Determining the magnitude of an earthquake

Magnitude -- measure of energy released during earthquake.

There are several different ways to measure magnitude.

Most common magnitude measure is **Richter Magnitude**, named for the renowned seismologist, Charles Richter.

Richter Magnitude

- Measure amplitude of largest S wave on seismograph record.
- Take into account distance between seismograph & epicenter.

Richter Scale

- Logarithmic numerical (NOT a physical) scale
- Increasing one whole unit on Richter Scale represents 10 times greater magnitude.
- Going up one whole unit on Richter Scale represents about a 30 times greater release of energy.

Intensity

- Intensity refers to the amount of damage done in an earthquake
- Mercalli Scale is used to express damage.

The Richter magnitude scale (for epicentral distance < 600 km) was originally devised for local earthquakes for Southern California region in 1935. This is the earliest magnitude scale for quantifying earthquakes. Richter recognized that these earthquakes originated at depths not much different from 15 km, so that the effects caused by focal depth variations could be ignored. Richter defined the local magnitude M_L of an earthquake observed at a station to be

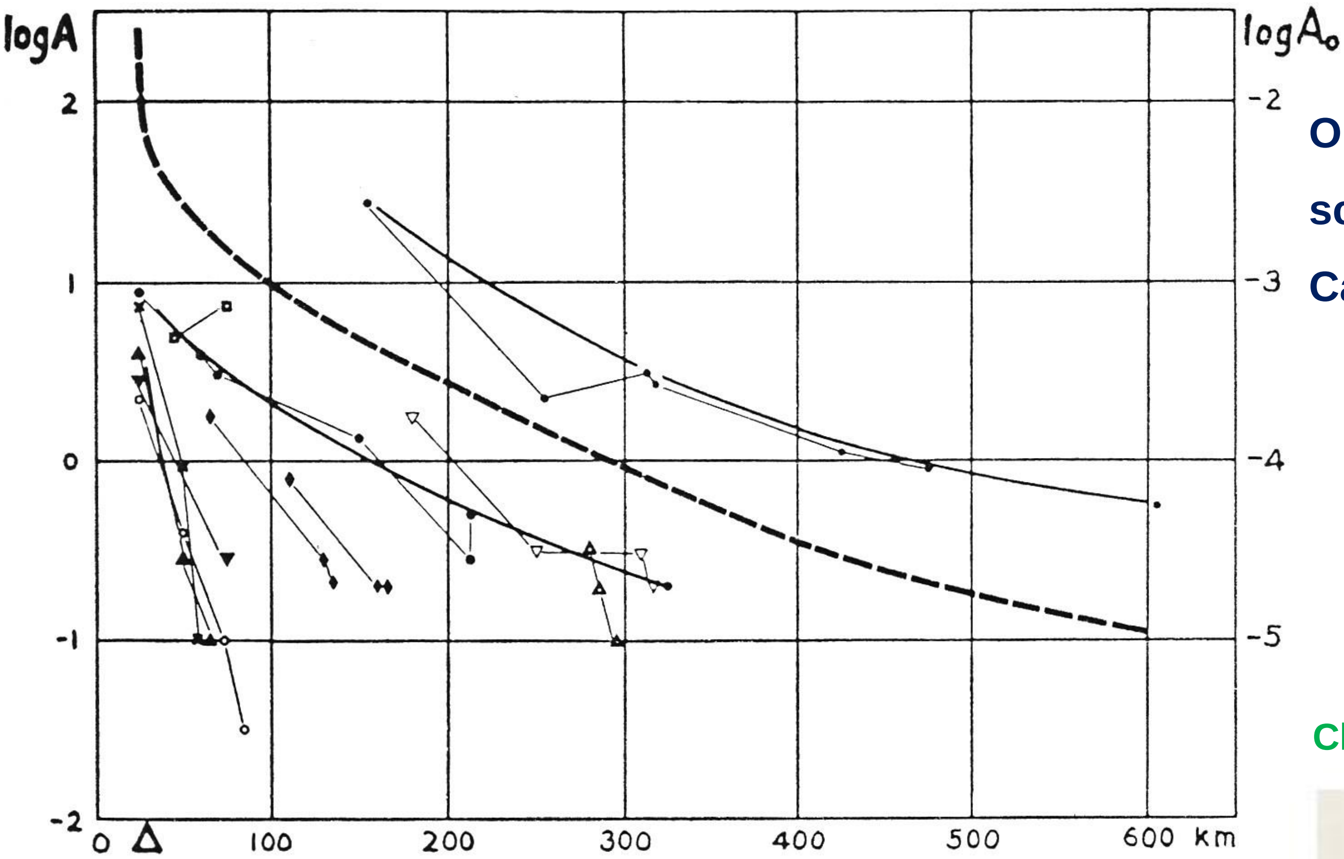
$$M_L = \log A(\Delta) - \log A_O(\Delta)$$

where A is the maximum amplitude in millimeters recorded by the Wood-Anderson seismograph (amplification 2800 and period 0.8 sec) for an earthquake at epicentral distance of Δ km, and $A_O(\Delta)$ is the maximum amplitude at Δ km for a standard earthquake. A_O corresponds to the amplitude that would be recorded at a given distance for an earthquake of magnitude $M_L = 0$. Richter calibrated the scale by assigning the value of $M_L = 3$ to a standard earthquake that, at a distance of 100 km, is recorded by a Wood-Anderson seismograph with a maximum amplitude of A = 1 mm ($\log A_O = -3$, for $\Delta = 100$ km).

In practice, we need to know the approximate epicentral distances of the recording stations. The maximum trace amplitude on a standard Wood-Anderson seismogram is then is measured in millimeters, and its logarithm to base 10 is taken. To this number we add the quantity tabulated as ($-\log A_O$) for the corresponding station distance from the epicenter. The sum is the value of local magnitude for that seismogram.

Richter magnitudes in their original form are no longer used because most earthquakes do not occur in California and Wood-Anderson seismograph is rare. However, local magnitudes are sometimes still reported because many buildings have resonant frequencies near 1 Hz, close to that of a Wood-Anderson seismograph, so ML is often a good indication of the structural damage an earthquake can cause.

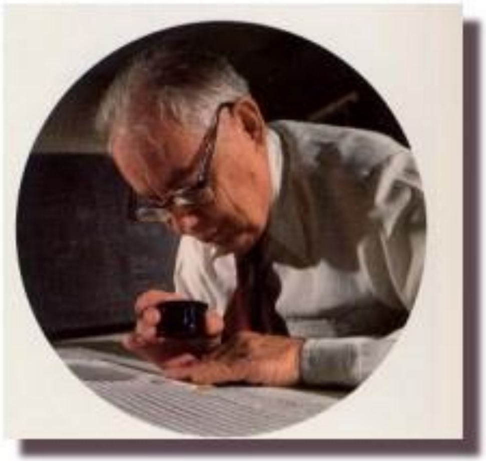
Earthquake Size – Magnitude



Epicentral Distance (km)

Origin of the magnitude scale. Data for Southern California earthquakes.

Charles Richter: 1900-1985



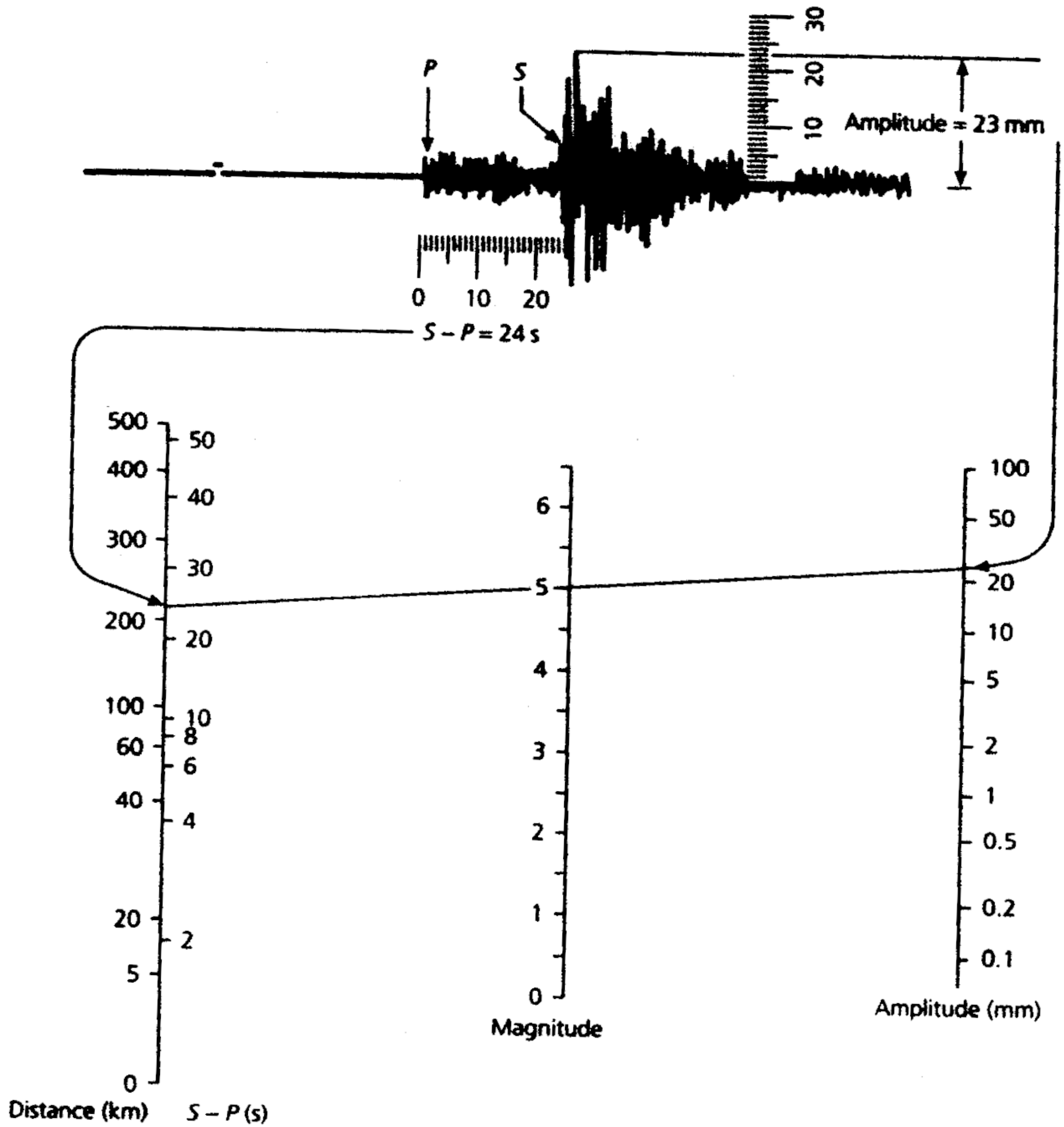
log of amplitude

Distance correction

$$M = \log A - \log A_0$$

The amplitude correction for local magnitude computation are given in the table for Calfornia and Tehri region.

Distance km	(-) log A_o Tehri	(-) log A_o California	Distance km	(-) log A_o Tehri	(-) log A_o California
0	2.60	1.4	160	3.37	3.3
10	2.65	1.5	170	3.41	3.3
20	2.70	1.7	180	3.46	3.4
30	2.75	2.1	190	3.50	3.5
40	2.80	2.4	200	3.55	3.5
50	2.85	2.6	210	3.59	3.6
60	2.90	2.8	220	3.64	3.6
70	2.95	2.8	230	3.68	3.7
80	2.99	2.8	240	3.73	3.7
90	3.04	3.0	250	3.77	3.8
100	3.09	3.0	260	3.81	3.8
110	3.14	3.1	270	3.86	3.9
120	3.18	3.1	280	3.90	3.9
130	3.23	3.2	290	3.94	4.0
140	3.32	3.3	300	3.98	4.0



P
S
Amplitude = 23 mm
0 10 20
S − P = 24 s
Distance (km)
S − P (s)
Magnitude
Amplitude (mm)

Magnitude of Microearthquakes

Lee and other in 1972 established an empirical formula for estimating magnitudes of local earthquakes recorded by the USGS Central California Microearthquakes Network using signal durations. For a set of 351 earthquakes, they computed the local magnitudes (as defined by Richter, 1935) from Wood-Anderson seismograpms or their equivalents. Correlating these local magnitudes with signal durations measured from seismograms recorded by the USGS network, they obtained the following empirical formula:

$$M_d = -0.87 + 2.00 \log \tau + 0.0035\,\Delta$$

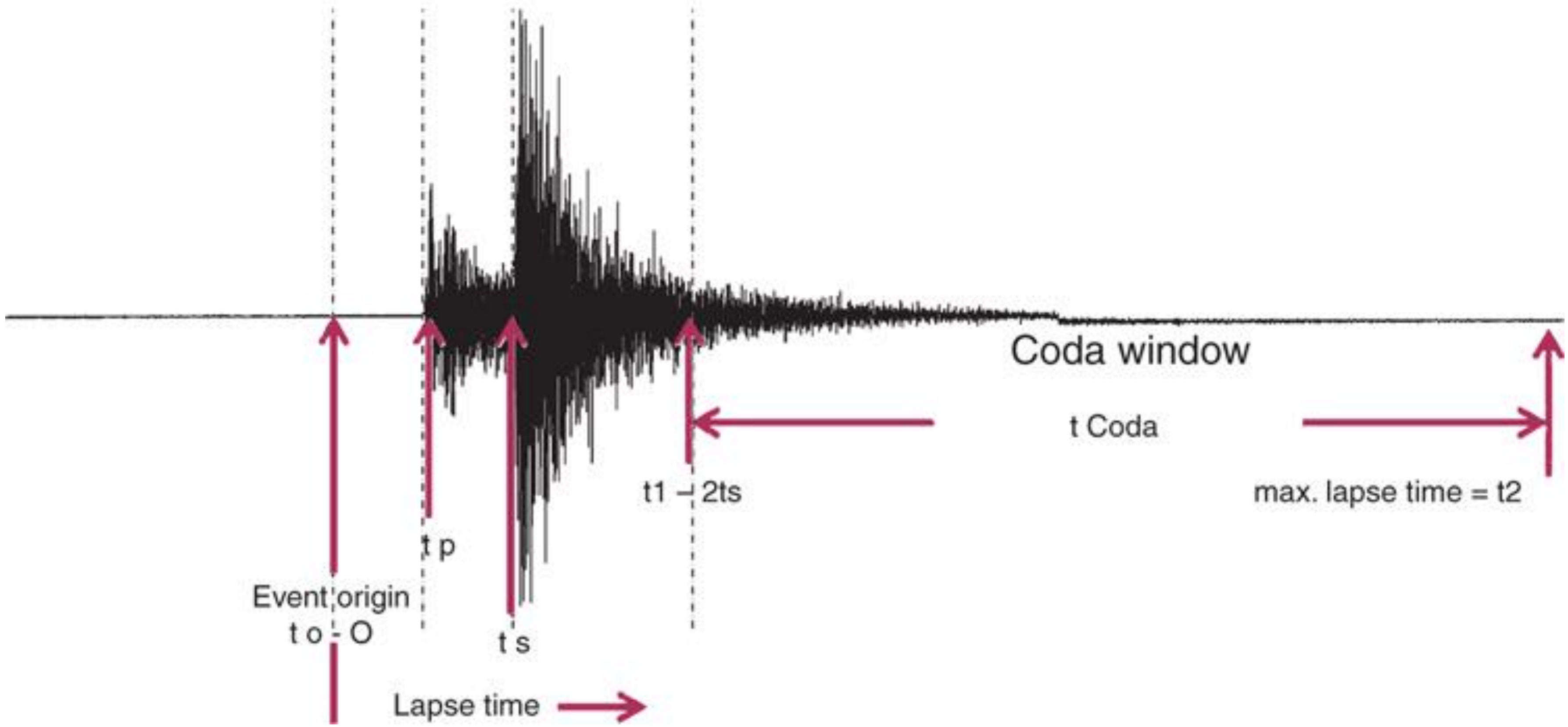
where M_d is an estimate of Richter magnitude, τ is signal duration in sec, and Δ is τicentral distance in km.

Duration magnitude M_d for a given station is usually given in the form

$$M_d = a_1 + a_2 \log \tau + a_3\Delta + a_4 h$$

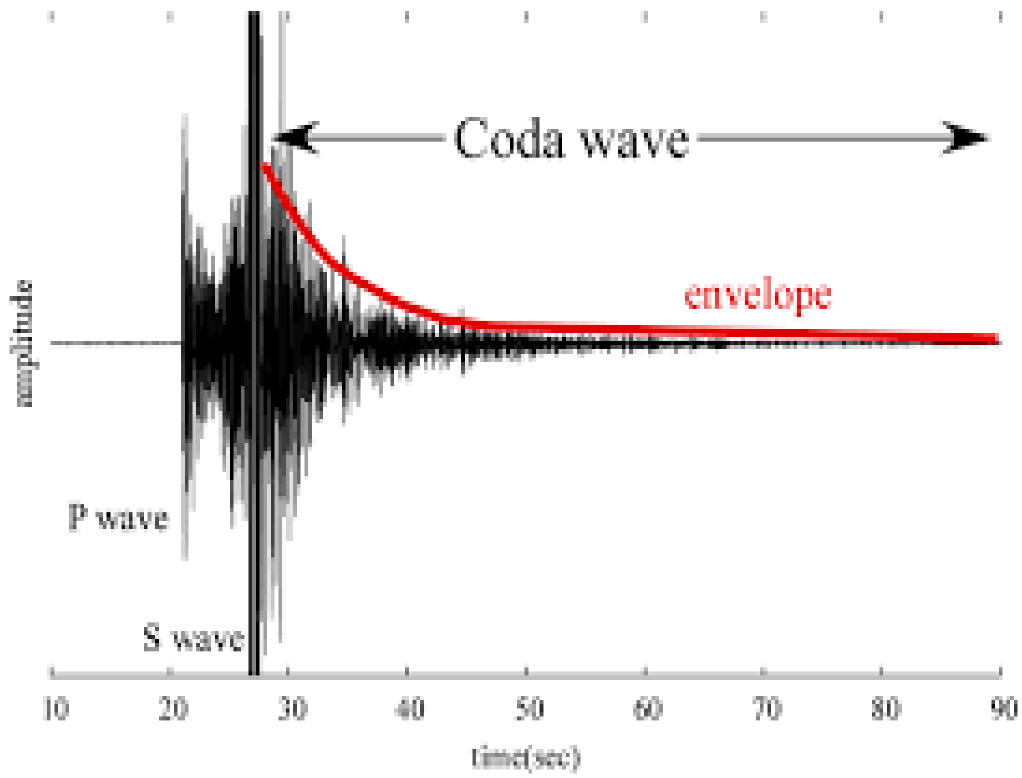
where τ is signal duration in sec, Δ is epicentral distance in km, h is focal depth in km, and a_1, a_2, a_3 and a_4 are empirical constants. These constants are usually determined by correlating signal duration with Richter magnitude for a set of selected earthquakes. Signal duration to be the time interval in sec. between the onset of first P-wave and the point where the seismic signal amplitude approximately twice that of the noise.

Coda energy window



Characteristics of seismogram of micro-earthquakes under local network

The incoming signals from a micro-earthquake network can have a wide range amplitudes, but local earthquakes are characterized generally by their impulsive onsets, high-frequency content, exponential envelop, and decreasing signal frequency with time.



With time, various local and global magnitude scales evolved. For global studies, the primary two were the body wave magnitude, m_b, and surface wave magnitude, M_S for epicentral distance > 600 km.

The body wave magnitude is given by

$$m_b = log_{10}\left(\frac{A_P}{T}\right) + Q(\Delta, h)$$

A_P is the maximum amplitude of the ground motion in microns associated with P-waves having a period (T) of less than 3 sec, usually about 1 sec estimated on vertical component. Where Q (Δ,h) is an empirical correction for signal attenuation due to epicentral distance (Δ) and focal depth (h) that is made by reading directly from a graph of table of values.

The International Association for Seismology and Physics of the Earth's Interior (IASPEI) has adopted the following definition of the surface-wave magnitude (M_S) of an earthquake:

$$M_S = log_{10}\left(\frac{A_S}{T}\right) + 1.66log_{10}(\Delta) + 3.3$$

where A_S is the vertical component of ground motion in μm determined from the maximum Raleigh-wave amplitude, T is the period of the wave (18-22 sec), Δ is the epicentral distance in degrees (> 15°), where the earthquake has a focal depth of less than 50 km.

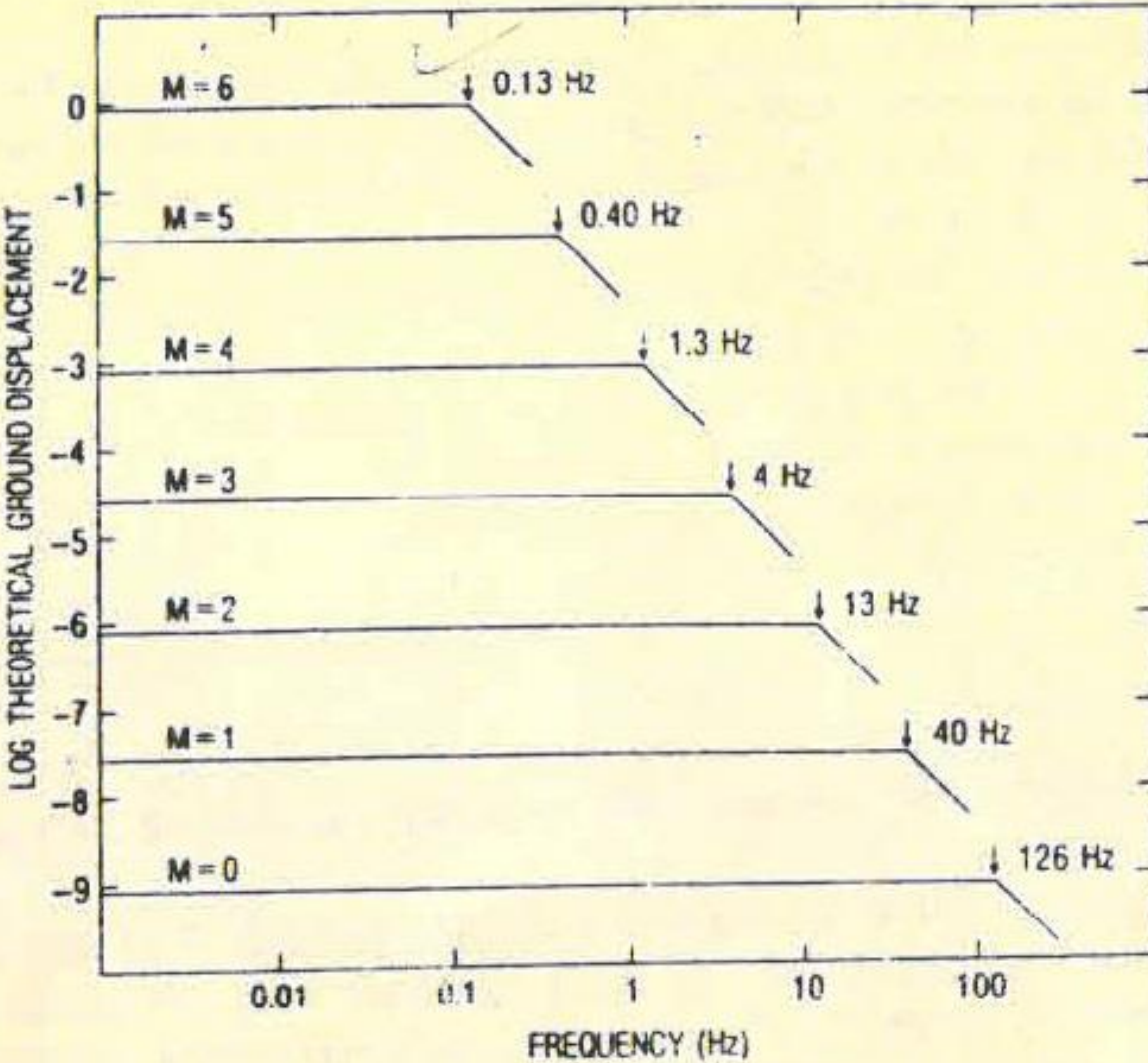
As measures of earthquake size, magnitudes have two major advantages. First, they are directly measured from seismograms without sophisticated signal processing. Second, they yield units of order 1 which are intuitively attractive: magnitude 5 earthquakes are moderate, magnitude 6 is strong, 7 are major, 8 are great, and 9 are mega.

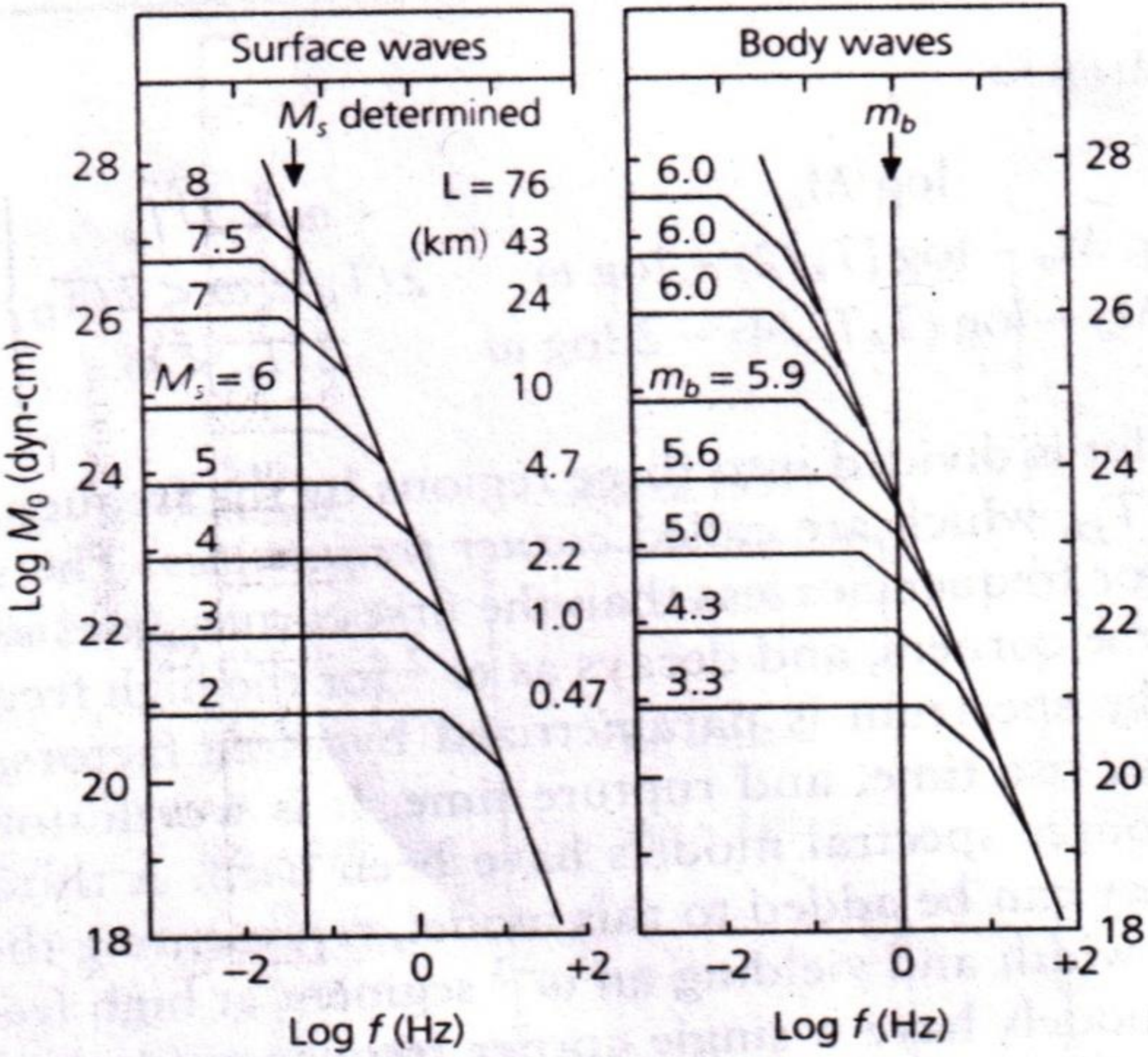
However, magnitudes have two related limitations. First, they are totally empirical and thus have no direct connection to the physics of earthquakes. Equations are dimensionally not correct. The different magnitude scales yield different values. Moreover, body and surface wave magnitudes do not correctly reflect the size of large earthquakes.

Saturation of Magnitude Scale

Table illustrates the variation of scalar moments for different earthquakes. As noted, m_b and M_S differ significantly. The earthquakes with moments greater than that of the San Fernando earthquake all have m_b 6.2, even as the moment increases by a factor of 20,000. Similarly, the earthquakes larger than the San Francisco earthquake have M_S about 8.3, even as the moment increases by a factor of 400. This effect, called magnitude saturation, is a general phenomenon for m_b above about 6.2 and M_S above about 8.3. This phenomenon is due to the fact that the amplitude spectrum is displaced toward low frequencies with increasing size of earthquakes. Large to mega-shocks produce fractures hundreds of kilometers long with displacements of several meters and waves of 20 sec are not representative of the energy radiated.

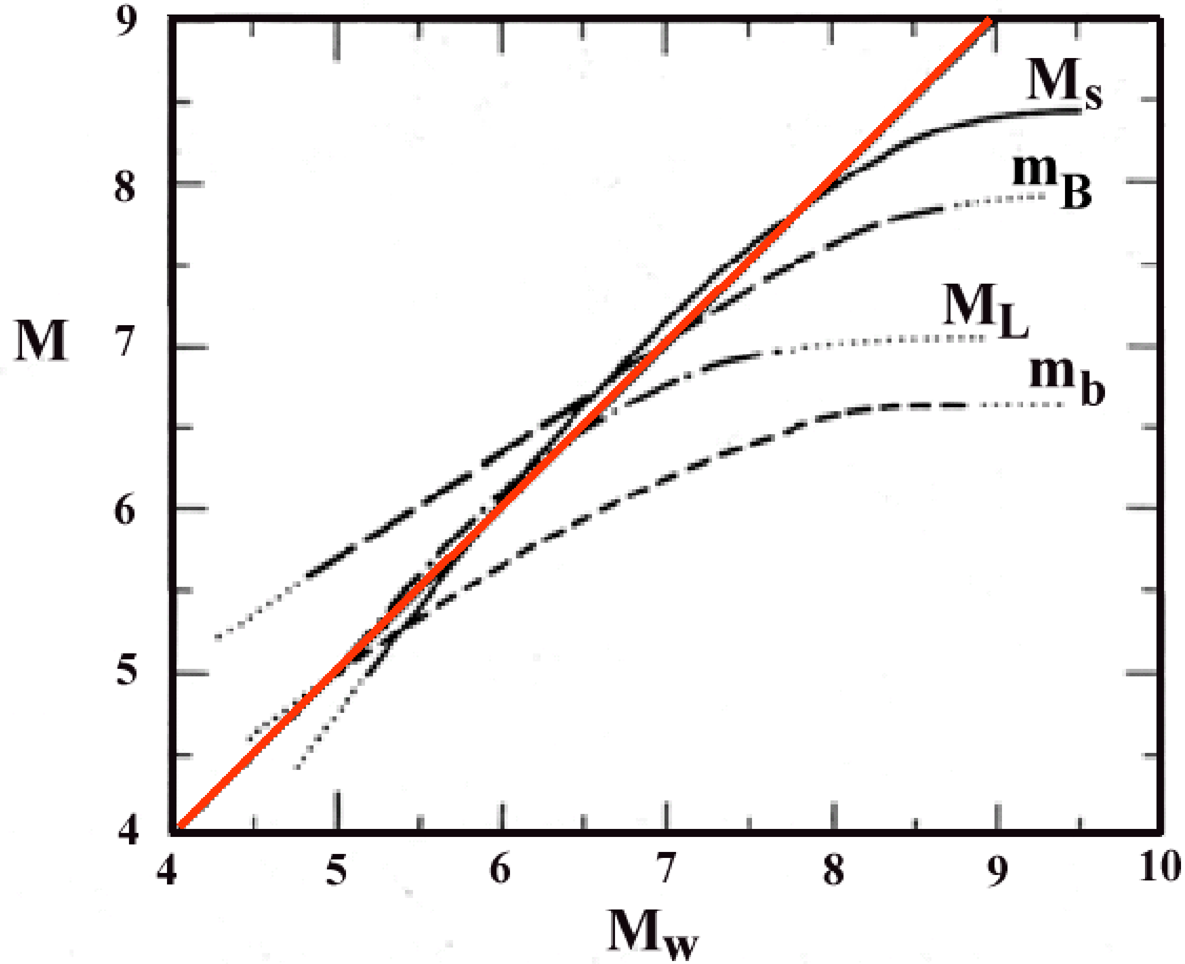
Earthquake	Body wave magnitude m_b	Surface wave magnitude M_s	Fault area (km^2) length × width	Average dislocation (m)	Moment (dyn-cm) M_0	Moment magnitude M_w
Truckee, 1966	5.4	5.9	10 × 10	0.3	8.3×10^{24}	5.8
San Fernando, 1971	6.2	6.6	20 × 14	1.4	1.2×10^{26}	6.7
Loma Prieta, 1989	6.2	7.1	40 × 15	1.7	3.0×10^{26}	6.9
San Francisco, 1906		8.2	320 × 15	4	6.0×10^{27}	7.8
Alaska, 1964	6.2	8.4	500 × 300	7	5.2×10^{29}	9.1
Chile, 1960		8.3	800 × 200	21	2.4×10^{30}	9.5


LOG THEORETICAL GROUND DISPLACEMENT
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
M = 6
0.13 Hz
M = 5
0.40 Hz
M = 4
1.3 Hz
M = 3
4 Hz
M = 2
13 Hz
M = 1
40 Hz
M = 0
126 Hz
0.01
0.1
1
10
100
FREQUENCY (Hz)

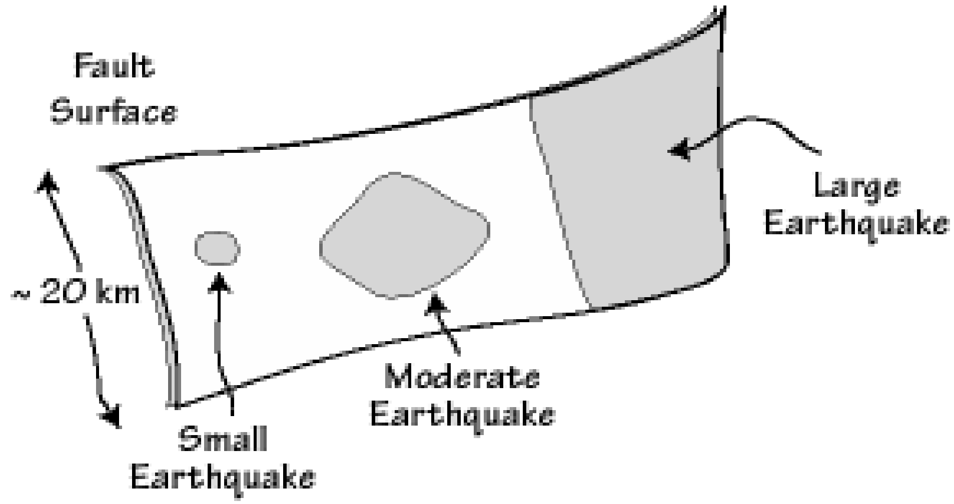


Surface waves
Body waves
M_s determined
m_b
L = 76
(km) 43
24
10
4.7
2.2
1.0
0.47
8
7.5
7
$M_s = 6$
5
4
3
2
6.0
6.0
6.0
$m_b = 5.9$
5.6
5.0
4.3
3.3
Log M_0 (dyn-cm)
28
26
24
22
20
18
−2
0
+2
Log f (Hz)

Relationship between different types of magnitudes



This problem is solved with Kanamori's moment magnitude M_W. This Scale does not depend on frequency and can be used for the whole range of sizes of earthquakes from very small to very large up to values of about 9.5. Kanamori in 1977 introduced to avoid the saturation of magnitude, and is given by



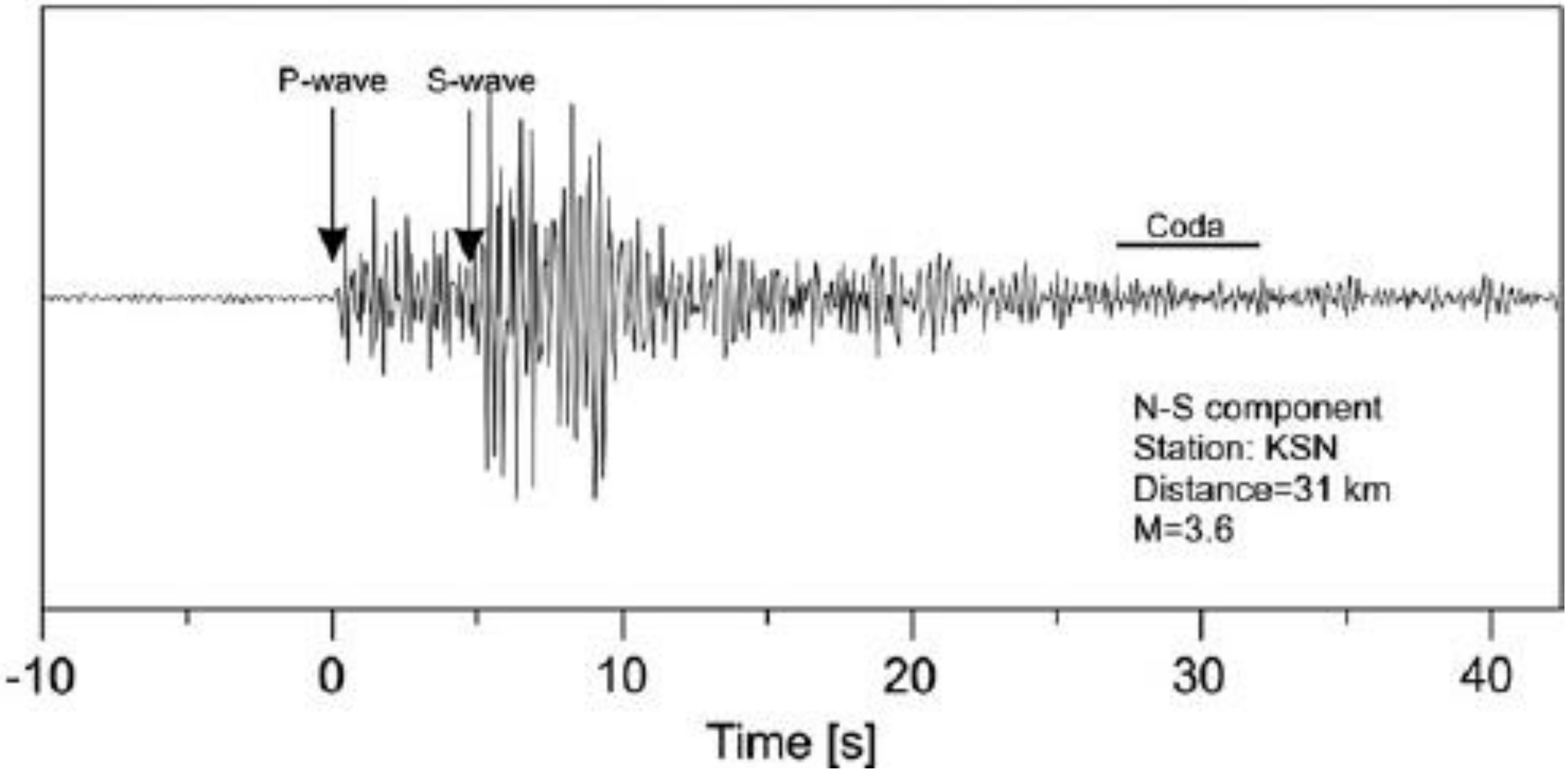
$$M_w = \frac{2}{3} log_{10}(M_O) - 10.7$$

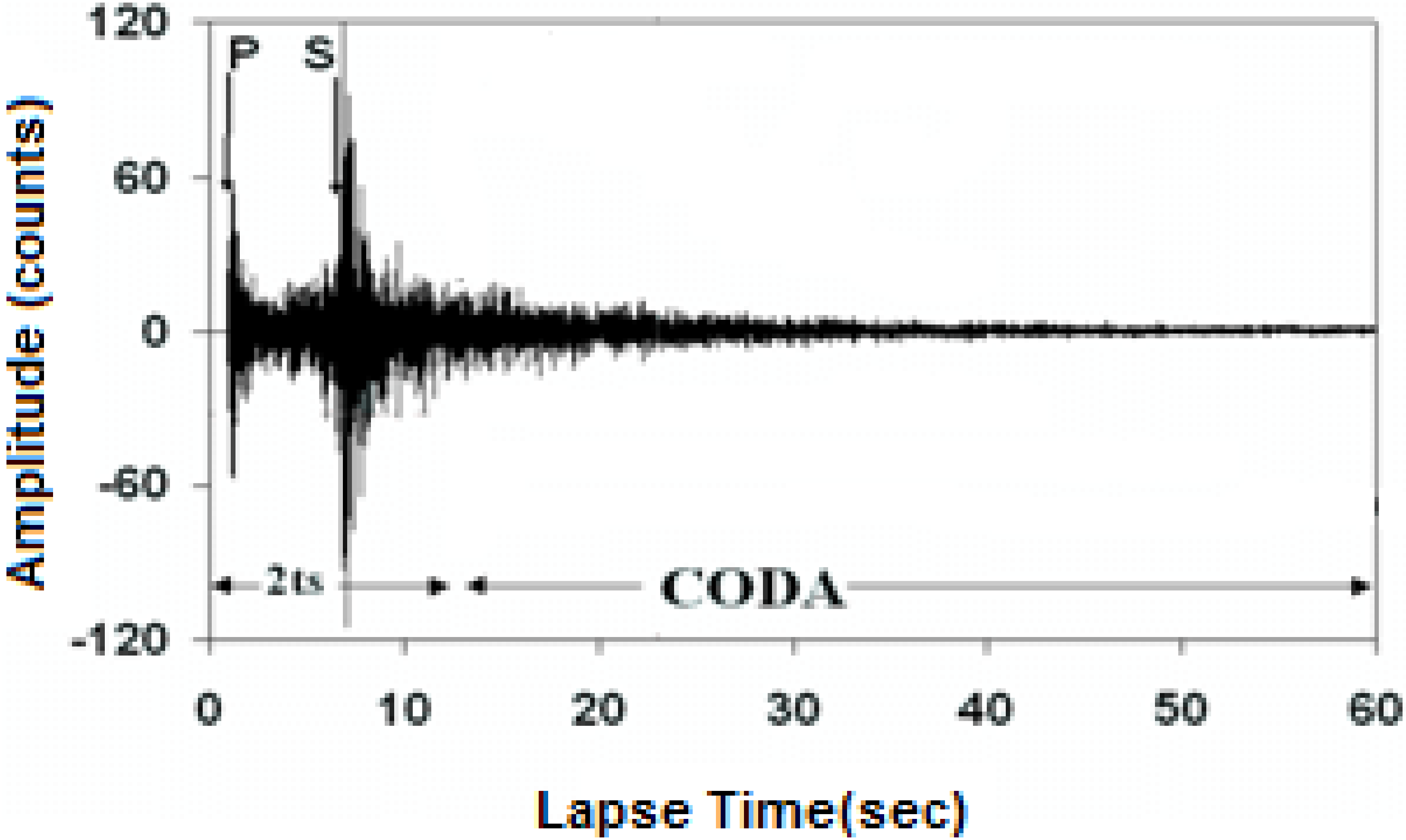
where M_O is scalar moment in dyne-cm.

Aki in 1966 introduced the seismic moment M_O as

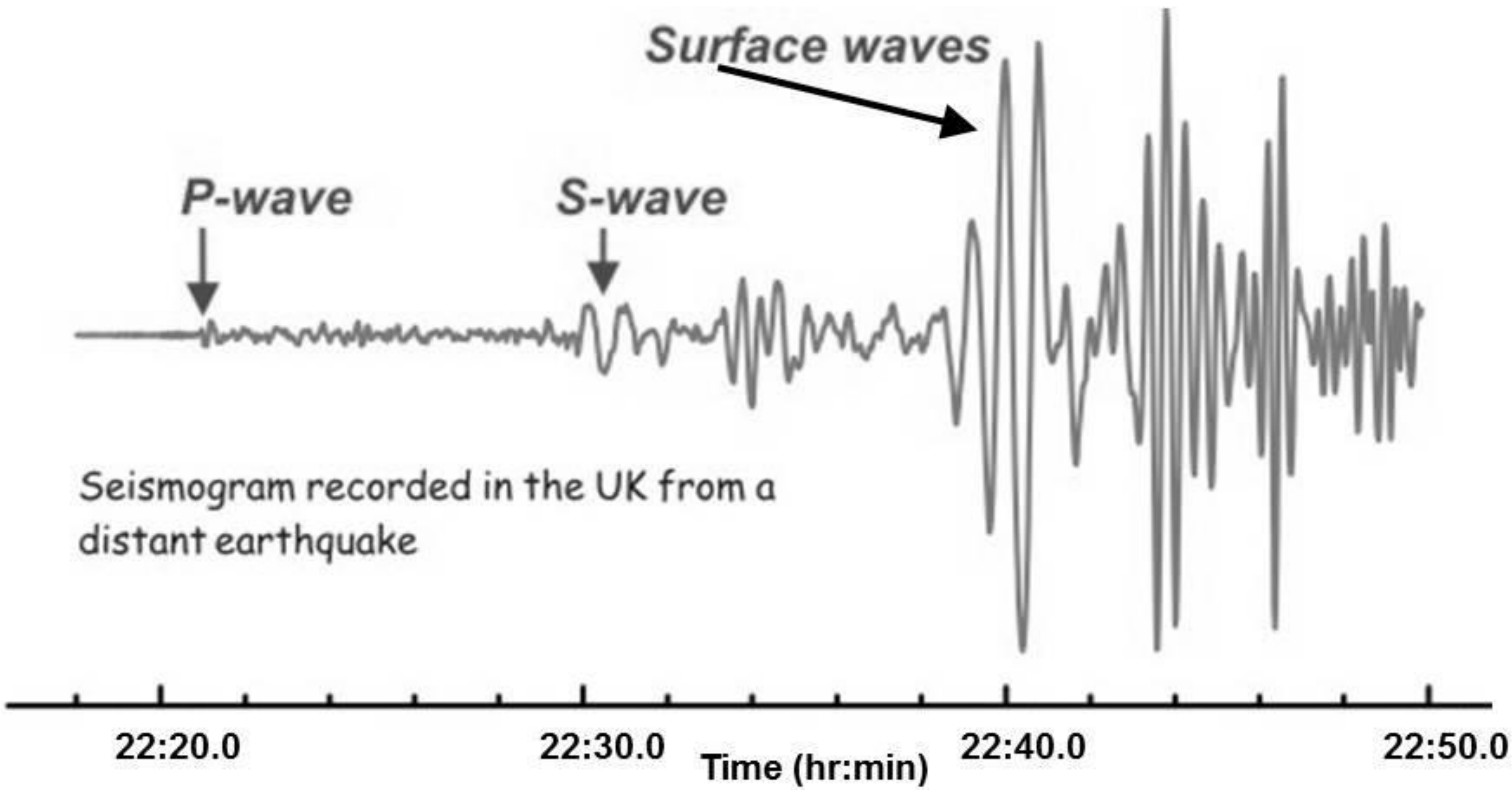
$$M_O = \mu DA$$

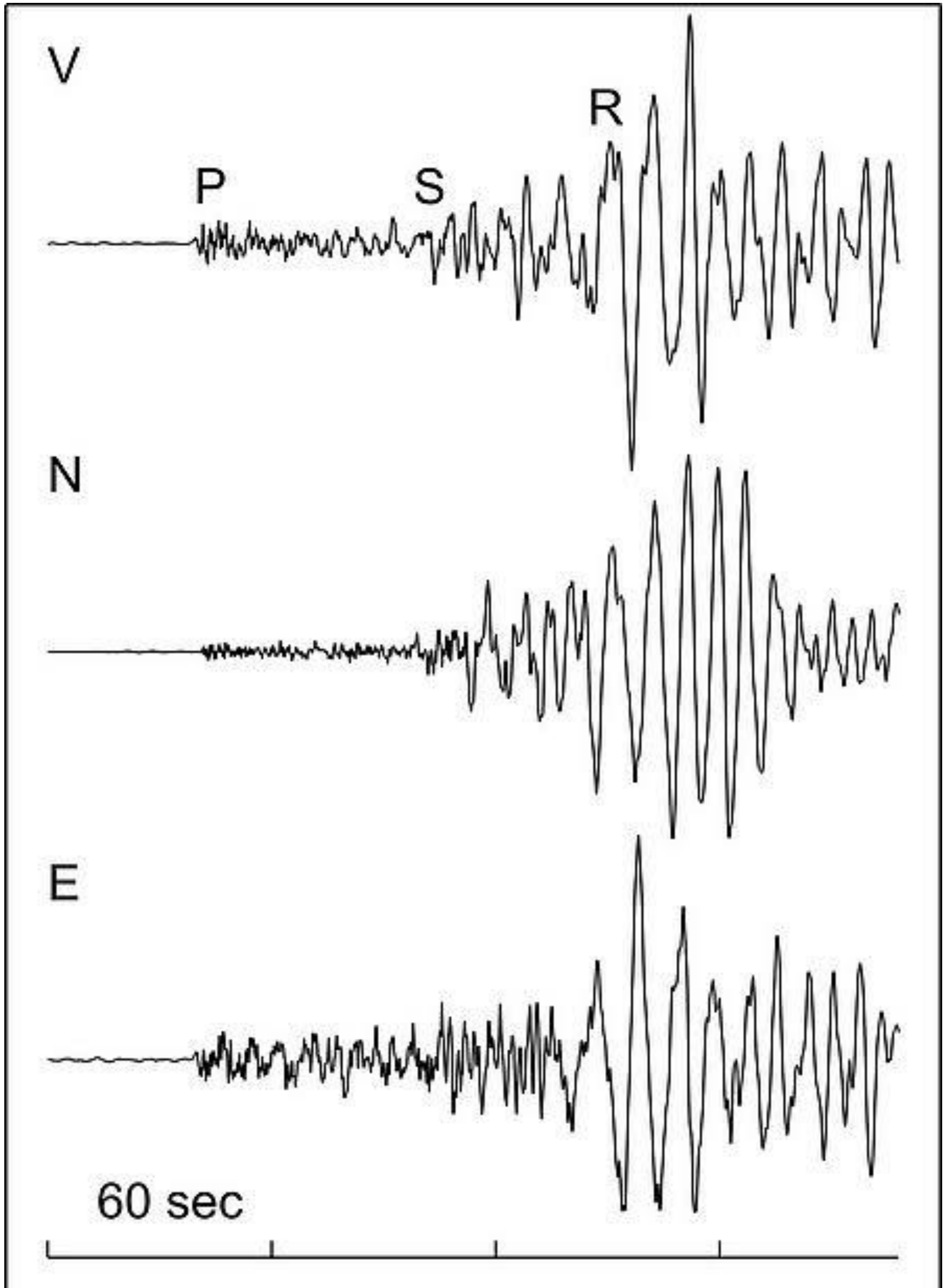
Where the μ is the shear modulus of the medium surrounding the source, A is the

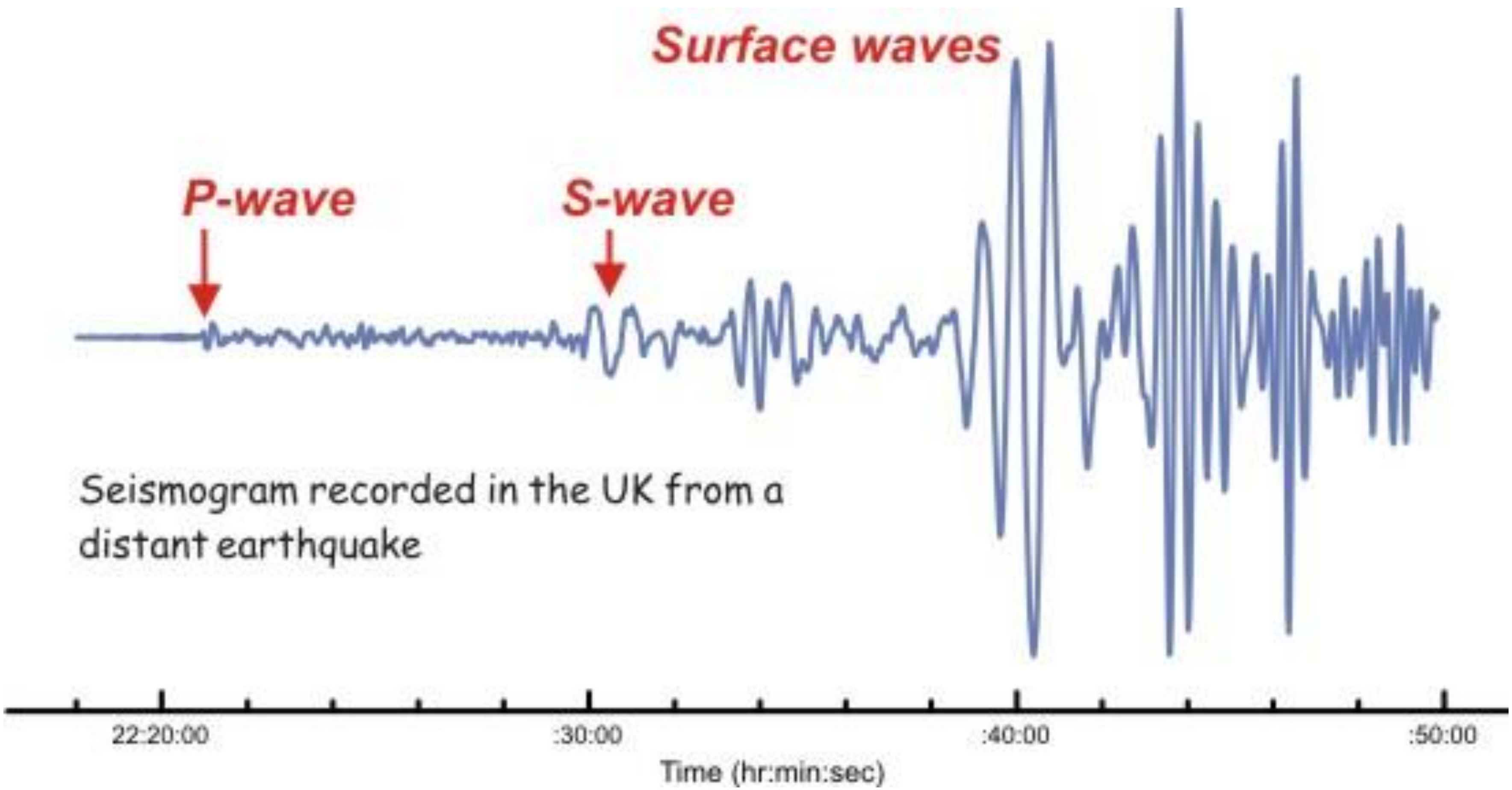

P-wave
S-wave
Coda
N-S component
Station: KSN
Distance=31 km
M=3.6
-10
0
10
20
30
40
Time [s]



120
60
0
-60
-120
P
S
2ts
CODA
0
10
20
30
40
50
60
Amplitude (counts)
Lapse Time(sec)


Surface waves
P-wave
S-wave
Seismogram recorded in the UK from a distant earthquake
22:20.0
22:30.0
Time (hr:min)
22:40.0
22:50.0


V
P
S
R
N
E
60 sec


Surface waves
P-wave
S-wave
Seismogram recorded in the UK from a distant earthquake
22:20:00
:30:00
:40:00
:50:00
Time (hr:min:sec)

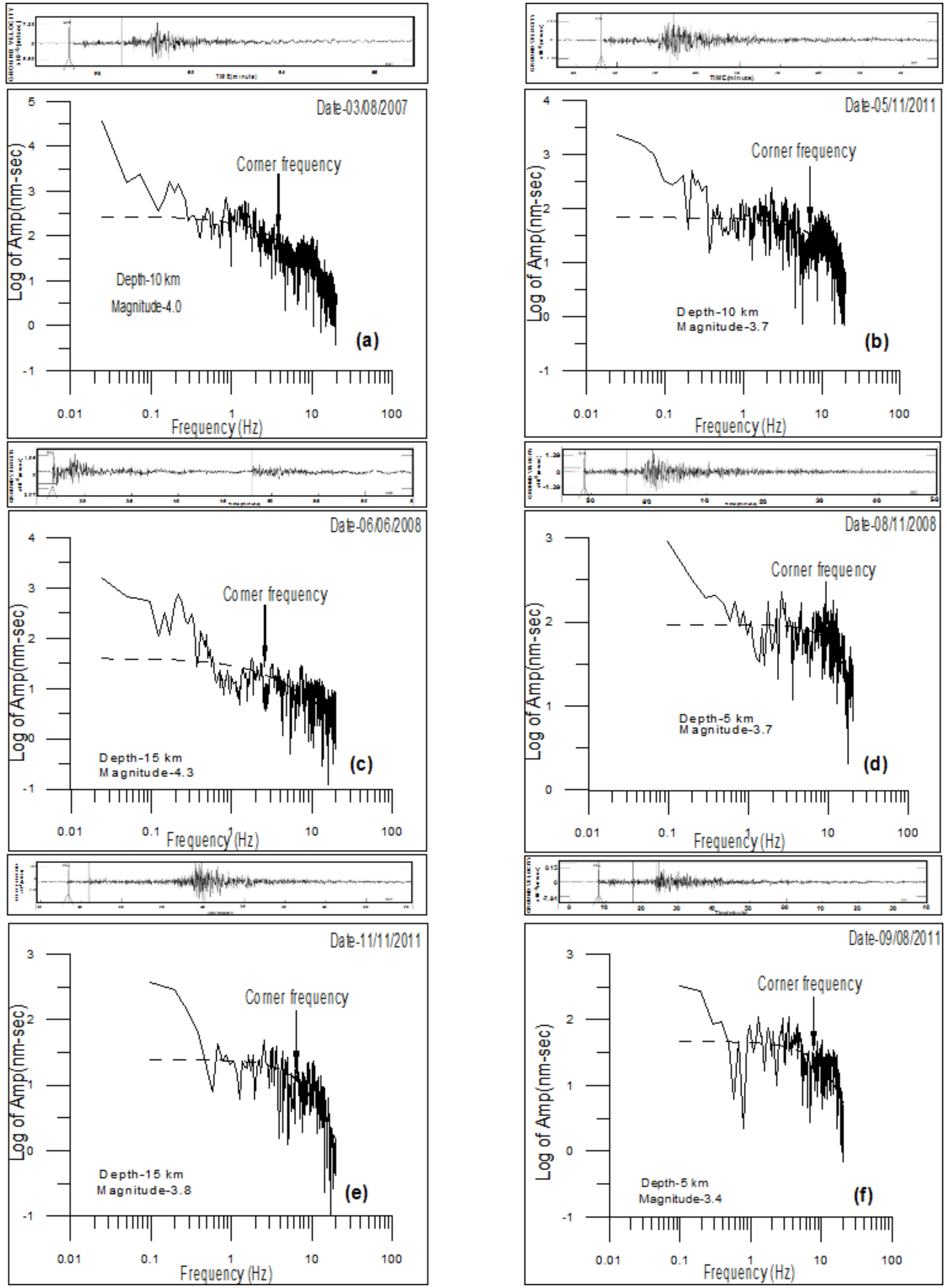
The following empirical relationships (Brune, 1970) were used for computing the seismic moment, source radius, and stress drop on a circular fault with a radius R.

$$M_O = \frac{4\pi\rho V_p^3 R\Omega_O}{\Psi}$$

$$r = \frac{2.3V_p}{2\pi f_O}$$

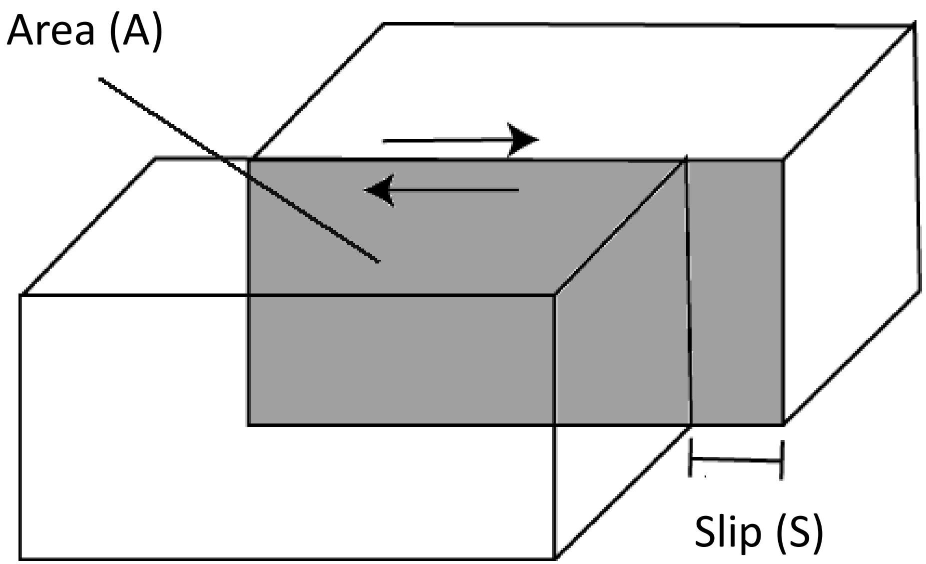
$$\Delta\sigma = \frac{7}{16}\frac{M_O}{r^3}$$

Where Ωo is the low-frequency displacement spectral level of either P or S waves, ψ is a function accounting for the radiation pattern, ρ is the density of the medium, R is the epicentral distance, V_P is the body wave velocity, r is the radius of the fault.



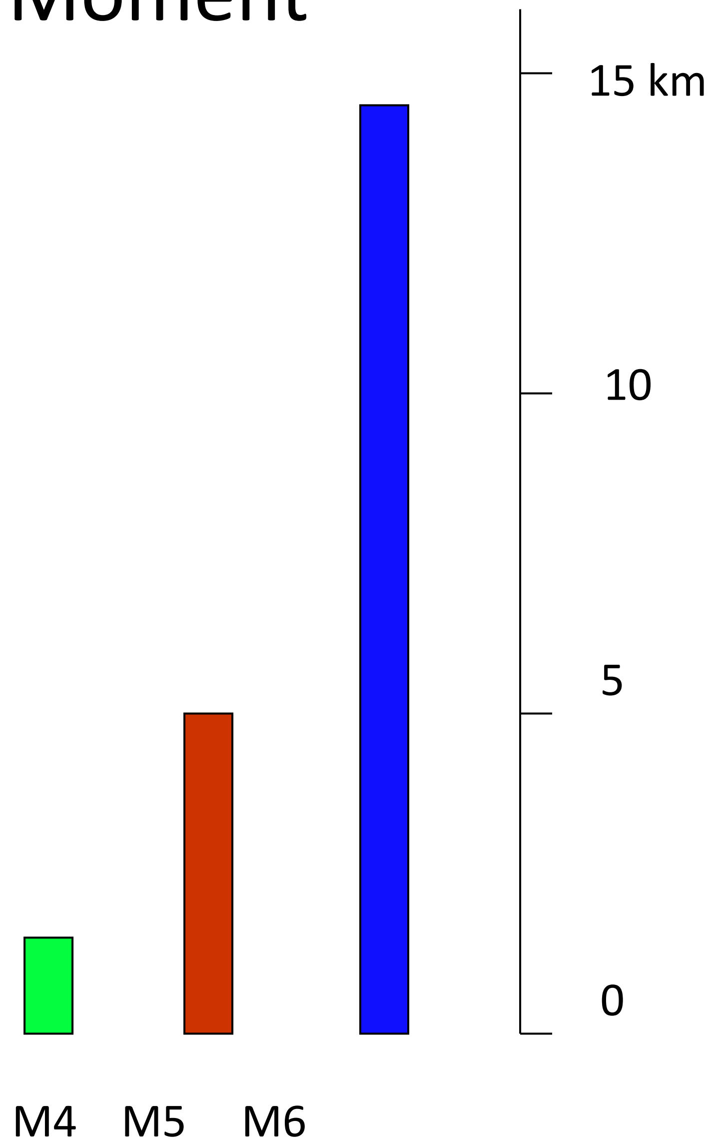
Date-03/08/2007
Corner frequency
Depth-10 km
Magnitude-4.0
(a)
Log of Amp(nm-sec)
Frequency (Hz)
Date-05/11/2011
Corner frequency
Depth-10 km
Magnitude-3.7
(b)
Log of Amp(nm-sec)
Frequency (Hz)
Date-06/06/2008
Corner frequency
Depth-15 km
Magnitude-4.3
(c)
Log of Amp(nm-sec)
Frequency (Hz)
Date-08/11/2008
Corner frequency
Depth-5 km
Magnitude-3.7
(d)
Log of Amp(nm-sec)
Frequency (Hz)
Date-11/11/2011
Corner frequency
Depth-15 km
Magnitude-3.8
(e)
Log of Amp(nm-sec)
Frequency (Hz)
Date-09/08/2011
Corner frequency
Depth-5 km
Magnitude-3.4
(f)
Log of Amp(nm-sec)
Frequency (Hz)

Earthquake size - Seismic Moment

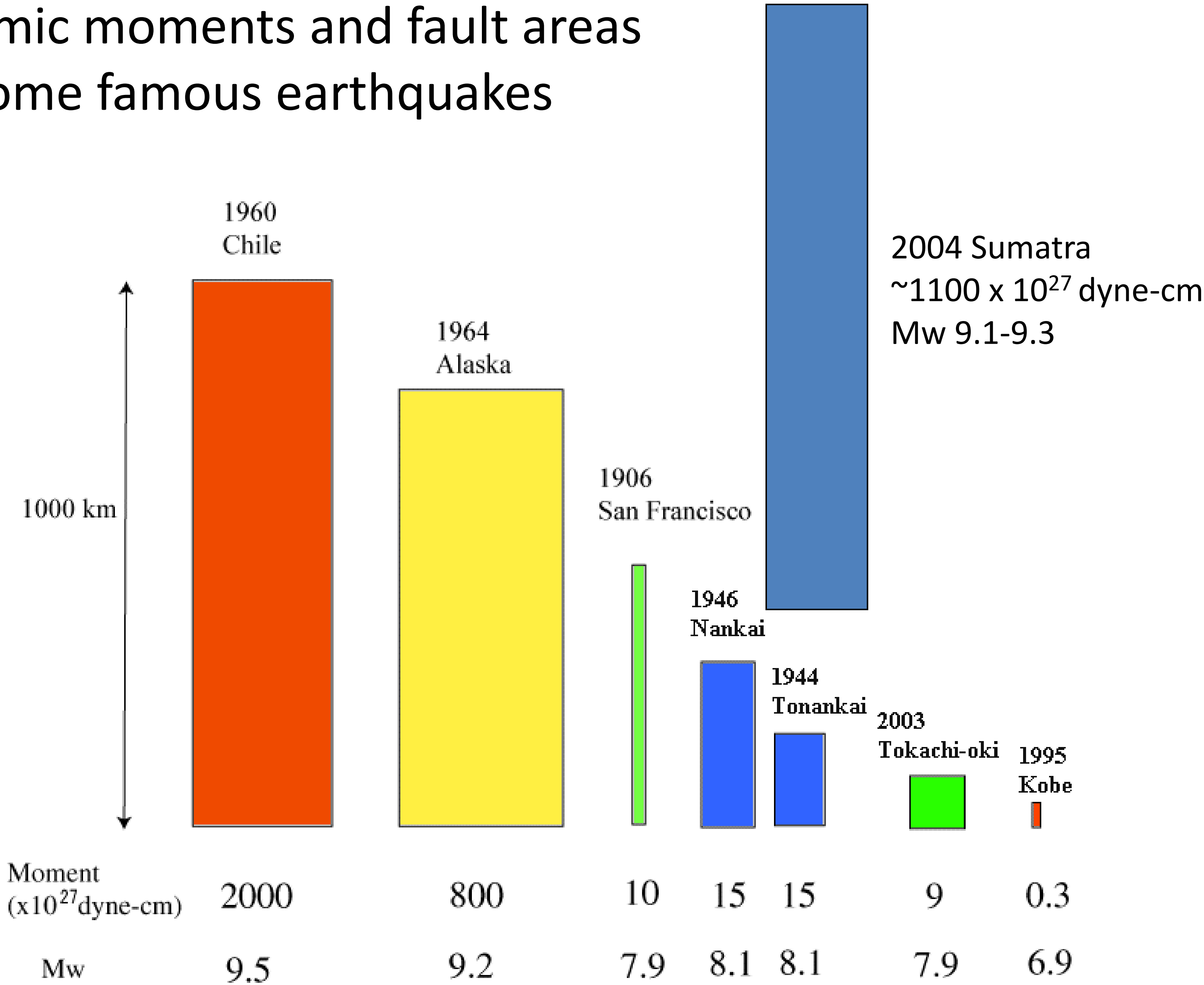


Seismic Moment = (Rigidity)(Area)(Slip)

$$M_0(t) = \mu \cdot S \cdot \Delta u(t)$$



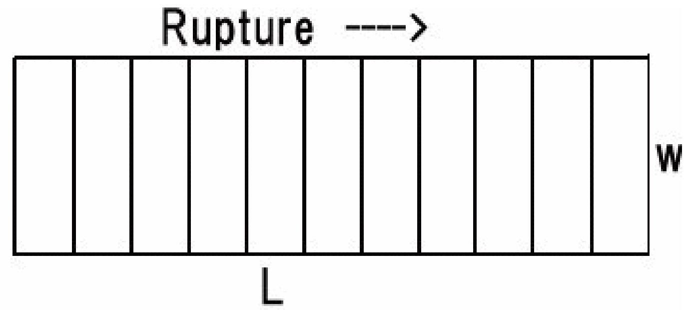
Seismic moments and fault areas of some famous earthquakes



The stress drop for a strike-slip on a rectangular fault with length L and width w yields

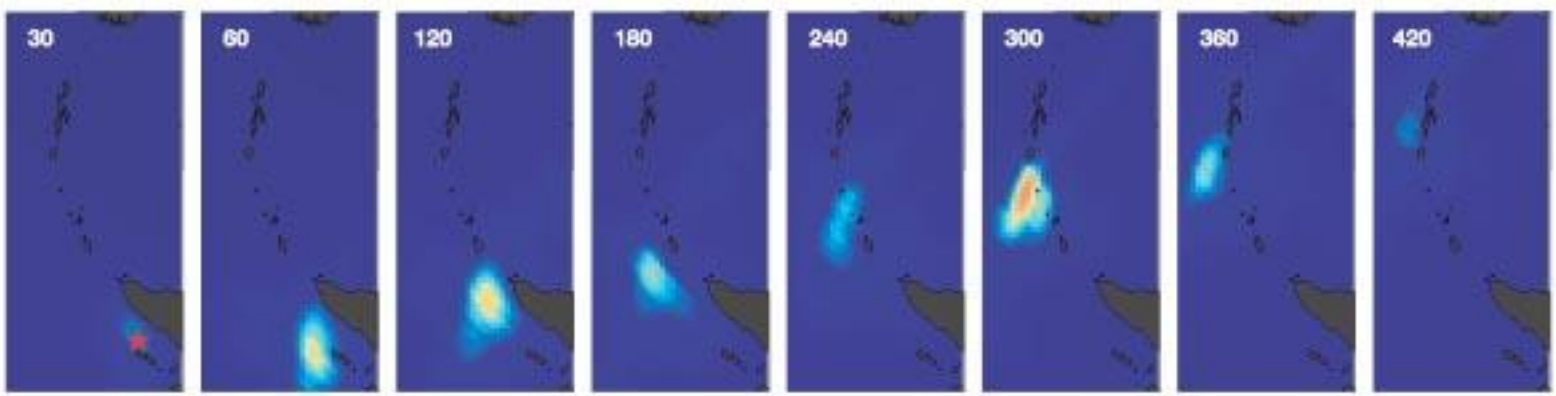
$$\Delta\sigma = \frac{7}{\pi}\frac{M_O}{w^2 L}$$

Haskell Line Source
Dislocation Source

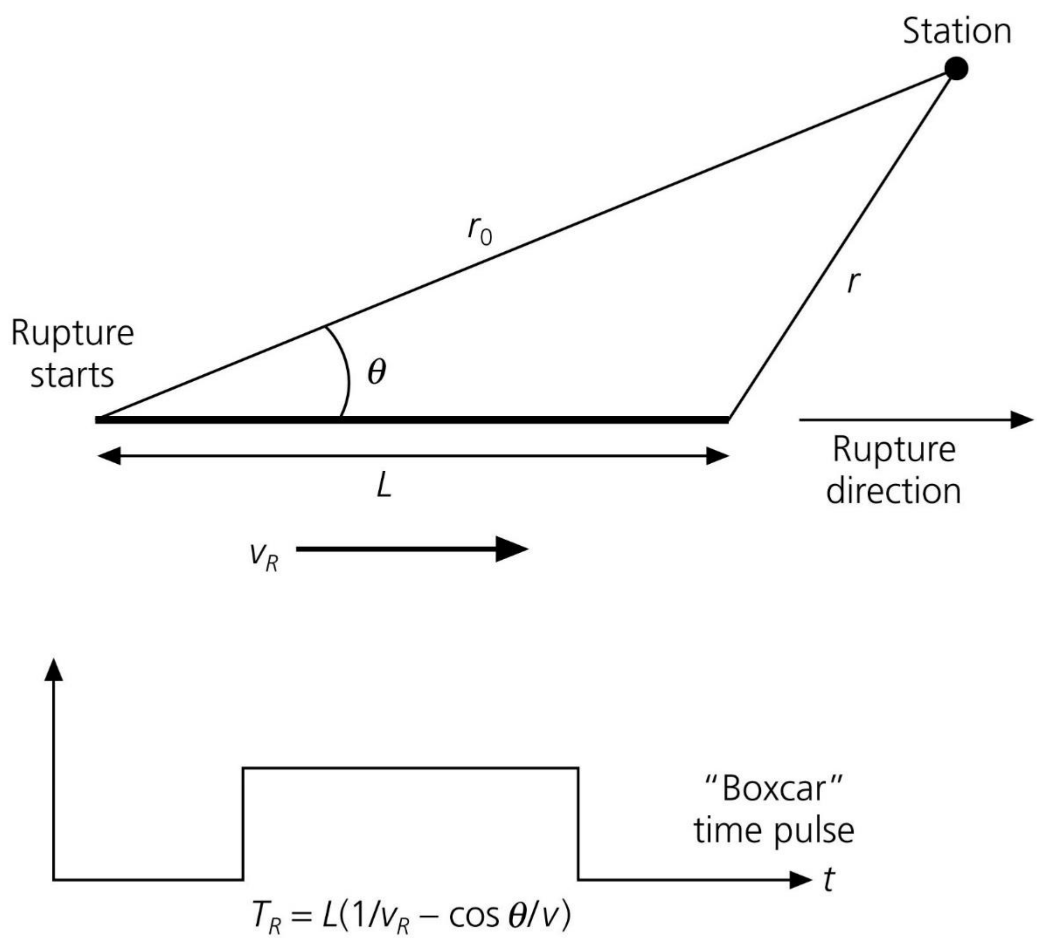


and dip-slip on a rectangular fault gives

$$\Delta\sigma = \frac{7}{3\pi}\frac{M_O}{w^2 L}$$



Sumatra earthquake Ishii et al., 2005



Source time function

V_R = rupture velocity

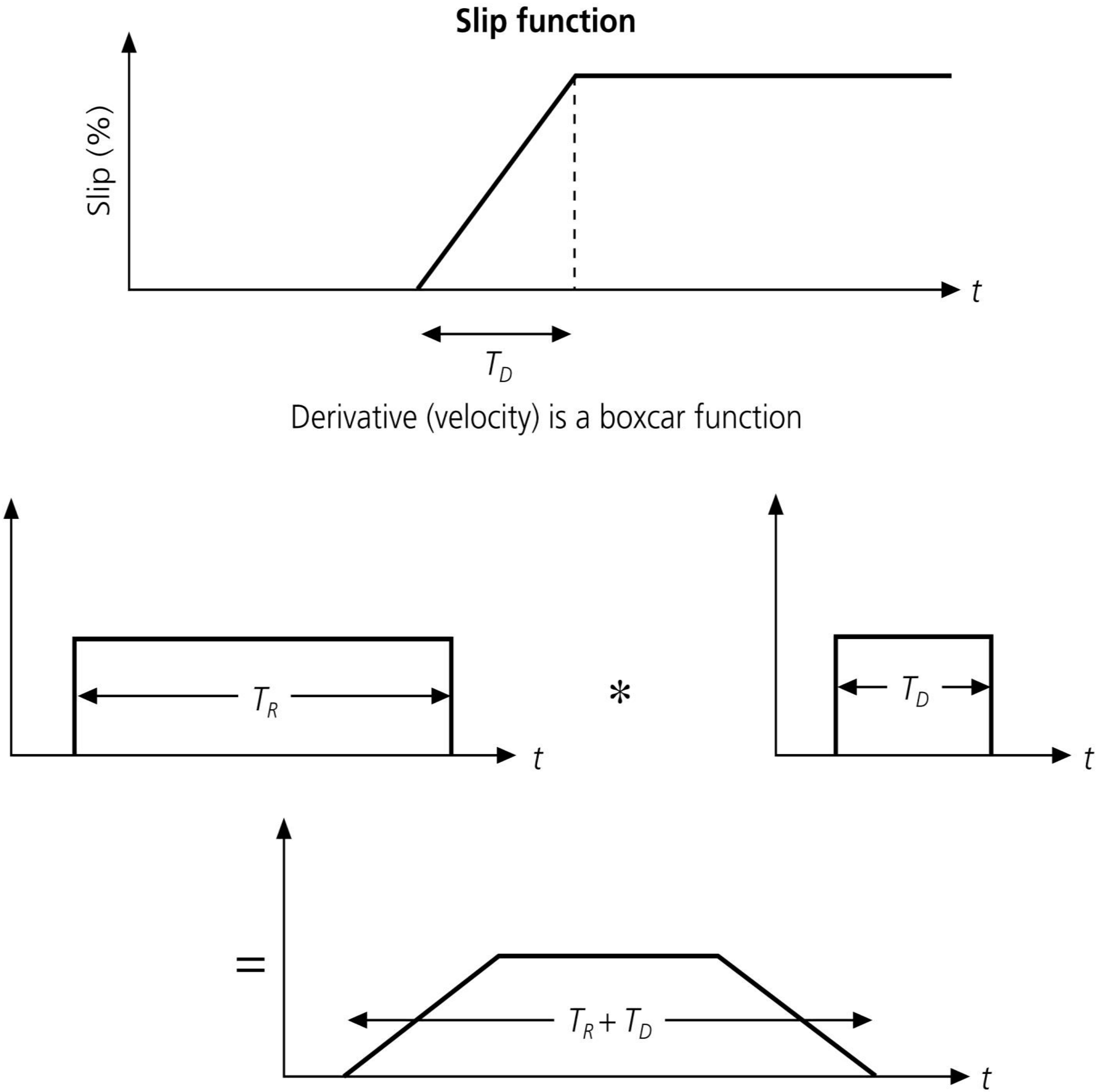
T_R = rupture time

$$r^2 = r_o^2 + L^2 - 2r_o L\cos\theta$$

$$r \approx r_o - L\cos\theta$$

$$T_R = L\ (1/V_R - \cos\theta/V)$$

$$= (L/V)\ (V/V_R - \cos\theta)$$

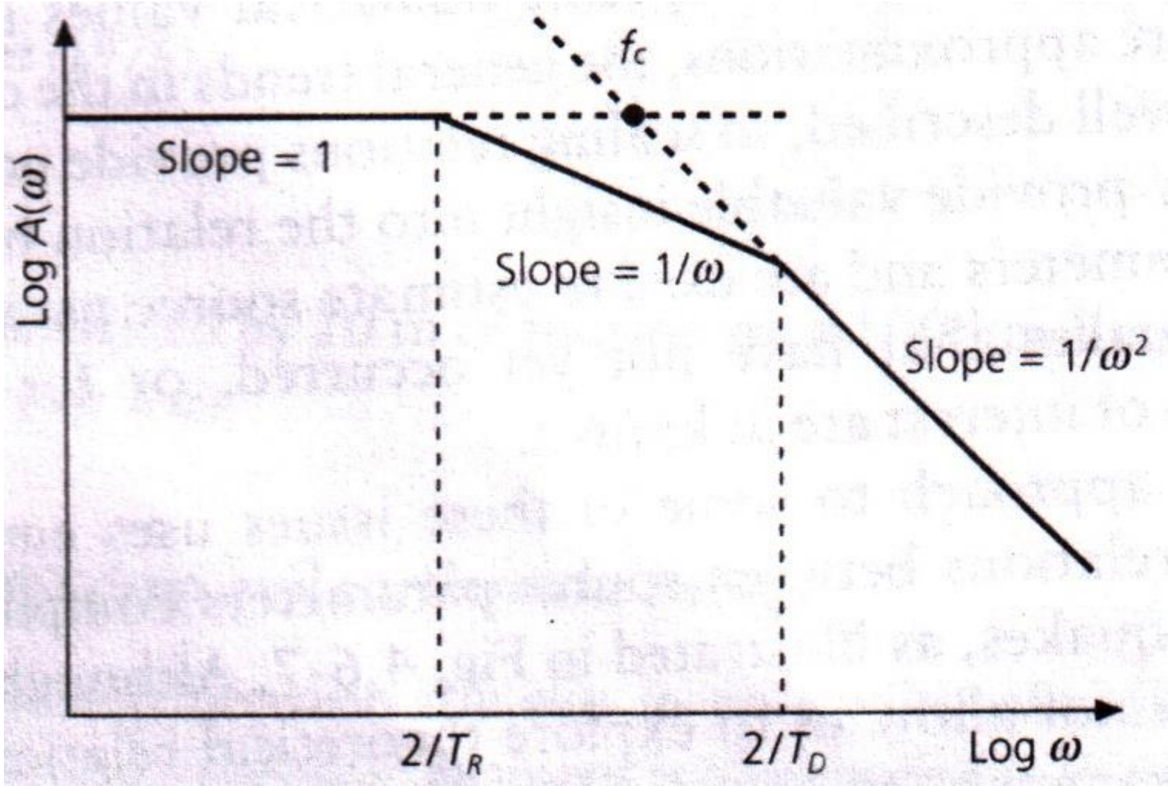


Slip function
Slip (%)
t
T_D
Derivative (velocity) is a boxcar function
T_R
t
*
T_D
t
=
$T_R + T_D$
t

The spectral amplitude of the source signal can be represented by the following equation:

$$logA(\omega) = logM_O + log\left[sinc\left(\frac{\omega T_R}{2}\right)\right] + log\left[sinc\left(\frac{\omega T_D}{2}\right)\right]$$

where T_R and T_D are the rupture and rise times, respectively. Assuming $T_R > T_D$, we have



Theoretical source spectrum of an earthquake, modeled as three regions with slopes of 1, ω^{-1}, and ω^{-2}, divided by angular frequencies corresponding to the rupture and rise times. Another common approximation uses a single corner frequency, f_c, at the intersection of the first and the third spectrum segments. The flat segment extending to zero frequency gives M_O.

Magnitude and Energy

$log_{10}(E_S) \approx 11.8+1.5M_S \approx 5.8 + 2.4m_b$

E_S is in ergs.

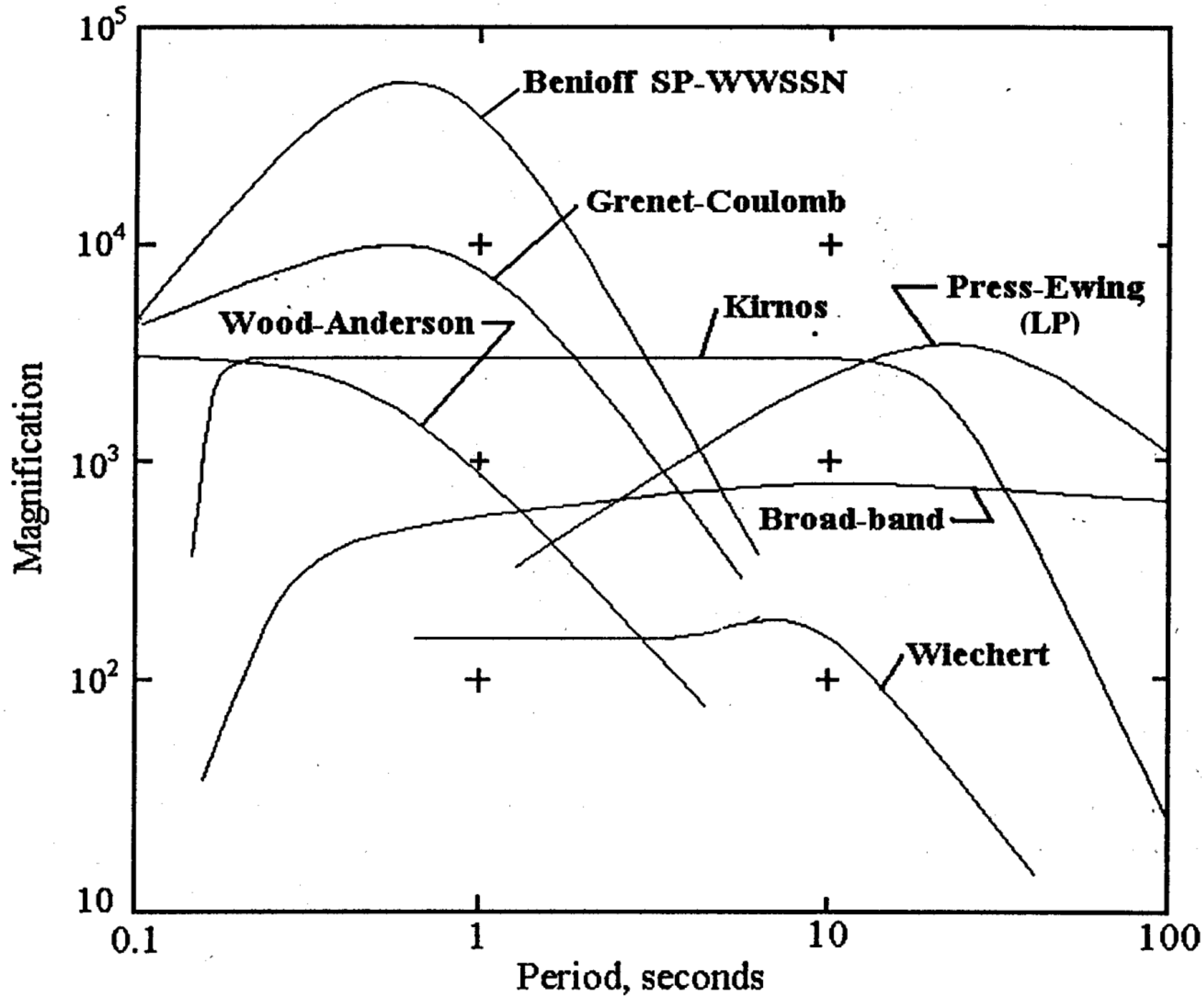


Figure 8. Magnification characteristics of several seismograph systems. (LP : Long Period, SP : Short Period; WWSSN : World Wide Standardised Seismograph Network.)

Intensity	Description of effects
I–IV	*light to moderate earthquakes*
I	**Not felt**
II	**Scarcely felt** Felt only by a few individual people at rest in houses.
III	**Weak** Felt indoors by a few people. People at rest feel a swaying or light trembling.
IV	**Largely observed** Felt indoors by many people; outdoors by very few. A few people are awakened. Windows, doors and dishes rattle.
V–VIII	*moderate to severe earthquakes*
V	**Strong** Felt indoors by most, outdoors by few. Many sleeping people awake. A few are frightened. Buildings tremble throughout. Hanging objects swing considerably. Small objects are shifted. Doors and windows swing open or shut.
VI	**Slightly damaging** Many people are frightened and run outdoors. Some objects fall. Many houses suffer slight non-structural damage like hair-line cracks and fall of small pieces of plaster.
VII	**Damaging** Most people are frightened and run outdoors. Furniture is shifted and objects fall from shelves in large numbers. Many well built ordinary buildings suffer moderate damage: small cracks in walls, fall of plaster, parts of chimneys fall down; older buildings may show large cracks in walls and failure of fill-in walls.
VIII	**Heavily damaging** Many people find it difficult to stand. Many houses have large cracks in walls. A few well built ordinary buildings show serious failure of walls, while weak older structures may collapse.
IX–XII	*severe to destructive earthquakes*
IX	**Destructive** General panic. Many weak constructions collapse. Even well built ordinary buildings show very heavy damage: serious failure of walls and partial structural failure.
X	**Very destructive** Many ordinary well built buildings collapse.
XI	**Devastating** Most ordinary well built buildings collapse, even some with good earthquake resistant design are destroyed.
XII	**Completely devastating** Almost all buildings are destroyed.

Seismic hazard, risk and vulnerability

Seismic hazard is defined as the probabilistic measure of ground shaking associated with the recurrence of earthquakes. This describes the potential for dangerous, earthquake-related natural phenomena such as ground shaking, fault rupture, or soil liquefaction. These phenomena could result in adverse consequences to the society such as the destruction of buildings or the loss of life.

Seismic risk is the probability of occurrence of these consequences. The assessment of seismic hazard is the first step in the evaluation of the seismic risk, obtained by convolving the seismic hazard with local site amplification tied to soil conditions and with the intrinsic value and vulnerability of the existing buildings and infrastructures. Frequent large events in remote areas result in high seismic hazard but pose no risk; on the contrary, moderate events in densely populated areas entail small hazard but high risk.

FROM SEISMIC HAZARD TO SEISMIC RISK