

Coding Part

Problem 1

(i).

```
Y = source("data8.txt")$value  
  
m = mean(Y)  
v = var(Y)  
  
theta1 = m - v / m  
theta2 = theta1 * v / m  
mme = c(theta1,theta2)  
print(mme)
```

```
## [1]      80.32726 11879.65785
```

(ii).

```
f <- function(Y){  
  function(para){  
    t1 = para[1]  
    t2 = para[2]  
    500 * log(gamma(t1^2/t2)) + 500 * log(t1) -  
    500 * t1^2 / t2 * log(t1/t2) -  
    (t1^2/t2 - 1) * sum(log(Y)) + t1 / t2 * sum(Y)  
  }  
}  
  
mle = optim(mme, f(Y))
```

```
## Warning in log(t1): NaNs produced  
## Warning in log(t1/t2): NaNs produced  
## Warning in log(t1): NaNs produced  
## Warning in log(t1/t2): NaNs produced  
## Warning in log(t1): NaNs produced  
## Warning in log(t1/t2): NaNs produced  
## Warning in log(t1): NaNs produced  
## Warning in log(t1/t2): NaNs produced  
print(mle$par)
```

```
## [1]      90.97896 12481.86544
```

(iii). The eigenvalues of hessian are all positive, therefore hessian is positive definite, so this is indeed the local minimum of minus log likelihood, which is the local maximum of likelihood.

```
mle = optim(mme, f(Y), hessian = T)  
eigenvalues = svd(mle$hessian)$d  
print(eigenvalues)
```

```
## [1] 2.693534e-02 9.405684e-07
```

(iv).

```
t1 = mle$par[1]
t2 = mle$par[2]
p_mle = qgamma(0.95, shape = t1^2 / t2 + 1, scale = t2 / t1)
print(p_mle)
```

```
## [1] 574.3776
```

(V).

```
estimator <- function(path){
  Y = source(path)$value
  m = mean(Y)
  v = var(Y)
  theta1 = m - v / m
  theta2 = theta1 * v / m
  mme = c(theta1,theta2)
  mle = optim(mme, f(Y))$par
  mle
}

theta12 = c()
for(i in 1:10){
  path = paste("data", i, ".txt",sep = "")
  theta12 = rbind(theta12, estimator(path))
}
print(theta12)
```

```
##           [,1]      [,2]
## [1,]  69.78007 11005.55
## [2,]  84.28554 13234.61
## [3,] 101.84094 13087.02
## [4,]  96.01610 12650.58
## [5,]  90.91716 11963.89
## [6,]  94.40069 12315.02
## [7,]  89.07779 12758.49
## [8,]  90.97896 12481.87
## [9,]  80.31894 10673.38
## [10,] 109.80055 13355.51
```

```
var1 = var(theta12[,1])
var2 = var(theta12[,2])
cov12 = cov(theta12[,1], theta12[,2])

print(var1)
```

```
## [1] 124.4669
```

```
print(var2)
```

```
## [1] 820024.6
```

```
print(cov12)
```

```
## [1] 7559.009
```

Problem 2

(i).

```
X = source("p2data")$value
```

```
u_mle = mean(X)
print(u_mle)
```

```
## [1] 3.56
```

(ii).

```
gu_mle = exp(-mean(X))
print(gu_mle)
```

```
## [1] 0.02843882
```

(iii).

```
X0 = (X == 0)
gu_ub = mean(X0)
print(gu_ub)
```

```
## [1] 0.06
```

(v).

```
gu_BW = (49 / 50)^(50 * mean(X))
print(gu_BW)
```

```
## [1] 0.02743099
```