Seismoscope

These instruments gave visible evidence of a seismic event but were unable to trace a permanent record of the seismic wave itself. They are classified as seismoscopes.

Seismograph and seismogram

The science of seismology dates from the invention of the seismograph by the English scientist John Milne in 1892. Its name derives from its ability to convert an unfelt ground vibration into a visible record. The seismograph consists of a receiver and a recorder. The ground vibration is detected and amplified by a sensor, called the seismometer or, in exploration seismology, the geophone. In modern instruments the vibration is amplified and filtered electronically. The amplified ground motion is converted to a visible record, called the seismogram.

Principle of the seismometer

Seismometers are designed to react to motion of the Earth in a given direction. Mechanical instruments record the amplified displacement of the ground; electromagnetic instruments respond to the velocity of ground motion. Depending on the design, either type may respond to vertical or horizontal motion. Some modern electromagnetic instruments are constructed so as to record simultaneously three orthogonal components of motion. Most designs employ variations on the pendulum principle.

The equation of the seismometer.

Answer: Seismometer, based on mass-spring system, consist of a mass M, attached to the frame of the seismometer through a spring in a manner that, in the absence of acceleration of the frame, the spring brings the mass back to some rest position with respect to the frame. For small displacements from the rest position, the spring exerts a force on the mass that is proportional to the displacement from the rest position. There is also a force of 'drag' on the mass, which is proportional to the instantaneous velocity of the mass with respect to the frame. This force is something like the damping provided by a dashpot (Fig. 1).

The frame of the seismometer is rigidly attached to the ground. Let the spring exerts a vertical restoring force equal to 'k' per unit displacement from the rest position, and let the dashpot provides a force of drag equal to 'c' for unit velocity of the mass with respect to the frame.

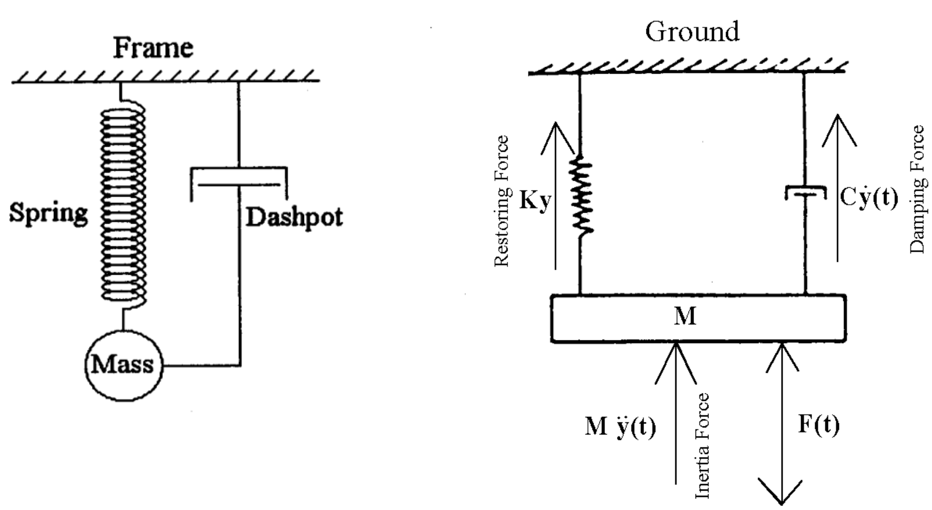

Figure 1

If with respect to an inertial frame of reference, the motion of the ground is represented by a function of time, denoted by u(t), and the motion of the mass (M) with respect to the ground is given by y(t), then the equation of motion can be written as

$$M\frac{d^2}{dt^2}[y(t)+u(t)]+c\frac{d}{dt}[y(t)]+ky(t)=0 \qquad (1)$$

$$M\frac{d^2y(t)}{dt^2}+c\frac{dy(t)}{dt}+ky(t)=-M\frac{d^2u(t)}{dt^2}$$

$$\frac{d^2y(t)}{dt^2}+\frac{c}{M}\frac{dy(t)}{dt}+\frac{k}{M}y(t)=-\frac{d^2u(t)}{dt^2} \tag{2}$$

Let the motion of the ground be represented by the following as

$$u(t)=F\cos pt$$

$$\therefore \frac{d^2u(t)}{dt^2}=-Fp^2\cos pt$$

Therefore, we can rewrite the equation (1.1) in the following form

$$\frac{d^2y}{dt^2}+\frac{c}{M}\frac{dy}{dt}+\frac{k}{M}y=Fp^2\cos pt$$

Let us put in (1.6) $\frac{c}{M}=2h\omega$ and $\frac{k}{M}=\omega^2$ (say), therefore,

$$\frac{d^2y}{dt^2}+2h\omega\frac{dy}{dt}+\omega^2y=Fp^2\cos pt \tag{3}$$

Let the recorded motion of the seismometer be

$$y = A\cos(pt-\varphi)$$

$$\therefore\ \dot{y}=-Apsin(pt-\varphi)$$

$$\text{or, } \ddot{y}=\ -Ap^2\cos(pt-\varphi)$$

$\therefore\ -Ap^2\cos(pt-\varphi)$-2Ahωp sin (pt-φ)+$A\omega^2\cos(pt-\varphi)$ = Fp^2 cos(pt-φ+φ)

or, (-Ap^2 + $A\omega^2$) $\cos(pt-\varphi)$ -2Ahωp sin (pt-φ) = Fp^2 cos(pt-φ) sinφ - Fp^2 sin(pt-φ) cosφ

Comparing both side, we have

$$(A\omega^2-Ap^2) = Fp^2\sin\varphi \tag{4}$$

$$-2Ah\omega p = Fp^2\cos\varphi \tag{5)}$$

Squaring eqns. 4 and 5, and further adding, we obtain

$$A^2\{(\omega^2-p2)^2+4h^2\omega^2p^2\} = F^2p^4$$

$$\therefore\ A=\frac{Fp^2}{\{(\omega^2-p^2)^2+4h^2\omega^2p^2\}^{\frac{1}{2}}} \tag{6}$$

Divided eqn. 4 by eqn. 5, we obtain

$$\varphi=\tan^{-1}\left(\frac{2h\omega p}{\omega^2-p^2}\right) \tag{7}$$

Dividing both numerator and denominator by ω^2 of eqn. 6, we obtain

$$A=\frac{F\frac{p^2}{\omega^2}}{\{(1-\frac{p^2}{\omega^2})^2+4h^2\frac{p^2}{\omega^2}\}^{\frac{1}{2}}}$$

Let, $\frac{p}{\omega}=\eta$, we have $\mu=\frac{\eta^2}{\{(1-\eta^2)^2+(2h\eta)^2\}^{\frac{1}{2}}}$ (8)

where, dynamic magnification of the system is given by

$$\mu=\frac{A}{F}$$

Equation 8 and 7 represents the amplitude and phase characteristics of a seismometer.

$$\therefore y = A\cos(\text{pt}-\varphi) = \mu F\cos(\text{pt}-\varphi) \tag{9}$$

When μ versus η for different values of h (pl. take h in place of ζ) is plotted, it is observed that at η = 1, μ is maximum. This is identified as resonance and the frequency ratio (η) at which it occurs is called the resonance frequency ratio.

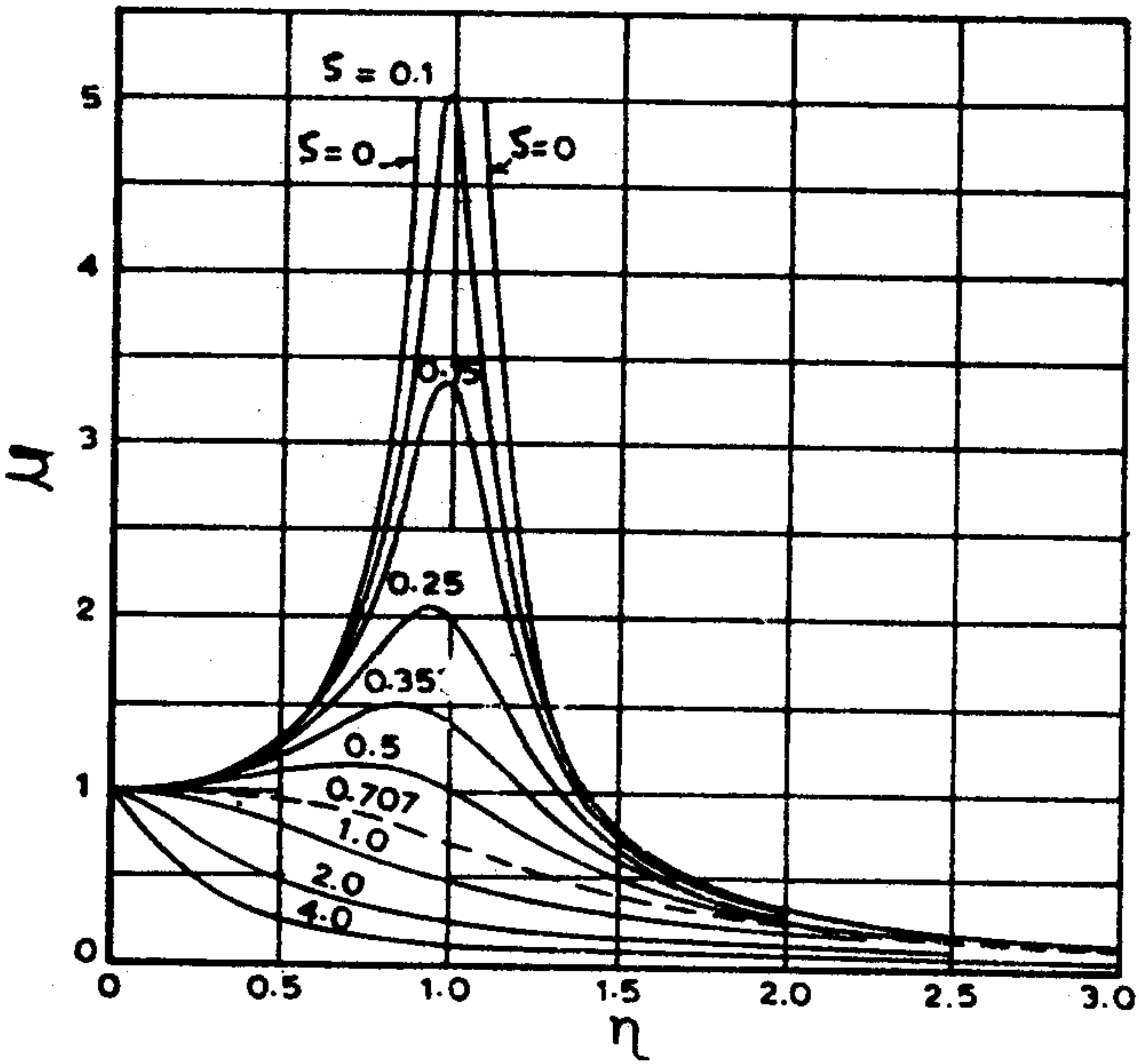

Figure 2. Effect of the damping factor h on the response of a seismometer to different signal frequencies. Critical damping corresponds to h = 1.

Considering the effect of h, it is seen that μ is the maximum for η less than 1, that is, the maximum magnification occurs for a frequency p slightly less than the undamped natural frequency ω of the system.

Effect of instrumental damping

The ground motion caused by a seismic wave contains a broad spectrum of frequencies. Equation 9 shows that the response of the seismometer to different signal frequencies is strongly dependent on the value of the damping factor h (Fig. 2). A completely undamped seismometer has h = 0, and for small values of h the response of the seismometer is said to be underdamped. An undamped or greatly underdamped seismometer preferentially amplifies signals near the natural frequency, and therefore cannot make an accurate record of the ground motion; the undamped instrument will resonate at its natural frequency ω. For all damping factors h < 1/√2, the instrument response function has a peak, indicating preferential amplification of a particular frequency.

The value h = 1 corresponds to critical damping, so called because it delineates two different types of seismometer response in the absence of a forcing vibration. If h < 1, the damped, free seismometer responds to a disturbance by swinging periodically with decreasing amplitude about its rest position. If h ≥ 1, the disturbed seismometer behaves aperiodically, moving smoothly back to its rest position. However, if the damping is too severe (h >> 1), the instrument is overdamped and all frequencies in the ground motion are suppressed.

The optimum behaviour of a seismometer requires that the instrument should respond to a wide range of frequencies in the ground motion, without preferential amplification or excessive suppression of frequencies.

This requires that the damping factor should be close to the critical value. It is usually chosen to be in the range 70% to 100% of critical damping (i.e., 1/√2 ≤ h < 1). At critical damping the response of the seismometer to a periodic disturbing signal with frequency ω is given by

$$y = \frac{Fp^2}{(\omega^2+p^2)}\cos(\omega t - \varphi) \qquad (10)$$

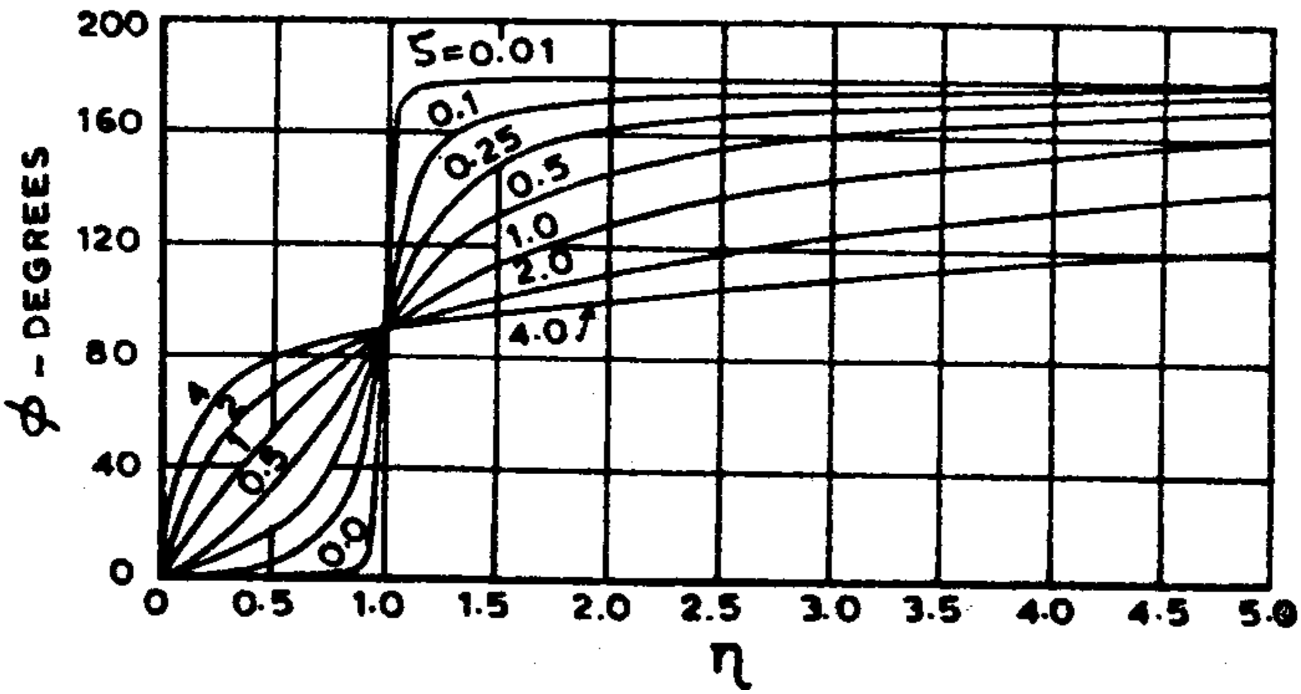

Figure 3. Phase characteristics of a seismometer to different signal frequencies.

$$\varphi = tan^{-1}(\frac{2h\omega p}{\omega^2 - p^2})$$

Now, putting, p/ω = η, we have the following:

$$\varphi = tan^{-1}(\frac{2\eta h}{1-\eta^2})$$

For low frequency of excitation, η<<1 and ϕ tends to zero and the motions are in phase as would be expected physically. For large frequencies, η>>1 and ϕ tends to 180° and the force and the motion are always in opposite directions. That is, the force is in phase with acceleration. At resonance, η = 1, and ϕ = 90°. That is, the force is in phase with the velocity.

Long-period and short-period seismometers

The natural period of a seismometer is an important factor in determining what it actually records. Two examples of special interest correspond to instruments with very long and very short natural periods, respectively.

The long-period seismometer is an instrument in which the resonant frequency ω_0 is very low. For all but the lowest frequencies we can write $\omega \ll p$. The phase lag ϕ between the seismometer and the ground motion becomes zero, and the amplitude of the seismometer displacement becomes equal to the amplified ground displacement y:

$$y = Fcospt = u(t)$$

The long-period seismometer is thus sometimes called a displacement meter. It is usually designed to record seismic signals with frequencies of 0.01–0.1 Hz (i.e., periods in the range 10–100 s).

The short-period seismometer is constructed to have a very short natural period and a correspondingly high resonant frequency ω, which is higher than most frequencies in the seismic wave. Under these conditions we have $\omega \gg p$, the phase difference ϕ is again small and eq. 10 becomes

$$y = \frac{Fp^2}{\omega^2}\cos(\omega t) = -\frac{1}{\omega^2}\ddot{u}$$

This equation shows that the displacement of the short-period seismometer is proportional to the acceleration of the ground, and the instrument is accordingly called an accelerometer. It is usually designed to respond to seismic frequencies of 1–10 Hz (periods in the range 0.1 - 1 s). An accelerometer is particularly suitable for recording strong motion earthquakes, when the amplitude of the ground motion would send a normal type of displacement seismometer off-scale.

Broadband seismometers

Short-period seismometers operate with periods of 0.1-1 s and long-period instruments are designed for periods greater than 10 s. The resolution of seismic signals with intermediate frequencies of 0.1-1 Hz (periods of 1-10 s) is hindered by the presence in this range of a natural form of seismic background noise. Short- and long-period seismometers have narrow dynamic ranges because they are designed to give optimum performance in limited frequency ranges, below or above the band of ground noise. This handicap was overcome by the design of broadband seismometers that have high sensitivity over a very wide dynamic range.

The broadband seismometer has basically an inertial pendulum-type design, with enhanced capability due to a force-feedback system. This works by applying a force proportional to the displacement of the inertial mass to prevent it from moving significantly. The amount of feedback force applied is determined using an electrical transducer to convert motion of the mass into an electrical signal. The force needed to hold the mass stationary corresponds to the ground acceleration. The signal is digitized with 24-bit resolution, synchronized with accurate time signals, and recorded on hard disk, or solid state memory. The feedback electronics are critical to the success of this instrument.

Broadband design results in a seismometer with great bandwidth and linear response. It is no longer necessary to avoid recording in the 1–10 s bandwidth of ground noise avoided by short-period and long-period seismometers. The recording of an earthquake by a broadband seismometer contains more useable information than can be obtained from the short-period or long-period recordings individually or in combination. Broadband seismometers can be used to register a wide range of signals. The dynamic range extends from ground noise up to the strong acceleration that would result from an earthquake with magnitude 9.5, and the periods that can be recorded range from high-frequency body waves to the very long period oscillations of the ground associated with bodily Earth-tides. The instrument is employed world-wide in modern standardized seismic networks, replacing short period and long-period seismometers.

Noise, Microseisms and Dynamic Range

The noise derives from a nearly continuous succession of small ground movements that are referred to as microseisms. Some microseismic noise is of local origin, related to such effects as vehicular traffic, rainfall, and wind action on trees, etc. However, an important source is the action of storm waves at sea, which is detectable on seismic records far inland. The drumming of rough surf on a shoreline and the interference of sea waves over deep water are thought to be the principal causes of microseismic noise. The microseismic noise has a low amplitude on a seismogram, but it may be as strong as a weak signal from a distant earthquake, which cannot be selectively amplified without also magnifying the noise. The problem is exacerbated by the limited dynamic range of short- or long-period seismometers. Short-period instruments yield records dominated by high frequencies while long-period devices smooth these out, giving a record with only a low-frequency content.

The range between the strongest and weakest signals that can be recorded without distortion by a given instrument is called its dynamic range. Dynamic range is measured by the power (or energy density) of a signal, and is expressed in units of decibels (dB). Dynamic range is expressed in dBs.

$$Dynamic\ Range\ in\ dB = 20log_{10}(\frac{A_{max}}{A_{min}})$$

where A is amplitude. A decibel is defined as 10 $\log_{10}$(signal power). Because power is proportional to the square of amplitude, a decibel is equivalent to 20 $\log_{10}$(amplitude). So, for example, a range of 20 dB in power corresponds to a factor 10 variation in acceleration, and a dynamic range of 100 dB corresponds to a 10^5 variation in amplitude. The dynamic range of a digital instrument depends on the number of bits per sample and is equal to 2^n, where n is the number of bits per sample. Thus, for a 24 bits digitizer, the dynamic range of the system is 2^{24}, which is equal to 144 dB.