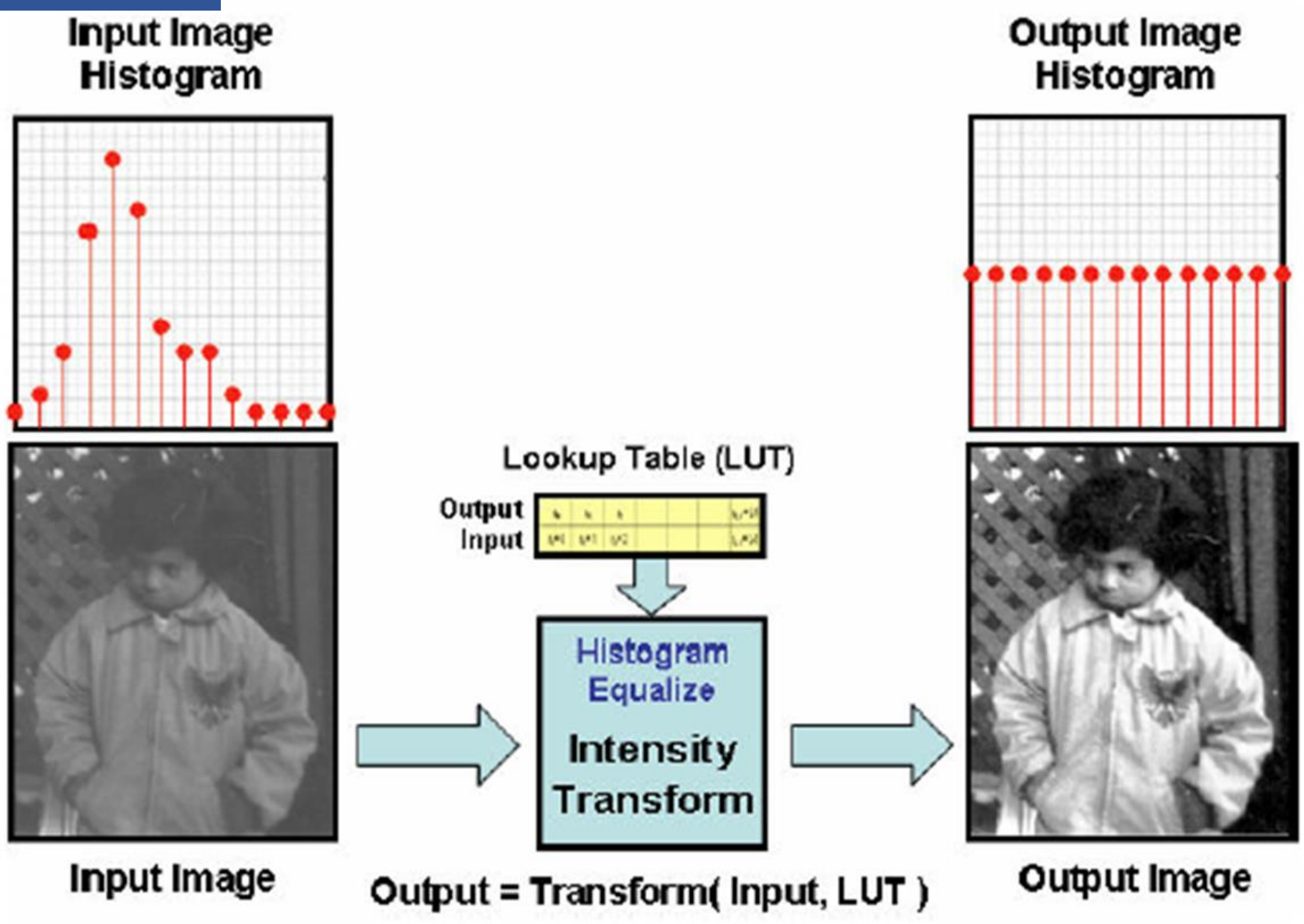
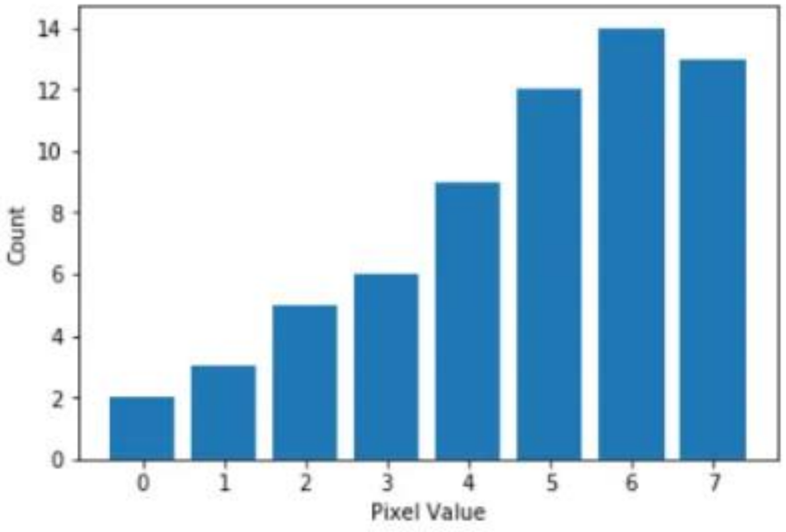
# Histogram Equalization




Histogram Equalization stretches the histogram to fill the dynamic range and at the same time tries to keep the histogram uniform as shown below

Suppose we have a 3-bit, 8×8 image whose pixel count and corresponding histogram is shown below

| $r_k$ | $n_k$ |
|---|---|
| 0 | 2 |
| 1 | 3 |
| 2 | 5 |
| 3 | 6 |
| 4 | 9 |
| 5 | 12 |
| 6 | 14 |
| 7 | 13 |




https://theailearner.com/2019/04/01/histogram-equalization/

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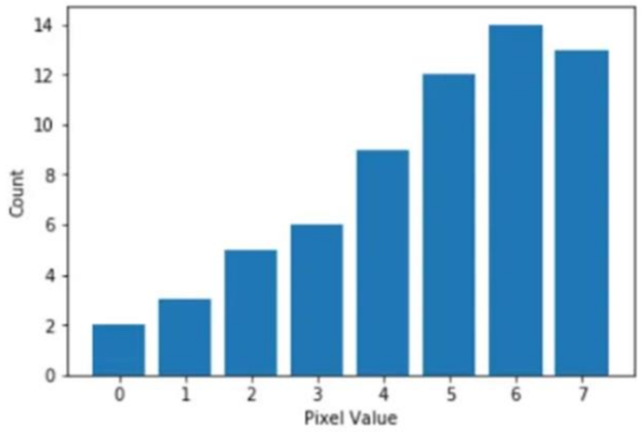



$$P_r(r_0) = \frac{n_0}{MN} = \frac{2}{64} = 0.03$$

$$s_0 = T(r_0) = (L-1)\sum_{j=0}^{0} P_r(r_j) = 7P_r(r_0) = 0.21$$

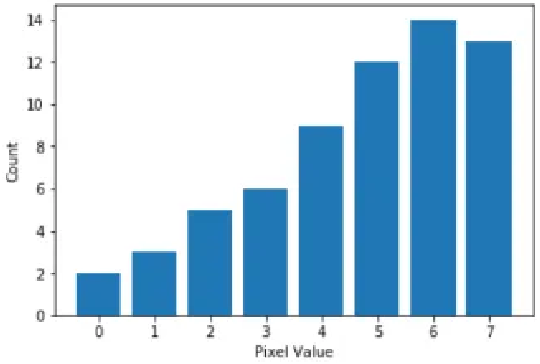
and;

$$P_r(r_1) = \frac{n_1}{MN} = \frac{3}{64} = 0.05$$

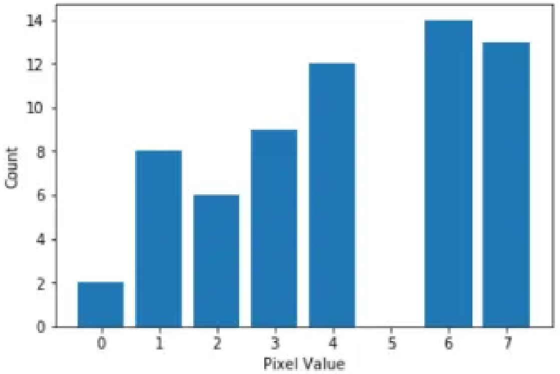
$$s_1 = (L-1)\sum_{j=0}^{1} P_r(r_j) = 7P_r(r_0) + 7P_r(r_1) = 0.56$$

https://theailearner.com/2019/04/01/histogram-equalization/

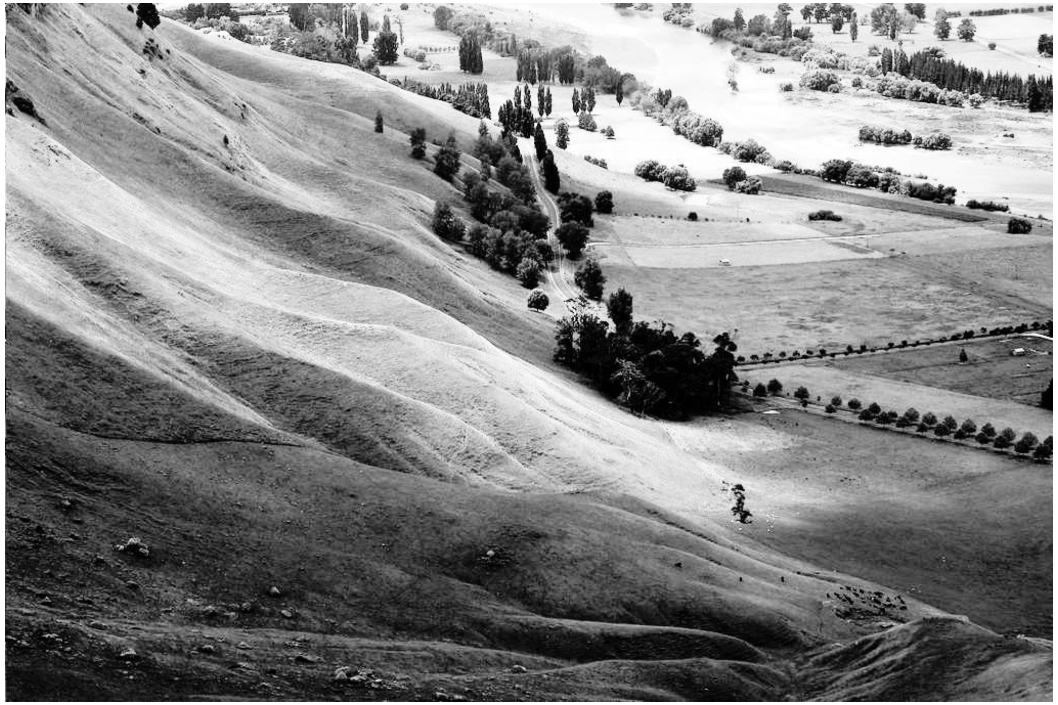
| $r_k$ | $n_k$ | $P_r(r_k)$ | $S_k$ | Round |
|---|---|---|---|---|
| 0 | 2 | 0.03 | 0.21 | 0 |
| 1 | 3 | 0.05 | 0.56 | 1 |
| 2 | 5 | 0.08 | 1.12 | 1 |
| 3 | 6 | 0.09 | 1.75 | 2 |
| 4 | 9 | 0.14 | 2.73 | 3 |
| 5 | 12 | 0.19 | 4.06 | 4 |
| 6 | 14 | 0.22 | 5.60 | 6 |
| 7 | 13 | 0.20 | 7.00 | 7 |




Original




Equalized



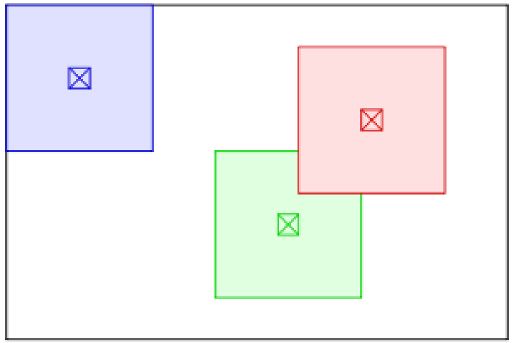
- Histogram Equalization apply the same transformation function to transform all the image pixels.

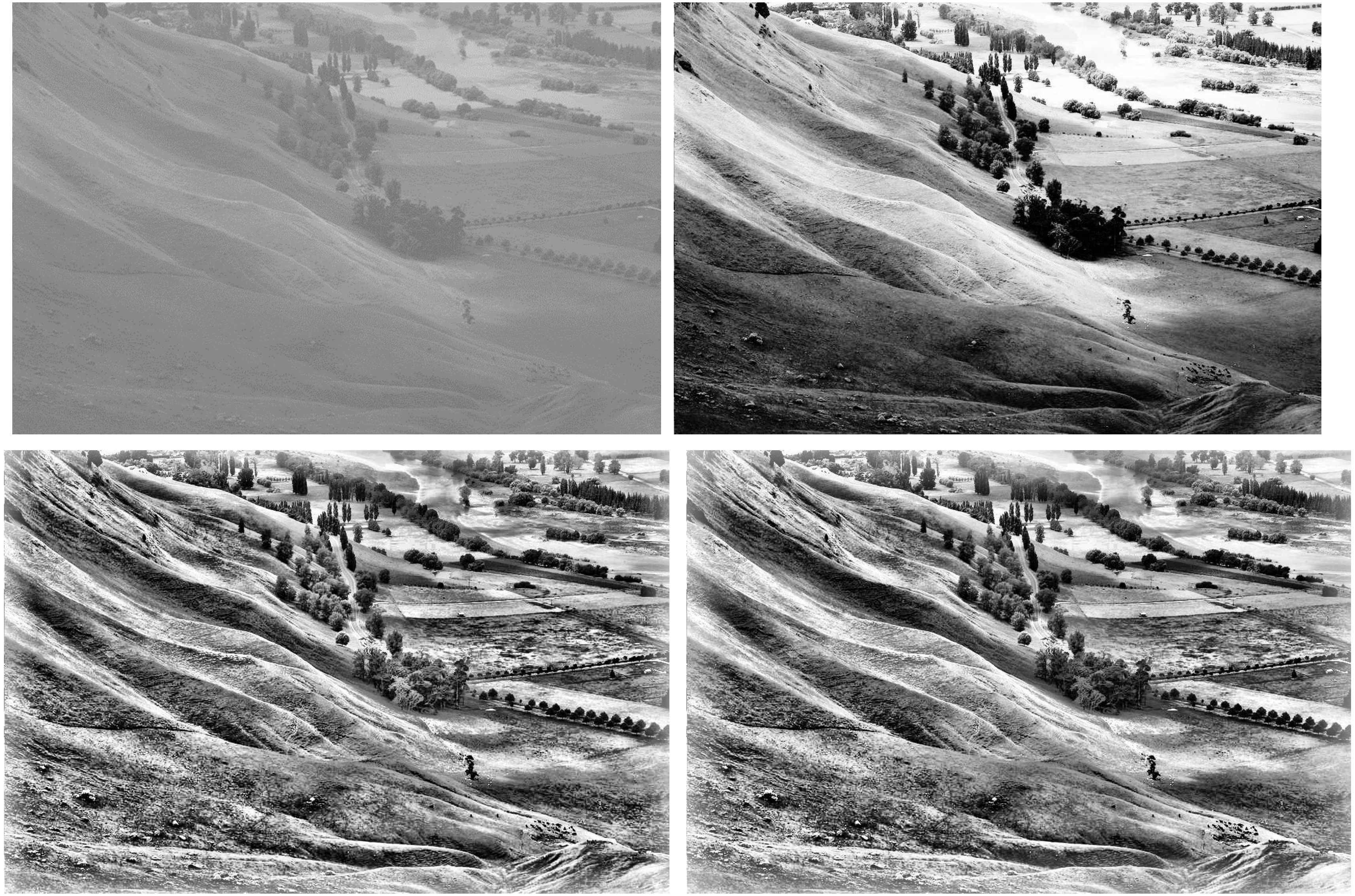
- This approach works well for most cases but when the image contains regions that are significantly lighter or darker than most of the image, the contrast in those regions will not be sufficiently enhanced.

- Sometimes we want to enhance details over small areas in an image rather than the whole image. This problem can be solved if we use a transformation function that is derived from the neighborhood of every pixel in the image. This is what Adaptive Histogram Equalization (AHE) do.

# Adaptive Histogram Equalization

Adaptive histogram equalization (AHE) improves contrast by transforming each pixel with a transformation function derived from a neighborhood region.

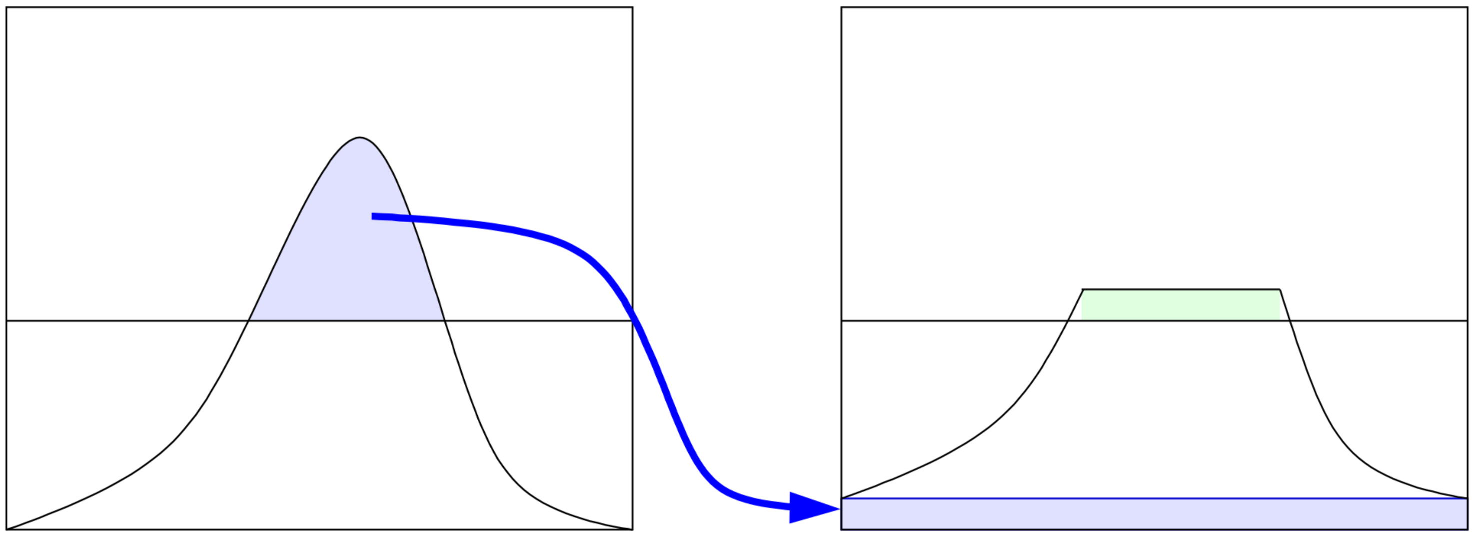




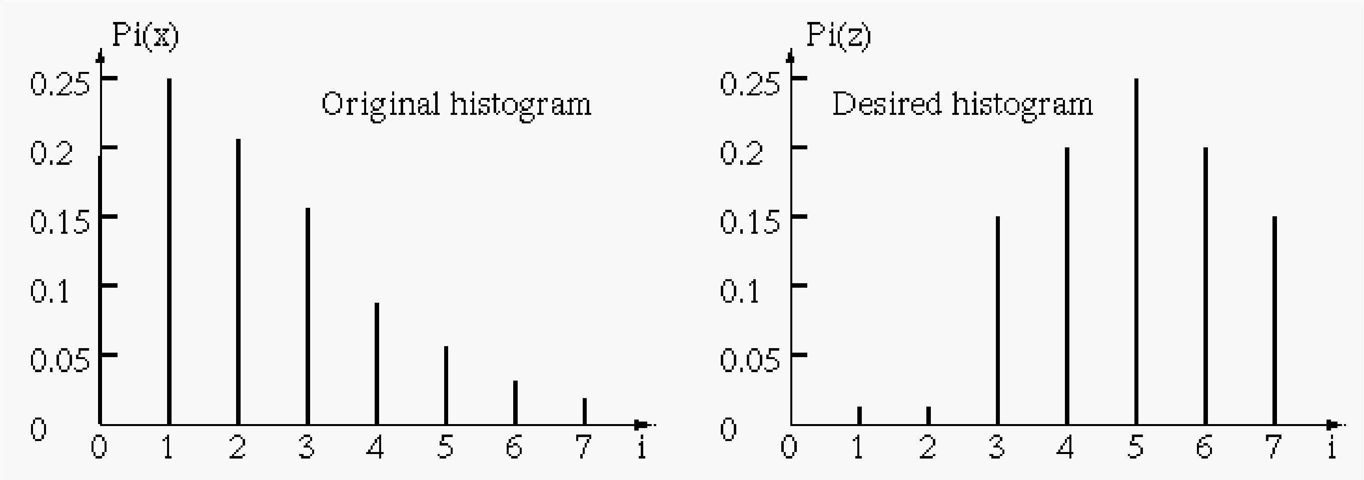
Ordinary AHE tends to overamplify the contrast in near-constant regions of the image, since the histogram in such regions is highly concentrated. As a result, AHE may cause noise to be amplified in near-constant regions.

Contrast Limited AHE (CLAHE) is a variant of adaptive histogram equalization in which the contrast amplification is limited, so as to reduce this problem of noise amplification.

CLAHE limits the amplification by clipping the histogram at a predefined value before computing the CDF.



# Histogram matching




| $x_i$ | $n_j$ | $h_x$ | $y = H_x$ |
|---|---|---|---|
| 0 | 790 | 0.19 | 0.19 |
| 1 | 1023 | 0.25 | 0.44 |
| 2 | 850 | 0.21 | 0.65 |
| 3 | 656 | 0.16 | 0.81 |
| 4 | 329 | 0.08 | 0.89 |
| 5 | 245 | 0.06 | 0.95 |
| 6 | 122 | 0.03 | 0.98 |
| 7 | 81 | 0.02 | 1.00 |

| $z_i$ | $p_z$ | $\underline{y' = H_z}$ |
|---|---|---|
| 0 | 0.0 | 0.0 |
| 1 | 0.0 | 0.0 |
| 2 | 0.0 | 0.0 |
| 3 | 0.15 | 0.15 |
| 4 | 0.20 | 0.35 |
| 5 | 0.30 | 0.65 |
| 6 | 0.20 | 0.85 |
| 7 | 0.15 | 1.0 |

| $x_i = i$ | $y_j = H_x$ | $\underline{y'_j = H_z}$ | $z_j = j$ |
|---|---|---|---|
| 0 | 0.19 | 0.0 | 3 |
| 1 | 0.44 | 0.0 | 4 |
| 2 | 0.65 | 0.0 | 5 |
| 3 | 0.81 | 0.15 | 6 |
| 4 | 0.89 | 0.35 | 6 |
| 5 | 0.95 | 0.65 | 7 |
| 6 | 0.98 | 0.85 | 7 |
| 7 | 1.0 | 1.0 | 7 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| i | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| j | 3 | 4 | 5 | 6 | 6 | 7 | 7 | 7 |