Fundamental relation between potential, Apparent resistivity, Resistivity transform and Layer distribution of a stratified earth (multiplayer)

3.5 Apparent Resistivity on a Horizontally Layered Earth

The equation of potential at the surface of the horizontally stratified earth is derived below with the following assumptions:

1. The subsurface is n–layered earth (Fig. 3.14).
2. The resistivity and thickness of each layer is ρ_i and h_j, where i = 1, 2, 3, …n and j = 1, 2, 3, … n –1.
3. The last (n^{th}) layer is infinitely thick.
4. Each layer is electrically homogeneous and isotropic.
5. The electric field is generated by a direct current (DC) point source placed on the surface of the earth and follows Ohm's law,

$$E = \rho J \tag{3.61}$$

The electrical potential (V) for a DC current source satisfies Laplace's equation and its differential form is given by,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

The above equation in the cylindrical coordinate system (r, θ, z) can be re-written as

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0$$

where z is the coordinate in the vertical direction.

Due to radial symmetry, the first and second derivatives of the potential with respect to θ will be zero. Thus, Eq. (3.63) reduces to,

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0 \qquad (3.64)$$

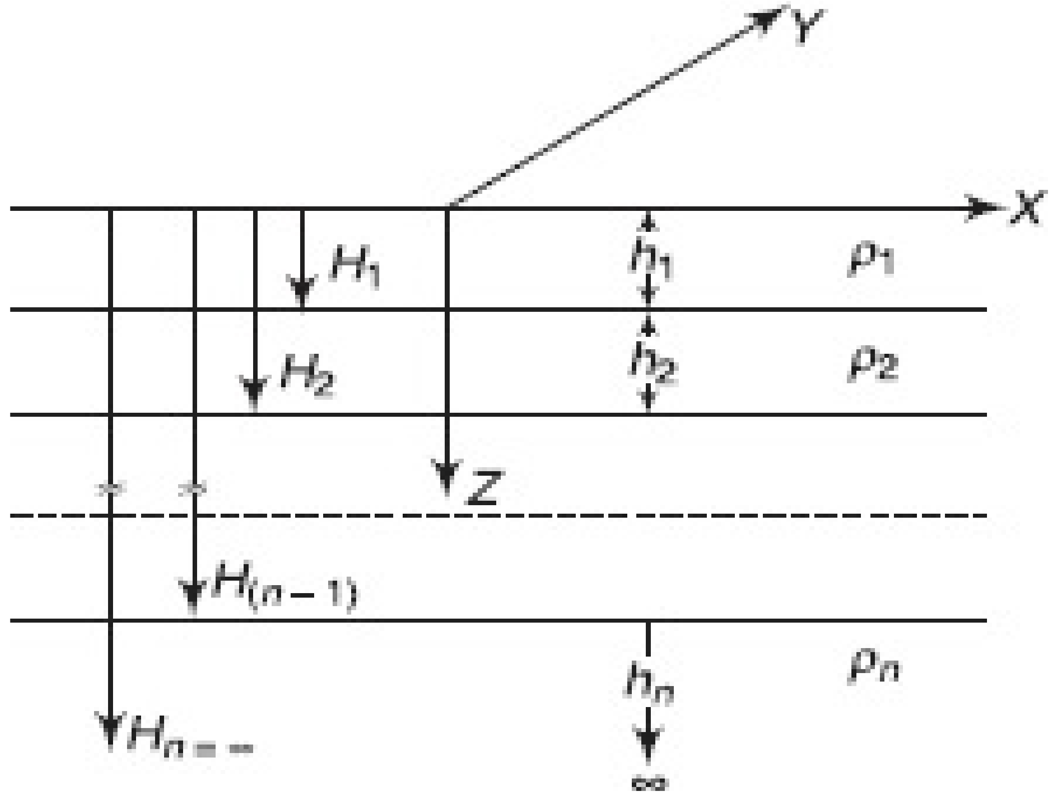

Fig. 3.14 Layer parameters of horizontal n-layered earth

The above partial differential has been treated in two different ways and we shall examine both the cases here. The general solution of Eq. (3.64) may be expressed as

$$V = \int_0^\infty \left[A(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}z} + B(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}z}\right] J_0(\boldsymbol{\lambda}r)\, d\boldsymbol{\lambda} \tag{3.65}$$

Potential (V_0) due to a point source placed on the surface of the earth is given by

$$V_0 = \frac{\rho I}{2\pi}\frac{1}{R} = \frac{\rho I}{2\pi}\frac{1}{(r^2 + z^2)^{1/2}}$$

The potential due to a point source on the surface of the layered earth is the sum of the potential in the homogeneous medium and the perturbation potential (V) due to the layered interfaces shown in Eq. (3.65). Thus, we get total potentials V_1, V_2, … V_n in various layers, i.e.,

$$V_1 = V_0 + V_1'$$
$$V_2 = V_0 + V_2'$$

$$V_i = V_0 + V_i'$$

$$V_n = V_0 + V_n'$$

where $V_1', V_2', \ldots, V_n'$ are the perturbation potentials contributed by the two series of infinite number of images on both sides of the layered boundaries (see Fig. 2.23 of Chapter 2). Thus, as a general case, the potential in the i[th] layer (Vi) may be expressed as

$$V_i = \frac{I\rho_1}{2\pi}\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \left[A_i(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda} z} + B_i(\boldsymbol{\lambda})e^{\boldsymbol{\lambda} z}\right] J_0(\boldsymbol{\lambda} r)\,d\boldsymbol{\lambda} \quad (3.66)$$

The constants Ai and Bi are obtained from the corresponding boundary conditions.

At the air –earth interface, $\frac{1}{\rho_1}\left[\frac{\partial V}{\partial z}\right]_{z=0}$ =0.

From Eq. (3.66), one can write the expression for potential in the first layer (V_1) as

$$V_1 = \frac{I\rho_1}{2\pi}\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \left[A_1(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda} z} + B_1(\boldsymbol{\lambda})e^{\boldsymbol{\lambda} z}\right] J_0(\boldsymbol{\lambda} r)\,d\boldsymbol{\lambda} \quad (3.67)$$

Therefore, its derivative with respect to z at the air–earth interface (z = 0) is expressed as

$$\left[\frac{\partial V}{\partial z}\right]_{z=0} = \left[\frac{Iz\rho_1}{2\pi}\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \left[-A_1(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda} z} + B_1(\boldsymbol{\lambda})e^{\boldsymbol{\lambda} z}\right] J_0(\boldsymbol{\lambda} r)\,d\boldsymbol{\lambda}\right]_{z=0}$$

$$= \left[\int_0^\infty \left[-A_1(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}z} + B_1(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}z}\right] J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}\right]_{z=0} \tag{3.68}$$

This equation has to be valid for any value of r, i.e.,

$-A_1(\boldsymbol{\lambda}) + B_1(\boldsymbol{\lambda}) = 0 \; or \; A_1(\boldsymbol{\lambda}) = B_1(\boldsymbol{\lambda})$

The potential (V1) in the first layer then reduces to,

$$V_1 = \frac{I\rho_1}{2\pi}\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \left[A_1(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda}z} + e^{\boldsymbol{\lambda}z})\right] J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} \tag{3.69}$$

In the last layer at z = ∞, the potential reduces to zero, i.e., $B_n = 0$, and the potential (Vn) in the n^{th} layer takes the form,

$$V_n = \frac{I\rho_1}{2\pi}\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \left[A_n(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}z}\right] J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} \tag{3.70}$$

The boundary conditions at any interface must satisfy

$$V_i = V_{i+1}$$

$$\frac{1}{\rho_i}\frac{\partial V_i}{\partial z} = \frac{1}{\rho_{i+1}}\frac{\partial V_{i+1}}{\partial z}_{at\ z=H_i} \tag{3.71}$$

Thus, one gets 2n equations to determine 2n unknowns. Therefore, the problem has a unique solution.

We know that $\frac{1}{R} = \frac{1}{(r^2+z^2)^{1/2}}$ and by using Weber's integral, one gets $\frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty e^{-\boldsymbol{\lambda}|z|} J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda}$, and, therefore, the potentials in various layers may be expressed as

$$V_1 = \frac{I\rho_1}{2\pi} \int_0^\infty e^{-\boldsymbol{\lambda}|z|} J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda} + \int_0^\infty \left[A_1(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda} z} + e^{\boldsymbol{\lambda} z})\right] J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda}$$

$$V_i = \frac{I\rho_1}{2\pi} \int_0^\infty e^{-\boldsymbol{\lambda}|z|} J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda} + \int_0^\infty \left[A_i(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda} z} + e^{\boldsymbol{\lambda} z})\right] J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda} \quad (3.72)$$

$$V_n = \frac{I\rho_1}{2\pi} \int_0^\infty e^{-\boldsymbol{\lambda}|z|} J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda} + \int_0^\infty \left[A_n(\boldsymbol{\lambda}) e^{-\boldsymbol{\lambda} z}\right] J_0(\boldsymbol{\lambda} r) d\boldsymbol{\lambda}$$

The following systems of equations are obtained from equation (3.71) and (3.72):

$$\int_0^\infty \left[A_1(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda}H_1}+e^{\boldsymbol{\lambda}H_1})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} = \int_0^\infty \left[A_2(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda}H_2}+B_2(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_1})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$-\frac{1}{\rho_1}\times\frac{I\rho_1}{2\pi}\int_0^\infty e^{-\boldsymbol{\lambda}H_1}J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} + \frac{1}{\rho_1}\left[A_1(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda}H_1}-e^{\boldsymbol{\lambda}H_1})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$= -\frac{1}{\rho_2}\times\frac{I\rho_1}{2\pi}\int_0^\infty e^{-\boldsymbol{\lambda}H_1}J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} + \frac{1}{\rho_2}\left[-A_2(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda}H_1}+B_2(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_1})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$\int_0^\infty \left[A_i(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_i} + B_i(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} = \int_0^\infty \left[A_{i+1}(\boldsymbol{\lambda})(e^{-\boldsymbol{\lambda}H_i}+B_{i+1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$-\frac{1}{\rho_1}\times\frac{I\rho_1}{2\pi}\int_0^\infty e^{-\boldsymbol{\lambda}H_i}J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} + \frac{1}{\rho_i}\left[-A_i(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_1} - B_i(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$= -\frac{1}{\rho_{i+1}}\times\frac{I\rho_1}{2\pi}\int_0^\infty e^{-\boldsymbol{\lambda}H_1}J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} + \frac{1}{\rho_{i+1}}\left[-A_{i+1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_1} + B_{i+1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

(3.73)

$$\int_0^\infty \left[A_{n-1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}} + B_{n-1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_{n-1}})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda} = \int_0^\infty \left[A_n(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$-\frac{1}{\rho_{n-1}} \times \frac{I\rho_1}{2\pi}\int_0^\infty \left[-A_{n-1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}} + B_{n-1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_{n-1}})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

$$= -\frac{1}{\rho_n} \times \frac{I\rho_1}{2\pi}\int_0^\infty \left[e^{-\boldsymbol{\lambda}H_{n-1}} + A_n(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}})\right]J_0(\boldsymbol{\lambda}r)d\boldsymbol{\lambda}$$

The relations expressed in the set of equations (3.73) are valid for all values of r, and, therefore

$$A_1(\boldsymbol{\lambda})(e^{\boldsymbol{\lambda}H_1}+e^{-\boldsymbol{\lambda}H_1}) - A_2(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_1} + B_2(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_1} = 0$$

$$A_1(\boldsymbol{\lambda})\rho_2(e^{\boldsymbol{\lambda}H_1} - e^{-\boldsymbol{\lambda}H_1}) + [A_2(\boldsymbol{\lambda})\rho_1 e^{-\boldsymbol{\lambda}H_1} - B_2\rho_1(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_1}] - \frac{I\rho_1}{2\pi}(\rho_2 - \rho_1)e^{-\boldsymbol{\lambda}H_1} = 0$$

$$A_i(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_i} + B_i(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i} - A_{i+1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_i} - B_{i+1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i} = 0$$

$$\rho_{i+1}[-A_i(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_1} + B_i(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i}] + \rho_i A_{i+1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_i} - \rho_i B_{i+1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_i} - \frac{I\rho_1}{2\pi}(\rho_2 - \rho_1)e^{-\boldsymbol{\lambda}H_i} = 0$$

$$A_{n-1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}} + B_{n-1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_{n-1}} - A_n(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}} = 0$$

$$-A_{n-1}(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}} + B_{n-1}(\boldsymbol{\lambda})e^{\boldsymbol{\lambda}H_{n-1}} + A_n(\boldsymbol{\lambda})e^{-\boldsymbol{\lambda}H_{n-1}} - \frac{I\rho_1}{2\pi}(\rho_n - \rho_{n-1})e^{-\boldsymbol{\lambda}H_{n-1}} = 0 \quad (3.74)$$

Thus, it is possible to find the potential and electric field in any medium. However, in practice, it is generally required to find the potential on the surface for VES and accordingly, the coefficients are to be determined depending on the different number of layer earth problems to be solved. Some special cases are considered below.

3.5.1 Homogeneous Earth

$$A_1(\lambda) = 0 \qquad (3.75)$$

And therefore,

$$V = V_0 = \frac{I\rho_1}{2\pi} \times \frac{1}{(r^2+z^2)^{1/2}} = \frac{I\rho_1}{2\pi} \times \frac{1}{R} \qquad (3.76)$$

3.5.2 Two-layer Earth

Substituting $h_2 = \infty$, one gets the following equations from the set expressed in equation (3.74):

$$A_1(\lambda)(e^{-\boldsymbol{\lambda} h_1}+e^{\boldsymbol{\lambda} h_1}) - A_2(\lambda)(e^{-\boldsymbol{\lambda} h_1}) = 0$$

$$A_1(\lambda)\rho_2(e^{-\boldsymbol{\lambda} h_1}+e^{\boldsymbol{\lambda} h_1}) + A_2(\lambda)\rho_1(e^{-\boldsymbol{\lambda} h_1}) - \frac{I\rho_1}{2\pi}(\rho_2 - \rho_1)e^{-\boldsymbol{\lambda} h_1} = 0 \qquad (3.77)$$

Solving the above two equation (3.77) and substituting the value of reflection coefficient (k_{12}) of the interface separating the top layer with resistivity ρ_1 with second layer of resistivity ρ_2 by $k_{12}=(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$, one gets

$$A_1(\lambda) = \frac{I\rho_1}{2\pi} \times \frac{k_{12}e^{-2\boldsymbol{\lambda} h_1}}{1-k_{12}e^{-2\boldsymbol{\lambda} h_1}} = \frac{I\rho_1}{2\pi} \times \left[k_{12}e^{-2\boldsymbol{\lambda} h_1} + k_{12}^2 e^{-4\boldsymbol{\lambda} h_1} + \cdots + k_{12}^n e^{-2\boldsymbol{\lambda} n h_1} + \right.$$

Therefore, the potential (V_1) in the first layer media is expressed as

$$V_1 = \frac{I\rho_1}{2\pi} \times \frac{1}{(r^2+z^2)^{1/2}} + \frac{I\rho_1}{2\pi} \sum_{n=1}^{\infty} \frac{k_{12}^n}{(r^2+(2nh_1-z)^2)^{1/2}} + \frac{I\rho_1}{2\pi} \sum_{n=1}^{\infty} \frac{k_{12}^n}{(r^2+(2nh_1+z)^2)^{1/2}} \qquad (2.79)$$

Equation (3.79) gives the potential at any point (r, z) in the first medium. Therefore, the potential at the surface will be obtained by substituting z = 0 in the above equation, i.e.,

$$V_1 = \frac{I\rho_1}{2\pi} \left[\frac{1}{r} + 2\sum_{n=1}^{\infty} \frac{k_{12}^n}{(r^2+(2nh_1)^2)^{1/2}}\right] \qquad (3.80)$$

The electric field (E) on the surface is obtained $E = \frac{\partial V}{\partial r}$

Therefore, $$E = \frac{I\rho_1}{2\pi} \left[\frac{1}{r^2} + 2\sum_{n=1}^{\infty} \frac{rk_{12}^n}{(r^2+(2nh_1)^2)^{3/2}}\right] \qquad (3.81)$$

The equations (3.80) and (3.81) can be used to obtain the apparent resistivity for any of the electrode arrays.

For Schlumberger array, i.e., MN → 0, the apparent resistivity equation is expressed as,

$$\rho_{aS} = 2\pi r^2 \frac{E}{I} = \rho_1 \left[1 + 2\sum_{n=1}^{\infty} \frac{k_{12}^n \left(\frac{AB}{2h_1}\right)^3}{\left(\left(\frac{AB}{2h_1}\right)^2 + 4n^2\right)^{3/2}}\right] \qquad (3.82)$$

For Wenner array, the potential difference (ΔV) on the surface of the earth is expressed as,

$$\Delta V = \frac{\rho_1 I}{2\pi}\left[\frac{1}{a} + 4\sum_{n=1}^{\infty}\frac{k_{12}^{n}\left(\frac{a}{h_1}\right)}{(a^2 + (2nh_1)^2)^{1/2}} - 4\sum_{n=1}^{\infty}\frac{k_{12}^{n}\left(\frac{a}{h_1}\right)}{((2a)^2 + (2nh_1)^2)^{1/2}}\right]$$

$$= \frac{\rho_1 I}{2\pi a}\left[1 + 4\sum_{n=1}^{\infty}\frac{k_{12}^{n}\left(\frac{a}{h_1}\right)}{[(a/h_1)^2+(2n)^2]^{1/2}} - 4\sum_{n=1}^{\infty}\frac{k_{12}^{n}\left(\frac{a}{h_1}\right)}{[(2a/h_1)^2+(2n)^2]^{1/2}}\right] \quad (3.83)$$

For Wenner array, the apparent resistivity (ρ_{aW}) equation is expressed as,

$$\rho_{aW} = 2\pi a\frac{\Delta V}{I}$$

$$= \rho_1\left[1 + 4\sum_{n=1}^{\infty}\frac{k_{12}^{n}\left(\frac{a}{h_1}\right)}{[(a/h_1)^2+(2n)^2]^{1/2}} - 4\sum_{n=1}^{\infty}\frac{k_{12}^{n}\left(\frac{a}{h_1}\right)}{[(2a/h_1)^2+(2n)^2]^{1/2}}\right] \quad (3.84)$$

3.5.3 Three-layer Earth

Substituting $h3 = \infty$ (Fig. 3.14), the set of equation (3.74) reduce to

$$A_1(\lambda)(e^{-\boldsymbol{\lambda} H_1}+e^{\boldsymbol{\lambda} H_1}) - A_2(\lambda)(e^{-\boldsymbol{\lambda} h_1}) - B_2(\lambda)e^{\boldsymbol{\lambda} H_1} = 0$$

$$A_1(\lambda)\rho_2(e^{-\boldsymbol{\lambda} H_1}+e^{\boldsymbol{\lambda} H_1}) + A_2(\lambda)\rho_1 e^{-\boldsymbol{\lambda} H_1} - \frac{I\rho_1}{2\pi}(\rho_2 - \rho_1)e^{-\boldsymbol{\lambda} H_1} = 0$$

$$A_2(\lambda)e^{-\boldsymbol{\lambda} H_2} - B_2(\lambda)e^{-\boldsymbol{\lambda} h_2} - A_3(\lambda)e^{\boldsymbol{\lambda} H_2} = 0$$

$$-A_2(\lambda)\rho_3 e^{-\boldsymbol{\lambda} H_2} + B_2(\lambda)\rho_3 e^{-\boldsymbol{\lambda} H_2} + A_3(\lambda)\rho_2 e^{-\boldsymbol{\lambda} H_2} - \frac{I\rho_1}{2\pi}(\rho_3 - \rho_2)e^{-\boldsymbol{\lambda} H_2} = 0 \quad (3.85)$$

Solving the set of equations in (3.85), one gets

$$A_1(\lambda) = \frac{I\rho_1}{2\pi} \times \frac{k_{12}e^{-2\boldsymbol{\lambda} H_1}+k_{23}e^{-2\boldsymbol{\lambda} H_2}}{1-k_{12}e^{-2\boldsymbol{\lambda} H_1}-k_{23}e^{-2\boldsymbol{\lambda} H_2}+k_{12}k_{12}e^{-2\boldsymbol{\lambda}(H_2-H_1)}} \quad (3.86)$$

Where $k_{12}=(\rho_2 - \rho_1)/(\rho_2 + \rho_1)$, and $k_{23}=(\rho_3 - \rho_2)/(\rho_3 + \rho_2)$,

$$V_1 = \frac{I\rho_1}{2\pi} \times \frac{1}{(r^2+z^2)^{1/2}} + \int_0^\infty \frac{k_{12}e^{-2\boldsymbol{\lambda} H_1}+k_{23}e^{-2\boldsymbol{\lambda} H_2}}{1-k_{12}e^{-2\boldsymbol{\lambda} H_1}-k_{23}e^{-2\boldsymbol{\lambda} H_2}+k_{12}k_{12}e^{-2\boldsymbol{\lambda}(H_2-H_1)}} \times (e^{-\boldsymbol{\lambda} Z} + e^{\boldsymbol{\lambda} Z}) J_0(\boldsymbol{\lambda} r)d\boldsymbol{\lambda} \quad (3.87)$$

Let us express H_1 and H_2 in terms of some fixed value H_0, i.e., $H_1 = p_1H_0$ and $H_2 = p_2H_0$, where p_1 and p_2 are whole numbers.

Substituting u = $e^{-2\lambda H0}$ in Eq. (3.87), one gets

$$A_1(\lambda) = \frac{\rho_1 I}{2\pi}\frac{k_{12}u^{p_1}+k_{23}u^{p_2}}{1-k_{12}u^{p_1}-k_{23}u^{p_2}+k_{12}k_{23}u^{(p_2-p_1)}} \qquad (3.88)$$

a rational function of u as p1 and p2 are whole numbers.

$$A_1(\lambda) = \frac{\rho_1 I}{2\pi}(b_1 u + b_2 u^2 + b_3 u^3 + \cdots) \qquad 3.88)$$

$$= \frac{\rho_1 I}{2\pi}\sum_{n=1}^{\infty} b_n u^n = \frac{\rho_1 I}{2\pi}\sum_{n=1}^{\infty} b_n e^{-2\boldsymbol{\lambda} n H_0} \qquad (3.88)$$

Comparing Eqs (3.88) and (3.89), one gets

$$k_{12}u^{p_1} + k_{23}u^{p_2} = \left[1 - k_{12}u^{p_1} - k_{23}u^{p_2} + k_{12}k_{23}u^{(p_2-p_1)}\right]\sum_{n=1}^{\infty} b_n u^n \qquad (3.90)$$

The requirement of Eq. (3.90) is that the coefficient of any order of u must be equal on both sides. Thus, the coefficient u greater than p_2 on the right-hand side must be zero as the highest order on the left hand side is up2. The coefficients of $u^{p_2+\lambda}$, where λ is a positive number, may be expressed as,

$$b_{p_2+\lambda}\text{-}k_{12}b_{(p_2-p_2)+\lambda} - k_{23}b_{\lambda} + k_{12}k_{23}b_{p_1+\lambda} = 0$$

which provides the following recurrence formula,

$$b_{p_2+\lambda}=k_{12}b_{(p_2-p_1)+\lambda} - k_{23}b_{\lambda} + k_{12}k_{23}b_{p_1+\lambda} \tag{3.91}$$

The $b_{p_2+\lambda}$on the left-hand side may be calculated once the values of $b_{(p_2-p_2)+\lambda}$, b_λ, and $b_{p_1+\lambda}$ are known. The coefficients up to the maximum value of b_{p_2} may be determined from Eq. (3.90). The rest of the coefficients are obtained from the recurrence formula shown in Eq. (3.91).

The potential at any point within the first layer may be expressed as,

$$V_1 = \frac{\rho_1 I}{2\pi}\left[\frac{1}{(r^2+z^2)^{1/2}} + \sum_{n=1}^{\infty}\frac{b_n}{[r^2+(2nH_0+z)^2]^{1/2}} + \sum_{n=1}^{\infty}\frac{b_n}{[r^2+(2nH_0-z)^2]^{1/2}}\right] \tag{3.92}$$

On the surface of the ground, i.e., at z = 0, the potential (V) reduces to

$$V_1 = \frac{\rho_1 I}{2\pi}\left[\frac{1}{r} + 2\sum_{n=1}^{\infty}\frac{b_n}{[r^2+(2nH_0)^2]^{1/2}}\right] \tag{3.93}$$

The Electric field (E) can be expressed as

$$E = \frac{\rho_1 I}{2\pi}\left[\frac{1}{r^2} + 2\sum_{n=1}^{\infty}\frac{b_n r}{[r^2+(2nH_0)^2]^{1/2}}\right] \tag{3.94}$$

The apparent resistivity for a Schlumberger array (ρaS) can be expressed as

$$\rho_{aS} = \rho_1\left[1 + 2\sum_{n=1}^{\infty}\frac{b_n r^3}{[r^2+(2nH_0)^2]^{3/2}}\right] \tag{3.95}$$

3.5.4 Four-layer Earth

In an analogous way, it can be shown that $A_1(\lambda)$ for a four-layer earth can be expressed as

$$\rho_{as} = \rho_1 \left[1 + 2\sum_{n=1}^{\infty} \frac{b_n \delta^3}{[\delta^2 + 4n^2]^{3/2}}\right] \qquad (3.97)$$

The potential in this case also can be expressed in the same way as that in Eq. (3.93) and the apparent resistivity expressed like Eq. (3.97). The computations become more complex with the increase in the number of layers.

The differential Eq. (3.64) shown earlier for n–layer earth may also be solved by using separation of variables. Let us assume,

$$V(r,z) = U(r)W(z) \qquad (3.99)$$

Substituting, Eq. (3.99) in (3.64) and dividing throughout by UW, one gets

$$\frac{1}{U}\frac{d^2U}{dr^2} + \frac{1}{Ur}\frac{dU}{dr} + \frac{1}{W}\frac{d^2W}{dz^2} = 0 \qquad (3.100)$$

Equation (3.100) is satisfied only if

$$\frac{1}{U}\frac{d^2U}{dr^2} + \frac{1}{Ur}\frac{dU}{dr} = -A^2 \qquad (3.101)$$

and

$$\frac{1}{W}\frac{d^2W}{dz^2} = A^2 \tag{3.102}$$

where A is a real constant. Solutions of Eqs (3.101) and (3.102) are

$$U = C_1 J_0(\lambda r) \tag{3.103}$$

$$W = C_1 \exp(-\lambda r) \text{ and } W = C_1 \exp(+\lambda r) \tag{3.104}$$

J_0 is the Bessel's function of zero order.

Thus, the solution of Eq. (3.99) can be obtained by combining Eqs (3.103) and (3.104).

$$V = C\exp(-\lambda r)\, J_0(\lambda r) \text{ and } \text{ and } V = C\exp(+\lambda r)\, J_0(\lambda r) \tag{3.105}$$

Any linear combination of solutions is also a solution of the differential equation. Therefore, varying λ from 0 to ∞ and making C dependent on λ, the general solution of Eq. (3.64) can be obtained as

$$V = \int_0^\infty [\emptyset(\lambda)\exp(-\lambda z) + \varphi(\lambda)\exp(+\lambda z)]\, J_0(\lambda r) d\lambda \tag{3.106}$$

Both $\Phi(\lambda)$ and $\psi(\lambda)$ in the above equation are arbitrary functions of λ.

Potential generated by a single point source of current I in a homogeneous isotropic half-space of resistivity ρ_1 is given as

$$V = \frac{I\rho_1}{2\pi\sqrt{(r^2+z^2)}} \tag{3.107}$$

The differential form of Eq. (3.107) can be rewritten in terms of the Lipschitz integral as

$$V = \frac{I\rho_1}{2\pi}\int_0^{\infty} \exp(-\lambda z) J_0(\lambda r) d\lambda \tag{3.108}$$

where the Lipschitz integral is

$$\frac{1}{\sqrt{(r^2+z^2)}} = \int_0^{\infty} \exp(-\lambda z) J_0(\lambda r) d\lambda \tag{3.109}$$

Thus, the general solution of Eq. (3.105) can be written as

$$V = \frac{I\rho_1}{2\pi}\int_0^{\infty}[exp(-\lambda z) + M(\lambda)exp(-\lambda z) + N(\lambda)exp(+\lambda z)]\, J_0(\lambda r) d\lambda \tag{3.110}$$

The potential in the i^th^ layer can be expressed as

$$V_i = \frac{I\rho_1}{2\pi}\int_0^{\infty}[exp(-\lambda z_i) + M_i(\lambda)exp(-\lambda z_i) + N_i(\lambda)exp(+\lambda z_i)]\, J_0(\lambda r) d\lambda \tag{3.111}$$

Adaptation of the Solution to the Boundary Conditions

Equation (3.110) is valid within the layer but at the boundary, it becomes discontinuous. The boundary conditions given below are applicable to determine the equation of potential at the surface of the n-layered earth.

a) At each of the boundary planes in the subsurface, the electrical potential must be continuous, i.e., Vi = Vi–1. Thus, the component of the potential along the interface (i.e., the tangential component of the electric field) is also continuous. Also the vertical component of the current density must be continuous, i.e., $J_{z_i} = J_{z_{i-1}}$.
b) At the ground surface (i.e., air–earth interface), the vertical component of the current density and, hence, the vertical component of the electrical field intensity must be zero everywhere except very close to the source.
c) Following Eq. (3.106), near the current source the potential must approach infinity.
d) Also, in accordance to Eq. (3.106), far away from the current source, the potential must approximate to zero.

Following the boundary condition (a) at the depth of the i^{th} plane z_i, the expressions for the potential Eq. (3.111) must be same. Thus,

$\int_0^\infty [exp(-\lambda z_i) + M_i(\lambda)exp(-\lambda z_i) + N_i(\lambda)exp(+\lambda z_i)]\, J_0(\lambda r)d\lambda = \int_0^\infty [exp(-\lambda z_i) +$

The above equation can only be satisfied for all values of r if the integrands on both sides of the equation are equal; thus,

$$M_i(\lambda)exp(-\lambda z_i) + N_i(\lambda)exp(+\lambda z_i) = M_{i+1}(\lambda)exp(-\lambda z_i) + N_{i+1}(\lambda)exp(+\lambda z_i) \quad (3.113)$$

Applying the boundary condition (a)

$$\frac{I}{\rho_{\mathrm{i}}}\int_0^{\infty}[\{1 + M_i(\lambda)\}exp(-\lambda z_i) - N_i(\lambda)exp(+\lambda z_i)]\,J_0(\lambda \mathrm{r})\mathrm{d}\lambda = \frac{I}{\rho_{\mathrm{i+1}}}\int_0^{\infty}[\{1 +$$

Again, this equation can only be satisfied for all values of r if the integrands on both sides of the equation be equal. One can obtain,

$$\frac{I}{\rho_{\mathrm{i}}}\int_{0}^{\infty}[\{1+M_i(\lambda)\}exp(-\lambda z_i)-N_i(\lambda)exp(+\lambda z_i)]$$

$$=\frac{I}{\rho_{\mathrm{i+1}}}\int_0^\infty[\{1+M_{i+1}(\lambda)\}exp(-\lambda z_i)-N_{i+1}(\lambda)exp(+\lambda z_i)] \qquad (3.115)$$

Boundary condition (c) can be satisfied only if the first-layer potential is differentiated w.r.t. z and further substitute z = 0. We thus obtain the equation

$$\left.\frac{\partial V_1}{\partial z}\right]_{z=0}=\int_0^\infty[\{-1-M_1(\lambda)\}+N_1(\lambda)]\,J_0(\lambda \mathrm{r})\mathrm{d}\lambda=0 \qquad (3.116)$$

The first term of Eq. (3.116), primary field, defines the field that would exist in a homogeneous isotropic earth, whereas the second and third terms of the above equation define the perturbing field.

As the vertical component of the electric field intensity of the perturbing field must be zero at the surface for all values of r.

$$M_1(\lambda) = N_1(\lambda) \tag{3.117}$$

Boundary condition (d) is automatically satisfied following equation of the potential. Further, following Eq. (3.106), the function $N_n(\lambda) = 0$, else the factor $\exp(\lambda z)$ would drive the potential to an infinite depth, leading to

$$M_n(\lambda)$$

3.6 Kernel Function and its Relationship with Layer Parameters

The potential V(x) over a layered earth, as the solution of Eq. (3.64) along with the use of relevant boundary conditions, can be expressed as

$$V = \frac{I\rho_1}{2\pi}\frac{1}{x}\left(1+2x\int_0^{\infty} K(\lambda, k, h)\, J_0(\lambda r)\mathrm{d}\lambda\right) \tag{3.118}$$

where ρ_1 is the resistivity of the first layer, I is the current injected, Jo is the Bessel's function of zero order, the kernel $K(\lambda, k, h)$ ranging from –1/2 to ∞ is a function of λ the integration variable having dimension of inverse of length and ranging from 0 to ∞, k is the reflection coefficient and h is the thickness of the layer.

$$\frac{\partial V(x)}{\partial x} = \frac{I\rho_1}{2\pi}\frac{1}{x^2}\left(1+2x^2\int_0^{\infty} K(\lambda, k, h)\, J_0(\lambda r)\mathrm{d}\lambda\right) \tag{3.119}$$

As

$$\frac{\partial J_x}{\partial x} = -J_1(x)$$

where $J_1(x)$ is the Bessel's function of first order.

The symmetrical Schlumberger arrangement employs two current electrodes; thus, at the sounding point, the electric field will be twice of Eq. (3.119) and substituting this in Eq.

$$\rho_{aS} = \frac{\pi}{4}\left(\frac{(L^2-a^2)}{a}\right)\frac{\Delta V}{I} \tag{3.6a}$$

$$\rho_{aS} \approx \frac{\pi L^2}{4a}\frac{\Delta V}{I} \approx \frac{\pi L^2}{4}\frac{\Delta V}{a}\frac{1}{I} \approx \frac{\pi L^2}{4}\frac{\mathrm{E}}{I} \tag{3.6b}$$

$$\rho_{aS} = \frac{\pi L^2}{I}\left(-\frac{dV}{dx}\right) \qquad (3.120)$$

For r = L/2, Eqs (3.120) and (3.6b) are same,

where L is the separation between the current electrodes A and B. One gets the value of ρ_{aS}. Substituting the derivative of potential as expressed in Eq. (3.118) in Eq. (3.120) and replacing x by L (=AB/2), one gets

$$\rho_{\mathrm{aS}} = \frac{I\rho_1}{2\pi}\frac{1}{x^2}\left(1+2L^2\int_0^{\infty} K(\lambda,k,h)\,J_1(\lambda r)\mathrm{d}\lambda\right) \qquad (3.121)$$

3.6.1 Pekeris Recurrence Relation

Equation (3.111) is valid in the i^{th} layer only. The potential or the apparent resistivity over a layered earth can be computed from two-layered earth where the electrodes, both current and potential, are on the top of the first layer. Subsequently, a new layer of resistivity, say, ρ_0, is added at the top and the electrodes are shifted to the top of this new added layer as shown in given Figure.

Adding $\exp(-\lambda z_i)$ to both sides of Eq. (3.113), one gets

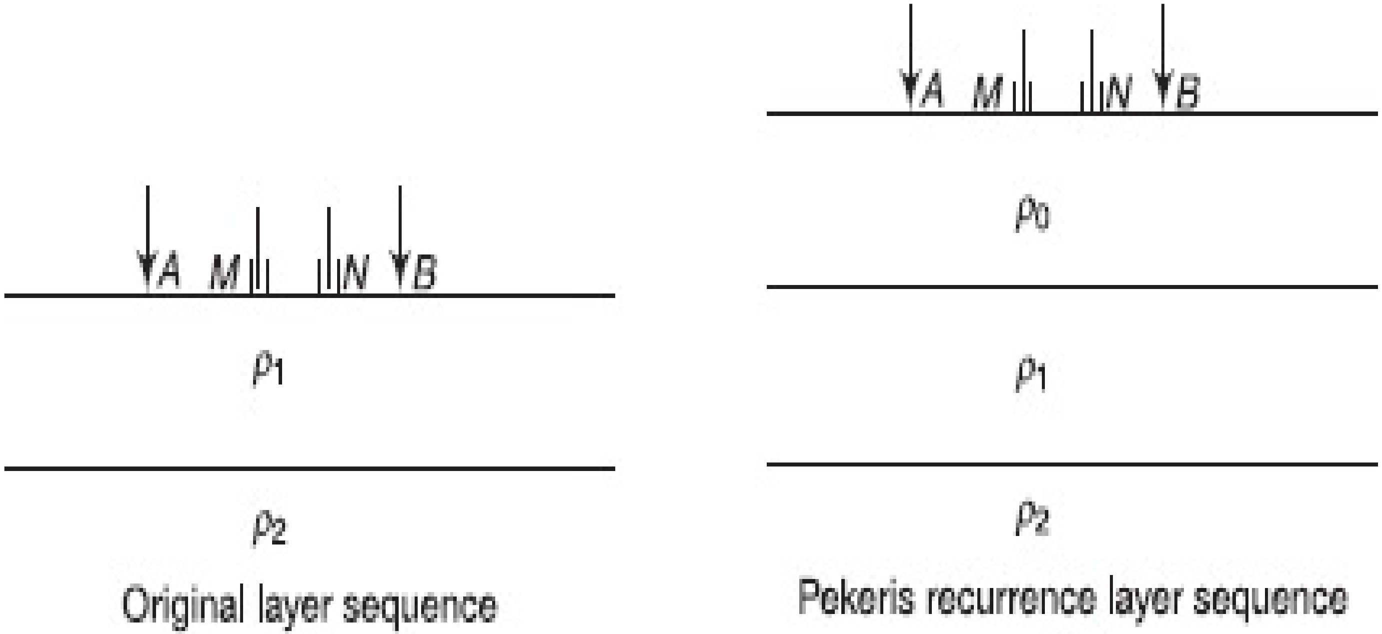

Illustration regarding Pekeris recurrence relation: (a) Original layer sequence (b) Layer of resistivity ρ_0 added at the top of the original sequence for Pekeris recurrence relation

$$[1+M_i(\lambda)\exp(-\lambda z_i)+N_i(\lambda)exp(+\lambda z_i)]=[1+M_{i+1}(\lambda)\exp(-\lambda z_i)+N_{i+1}(\lambda)exp(+\lambda z_i)] \quad (3.122)$$

Dividing Eq. (3.122) by Eq. (3.115),

$$\rho_{\mathrm{i}}\left[\frac{\left(1+M_i(\lambda)\right)\exp(-\lambda z_i)+N_i(\lambda)exp(+\lambda z_i)}{\left(1+M_i(\lambda)\right)\exp(-\lambda z_i)-N_i(\lambda)exp(+\lambda z_i)}\right]=\rho_{\mathrm{i+1}}\left[\frac{\left(1+M_{i+1}(\lambda)\right)\exp(-\lambda z_i)+N_{i+1}(\lambda)exp(+\lambda z_i)}{\left(1+M_{i+1}(\lambda)\right)\exp(-\lambda z_i)-N_{i+1}(\lambda)exp(+\lambda z_i)}\right] \quad (3.123)$$

Multiplying each term of Eq. (3.123) by exp (+lz_i) leads to

$$\rho_{\mathrm{i}}\left[\frac{1+M_i(\lambda)+N_i(\lambda)exp(+2\lambda z_i)}{1+M_i(\lambda)-N_i(\lambda)exp(+2\lambda z_i)}\right]=\rho_{\mathrm{i+1}}\left[\frac{1+M_{i+1}(\lambda)+N_{i+1}(\lambda)exp(+2\lambda z_i)}{1+M_{i+1}(\lambda)-N_{i+1}(\lambda)exp(+2\lambda z_i)}\right] \quad (3.124)$$

$$\rho_{\mathrm{i}}\left[\frac{\left(\frac{1+M_i(\lambda)}{N_i(\lambda)}\right)+exp(+2\lambda z_{i-1})}{\left(\frac{1+M_i(\lambda)}{N_i(\lambda)}\right)-exp(+2\lambda z_{i-1})}\right]=\rho_{\mathrm{i+1}}\left[\frac{\left(\frac{1+M_{i+1}(\lambda)}{N_{i+1}(\lambda)}\right)+exp(+2\lambda z_i)}{\left(\frac{1+M_{i+1}(\lambda)}{N_{i+1}(\lambda)}\right)-exp(+2\lambda z_i)}\right] \quad (3.125)$$

Let us assume,

$$K_i=\left[\frac{\left(\frac{1+M_i(\lambda)}{N_i(\lambda)}\right)+exp(+2\lambda z_{i-1})}{\left(\frac{1+M_i(\lambda)}{N_i(\lambda)}\right)-exp(+2\lambda z_{i-1})}\right] \quad (3.126)$$

Equation (3.126) can be rewritten as

$$\left(\frac{1+M_i(\lambda)}{N_i(\lambda)}\right) = exp(+2\lambda z_{i-1})\left(\frac{K_i+1}{K_i+1}\right) \tag{3.127}$$

For i = i + 1, Eq. (3.126) reduces to

$$K_{i+1} = \left[\frac{\left(\frac{1+M_{i+1}(\lambda)}{N_{i+1}(\lambda)}\right)+exp(+2\lambda z_i)}{\left(\frac{1+M_{i+1}(\lambda)}{N_{i+1}(\lambda)}\right)-exp(+2\lambda z_i)}\right] \tag{3.128}$$

Substituting Eqs (3.126), (3.127), and (3.128) in Eq. (3.125), one gets

$$\rho_{\mathrm{i}}\left[\frac{ex\,p(+2\lambda z_{i-1})\left(\frac{K_i+1}{K_i-1}\right)+ex\,p(+2\lambda z_i)}{ex\,p(+2\lambda z_{i-1})\left(\frac{K_i+1}{K_i-1}\right)-ex\,p(+2\lambda z_i)}\right] = \rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.129}$$

$$\rho_{\mathrm{i}}\left[\frac{ex\,p(+2\lambda z_{i-1})(K_i+1)+(K_i-1)ex\,p(+2\lambda z_i)}{ex\,p(+2\lambda z_{i-1})(K_i+1)-(K_i-1)ex\,p(+2\lambda z_i)}\right] = \rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.130}$$

$$\rho_{\mathrm{i}}\left[\frac{(K_i+1)+(K_i-1)\frac{ex\,p(+2\lambda z_i)}{ex\,p(+2\lambda z_{i-1})}}{(K_i+1)-(K_i-1)\frac{ex\,p(+2\lambda z_i)}{ex\,p(+2\lambda z_{i-1})}}\right] = \rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.131}$$

$$\rho_{\mathrm{i}}\left[\frac{(K_i+1)+(K_i-1)ex\,p(+2\lambda h_i)}{(K_i+1)-(K_i-1)ex\,p(+2\lambda h_i)}\right] = \rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.132}$$

as, $z_i - z_{i-1} = h_i$, thickness of the i^{th} layer

$$\rho_{\mathrm{i}}\left[\frac{(K_i+1)+(K_i exp(+2\lambda h_i)-exp(+2\lambda h_i))}{(K_i+1)-(K_i exp(+2\lambda h_i)-exp(+2\lambda h_i))}\right]=\rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.133}$$

$$\rho_{\mathrm{i}}\left[\frac{(K_i exp(+2\lambda h_i)+1)-exp(+2\lambda h_i)-1}{(exp(+2\lambda h_i)+1)-K_i(exp(+2\lambda h_i)-1)}\right]=\rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.134}$$

$$\rho_{\mathrm{i}}\left[\frac{\left(K_i\frac{[exp(+2\lambda h_i)+1]}{[exp(+2\lambda h_i)+1]}\right)-1}{\left(1-K_i\frac{[exp(+2\lambda h_i)+1]}{[exp(+2\lambda h_i)+1]}\right)}\right]=\rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.135}$$

As, $\frac{[exp(+2\lambda h_i)+1]}{[exp(+2\lambda h_i)+1]}=\tanh(\lambda h_i)$, thus Eq.(3.135) reduces to

$$\rho_{\mathrm{i}}\left[\frac{K_i-\tanh(\lambda h_i}{(1-K_i\tanh(\lambda h_i)}\right]=\rho_{\mathrm{i+1}}K_{\mathrm{i+1}} \tag{3.136}$$

Thus,

$$K_{\mathrm{i+1}}=\frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{i+1}}}\left[\frac{K_i-\tanh(\lambda h_i}{(1-K_i\tanh(\lambda h_i)}\right] \tag{3.137}$$

$$K_{\mathrm{i+1}}=p_{\mathrm{i}}\left[\frac{K_i-\tanh(\lambda h_i}{(1-K_i\tanh(\lambda h_i)}\right] \tag{3.138}$$

Where, $p_{\mathrm{i}}=\frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{i+1}}}$

$$K_{\mathrm{i}}=\left[\frac{K_{i+1}+p_{\mathrm{i}}\tanh(\lambda h_i}{(p_{\mathrm{i}}+K_{i+1}\tanh(\lambda h_i)}\right] \tag{3.139}$$

3.6.2 Resistivity Transform

In terms of resistivity transform (Ti(λ) = ρiKi), Eq. (3.138) can be rewritten as

$$T_{i+1} = \left[\frac{T_i - \rho_i \tanh(\boldsymbol{\lambda} h_i)}{1 + \frac{T_i \tanh(\boldsymbol{\lambda} h_i)}{\rho_i}}\right] \qquad \textit{(3.140)}$$

$$T_i = \left[\frac{T_{i+1} + \rho_i \tanh(\boldsymbol{\lambda} h_i)}{1 + \frac{T_{i+1} \tanh(\boldsymbol{\lambda} h_i)}{\rho_i}}\right] \qquad (3.141)$$

Resistivity transform has the dimension of resistivity. Let us compare the analogies between T(1/λ) and $\rho_a\left(s = \frac{AB}{2} = \frac{L}{2}\right)$ =(i.e., both are compared as a function of length).

Following Eq. (3.141), $\frac{1}{\boldsymbol{\lambda}} \to \infty$, $tanh \to 0$ and, thus, $T_i \to T_{i+i}$ and $\frac{1}{\boldsymbol{\lambda}} \to 0$, $tanh \to 1$ and, thus, $T_i \to \rho_i$. Consequently, for small values of $\left(\frac{1}{\boldsymbol{\lambda}}\right)$, $T_i \to \rho_i$.

We find that the resistivity transform function, both for large and small values of , shows the same asymptotic behaviour as shown by apparent resistivity, i.e., ρ_a for $s \rightarrow 0$, $\rho_i \rightarrow \rho_a$, and ρ_a for $s \rightarrow \infty$, $\rho_a \rightarrow \rho_n$. However, resistivity transform curves cannot have slopes greater than 45° but apparent resistivity curves cannot have slopes greater than 45° on ascending branches but on descending branches, their slopes can be considerably larger.

3.6.3 Apparent Resistivity for a Schlumberger Array

By substituting, $\frac{1}{x} = \int_0^\infty J_0(\boldsymbol{\lambda} x) d\boldsymbol{\lambda}$ in equation 3.118), we get

$$V(x)= \frac{I\rho_1}{2\pi}\left(\int_0^\infty J_0(\boldsymbol{\lambda} x)d\boldsymbol{\lambda}\right)+\left(\int_0^\infty K(\boldsymbol{\lambda},k,h)J_0(\boldsymbol{\lambda} x)d\boldsymbol{\lambda}\right) \tag{3.142}$$

$$V(x) = \frac{I}{2\pi}\int_0^\infty T(\boldsymbol{\lambda},k,h)J_0(\boldsymbol{\lambda} x)d\boldsymbol{\lambda} \tag{3.143}$$

Where, $T(\boldsymbol{\lambda},k,h)= \rho_1\left(1+2K(\boldsymbol{\lambda},k,h)\right)$ (3.144)

As $\frac{1}{x^2} = \int_0^\infty J_1(\boldsymbol{\lambda} x)\, d\boldsymbol{\lambda}$ (3.145)

Equation 3.119 can be written as

$$-\frac{dV(x)}{dx} = \frac{I\rho_1}{2\pi}\left(\int_0^\infty J_1(\boldsymbol{\lambda} x)d\boldsymbol{\lambda} + 2\int_0^\infty K(\boldsymbol{\lambda},k,h)J_1(\boldsymbol{\lambda} x)\boldsymbol{\lambda} d\boldsymbol{\lambda}\right) \tag{3.146}$$

$$= \frac{I\rho_1}{2\pi}\left(1+2\int_0^\infty K(\boldsymbol{\lambda},k,h)\, d\boldsymbol{\lambda}\right)\int_0^\infty J_1(\boldsymbol{\lambda} x)\boldsymbol{\lambda} d\boldsymbol{\lambda} \tag{3.147}$$

Using equations (3.144) and (3.147) can be expressed as

$$-\frac{dV(x)}{dx} = \frac{I}{2\pi}\int_0^\infty T(\boldsymbol{\lambda},k,h)J_1(\boldsymbol{\lambda} x)\boldsymbol{\lambda} d\boldsymbol{\lambda} \tag{3.148}$$

Equation (3.121) in terms of $T(\boldsymbol{\lambda},k,h)$ can be expressed as

$$\rho_{aS} = \frac{I}{2\pi}\int_0^\infty T(\boldsymbol{\lambda},k,h)J_1(\boldsymbol{\lambda} L)\boldsymbol{\lambda} d\boldsymbol{\lambda} \tag{3.149}$$

3.6.4 Apparent Resistivity for a Wenner Array

The potential difference (ΔV_W) between the potential electrodes M and N with electrode separation for Wenner array can be expressed as

$$\Delta V_W = V_a - V_{2a} = \frac{I}{2\pi}\int_0^{\infty} T(\lambda)\{J_0(\lambda a) - J_0(\lambda \times 2a)\}d\lambda \qquad (3.150)$$

Apparent resistivity for the Wenner array, $\rho_{aW} = \frac{2\pi}{I} \times \Delta V_W$ (3.150)

$$\rho_{aW} = a\int_0^{\infty} T(\lambda)\{J_0(\lambda a) - J_0(\lambda \times 2a)\}d\lambda$$