which value of `alpha` has the smalled RMSE on the training set?

In [ ]:
```
alpha_training_min = rmse_training.min()

# YOUR CODE HERE
raise NotImplementedError()
```

In [ ]:
```
# TEST
assert alpha_training_min in alphas
```

In [ ]:

Which value of `alpha` has the smallest RMSE on the validating set?

In [55]:
```
alpha_validating_min = rmse_validating.min()

# YOUR CODE HERE
raise NotImplementedError()
```

```
---------------------------------------------------------------------------
AttributeError                            Traceback (most recent call last)
<ipython-input-55-0977195682cd> in <module>
----> 1 alpha_validating_min = rmse_validating.min()
      2 
      3 # YOUR CODE HERE
      4 raise NotImplementedError()

AttributeError: 'dict' object has no attribute 'min'
```

In [ ]:
```
# TEST
assert alpha_validating_min in alphas
```

In [ ]:

Question 8b

Using the `alpha` from Question 8a with the smallest RMSE on the validating set, predict `SalePrice` on the testing set.

In [ ]:
```
y_predict = ...

# YOUR CODE HERE
raise NotImplementedError()
```

One way of understanding the appropriateness of a model is through a residual plot. Run the cell below to plot the actual sale prices against the residuals of the model for the test data.

In [ ]:
```
# RUN

residuals = y_test - y_predict
ax = sns.regplot(y_test, residuals)
ax.set_xlabel('Sale Price (Test Data)')
ax.set_ylabel('Residuals (Actual Price - Predicted Price)')
ax.set_title("Residuals vs. Sale Price on Test Data")
plt.savefig('residuals.png')
```

In [ ]:
```
# RUN

Image('residuals.png')
```

Question 8c (Optional)

Ideally, we would see a horizontal line of points at 0. The next best thing would be a set of points centered at 0. However the most expensive homes are always more expensive than our prediction.

What changes could you make to your linear model to improve its accuracy and lower the test error? Suggest at least two things you could try in the cell below, and carefully explain how each change could potentially improve your model's accuracy.

Your response here...

We could potentially add more columns or features that would correlate with an expensive home. For instance, we could add columns for number of bedrooms, distance from the downtown commercial area, etc. We could also remove features that we believe might not have a ver big impact in determining the price. Another change is that we should account for outliers. For example, in the beginning, we called the training_data['SalePrice'].describe() method. It showed us the maximum sale price was around 740,000 dollars. However, the mean saleprice of the dataset was 180,000 dollars. If we cleaned the data to account for outliers our model would possibly be more accurate.

In [ ]:
```
```