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Project 2

Expectation Decider

(1) Understanding the Basics:

a. Probability is the likelihood of an event occurring
- Value lies between 0 and 1

Eg: Probability ~~of~~ a student randomly selected passes the final exam means how likely students are to pass the exam.

b. Key Probability Terminology

- Experiment : An action with uncertain results
- Outcome : Possible result of an experiment
- Sample Space : All possible outcomes
- Event : One or more outcomes
- Favourable outcomes : outcomes that match the desired event

c. Probability Event Examples:

(i) P(student passes final exam) =

$$\frac{\text{Students who passed}}{200}$$

(ii) P(student with Attendance > 80%) =

$$\frac{\text{Student with } > 80\% \text{ attendance}}{200}$$

(iii) P(Students who participate in group discussions):

$$= \frac{\text{Student who participate}}{200}$$

(2) Empirical & Theoretical Probability

a. Emperical (Experimental)

$$P(E) = \frac{\text{No. of times event occurred}}{\text{Total observations}}$$

eg. P(Student Passes final exam)

$$= \frac{\text{No. of Passing students}}{200}$$

b. Theoretical

$$P(E) = \frac{\text{No. of Favourable outcomes}}{\text{Total Possible outcomes}}$$

eg. Final result has 2 outcomes
Pass or Fail.

P(Student passes final exams)

$$= \frac{1}{2} = 0.5$$

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(3) Random Variable & Probability Distribution:

a. Define Random Variable

X = Students who pass final exam out of 3 randomly selected students

$X \in \{0, 1, 2, 3\}$

~~pcfbo~~ $P(\text{pass}) = 0.755$

$P(\text{fail}) = 1 - P(\text{pass}) = 0.245$

$P(X=k) = \binom{3}{k} p^k q^{3-k}$

b.

X (Passed students)	P (x)
0	0.0147
1	0.1361
2	0.4206
3	0.4286

c. Mean of R.V. : $\mu = n \times p$

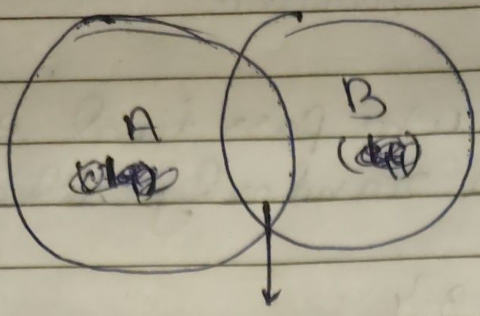
$\mu = 3 \times 0.755 = 2.265$

Variance of R.V. : $\sigma^2 = n \times p \times q$

$= 3 \times 0.755 \times 0.245$

$= 0.555$

(4) Venn Diagram.



$n(A \cap B) \Rightarrow 37$

$n(A) = 79$

$n(B) = 86$

(5) Contingency Table & Probability Calculation

	Pass	Fail	
Yes	91	14	105
No	60	35	95
	151	49	200

$$P(\text{Yes AND Pass}) = \frac{91}{200} = 0.455$$

$$P(\text{Pass}) = \frac{151}{200} = 0.755$$

$$P(\text{Pass} \mid \text{Yes}) = \frac{91}{105} = 0.867$$

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(6) Understanding Relationships :

a. Conditional Probability tells us how likely a student is to pass the final exam if they actively participate in group discussions.

As Conditional Probability is higher than marginal probability of passing, we can say that group discussions are beneficial.

b) Events are independent if,

$$P(B|A) = P(B)$$

Here, $P(\text{Pass}|\text{yes}) \neq P(\text{pass})$

$0.867 \neq 0.755$

Hence, both the events are dependent

⇒ These events are not Mutually Exclusive, as they can occur together at the same time.

(7) Bayes Theorem :

P = Pass F = Fail H = High Attendance

Given,

$P(H|P) = 0.70$ $P(H) = 0.60$

$P(H|F) = 0.40$

$P(H) = P(H|P)[P(P)] + P(H|F)[P(F)]$ ①

$P(F) = 1 - P(P)$ ②

② in ①

$0.60 = 0.70[P(P)] + 0.40[1 - P(P)]$

$0.60 = 0.70\,P(P) + 0.40 - 0.40\,P(P)$

$0.60 - 0.40 = 0.30\,P(P)$

$0.20 = 0.30\,P(P)$

$$\boxed{P(P) = \frac{0.20}{0.30} = \frac{2}{3}}$$

$$\boxed{P(F) = 1 - \frac{2}{3} = \frac{1}{3}}$$

Bayes Theorem

$$P(P|H) = \frac{P(H|P)[P(P)]}{P(H)}$$

$$= \frac{0.70 \times \frac{2}{3}}{0.60}$$

$$\boxed{P(P|H) = 0.778}$$