Homework 1

1. A hospital administrator codes incoming patients suffering from gunshot wounds according to whether or not they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), and serious (s). Consider an experiment that consists of the coding of such a patient.

   (a) Give the sample space of this experiment

   (b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.

   (c) Let B be the event that the patient is uninsured. Specify the outcomes in B

   (d) Give all outcomes in the event $B^c \cup A$.

   (e) Give all outcomes in the event $B^c \cap A$.

2. Let A, B, and C be three arbitrary events. Find expressions for the following events:

   (a) Only A occurs

   (b) Both A and B occur, but not C

   (c) All three events occur

   (d) At least one occurs

   (e) Exactly 2 occur

   (f) None occurs

3. We roll two fair 6-sided dice (each outcome is assumed to be equally likely). Find:

   (a) The probability that at least one die rolls a 6.

   (b) The probability of that the same number comes up on both dice.

4. Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

5. Suppose you draw a sample of size two without replacement from the set $\{1, 2, 3, 4, 5\}$. Find the probability that an odd digit will be selected:

   (a) first?

   (b) second?

   (c) both times?

6. How many different sets of initials can be formed if the every person has one last name and

(a) exactly 2 given names?

(b) at most 2 given names?

(c) at most 3 given names?

7. The numbers $1, 2, \ldots, n$ are arranged in random order. Find the probability that the digits 1,2, and 3 appear as neighbors in the order.

8. 7 gifts are to be distributed among 10 children. How many distinct results are possible if no child is to receive more than 1 gift?

9. Determine the number of vectors $(x_1, x_2, \ldots, x_n)$ such that x_i is either 0 or 1 and $\sum_{i=1}^{n} x_i \geq k$?

10. A car is parked among N other cars in a row (not at either end). On his return, the owner finds that exactly r of the N spaces are still occupied. What is the probability that both neighboring places are empty?

11. We roll two fair 6-sided dice. Each outcome is assumed to be equally likely. Find:

    (a) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.

    (b) Given that the two dice land on different numbers, find the conditional probability that at least one die rolled is a 6.

12. You are given three coins: one has heads on both faces, the second has tails on both faces, and the third has a head on one face and a tail on the other. You choose a coin at random, toss it, and it comes up heads. What is the probability that the opposite face is tails?

13. Suppose that you continually collect stamps, and that there are m different types of stamps. Suppose that each time you get a new stamp it is type i with probability p_i, $i = 1, 2, ...m$. Suppose you have just collected your n^{th} stamp. What is the probability that it is a new kind? (Hint: condition on the type of stamp.)

Extra Credit (worth one point added to your average at the end of the term; no partial credit):

Suppose n balls are randomly placed in r urns. What's the probability that exactly one urn has k balls, $0 < k < n$? (Credit for providing an algorithm, a formula, or a proof that one doesn't exist.)