Coding Part

Problem 1

(i).

```r
Y = source("data8.txt")$value

m = mean(Y)
v = var(Y)

theta1 = m - v / m
theta2 = theta1 * v / m
mme = c(theta1,theta2)
print(mme)
```

```
## [1]    80.32726 11879.65785
```

(ii).

```r
f <- function(Y){
    function(para){
        t1 = para[1]
        t2 = para[2]
        500 * log(gamma(t1^2/t2)) + 500 * log(t1) -
        500 * t1^2 / t2 * log(t1/t2) -
        (t1^2/t2 - 1) * sum(log(Y)) + t1 / t2 * sum(Y)
    }
}

mle = optim(mme, f(Y))
```

```
## Warning in log(t1): NaNs produced

## Warning in log(t1/t2): NaNs produced

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```

```r
print(mle$par)
```

```
## [1]    90.97896 12481.86544
```

(iii). The eigenvalues of hessian are all positive, therefore hessian is positive definite, so this is indeed the local minimum of minus log likelihood, which is the local maximum of likelihood.

```r
mle = optim(mme, f(Y), hessian = T)
eigenvalues = svd(mle$hessian)$d
print(eigenvalues)
```

```
## [1] 2.693534e-02 9.405684e-07
```

(iv).

```r
t1 = mle$par[1]
t2 = mle$par[2]
p_mle = qgamma(0.95, shape = t1^2 / t2, scale = t2 / t1)
print(p_mle)
```

```
## [1] 315.7731
```

(V).

Problem 2

(i).

```r
X = source("p2data")$value

u_mle = mean(X)
print(u_mle)
```

```
## [1] 3.56
```

(ii).

```r
gu_mle = exp(-mean(X))
print(gu_mle)
```

```
## [1] 0.02843882
```

(iii).

```r
X0 = (X == 0)
gu_ub = mean(X0)
print(gu_ub)
```

```
## [1] 0.06
```

(v).

```r
gu_BW = (49 / 50)^(50 * mean(X))
print(gu_BW)
```

```
## [1] 0.02743099
```