Statistical Theory Homework 1

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1. (a) $\{(0,g),(0,f),(0,s),(1,g),(1,f),(1,s)\}$
   (b) $\{(0,s),(1,s)\}$
   (c) $\{(0,g),(0,f),(0,s)\}$
   (d) $\{(0,s),(1,s),(1,g),(1,f)\}$
   (e) $\{(1,s)\}$

2. (a) $A \cap B^c \cap C^c$
   (b) $A \cap B \cap C^c$
   (c) $A \cap B \cap C$
   (d) $A \cup B \cup C$
   (e) $(A \cap B \cap C^c) \cup (A \cap B^c \cap C) \cup (A^c \cap B \cap C)$
   (f) $A^c \cap B^c \cap C^c$

3. (a) $\mathbf{P}(\text{ at least a 6 }) = \frac{6+6-1}{6\times 6} = \frac{11}{36}$
   (b) $\mathbf{P}(\text{ same number }) = \frac{6}{6\times 6} = \frac{1}{6}$

4. It's the same number as to choose 2 people out of 20 to shake hands, and that's $\binom{20}{2} = 190$.

5. (a) $\frac{3\times 4}{\binom{5}{2}\times 2!} = \frac{3}{5}$
   (b) $\frac{2\times 3+3\times 2}{\binom{5}{2}\times 2!} = \frac{3}{5}$
   (c) $\frac{\binom{3}{2}\times 2!}{\binom{5}{2}\times 2!} = \frac{3}{10}$

6. I think it's to choose n ordered character from a-z , where n is the sum of the numbers of one's last and given name. Then:

   (a) $26 \times 26 \times 26 = 17576$
   (b) $26 \times 26 + 26 \times 26 \times 26 = 18252$
   (c) $26 \times 26 + 26 \times 26 \times 26 + 26 \times 26 \times 26 \times 26 = 475228$

7. We can deal $\{1,2,3\}$ as one number and arrange the $n-2$ numbers in order first, then we arrange $1,2,3$ in order. Therefore: $\mathbf{P} = \frac{(n-2)!\times 3!}{n!} = \frac{6}{n(n-1)}$, where n must no less than 3.

8. It's equivalent to choose 7 children from 10 to have one and only one gift, therefore the number of results is $\binom{10}{3} = 120$.

9. It's the sum of ways to choose t elements in $(x_1, x_2, \cdots, x_n)$ to be 1 , where t ranges from k to n. And that's $\sum_{t=k}^{n} \binom{n}{t}$

10. $\mathbf{P} = \frac{|\text{choose } r \text{ spaces occupied out of the rest } N-2 \text{ spaces}|}{|\text{choose } r \text{ occupied out of N}|} = \frac{\binom{N-2}{r}}{\binom{N}{r}} = \frac{(N-r)(N-r-1)}{N(N-1)}$

11. I'm a little confused about the meaning of "double" here, I think it means 2 dices are in same number.

    (a) $\mathbf{P} = \frac{2}{6\times6} / \frac{6}{6\times6} = \frac{1}{3}$

    (b) $\mathbf{P} = \frac{5+5}{6\times6} / \frac{6\times6-6}{6\times6} = \frac{1}{3}$

12. $\mathbf{P}(\text{coin with heads and tails}|\text{show up heads}) =$
    $\mathbf{P}(\text{choosed third coin and tossed to be heads})/\mathbf{P}(\text{tossed to be heads}) =$
    $\frac{1/3\times1/2}{1/3\times1/2+1/3\times1+1/3\times0} = \frac{1}{3}$

13. if the n^{th} stamp is of type i, then it's new means the first $n-1$ stamps are of types other than i, whose probability is $(1-p_i)^{n-1}$. Therefore,

$$\mathbf{P}(\text{new}) = \sum_{1}^{m} \mathbf{P}(\text{new}|n^{th} \text{ of type i})\mathbf{P}(n^{th} \text{ of type i}) = \sum_{1}^{m} p_i(1-p_i)^{n-1}$$

Extra Credit:

Denote $A_{n,r,k}$ to be