E0 259 Data Analytics Assignment-1
Mars Orbit Module

Name : Bhushan Deo
Sr. No. : 18224

Output of Best Orbit Mars Parameter (all parameters):

I obtained the following best optimal parameters for the variables. The errors for the oppositions and the maxError is also given below :

Parameter	Best Value found by the function
r	10.96
s	360/686.91
c	146.1
e1	2.0388
e2	92.9
z	55.855

The error vector for the 12 oppositions : (angle discrepancy is in minutes) :
[3.1, 6.36, 2.53, 6.96, 4.125, **7.25**, 1.90, 1.91, 6.36, 5.55, 4.61, 6.58]
All values are rounded to 2 decimal places for better readability.

The maximum angular discrepancy : **7.25** minutes

Implementation approach/Code Architecture:

The approach followed was as prescribed by the course instructor. It is summarised as follows:

1.) The data is extracted to a dataframe and the times and angles for the longitude oppositions are noted.
2.) Without loss of generality we can assume equant-0 intersects longitude-1. (Otherwise permute the numbers given to the dotted lines to get this).
3.) Once the parameters are fixed, we compute the dotted lines, the intersections of dotted lines and the longitudes with the circle. We DO NOT explicitly compute the intersections of the equant dotted lines and the longitudes as we are only interested in the angular error. The intersections with the circle are solved after setting up a standard quadratic equation and using the positive root for the distance.
4.) Then the angle between the lines is computed (lines joining intersection and the SUN).

5.) First a very **rough cut** estimate of the parameters was obtained, r was estimated from the eccentricity (as suggested by Prof. Rajesh). Time period of Mars was approximated as 687. The distance of the equant from the SUN was assumed to be within 4 units, relative to the SUN-center distance of 1 unit, as all points lie in a close interval. Initially all the three were assumed to be on the same line, this helped us to correlate e2 and c. As we assumed WLOG equant-0 intersects longitude-1 and as the equant lies close to the SUN compared to the radius of the orbit we could guess z within [45,60] as the first longitude angle was 66 degrees. Then a full fledged grid search with large steps was carried out to get these ROUGH CUT(CRUDE) parameter guess :

r ~ 11 :: s ~ 360/687 :: z ~ 54 :: e1 ~ 2 :: e2 ~ 94 :: c ~ 148

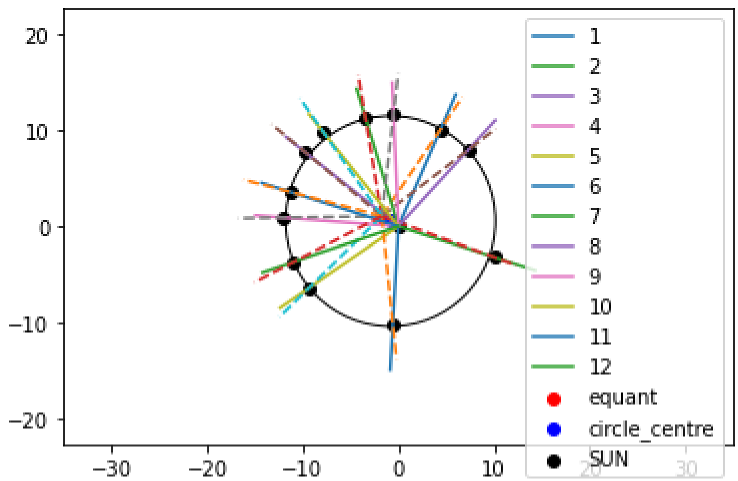
6.) Then as suggested the parameters were fine-tuned one after the other repeatedly across multiple runs via running the functions :
a.) bestOrbitInnerParams
b.)bestR
c.)bestS

7.) Once we got finer ranges for the parameters the final wrapper function was run with precision of 0.01 for each parameter and the final best parameters were computed (detailed at the start of this report). This code takes less than 10 minutes to run. We got an error of 7.25 minutes which after many experiments wasn't improving, suggesting we might be stuck in a local minima. (if there indeed exists a global minima with a lesser objective function value).

8.) Per iteration running time of MarsEquantModel function was of the order of 0.002 seconds.

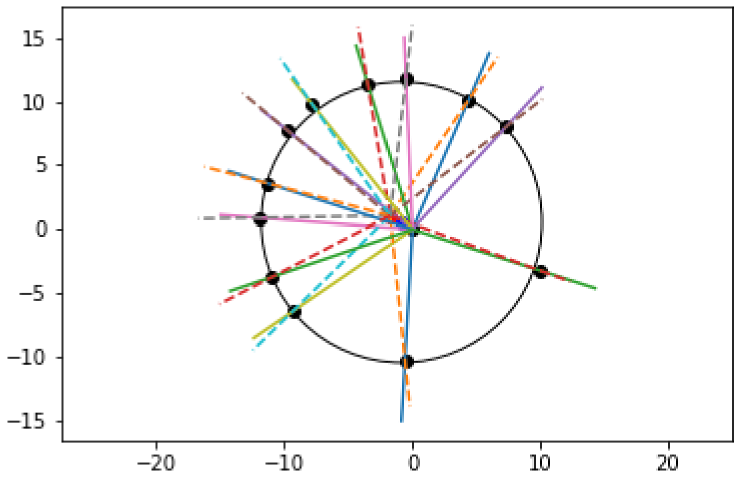
Plot with final parameters :

The plot of the equant lines, longitudes and the fitted circle for the final parameter



set, as shown in the table is as follows:

With Legend



Without Legend

Libraries used :

a.) numpy : computations
b.) matplotlib : plots
c.) pandas : dataframe op.
d.) time : measure time
e.) datetime : get accurate days between dates

Conclusion : The module was very interesting as everything was devised from first principles. Initially in the code I was computing intersections of equant dotted and SUN longitude lines, but as we are only interested in the angular discrepancy it becomes redundant. The main challenge was narrowing down to very fine ranges for the parameters so that neighborhood search is efficient.

The results were finally very intuitive, as the output major axis (approx. equant-center-SUN) along with the input data points us to the same aphelion and perihelion regions of the orbit. The found out equant longitudes were also close to the average longitudes provided in the data as observed by Kepler. The utility of the given latitude data was what perplexed me, as it infers about the out of plane component of the orbit. Modifications to this assignment could include fitting elliptical orbits, using triangulation data, and finally estimating an out of plane (w.r.t ecliptic) ellipse with the longitude information.