Probability Theory II - Homework #1

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1. Let F be the cdf of some random variable with density f. Compute

$$\int_{-\infty}^{\infty}[F(x+5)-F(x)]dx.$$

   Justify your steps.

2. Suppose that $R_1, ..., R_n$ are independent, where $R_i = \sigma(X_{i,j}, 1 \leq j \leq n)$, and $C_1, ..., C_n$ are independent, where $C_j = \sigma(X_{i,j}, 1 \leq i \leq n)$. Prove that then the whole family of random variables $X_{i,j}, 1 \leq i, j \leq n$, is independent.

3. A generalization of the L^2 weak law to certain dependent sequences. Suppose $EX_n = 0$ and $EX_nX_m \leq r(n-m)$ for $m \leq n$, with $r(k) \to 0$ as $k \to \infty$. Show that $(X_1 + ... + X_n)/n \to 0$ in L^2 and in probability.