# Homework 6

1. You are called on to analyze the results of an experiment in which cells are exposed to low level radiation and scientists are interested in the rate of resulting mutations. The experiment is conducted using a 96-well plate. Let $X_i$ be the number of mutated cells in well $i$. Suppose that $X_i \sim Poi(\lambda)$. It's not possible to count the number of mutated cells in each well, but it is possible to detect, for each well, whether there are **any** mutated cells in that well.

   (a) Find the MLE ($\hat{\lambda}$) and MME ($\tilde{\lambda}$) for $\lambda$.

   (b) Find their asymptotic distributions.

2. Let $X_1, \ldots, X_n$ be iid random variables with pdf

$$f(x \mid \theta) = \theta(1+\theta)x^{\theta-1}(1-x) \; for \; x \in (0,1), \theta > 0.$$

   Find the method of moments estimator of $\theta$. Is it unbiased? Does it achieve the CRLB?

3. Assume that $X_1, \ldots, X_n$ are iid random variables following the truncated exponential distribution with parameters $\lambda$ and $\beta$. The density function is $f(x \mid \lambda, \beta) = \lambda e^{-\lambda(x-\beta)} 1\{x > \beta\}$.

   (a) Find the MLE for $\lambda$ and $\beta$.

   (b) What is its limiting distribution?

4. Suppose $X_1, ..., X_n$ are iid and follow a uniform distribution on$[0, \theta]$. Show that $2\bar{X}$ and $(n+1)\frac{X_{(n)}}{n}$ are both consistent estimators of $\theta$ and compare their variances.