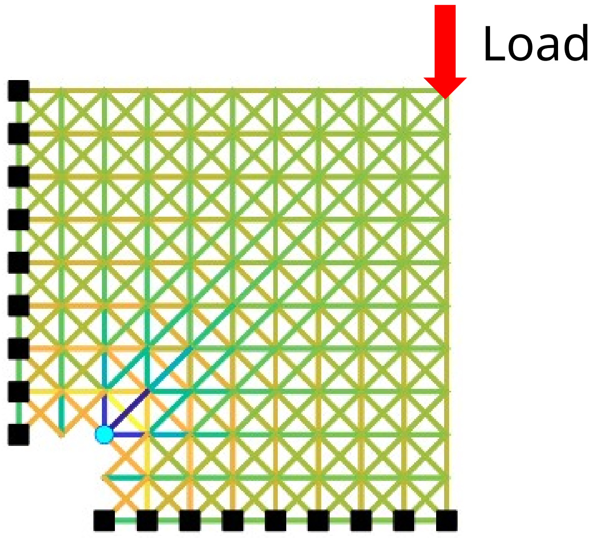
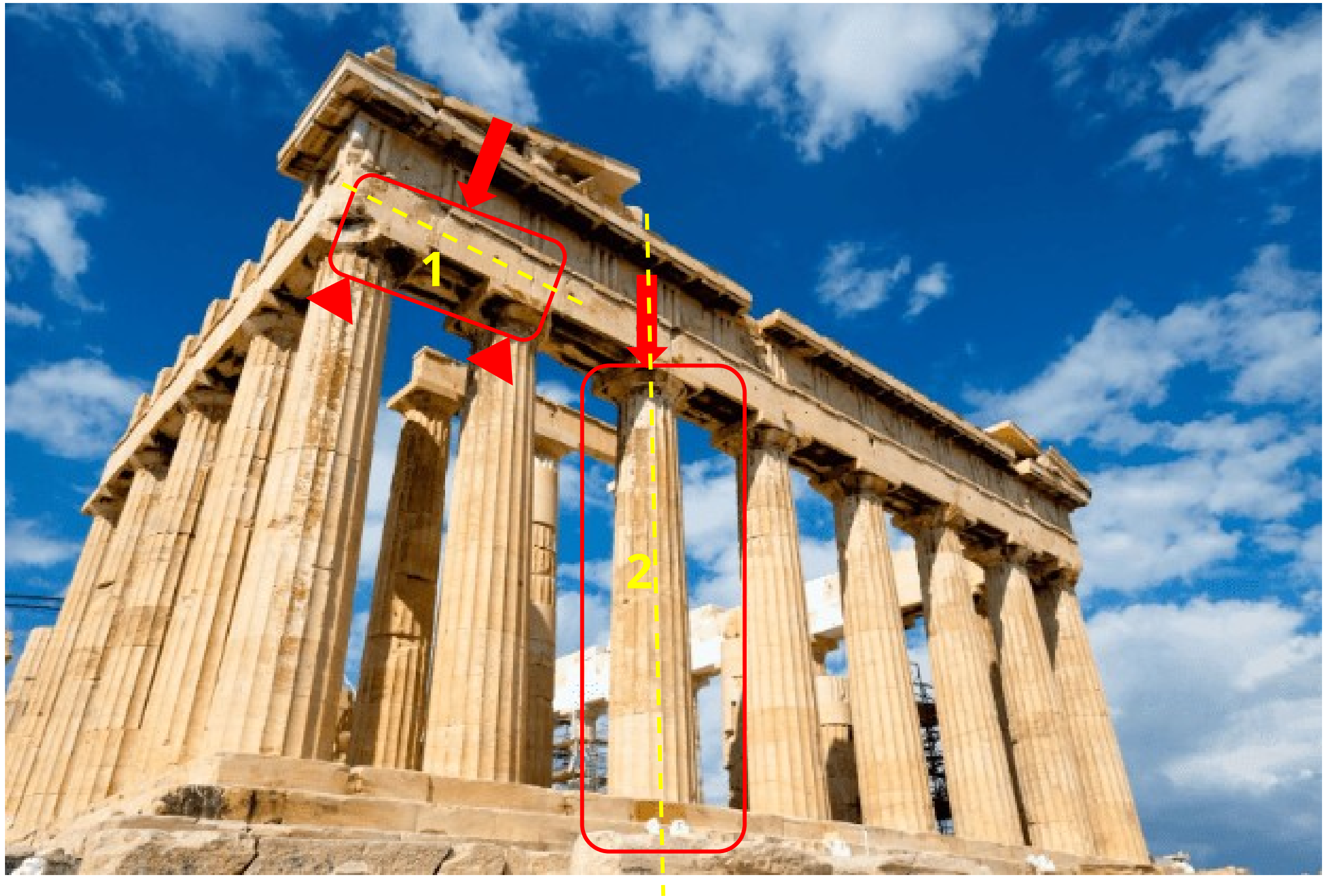
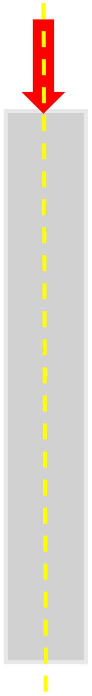
Columns: Euler Buckling





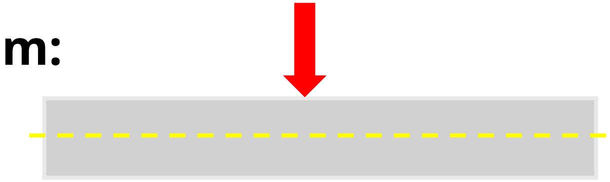
Parthenon: Greece

2 = Column:

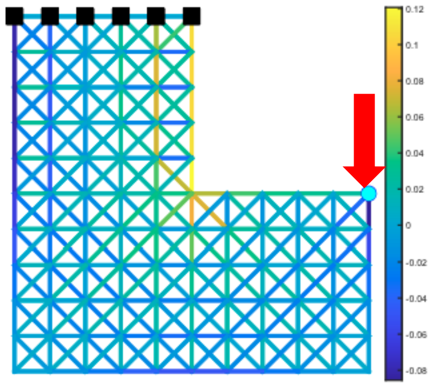


- Loads act along the axis of the structure

1 = Beam:

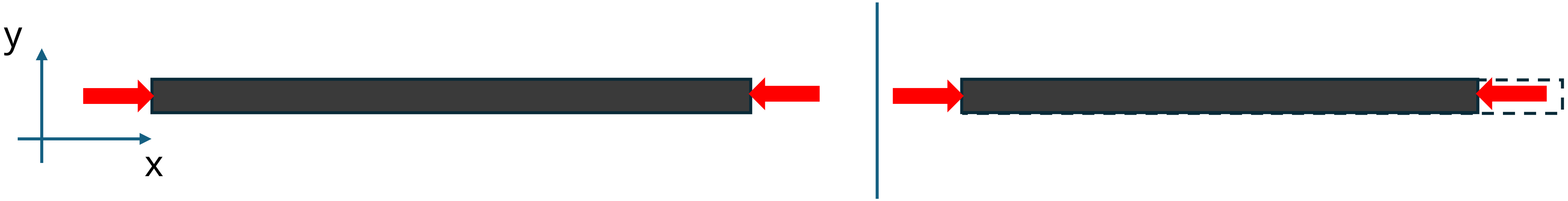


- Loads act perpendicular to the axis of the structure



Case 1: Low levels of load

- The deformations are axial only

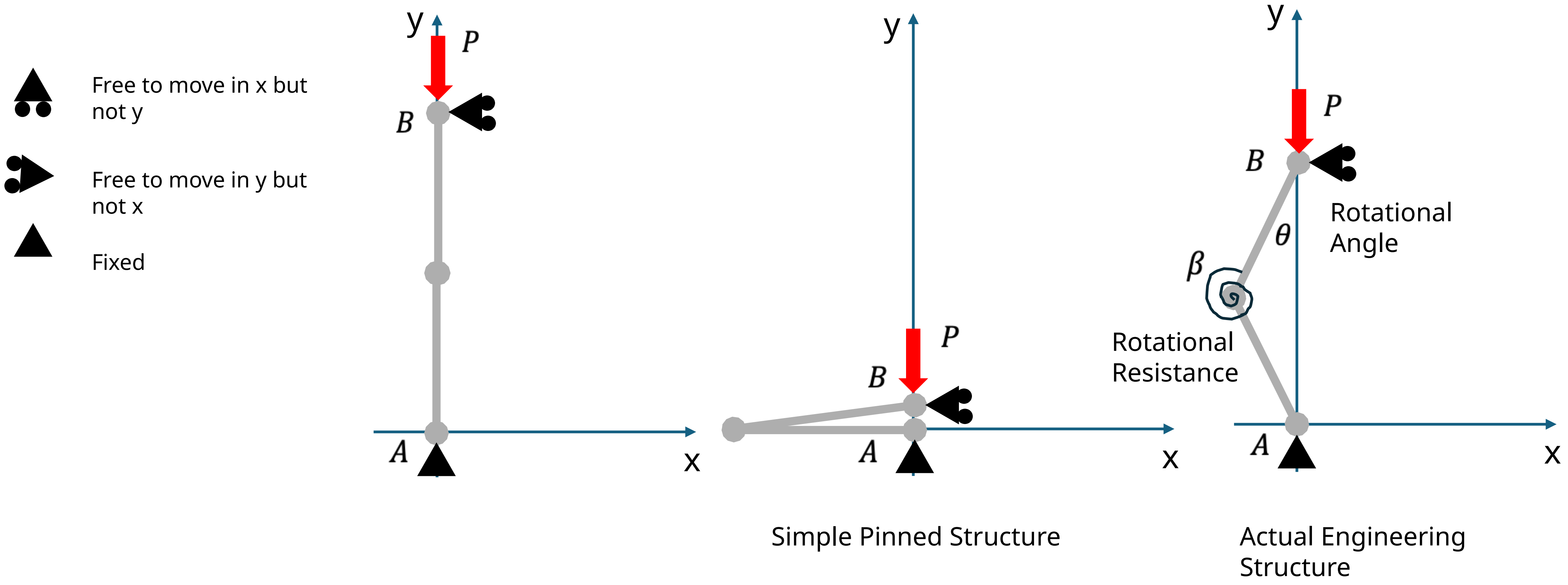


Case 2: High levels of load

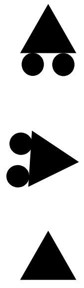
- The deformations/deflections are governed by the bending with large deformations

Buckling

Buckling: Simple Case Study



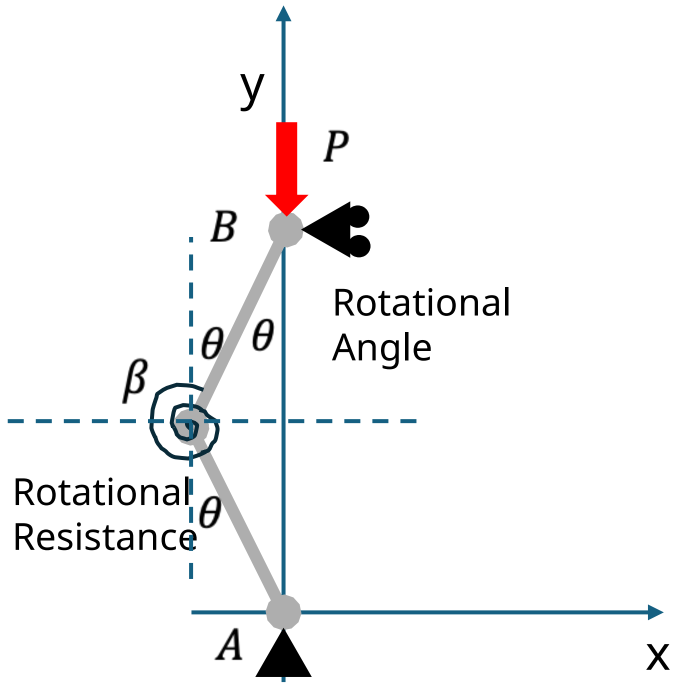
Buckling: Simple Case Study

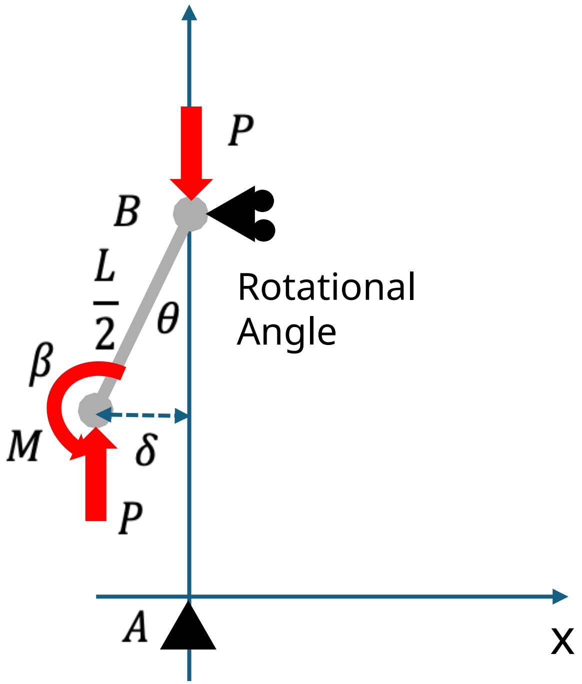


Free to move in x but not y

Free to move in y but not x

Pinned





Actual Engineering Structure

Moment Balance: $P\delta = M = 2\beta\theta$

Small Angle Assumption: $\delta = \frac{L}{2}\theta$

Load at which the structure is in equilibrium: $\boxed{P = \frac{4\beta}{L}}$

Force Balance, for structure to be in equilibrium:

$$P\delta = M = 2\beta\theta$$

Case #1: Restoring moment $(2\beta\theta)$ is GREATER than the moment of due to the applied load $(P\delta)$

$$2\beta\theta > P\delta \longrightarrow 2\beta\theta > P\frac{L}{2}\theta$$

$$P < \frac{4\beta}{L}$$

STABLE EQUILIBRIUM

Case #2: Restoring moment $(2\beta\theta)$ is LESS than the moment of due to the applied load $(P\delta)$

$$2\beta\theta < P\delta$$

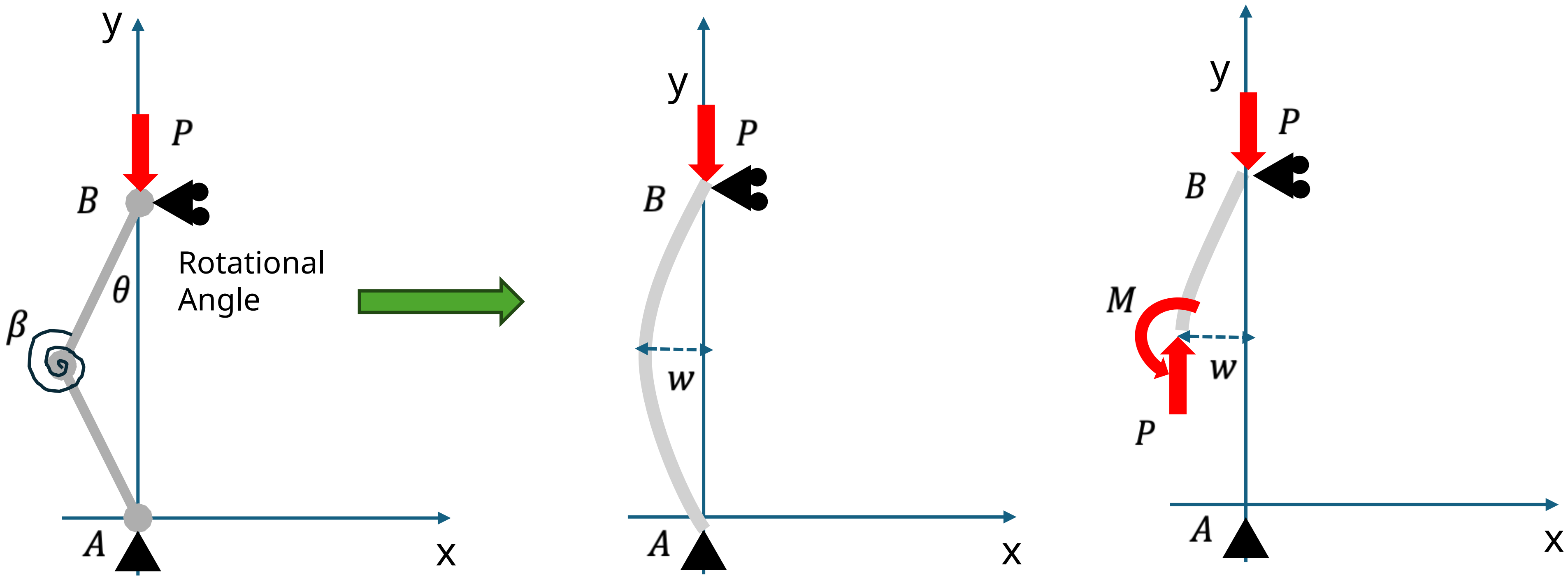
$$P > \frac{4\beta}{L}$$

UNSTABLE EQUILIBRIUM

Rough Work

$$\delta = \frac{L}{2}\theta$$

Pinned-Pinned Columns



Moment Balance:

$$M - P(-w) = 0$$

$$M + Pw = 0$$

Bending Moment Equation:

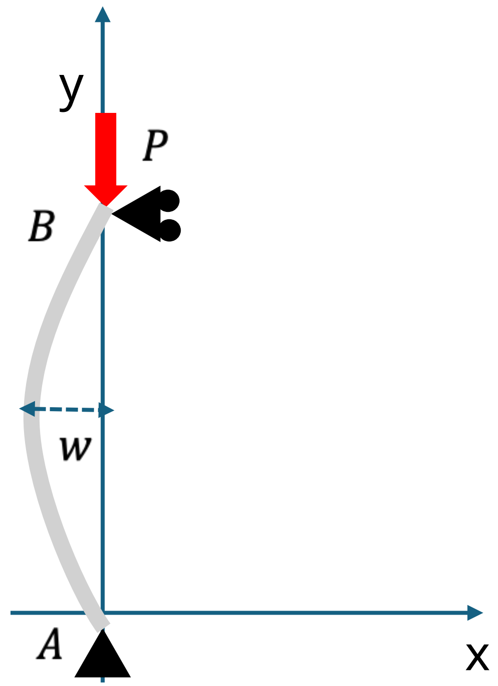
$$\frac{d^2w}{dy^2} = \frac{M}{EI}$$

Differential Equation of Deflection Curve:

$$\boxed{EI\frac{d^2w}{dy^2} + Pw = 0}$$

$$\frac{d^2w}{dy^2} + k^2w = 0 \qquad where, k^2 = \frac{P}{EI}$$

$$\frac{d^2w}{dy^2} + k^2w = 0 \qquad where, k^2 = \frac{P}{EI}$$



General solution of the differential equation:

$$w = C_1 \sin(ky) + C_2 \cos(ky)$$

Boundary Conditions:

$w = 0\ at\ y = 0,$	$y = L$
$C_2 = 0$	$C_1 \sin(kL) = 0$
	Either : $C_1 = 0$ or $\sin(kL) = 0$

Trivial Solution: Both $C_1\ and\ C_2$ are zero

Buckling Equation:

$$\sin(kL) = 0$$

$$\sin(kL) = 0 \Rightarrow kL = 0, \pi, 2\pi, \ldots$$

$$kL = 0, \pi, 2\pi, \ldots \qquad where, k^2 = \frac{P}{EI}$$

Case #1: $kL = 0$ ➡ $P = 0$

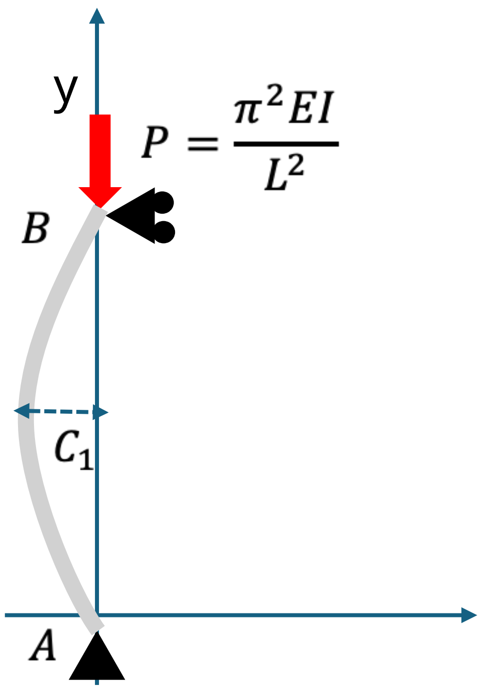
Case #2: $kL = n\pi$ ➡ $P = \frac{n^2\pi^2 EI}{L^2}$

Critical loads on the structure

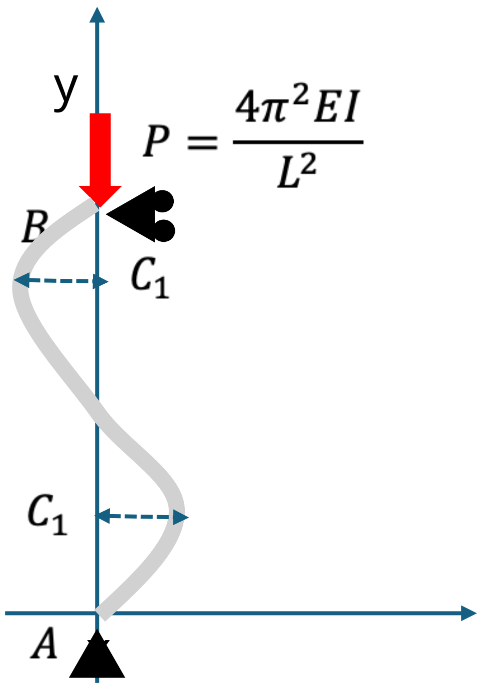
Deflection Equation: ➡ $w = C_1 \sin(\frac{n\pi}{L} y)$

For $n = 1$, Fundamental Case of column buckling:

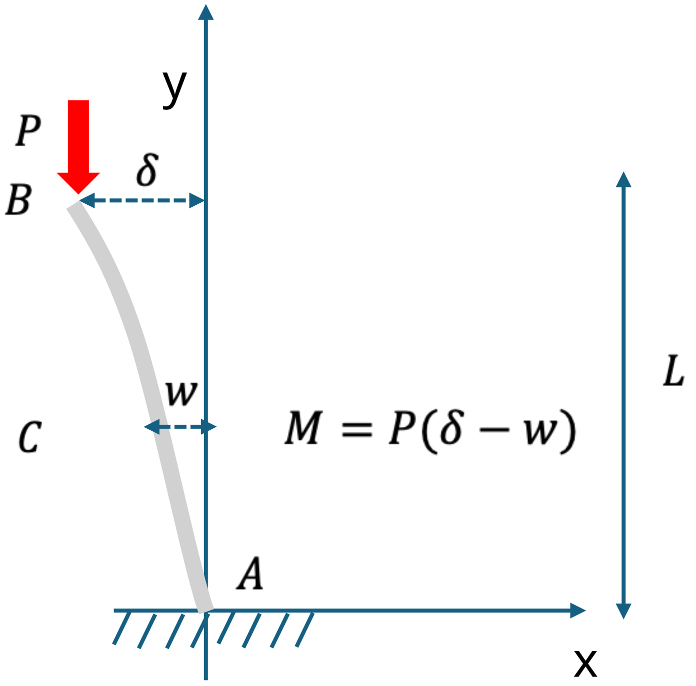
Load	Deflection
$P = \frac{\pi^2 EI}{L^2}$	$w = C_1 \sin(\frac{\pi}{L} y)$



Deflection shape for n=1



Deflection shape for n=2



Fixed-Free Columns

$$\frac{d^2w}{dy^2} = \frac{M}{EI}$$

$$EI\frac{d^2w}{dy^2} = P(\delta - w)$$

$$\frac{d^2w}{dy^2} + k^2w = k^2\delta$$

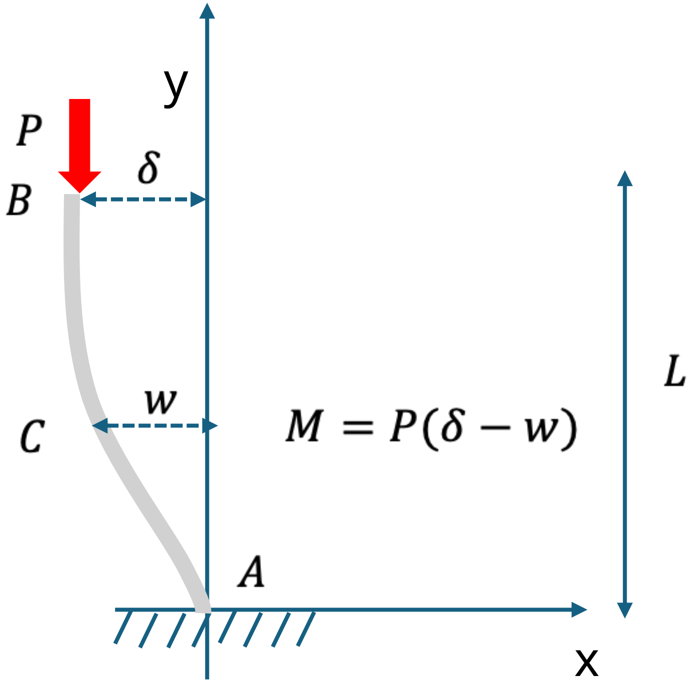
$$w = C_1\sin(ky) + C_2\cos(ky) + \delta$$

Boundary Conditions: $w = 0 \ and \ w' = 0 \ at \ y = 0$

$C_2 = -\delta$ | $C_1 = 0$

$$w = \delta(1 - \cos(ky))$$

Boundary Conditions: $w = \delta \ at \ y = L$ → $\cos(kL) = 0$



$$\cos(kL) = 0$$

$$kL = \frac{n\pi}{2} \qquad n = 1, 3, 5 \ldots$$

$$P = \frac{n^2\pi^2 EI}{4L^2}$$

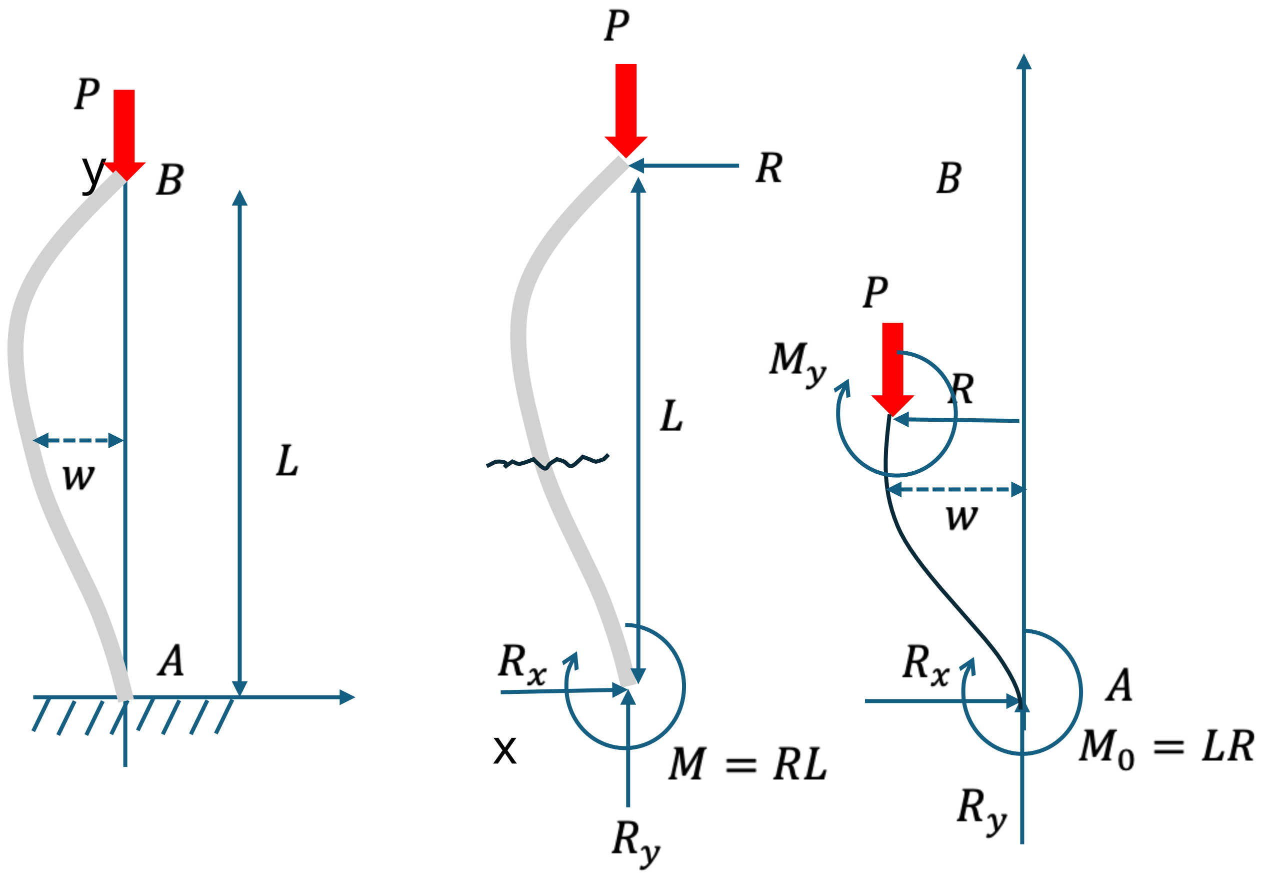
Deflection Equation: $\quad w = \delta\left(1 - \cos\left(\frac{n\pi}{2L}y\right)\right)$

For $n = 1$, Fundamental Case of column buckling:

Load	Deflection
$P_{cr} = \frac{\pi^2 EI}{4L^2}$	$w = \delta\left(1 - \cos\left(\frac{\pi}{2L}y\right)\right)$

Fixed-Pinned Columns

$$\boldsymbol{M} = [\boldsymbol{r}]\times[\boldsymbol{F}]$$



Force Balance

$$R_x = R$$

$$R_y = P$$

Moment Balance about A:

$$M_P = [-w\hat{\imath}]\times[-P\hat{\jmath}] = wP\hat{k}$$

$$M_R = [y\hat{\jmath}]\times[-R\hat{\imath}] = yR\hat{k}$$

$$M_y + wP + yR = RL$$

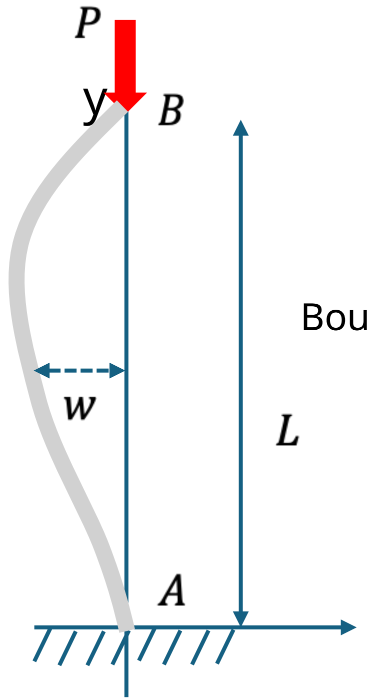
$$M_y = RL - Ry - Pw$$

$$\frac{d^2w}{dy^2} = \frac{M}{EI}$$

Differential Equation of Deflection Curve:

$$\frac{d^2w}{dy^2} + k^2w = \frac{R}{EI}(L - y)$$

Deflection curve: $$w = C_1\sin(ky) + C_2\cos(ky) + \frac{R}{P}(L - y)$$



$$w = C_1 \sin(ky) + C_2 \cos(ky) + \frac{R}{P}(L - y)$$

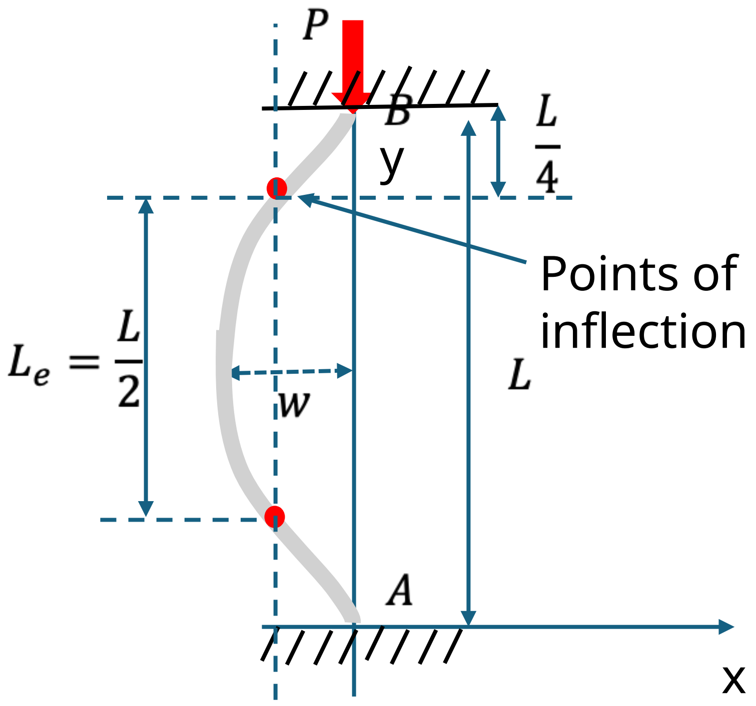
Boundary Conditions: $w = 0 \; and \; w' = 0 \; at \; y = 0 \quad w = 0 \; at \; y = \mathrm{L}$

$$\left.\begin{aligned} C_2 + \frac{RL}{P} &= 0 \\ C_1 k - \frac{R}{P} &= 0 \end{aligned}\right\} \quad C_2 = -C_1 kL$$

$$C_1 tan(kL) + C_2 = 0 \quad \Rightarrow \quad kL = \tan(kL)$$

$$\boxed{\begin{gathered} kL = 4.49 \\ P_{cr} = \frac{20.19EI}{L^2} \Rightarrow P_{cr} = \frac{2.046\pi^2 EI}{L^2} \\ w = C_1[\sin(ky) - kL\cos(ky) + k\,(L - y)] \end{gathered}}$$

Fixed-Fixed Columns

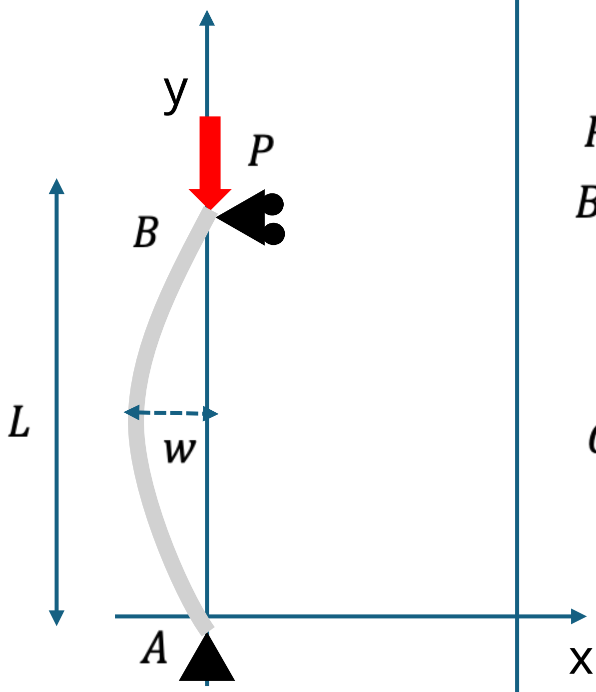


1. Points of inflection: Points at which the sign of the curvature changes
2. Moment at the points of inflection is zero
3. Deflection is symmetrical with slope of deflection curve zero at top, base and middle

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \Rightarrow P_{cr} = \frac{4\pi^2 EI}{L^2}$$

The critical load for a column with fixed ends is 4 times the critical load for column with pinned ends

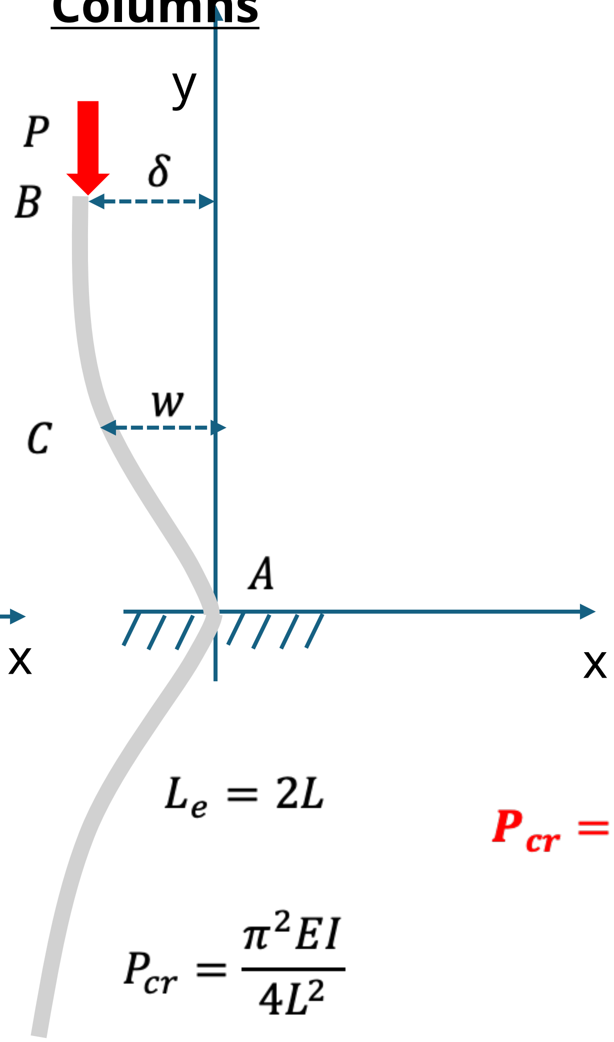
Pinned-Pinned Columns



$$L_e = L$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Fixed-Free Columns

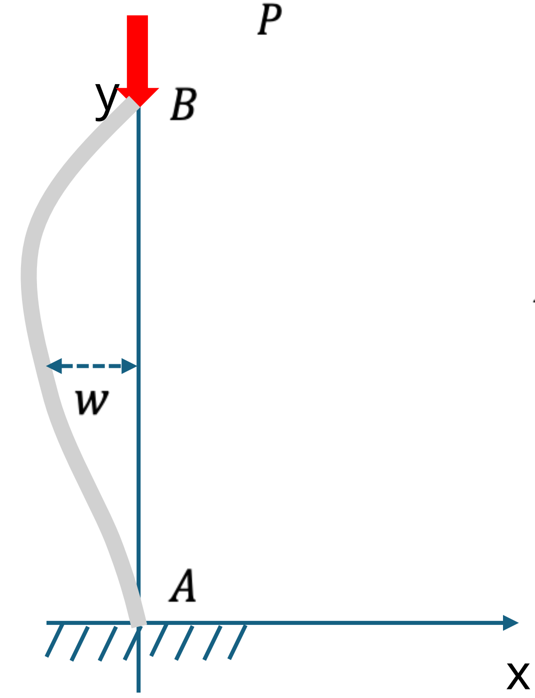


$$L_e = 2L$$

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

$$\boldsymbol{P_{cr} = \frac{\pi^2 EI}{L_e^2}}$$

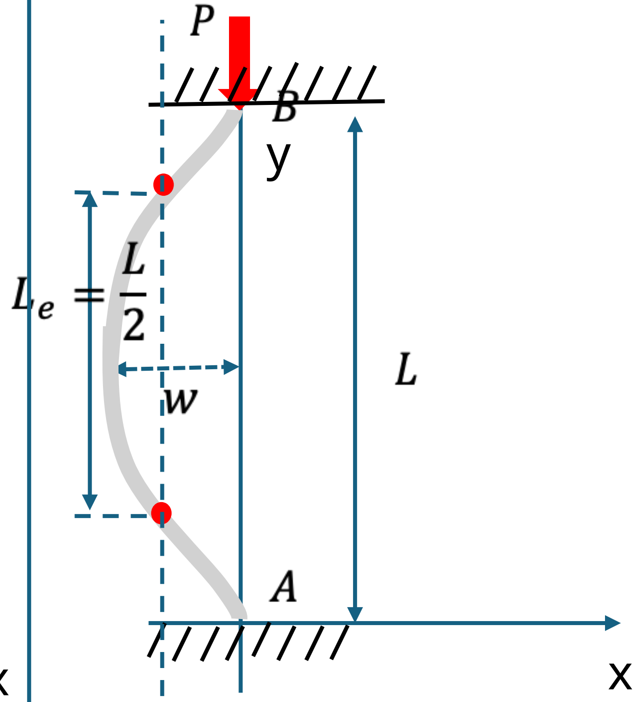
Fixed-Pinned Columns



$$L_e = 0.699L$$

$$P_{cr} = \frac{2.046\pi^2 EI}{L^2}$$

Fixed-Fixed Columns



$$L_e = 0.5L$$

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

THANK YOU!

Email:
1. bhattacharyyanurag@gmail.com
2. anurag.bhattacharyya@sri.com