Homework 4

Before you do problems 5 and 6, read Section 4.6 of Casella & Berger.

1. Does a distribution exist for which $M_X(t) = \frac{t}{1-t}, |t| < 1$? If yes, find it; if not, explain why not.
2. Let $X \sim exp(1)$ and let Y be $X + 1$ rounded down to the nearest integer.
   (a) What is the distribution of Y?
   (b) What is the distribution of $X - 4$ conditional on $Y \geq 5$?
3. Let X and $Y \sim f(x, y)$, and let $U = aX + b$ and $V = cY + d$ where a and c are positive. What is $f_{U,V}(u, v)$?
4. Let X, Y, and Z be independent $unif(0, 1)$ random variables. Find $P(\frac{XY}{Z} < t)$.
5. Let $X_1, ..., X_n$ be independent $exp(\lambda)$ random variables. Find the distribution of $Y = X_1 + X_2 + ... + X_n$ by finding the joint distribution of $Z_1 = X_1$, $Z_2 = X_1 + X_2$, ..., $Z_n = Y$ and then finding the marginal of Z_n.
6. The random vector $X = (X_1, X_2, X_3)$ is distributed according to $f_{\mathbf{X}}(\mathbf{x}) = \frac{2}{2e-5} x_1^2 \cdot x_2 \cdot e^{x_1 x_2 x_3} I\{0 < x_1, x_2, x_3 < 1\}$. What is the marginal distribution of X_1? What is the conditional distribution of X_2 and X_3 given X_1? Are X_1, X_2, and X_3 mutually independent? Are X_2 and X_3 independent conditional on X_1?