Untitled

February 16, 2019

1 Epidemiology Model (SIRS Model)

Below is coded to help myself study Math 113B (Mathematic modles in Biology) in UCI. I refered to my professor and TA's teaching.

This code is to analyze one example of Non-linear systems, Epidemiology Models.

$$\frac{dS}{dt} = -\beta SI + \gamma R$$

$$\frac{dI}{dt} = \beta SI - vI$$

$$\frac{dR}{dt} = vI - \gamma R$$

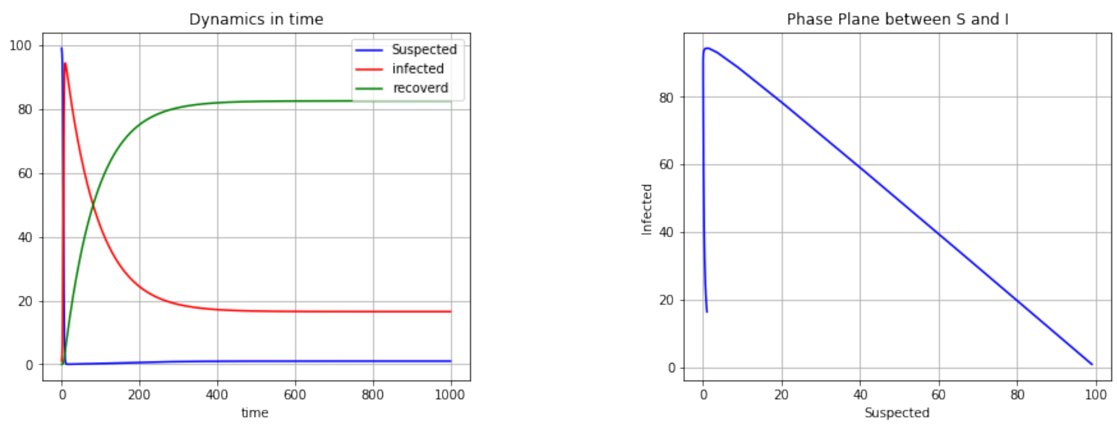
Because the number of variables is three, I will not draw nullclines for SIRS model. I will draw phase plane, find steady states & eigen values to linearize SIRS model at steady states.

The results will be different if reproductive number is greater than 1 or not. (reproductive number = $\frac{N\beta}{v}$)

```
In [2]: # import the relevant modules
        import matplotlib.pyplot as plt
        %matplotlib inline
        import numpy as np
        from scipy.integrate import odeint
```

```
In [110]: '''
              Case 1 reproductive number > 1
              S = 99, I= 1, R=0, beta = 0.1, v = 0.1, gamma = 1./50 = 0.02
          '''
          beta = 0.1
          v = 0.1
          gamma = 1./50
          endtime = 100

          # Draw Phase Plane
          def ode_system(X, t=0):
              # X[0] = S, X[1] = I, X[2] = R
              S, I, R = X
              return np.array([-beta*S*I + gamma*R, beta*S*I - v*I, v*I - gamma*R])
```

```
t = np.linspace(0, endtime, 1001)

# initial value: S=99, I=1, R=0
initial_value = np.array([99, 1, 0])

# call odeint()
Z = odeint(ode_system, initial_value, t)

s, i, r = Z.T

# Plot
fig = plt.figure(figsize=(15,5))
fig.subplots_adjust(wspace=0.5, hspace=0.3)
sub1 = fig.add_subplot(1,2,1)
sub2 = fig.add_subplot(1,2,2)

sub1.plot(s, 'b-', label="Suspected")
sub1.plot(i, 'r-', label="infected")
sub1.plot(r, 'g-', label="recoverd")
sub1.set_title("Dynamics in time")
sub1.set_xlabel("time")
sub1.grid()
sub1.legend(loc="best")

sub2.plot(s, i, color="blue")
sub2.set_title("Phase Plane between S and I")
sub2.set_xlabel("Suspected")
sub2.set_ylabel("Infected")
sub2.grid()
```



In [111]:
```
'''
    Case 2 reproductive number < 1
```

```python
    S = 99, I= 1, R=0, beta = 0.1, v = 15, gamma = 1./50
'''
beta = 0.1
v = 15
gamma = 1./50
endtime = 100

# Draw Phase Plane
def ode_system(X, t=0):
    # X[0] = S, X[1] = I, X[2] = R
    S, I, R = X
    return np.array([-beta*S*I + gamma*R, beta*S*I - v*I, v*I - gamma*R])

t = np.linspace(0, endtime, 1001)

# initial value: S=99, I=1, R=0
initial_value = np.array([99, 1, 0])

# call odeint()
Z = odeint(ode_system, initial_value, t)

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sub1.grid()
sub1.legend(loc="best")

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sub2.set_ylabel("Infected")
sub2.grid()
```

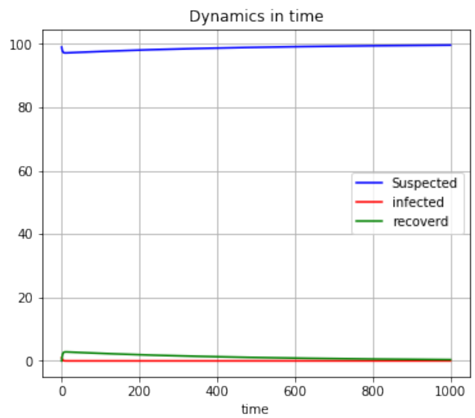

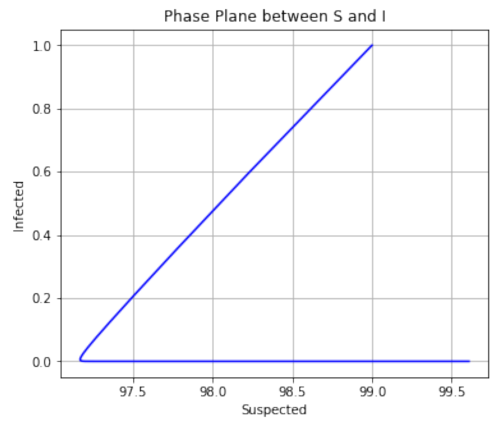

1.1 Steady States

To find steady states, I will use sympy library.

Assume

$$\beta = 0.1, v = 0.1, \gamma = 0.02, N = 100$$

In [148]:
```
# Find Steady States

import sympy as sm

beta = 0.1
v = 0.1
gamma = 1./50

# Define system and this system has no negative value
x,y,z = sm.symbols('S, I, R', negative=False)
x_prime = -beta*x*y + gamma*z
y_prime = beta*x*y - v*y
z_prime = v*y - gamma*z

XEqual = sm.Eq(x_prime,0)
YEqual = sm.Eq(y_prime,0)
ZEqual = sm.Eq(z_prime,0)

steady_states = sm.solve( (XEqual, YEqual, ZEqual), x,y,z)

# Expected steady states 1: { N,0,0 }, 2:{ 1, 0.2*(N-1), 5I(=N-1) } or { 1, 0.2*R, R}
print(steady_states)
```

```
[{I: 0.0, R: 0.0}, {I: 0.2*R, S: 1.00000000000000}]
```

1.2 Eigen Values

To get eigen values, We will use quadratic formula.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\lambda = (trace/2) \pm \sqrt{trace^2/4 + \delta}$$

$$trace = a_{11} + a_{22}$$

$$\Delta = a_{11}a_{22} - a_{12}a_{21}$$

But, this system has three variables. So I will use sympy library.
Assume

$$\beta = 0.1, v = 0.1, \gamma = 0.02, N = 100$$

Then first steady state is

$$(100, 0, 0)$$

Second state can be gotten through

$$(\frac{v}{\beta}, \frac{\gamma(N-S)}{(v+\gamma)}, \frac{vI}{\gamma}) = (1, 16.5, 82.5)$$

```
In [149]: eq_mat = sm.Matrix([x_prime, y_prime, z_prime])
          mat = sm.Matrix([x,y,z])
          jacobian_mat = eq_mat.jacobian(mat)

          print(f'Jacobian {jacobian_mat}')
          print('-----------------------------------------------------------------------------

          # Assume first S.S. = (100,0,0), Second S.S = (1, 16.5, 82.5)
          eq_mat = jacobian_mat.subs([ (x, 100), (y, 0), (z, 0) ])
          print(f'The eigenvalues for the first steady states are {list(eq_mat.eigenvals().keys(
          print('-----------------------------------------------------------------------------
          eq_mat = jacobian_mat.subs([ (x, 1), (y, 16.5), (z, 82.5) ])
          print(f'The eigenvalues for the second steady states are {list(eq_mat.eigenvals().keys
          print('-----------------------------------------------------------------------------
```

```
Jacobian Matrix([[-0.1*I, -0.1*S, 0.0200000000000000], [0.1*I, 0.1*S - 0.1, 0], [0, 0.1000000000
-------------------------------------------------------------------------------------
The eigenvalues for the first steady states are [99/10, -1/50, 0]
-------------------------------------------------------------------------------------
The eigenvalues for the second steady states are [-167/200 - sqrt(19969)/200, -167/200 + sqrt(19
-------------------------------------------------------------------------------------
```

1.3 Result

Each steady states has 0 as one of eigenvalues. Then jacobian matrices are not Hyperbolic. We can't linearize this non-linear system.