Capstone Thesis Presentation – Spring 2025

Ring Trapdoor Functions: A Lattice-Based Framework for Secure Ring Signatures

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Thesis Supervisors

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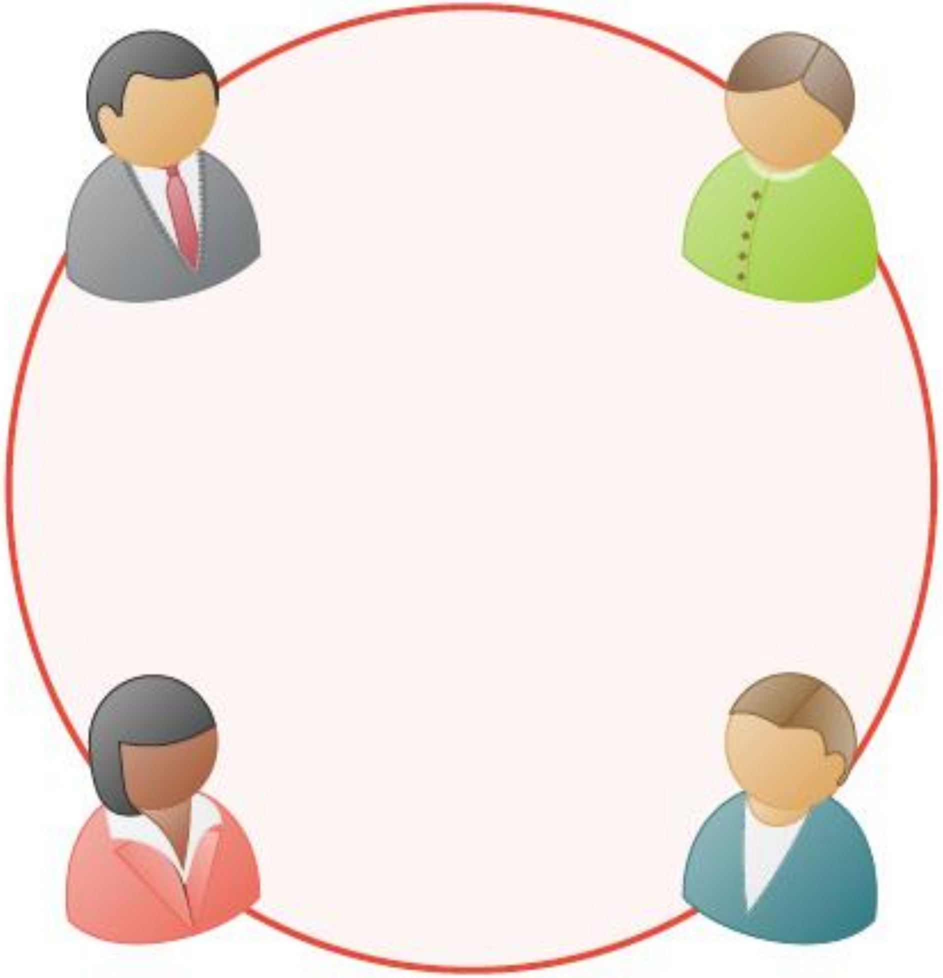
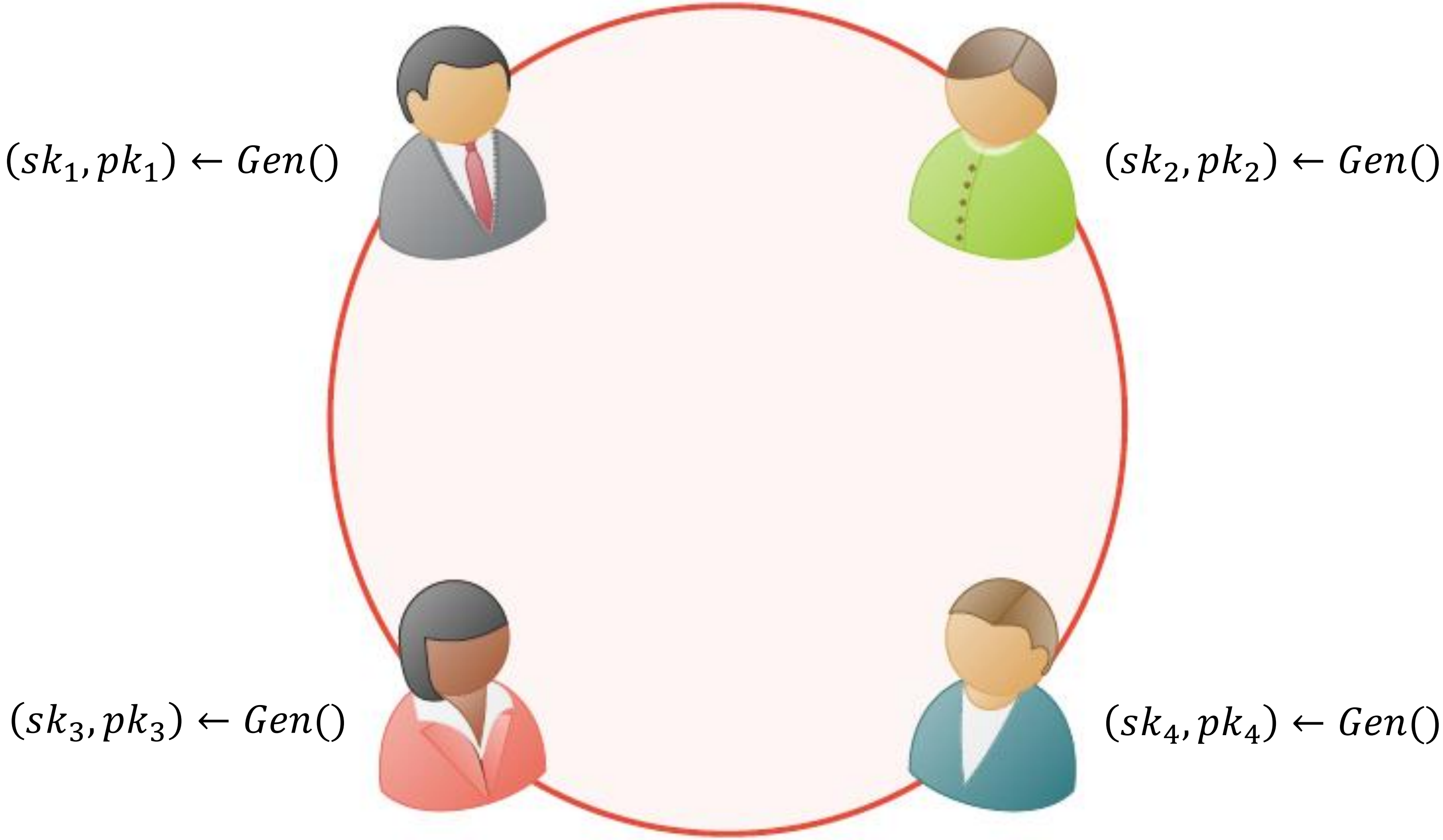
Ashoka University

A generic framework for constructing efficient ring signature schemes through novel cryptographic abstractions.

Ring Signatures

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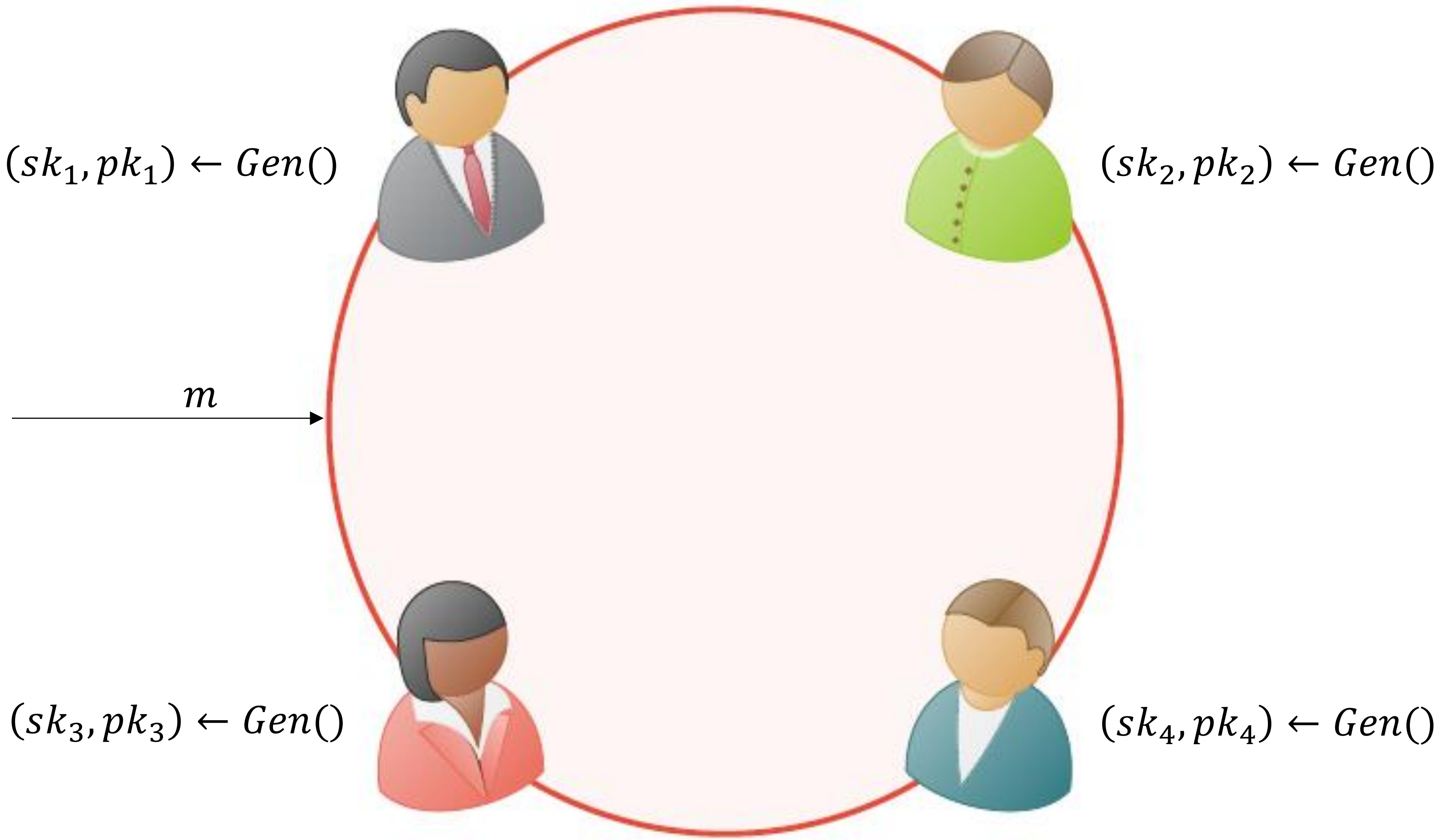
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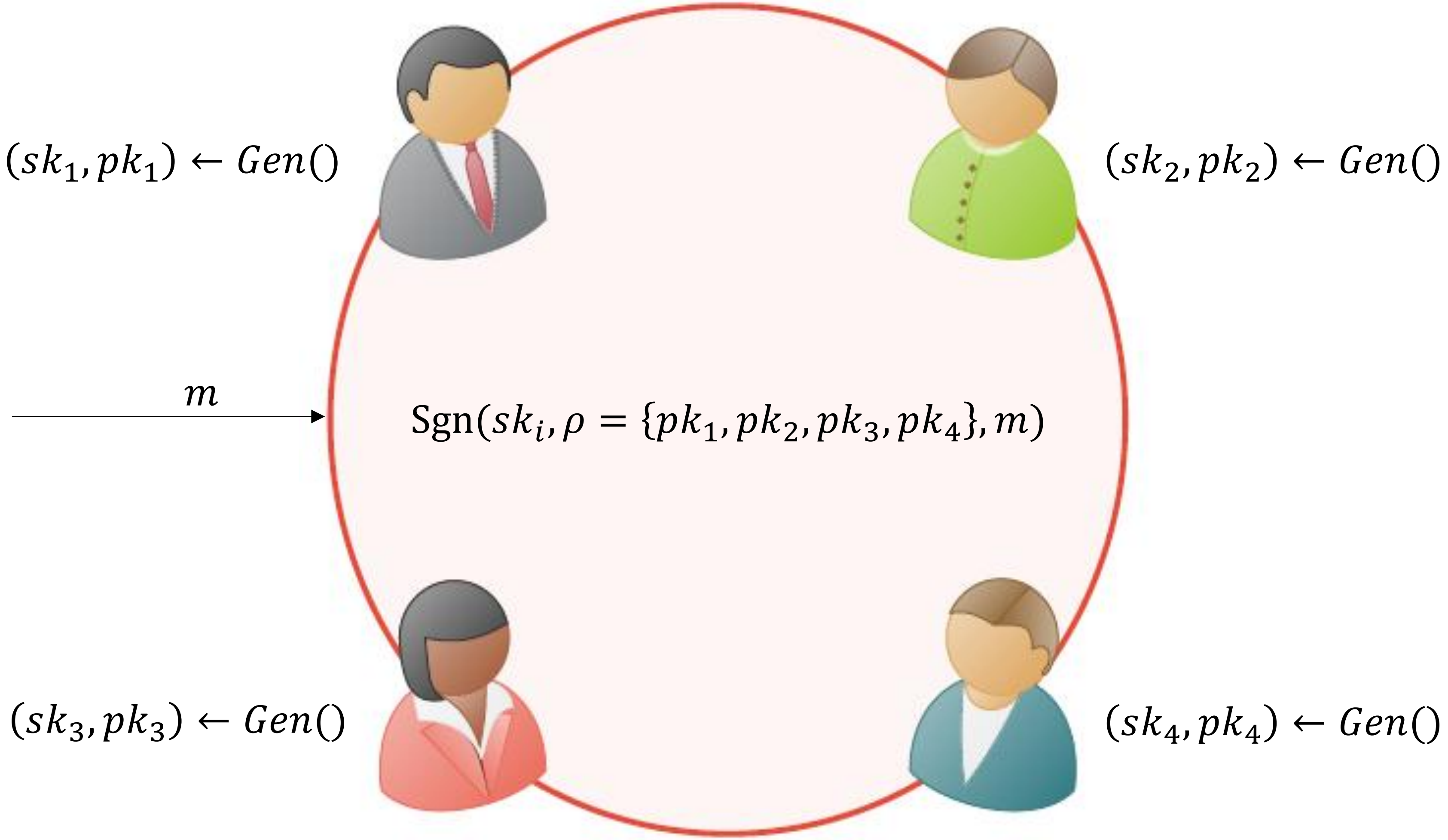
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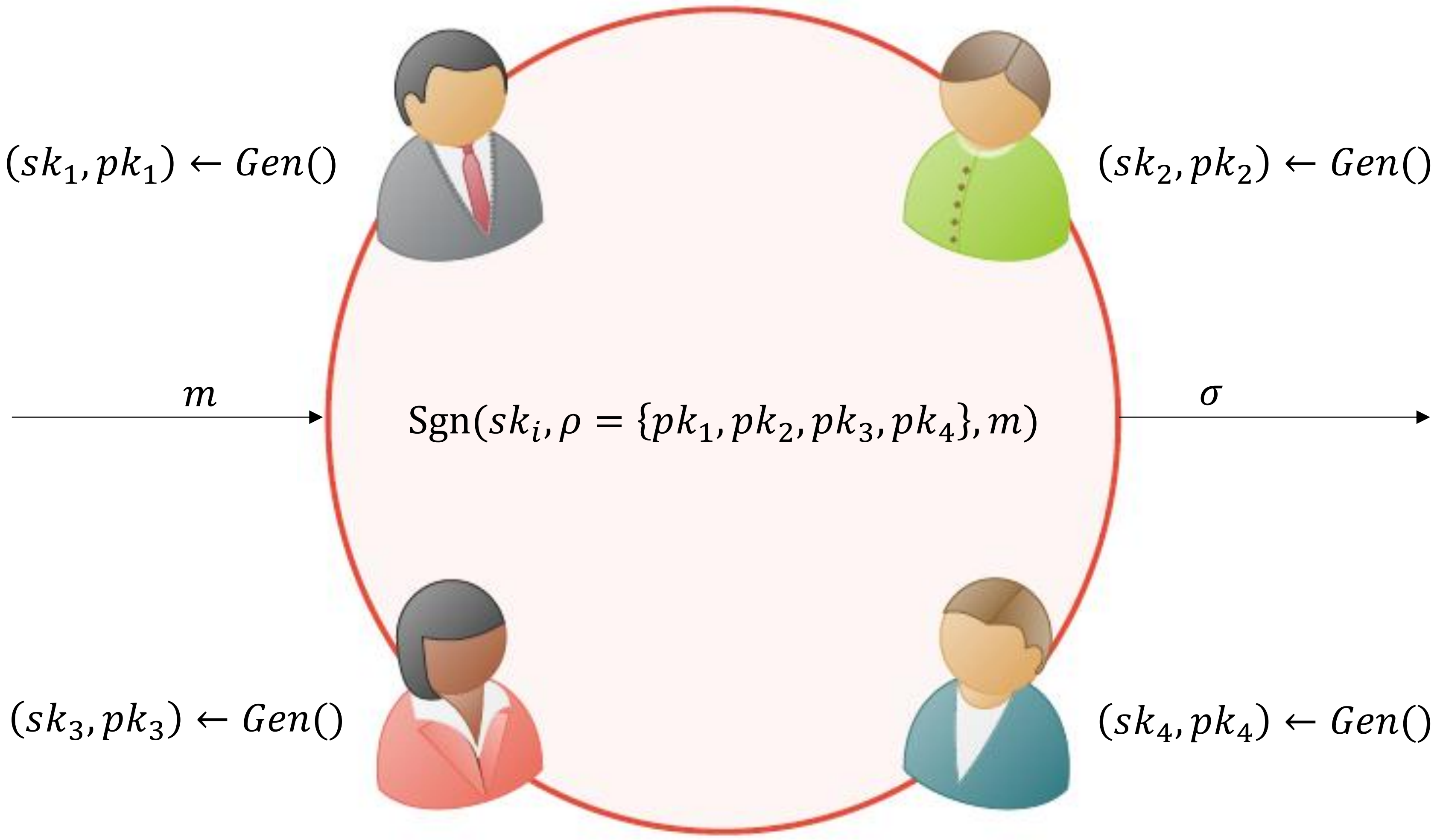
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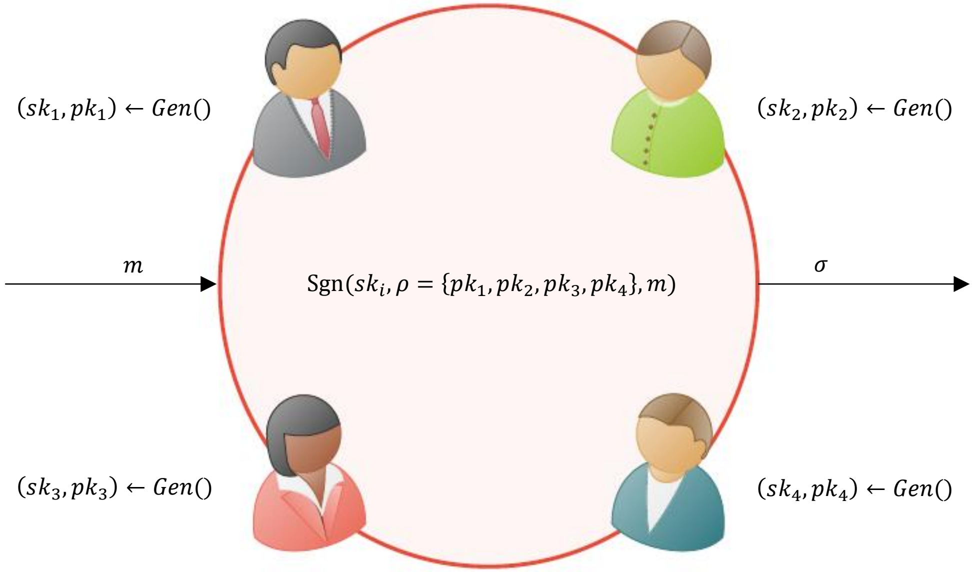
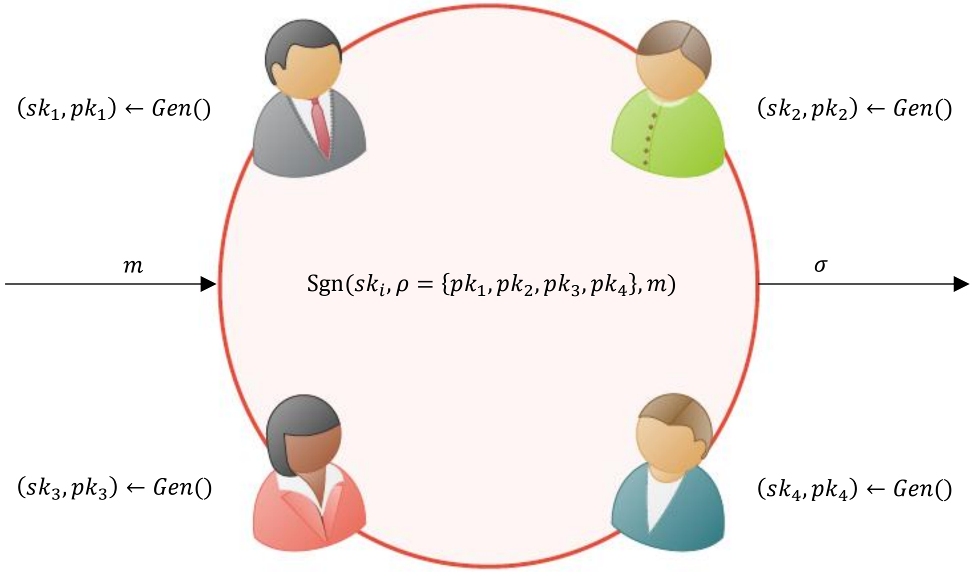


- **Unforgeability**: One sk_i is needed to generate σ.



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- **Anonymity**: Given σ and ρ, it is not possible to identify who signed.

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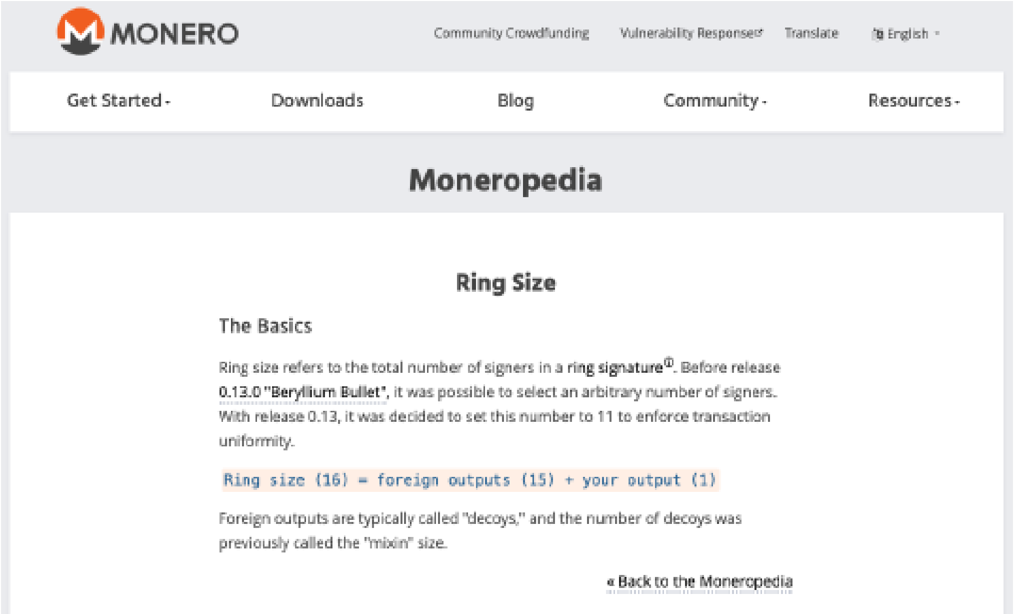

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Moneropedia

Ring Size

The Basics

Ring size refers to the total number of signers in a ring signature. Before release 0.13.0 "Beryllium Bullet", it was possible to select an arbitrary number of signers. With release 0.13, it was decided to set this number to 11 to enforce transaction uniformity.

```
Ring size (16) = foreign outputs (15) + your output (1)
```

Foreign outputs are typically called "decoys," and the number of decoys was previously called the "mixin" size.

« Back to the Moneropedia

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- Currently used in electronic voting systems.
- Primitive behind cryptocurrencies such as Dash & Monero.

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- $b \leftarrow Ver(\sigma, \rho, m)$: Given a signature σ, a ring ρ, and a message m, the deterministic verification algorithm Ver returns a bit $b \in \{0, 1\}$.

RSig Properties—Correctness

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RSig is $\delta(\kappa)$-correct if $\forall\kappa \in \mathbb{N}, \{(pk_i, sk_i)\}_{i\in[k]} \in \sup(Gen)$, and for any $i \in [k]$ with $k \le \kappa$,

$$\Pr[\mathrm{Ver}\,(\mathrm{Sgn}(sk_i, \rho, m), \rho, m) \neq 1] \le \delta(\kappa)$$

where $\rho := \{pk_1, \cdots, pk_k\}$.

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Given a target set of public keys $\rho = \{pk_1, pk_2, \ldots, pk_n\}$, there doesn't exist an adversary that can forge a signature σ^* on a message m^* and a ring $\rho^* \subseteq \rho$.

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The adversary is allowed to make **adaptive signing queries** on a message m_i and ring ρ_i, as long as the ring contains at least one of the supplied keys from ρ.

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Guess the index of the real signer.

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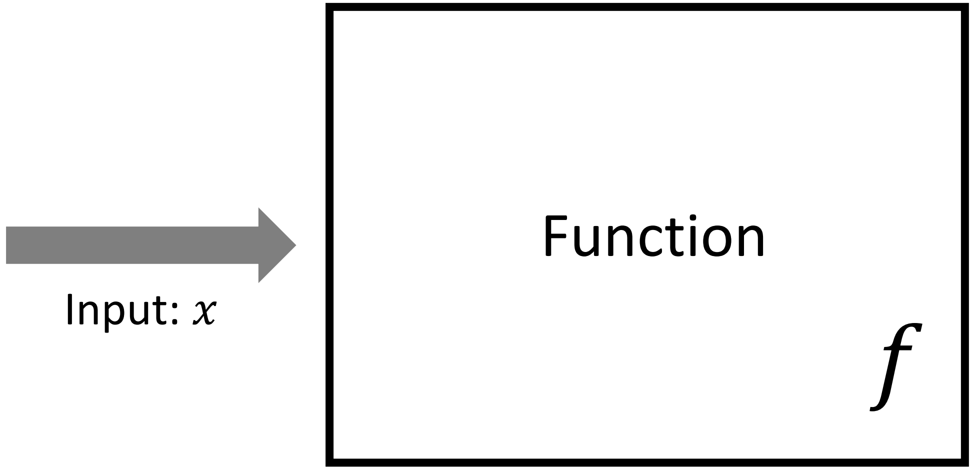
   Permits black-box enhancements (e.g., ring size scaling) without protocol redesign.

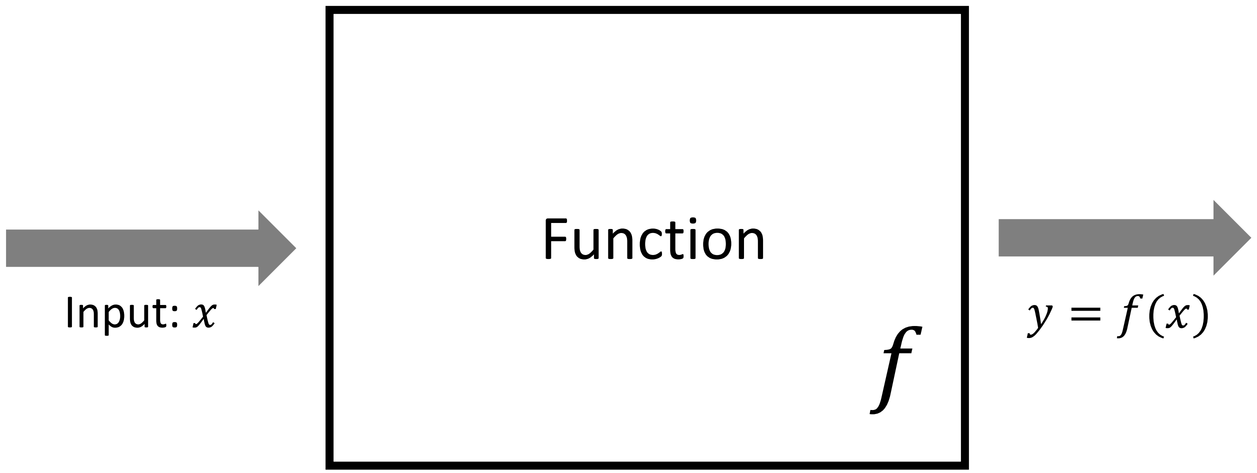
3. **Primitive Reusability**
   Provides building blocks for other anonymity-preserving cryptosystems

Trapdoor Functions

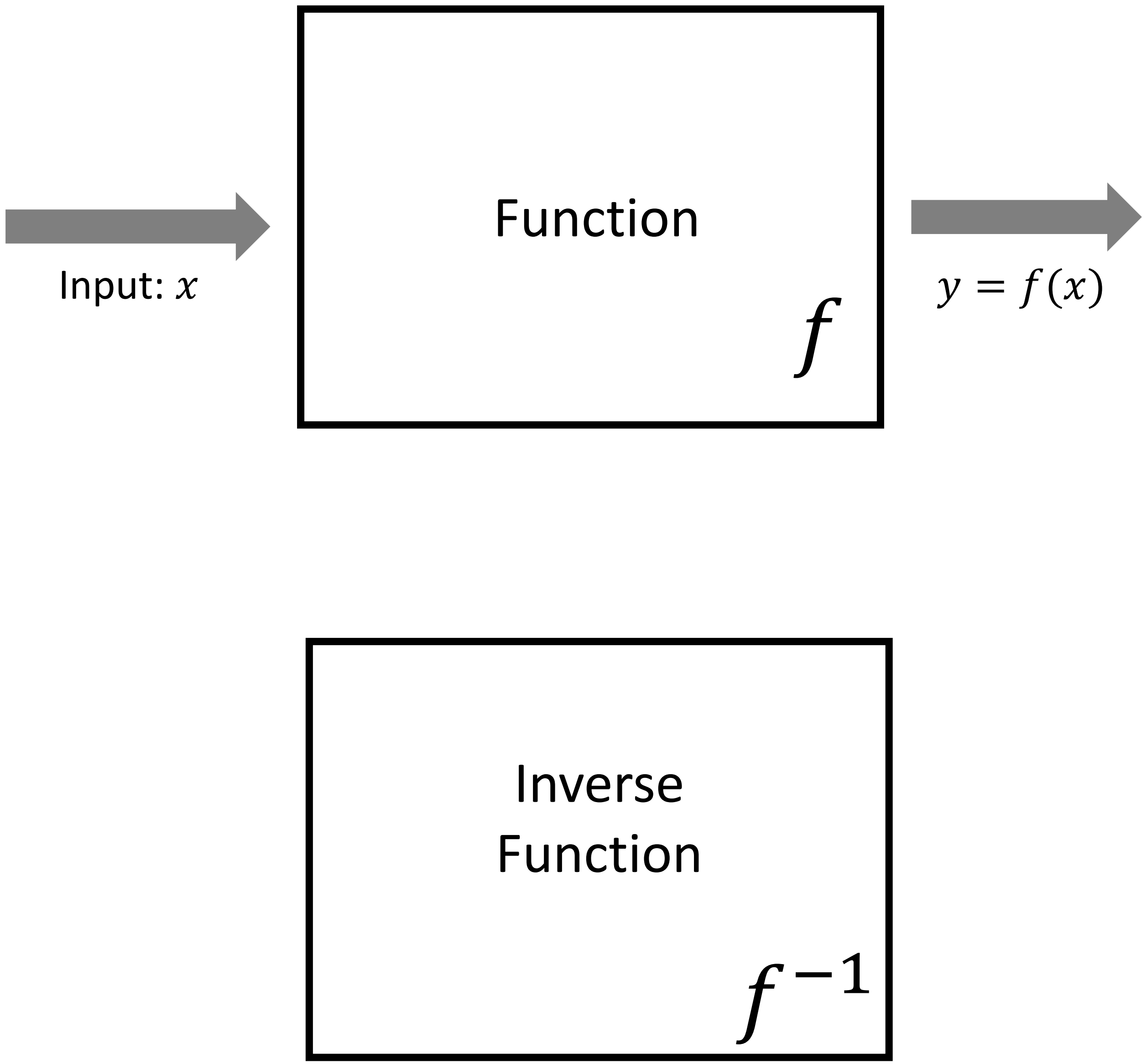
Function

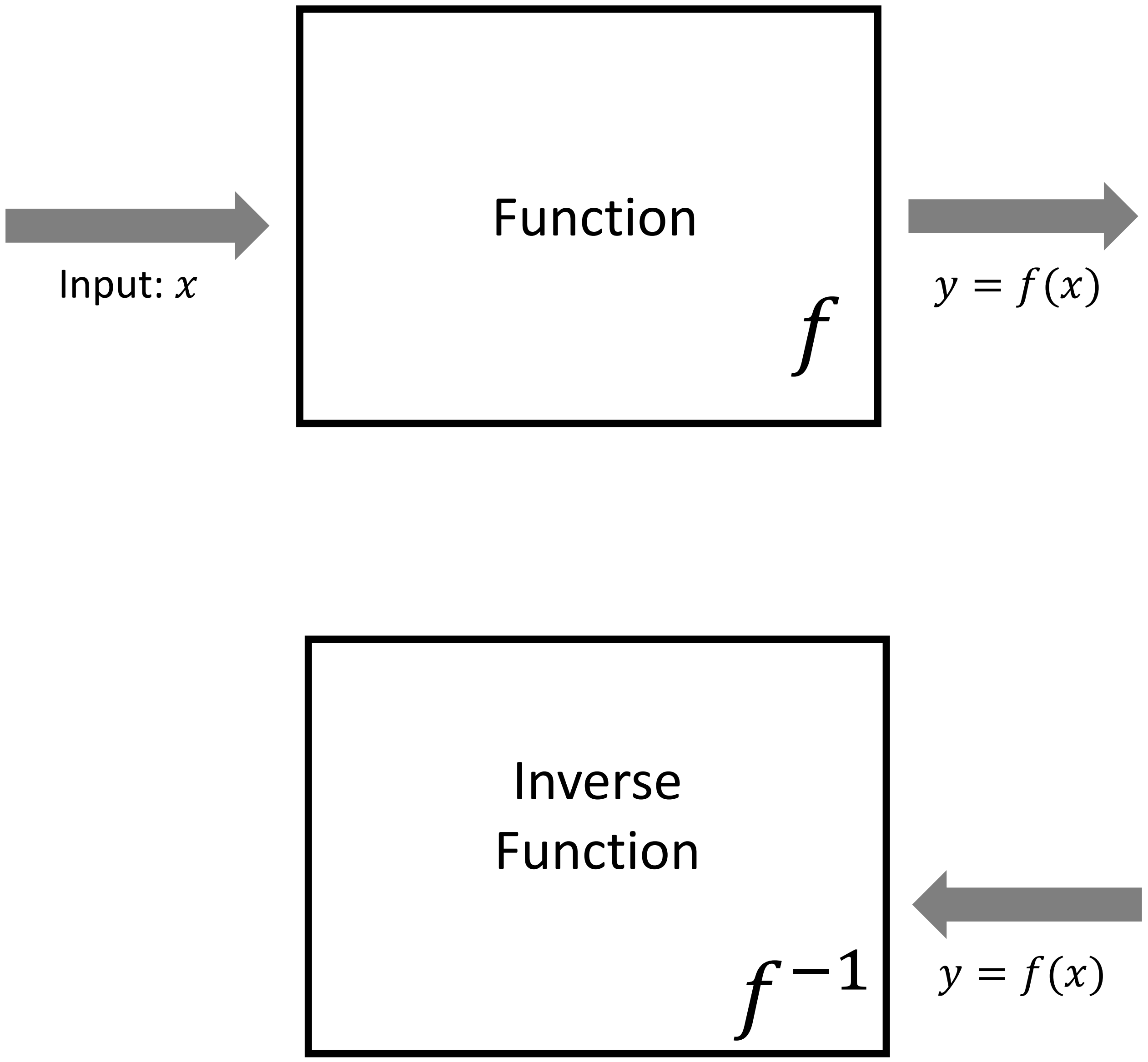
f


Function
Input: x
f



Function
Input: x
f
$y = f(x)$


Input: x
Function
f
$y = f(x)$
Inverse
Function
f^{-1}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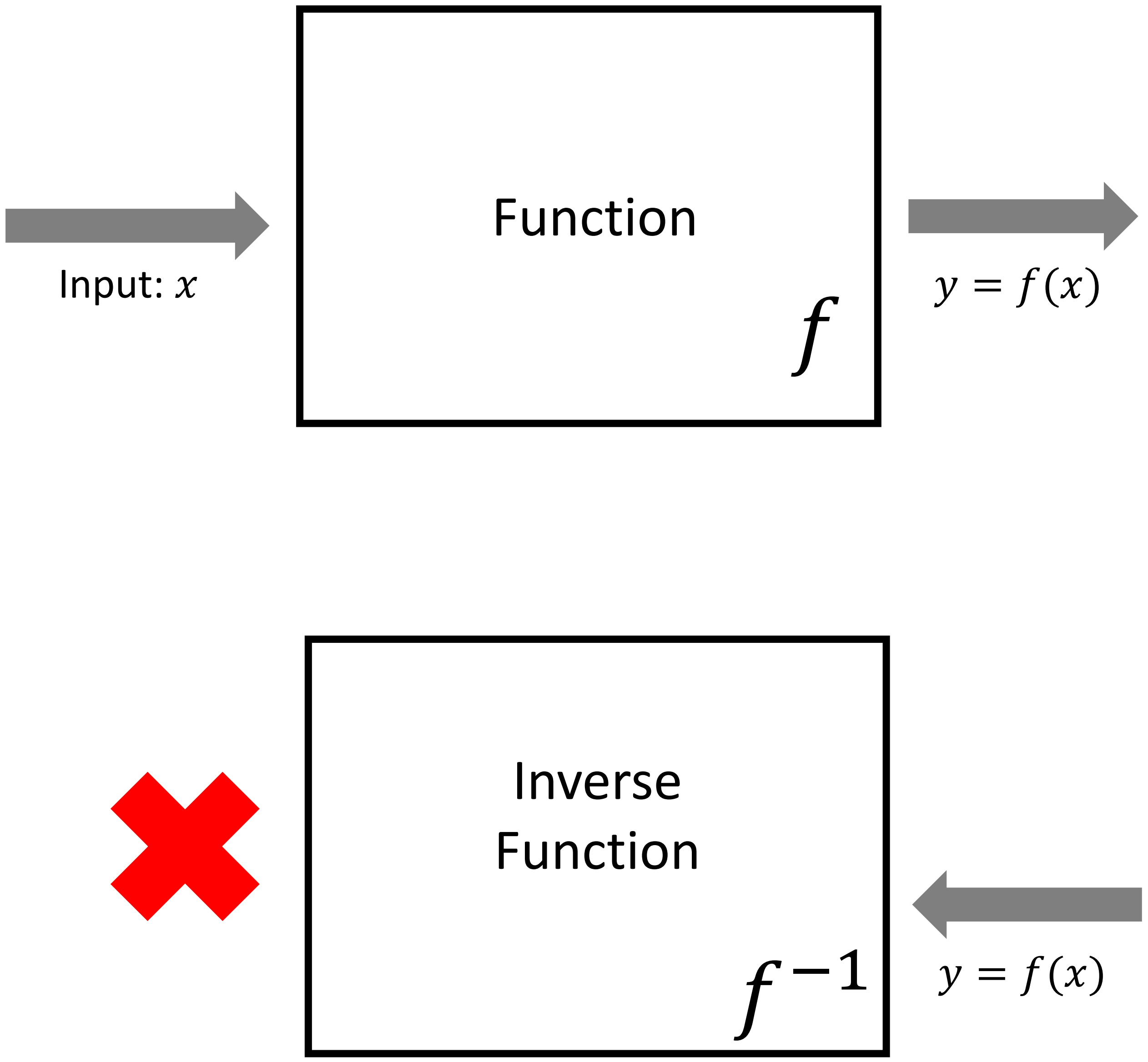
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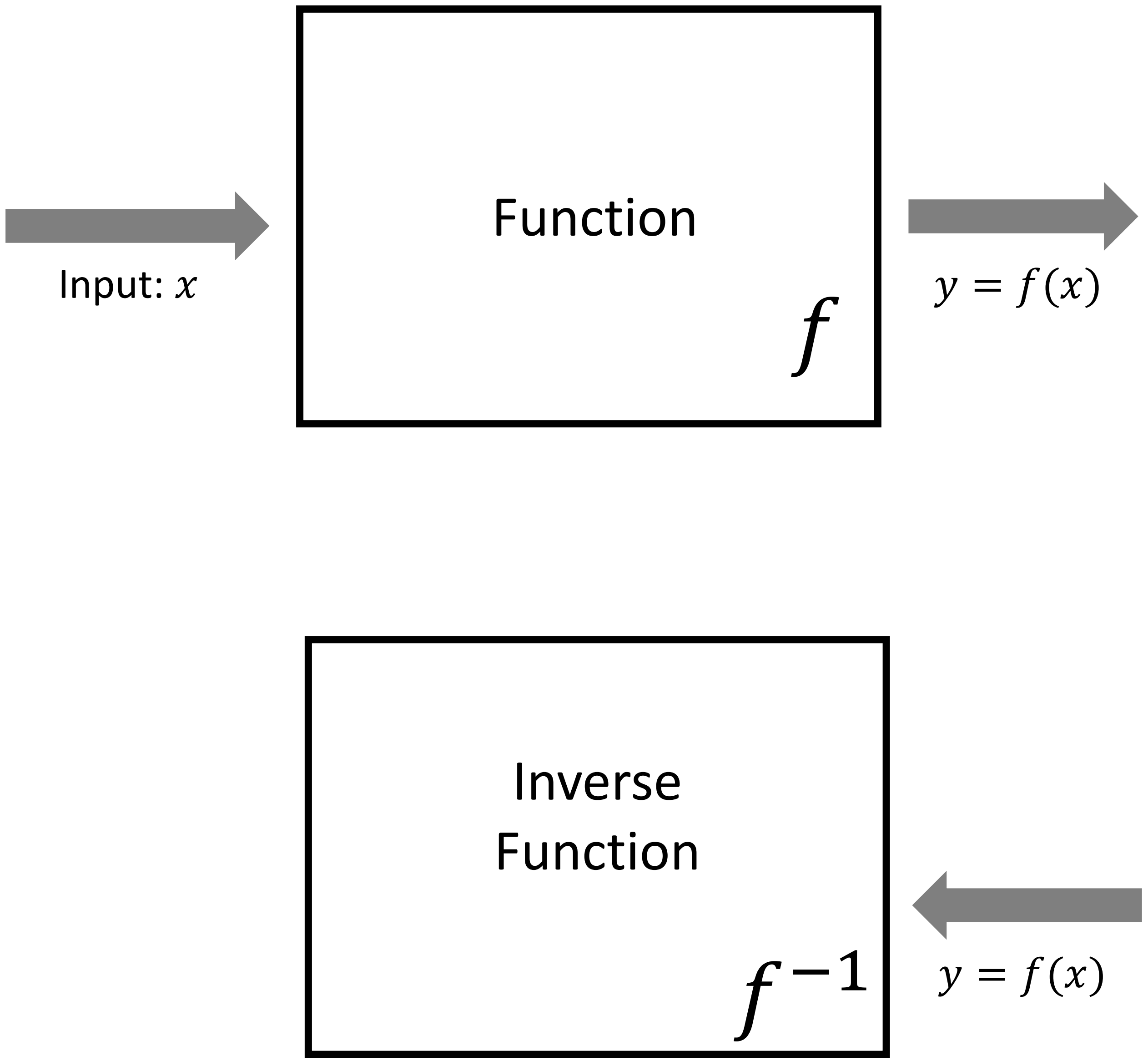
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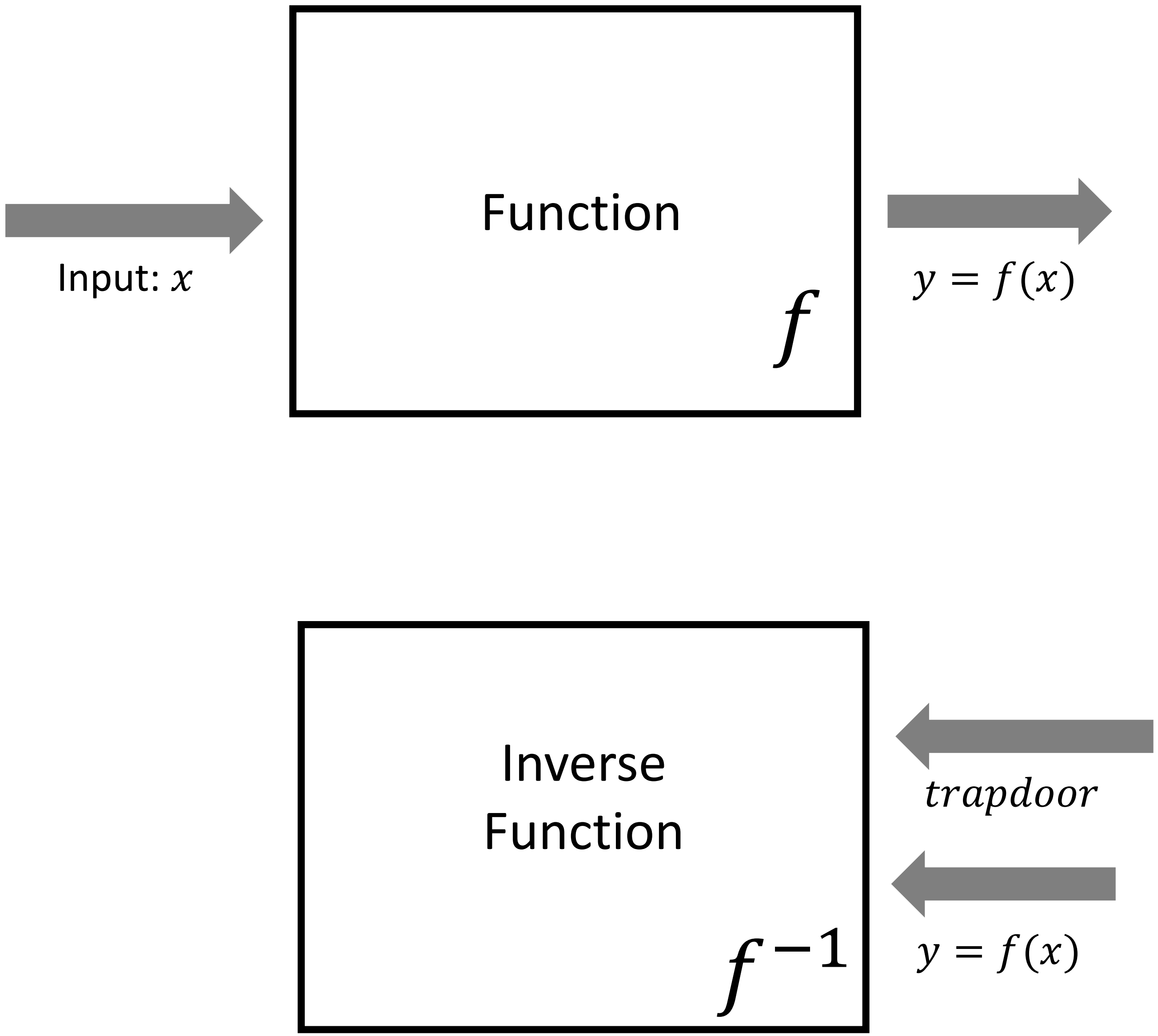
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$trapdoor$
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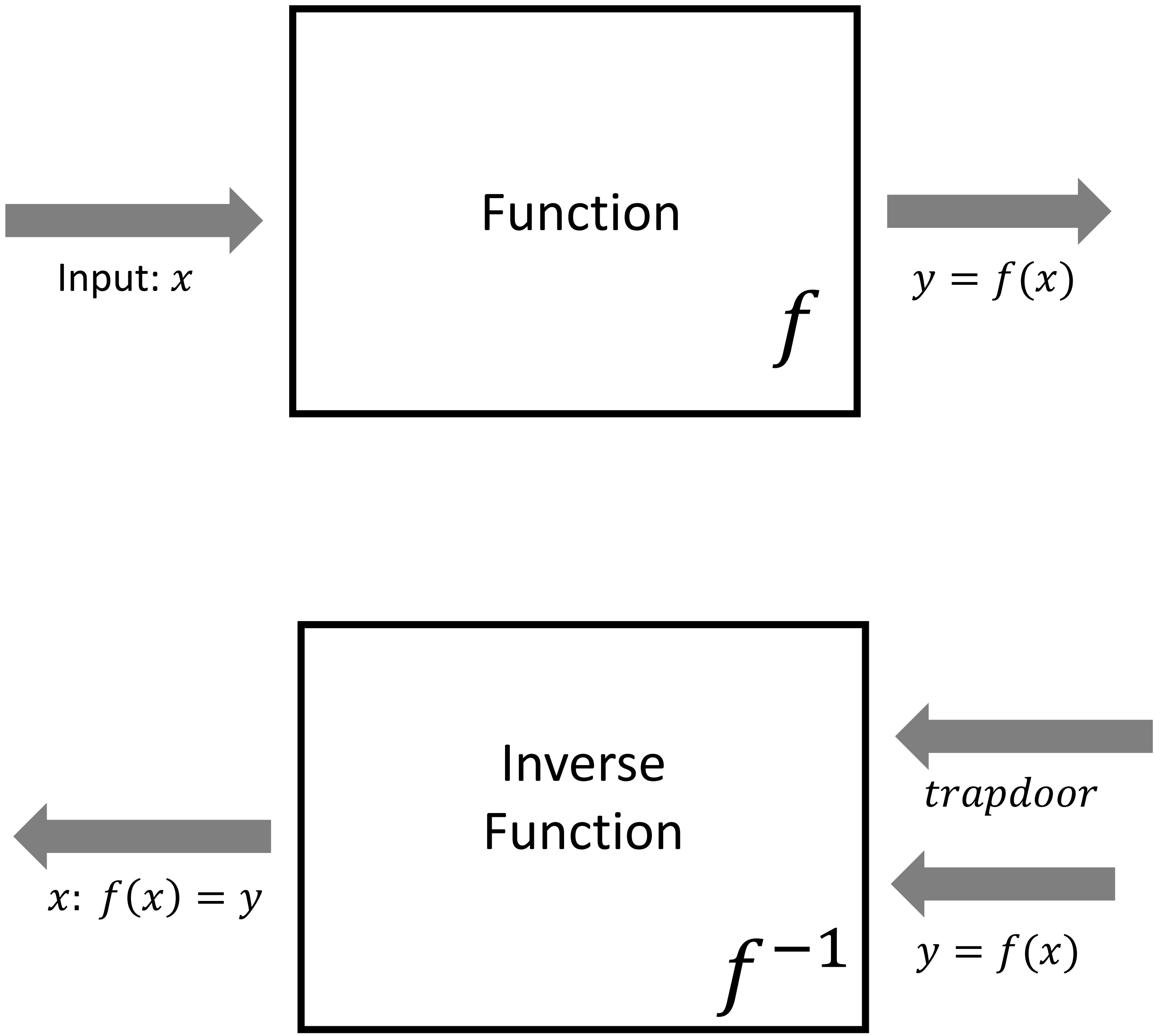
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x: $f(x) = y$

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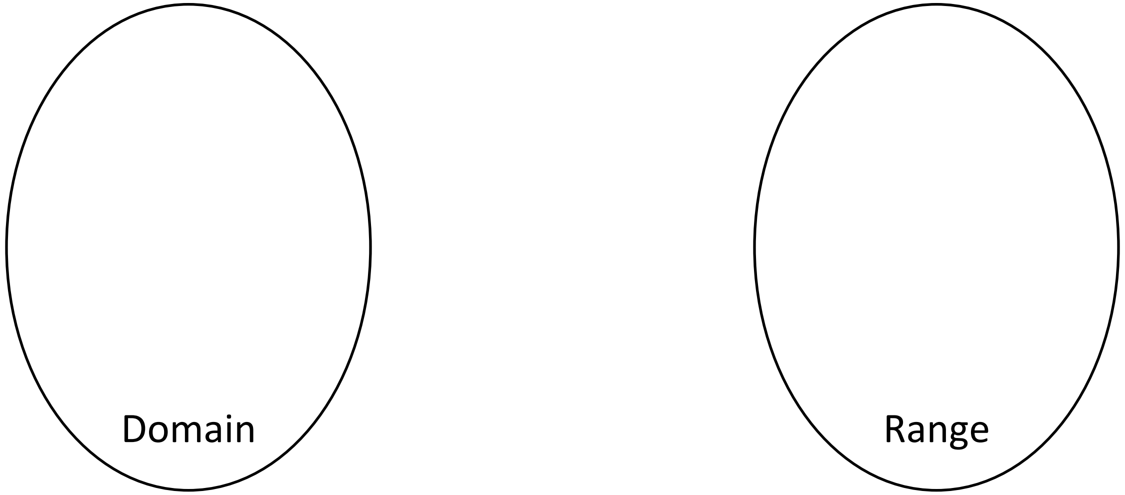

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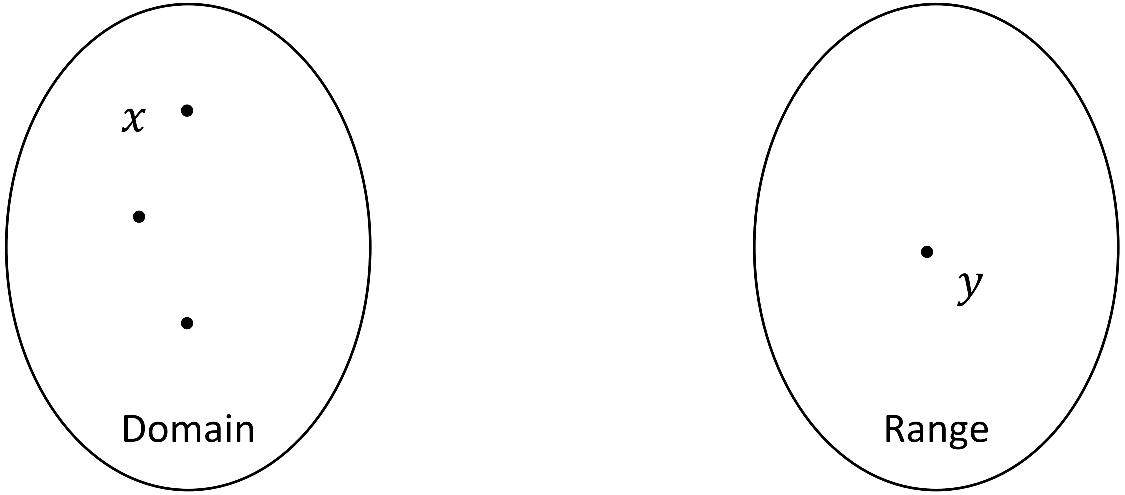

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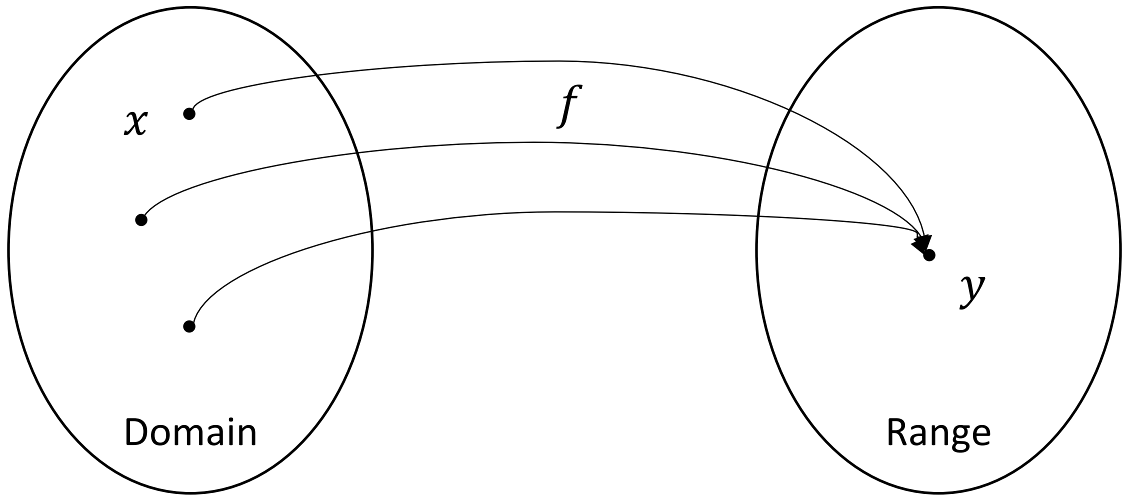

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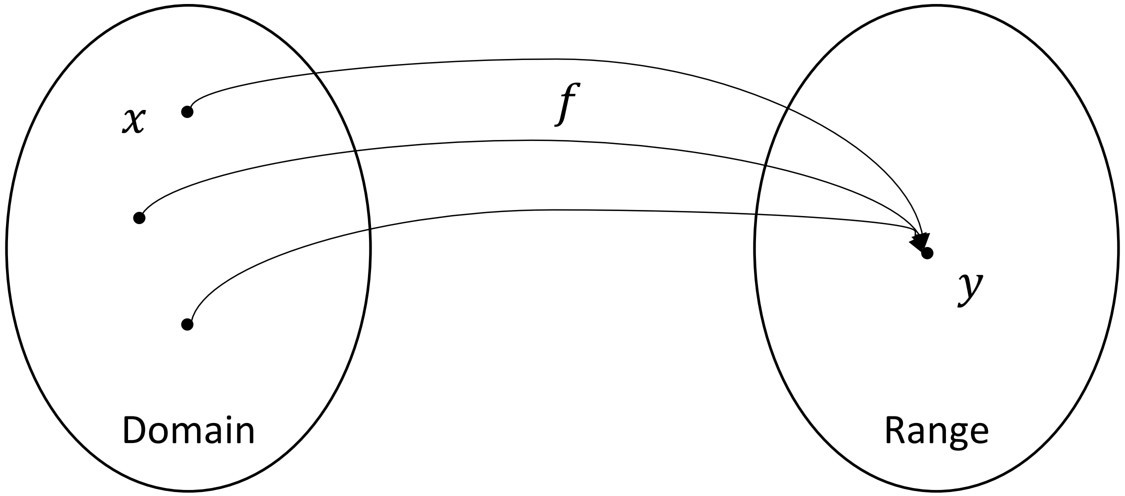

- Sample inputs so their outputs look random.

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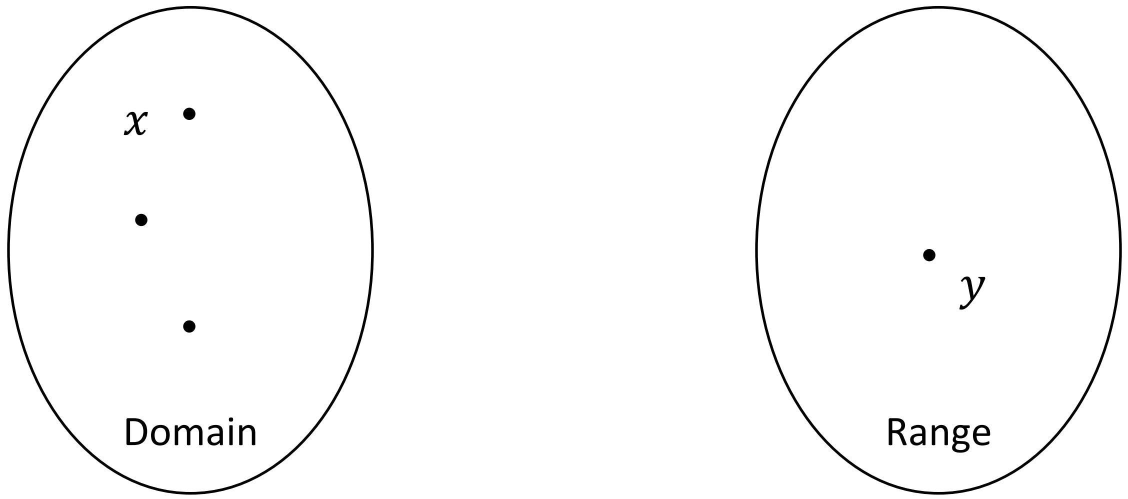

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- Given an output and the trapdoor, produce a matching input that looks like a real, random input.

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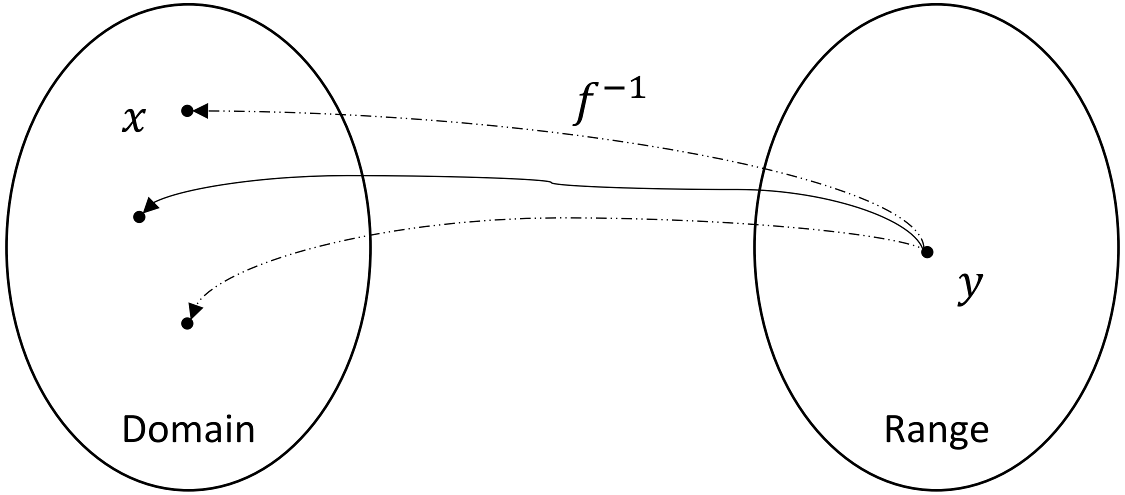

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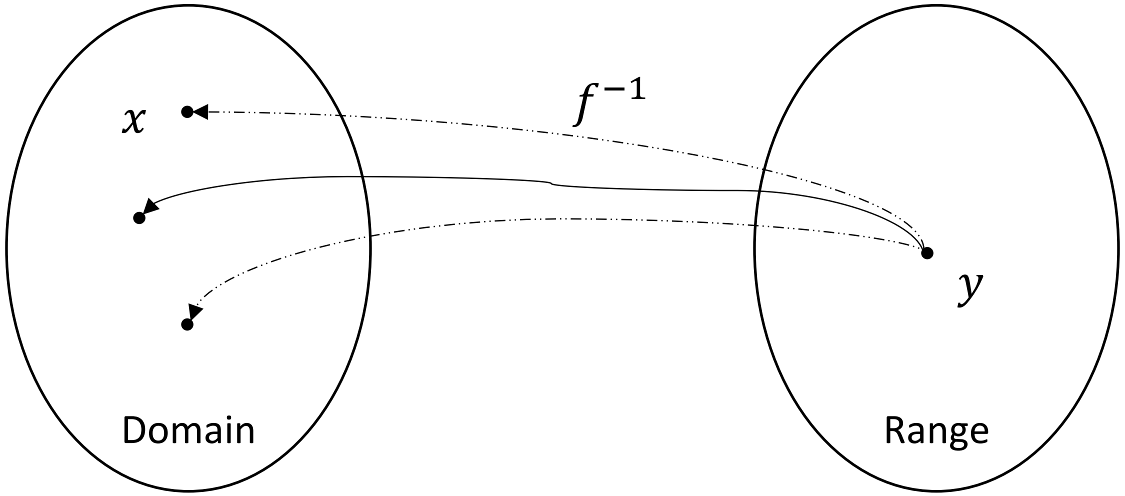

- Sample inputs so their outputs look random.
- Given an output and the trapdoor, produce a matching input that looks like a real, random input.
- But without the trapdoor, you can't figure out the input — it's still a one-way function.

Ring Preimage Sampleable Trapdoor Function

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- $x \leftarrow Smp(\rho = \{a_1, \ldots, a_k\})$: Given ring ρ, returns a sampled $x \in D$.

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5. **Anonymity**: Even if the adversary knows all trapdoors and can make inverse queries, they shouldn't be able to tell which trapdoor was used to invert the function.

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One-wayness without trapdoor

$$\mathsf{Adv}^{\mathsf{OW}(\kappa,k)}_{\mathsf{RTDF},\mathcal{A}} := \max_{k} \left(\Pr\left[\mathsf{OW}^{\mathsf{RTDF}}_{\mathcal{A}}(\kappa,k) = 1\right]\right)$$

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Game $\mathsf{OW}^{\mathsf{RTDF}}_{\mathcal{A}}(\kappa,k)$

1: $par \xleftarrow{\$} \mathsf{Stp}(\kappa)$
2: **for** $i \in [k]$ **do**
3: $\quad (a_i, t_i) \xleftarrow{\$} \mathsf{Gen}()$
4: **end for**
5: $\rho \leftarrow \{a_1, \ldots, a_k\}$
6: $y \xleftarrow{\$} \mathcal{R}$
7: $x' \xleftarrow{\$} \mathcal{A}(par, \rho, y)$
8: **return** 1 if $\mathsf{Eval}(\rho, x') = y$, else 0

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5: $b \xleftarrow{\$} \{0,1\}$
6: $b' \xleftarrow{\$} \mathcal{A}^{\mathsf{Sample}}(par, \{a_1, \ldots, a_n\})$
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Oracle $\mathsf{Sample}(\rho = \{a_{i_1},\dots,a_{i_k}\})$

if $k > \kappa$ **or** $\exists a(a \in \rho \wedge a \notin \{a_1,\dots,a_n\})$ **then**
  return $\perp$
end if
if $b = 0$ **then**
  $x \xleftarrow{\$} \mathsf{Smp}(\rho)$
  $y \leftarrow \mathsf{Eval}(\rho, x)$
else
  $y \xleftarrow{\$} \mathcal{R}$
end if
return y

An adversary should not be able to tell whether a value came from the ring function's real input-output process or was just randomly chosen from the output space.

Preimage Sampling

An adversary shouldn't be able to tell whether a preimage was generated using the trapdoor or sampled directly and conditioned to match the output.

Preimage Sampling

$$\mathsf{Adv}^{\mathsf{PS}(n,\kappa)}_{\mathsf{RTDF},\mathcal{A}} := \left|\Pr\left[\mathsf{PS}^{\mathsf{RTDF}}_{\mathcal{A}}(n,\kappa) = 1\right] - \frac{1}{2}\right|$$

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Game $\mathsf{PS}^{\mathsf{RTDF}}_{\mathcal{A}}(n,\kappa)$

$par \xleftarrow{\$} \mathsf{Stp}(\kappa)$
for $i \in [n]$ **do**
    $(a_i, t_i) \xleftarrow{\$} \mathsf{Gen}()$
end for
$b \xleftarrow{\$} \{0,1\}$
$b' \xleftarrow{\$} \mathcal{A}^{\mathsf{Challenge}}(par, \{a_1, \ldots, a_n\})$
return $[b' = b]$

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Oracle $\mathsf{Challenge}(\rho = \{a_{i_1}, \ldots, a_{i_k}\}, y \in \mathcal{R})$

if $k > \kappa$ **or** $\exists a(a \in \rho \wedge a \notin \{a_1, \ldots, a_n\})$ **then**
  return $\perp$
end if
if $b = 0$ **then**
  $t \xleftarrow{\$} \{\mu(a) \mid a \in \rho\}$
  $x \xleftarrow{\$} \mathsf{Inv}(t, \rho, y)$
else
  Sample $x \xleftarrow{\$} \mathsf{Smp}(\rho)$ until $f_\rho(x) = y$
end if
return x

An adversary shouldn't be able to tell whether a preimage was generated using the trapdoor or sampled directly and conditioned to match the output.

Anonymity

Even if the adversary knows all trapdoors and can make inverse queries, they shouldn't be able to tell which trapdoor was used to invert the function.

Anonymity

$$\mathsf{Adv}_{\mathrm{RTDF},\mathcal{A}}^{(n,\kappa,Q_{\mathsf{OInv}})\text{-MC-Ano}} := \left|\Pr[(n,\kappa,Q_{\mathsf{OInv}})\text{-MC-Ano}_{\mathrm{RTDF}}(\mathcal{A}) \to 1] - \frac{1}{2}\right|$$

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for $i \in [n]$ **do**
    $(a_i, t_i) \xleftarrow{\$} \mathrm{Gen}$
end for
$b \xleftarrow{\$} \{0,1\}$
$b' \xleftarrow{\$} \mathcal{A}^{\mathrm{OInv}}(par, (a_1,t_1),\ldots,(a_n,t_n))$
return $[b = b']$

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return $[b = b']$

Oracle $\mathrm{OInv}(i_0 \in [n], i_1 \in [n], \rho = \{a_1, \ldots, a_{k \le n}\}, x)$

if $(\rho \subseteq \{a_1, \ldots, a_n\}) \wedge (a_{i_0} \in \rho) \wedge (a_{i_1} \in \rho)$ **then**
  $y \leftarrow \mathrm{Eval}(\rho, x)$
  return y
else
  return $\perp$
end if

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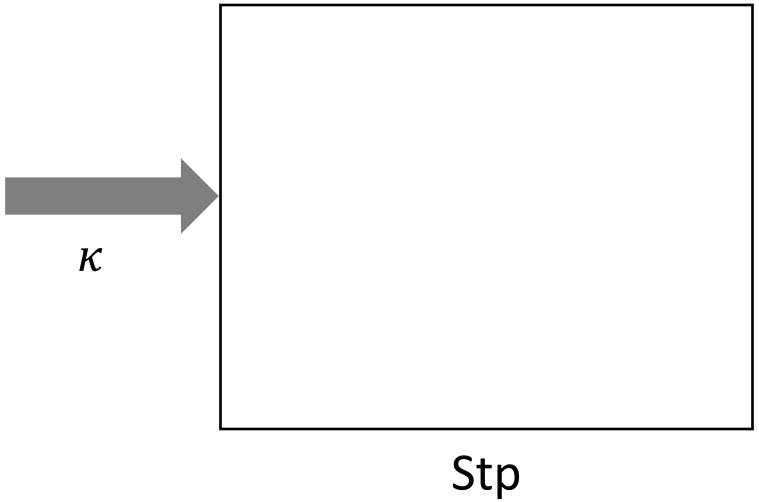
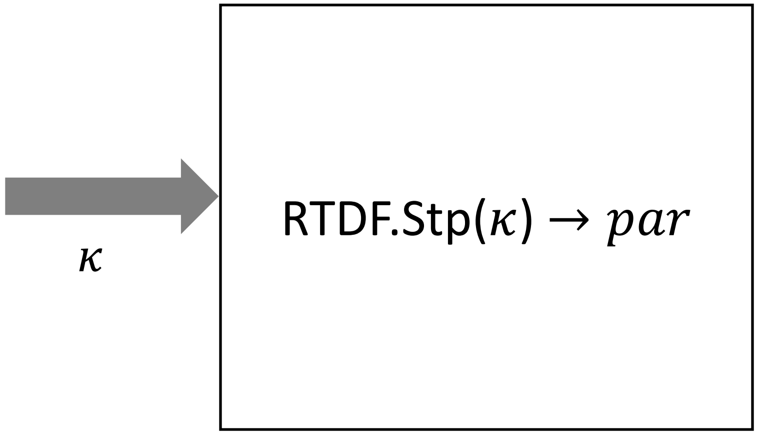
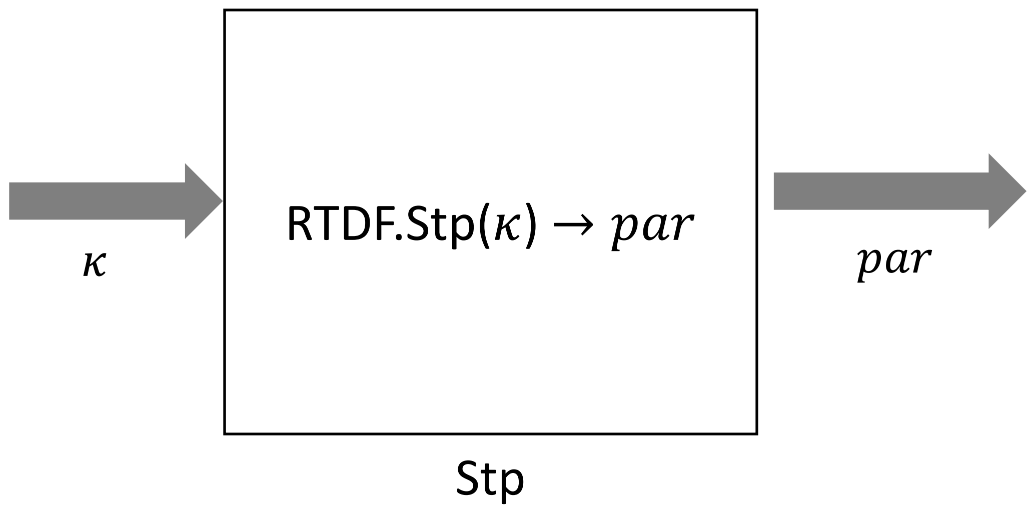
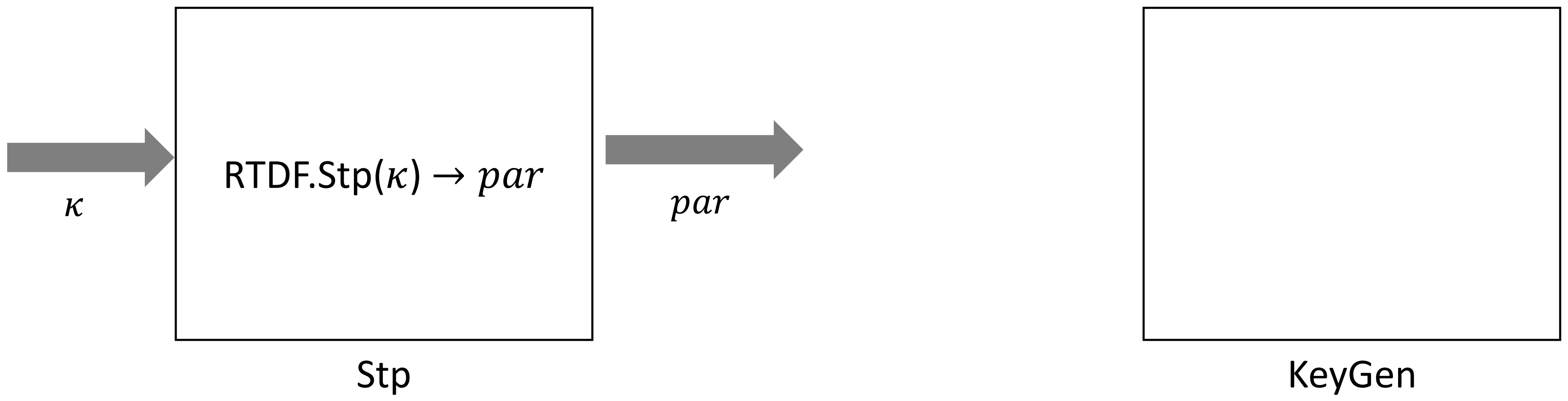
RTDF → RSig


κ
Stp



Stp


κ
RTDF.Stp(κ) $\rightarrow par$
par
Stp


κ
RTDF.Stp$(\kappa) \rightarrow par$
par
Stp
KeyGen

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$

KeyGen

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$

KeyGen

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$

KeyGen

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$
4. s$k \coloneqq t$

KeyGen

κ → RTDF.Stp(κ) $\rightarrow par$ → par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$
4. sk $\coloneqq t$

→ (pk, sk)

KeyGen

κ

RTDF.Stp(κ) $\to par$

par

Stp

1. Stp(κ) $\to par$
2. RTDF.Gen() $\to (a, t)$
3. $pk \coloneqq a$
4. sk $\coloneqq t$

(pk, sk)

KeyGen

Sign

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$
4. sk $\coloneqq t$

(pk, sk)

KeyGen

sk

ρ

m

Sign

κ → **Stp** → par

RTDF.Stp$(\kappa) \rightarrow par$

Stp

$\rightarrow (pk, sk)$

1. Stp$(\kappa) \rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$
4. sk $\coloneqq t$

KeyGen

Inputs: sk, ρ, m

1. $H(m) \rightarrow y \in R$

Sign

κ

RTDF.Stp(κ) $\rightarrow$ par

par

Stp

1. Stp(κ) $\rightarrow$ par
2. RTDF.Gen() $\rightarrow$ (a, t)
3. $pk \coloneqq a$
4. sk $\coloneqq$ t

(pk, sk)

KeyGen

sk

ρ

m

1. $H(m) \rightarrow y \in R$
2. RTDF.Inv(sk, ρ, y) $\rightarrow$ x

Sign

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$
4. sk $\coloneqq t$

(pk, sk)

KeyGen

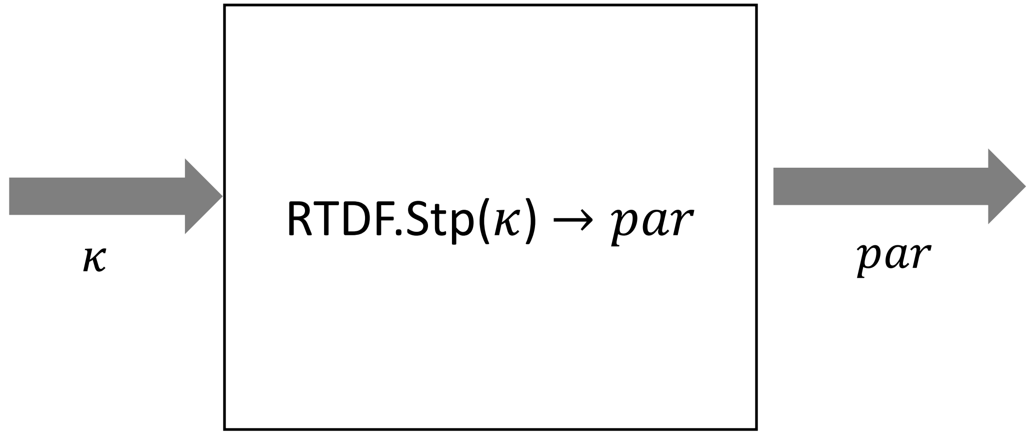
sk

ρ

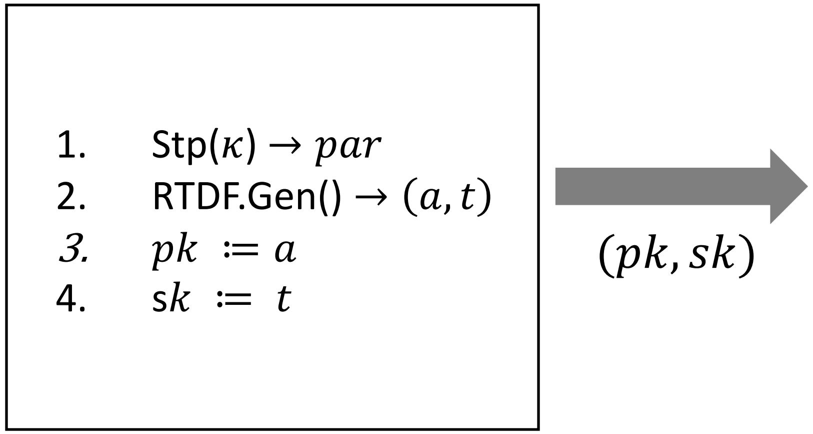
m

1. $H(m) \rightarrow y \in R$
2. RTDF.Inv(sk, ρ, y) $\rightarrow x$
3. $\sigma \coloneqq x$

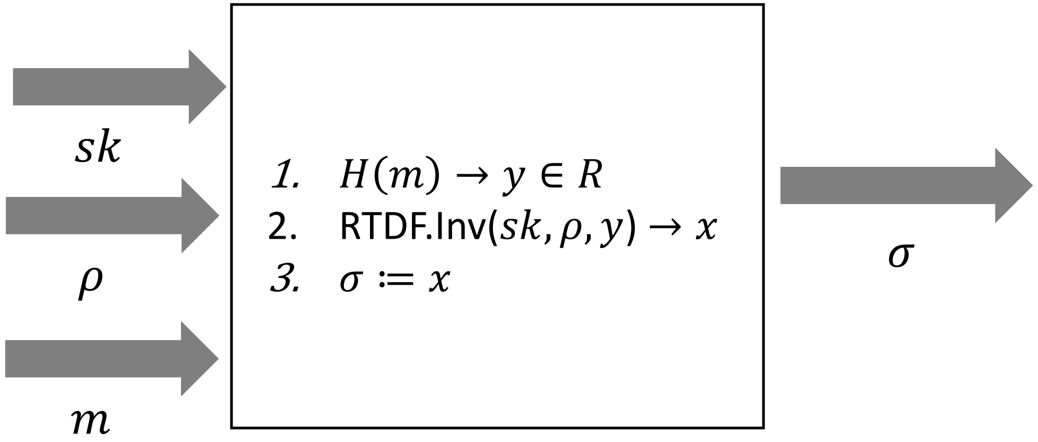
Sign



Stp



KeyGen



Sign

κ

RTDF.Stp(κ) $\rightarrow par$

par

Stp

1. Stp(κ) $\rightarrow par$
2. RTDF.Gen() $\rightarrow (a, t)$
3. $pk \coloneqq a$
4. sk $\coloneqq t$

(pk, sk)

KeyGen

sk

ρ

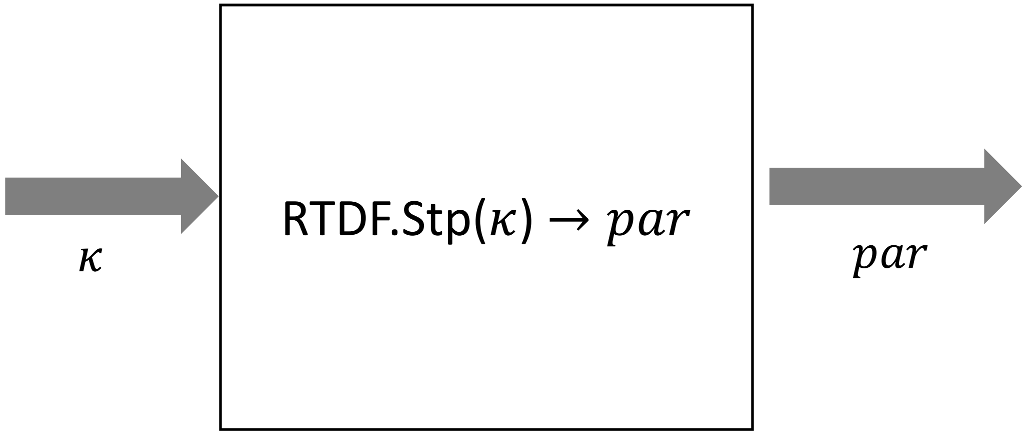
m

1. $H(m) \rightarrow y \in R$
2. RTDF.Inv(sk, ρ, y) $\rightarrow x$
3. $\sigma \coloneqq x$

σ

Sign

Sign



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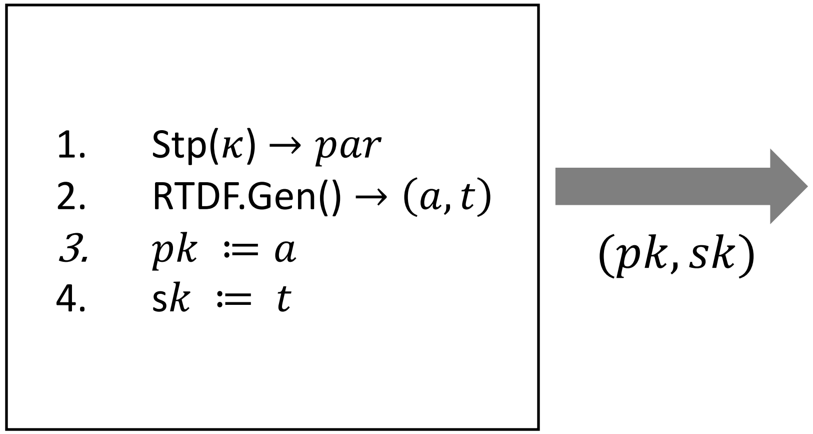
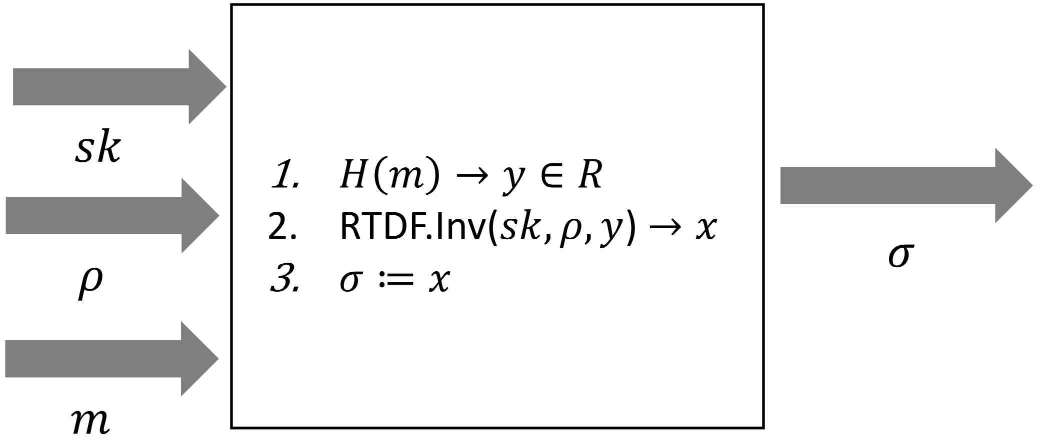


KeyGen



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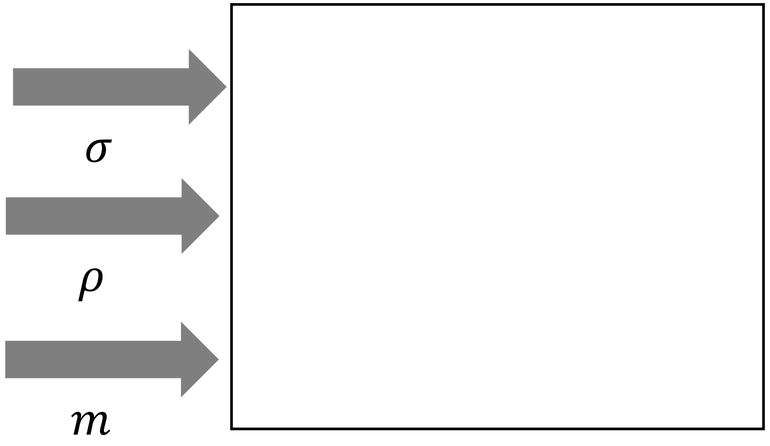

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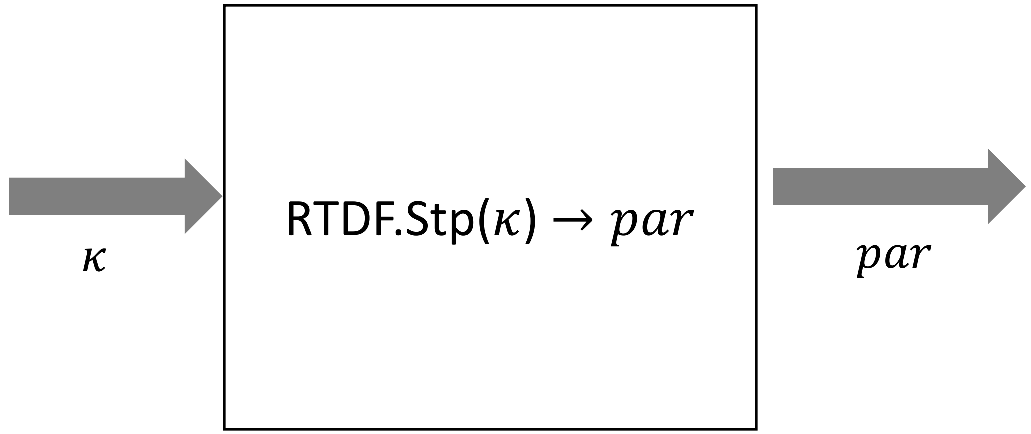

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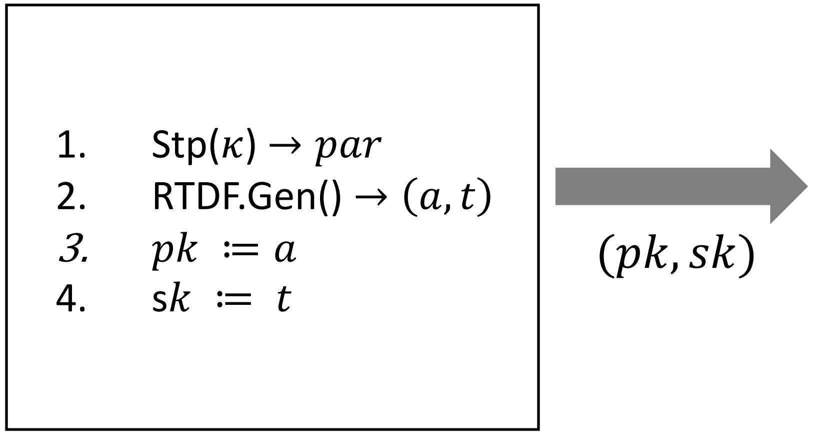

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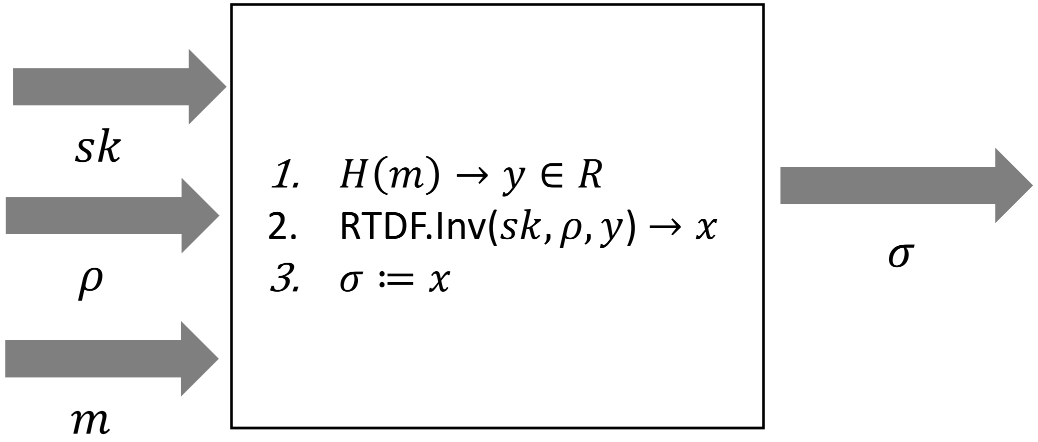

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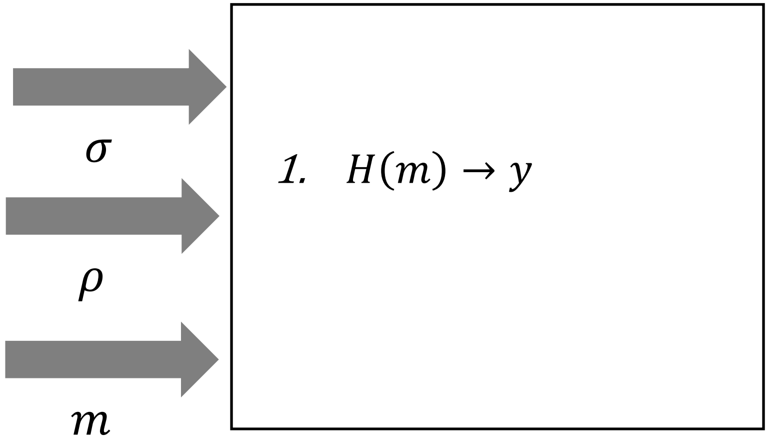

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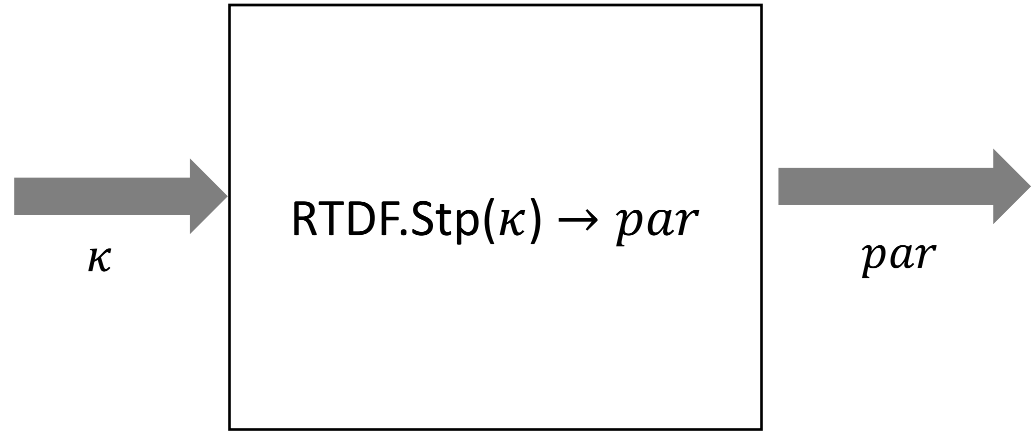

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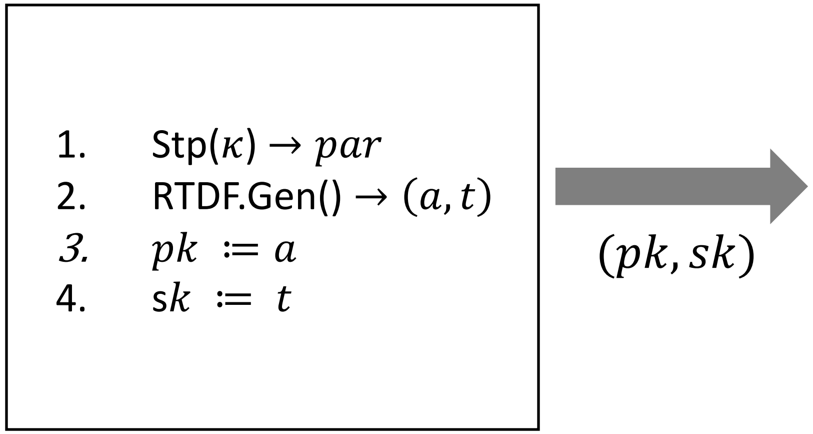

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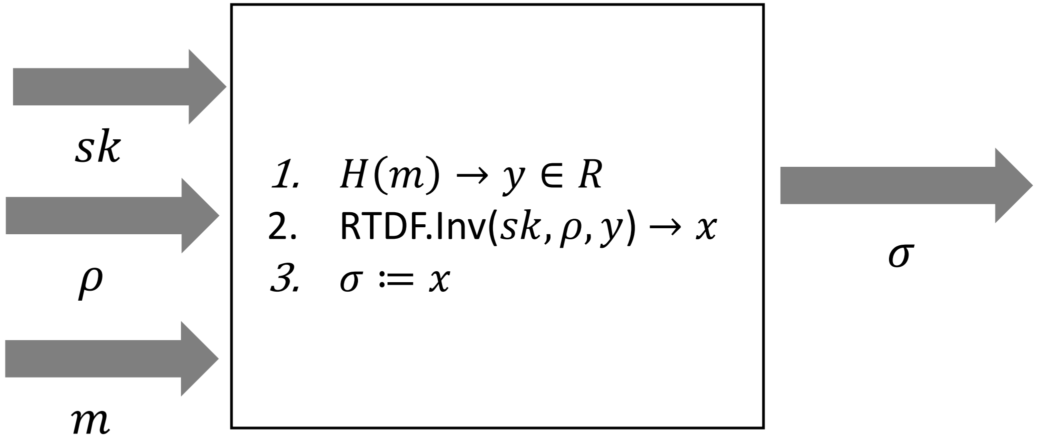

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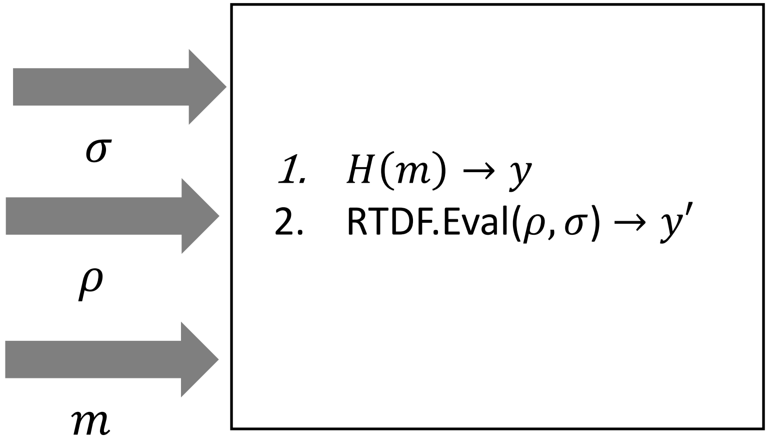

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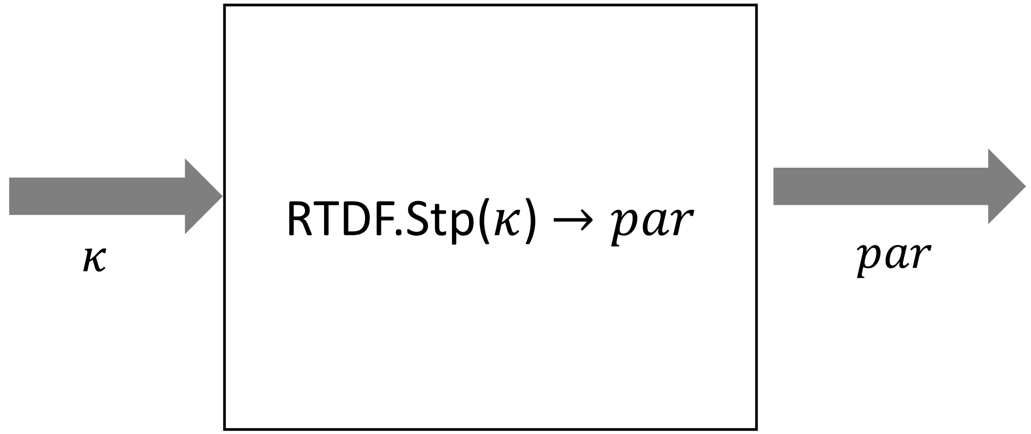

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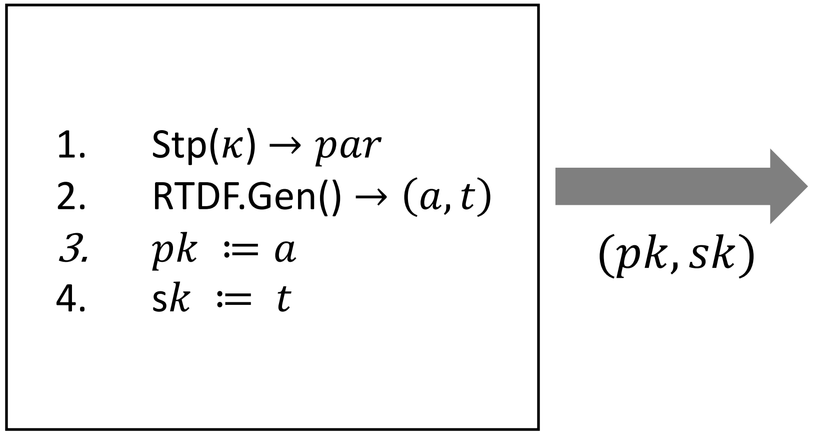

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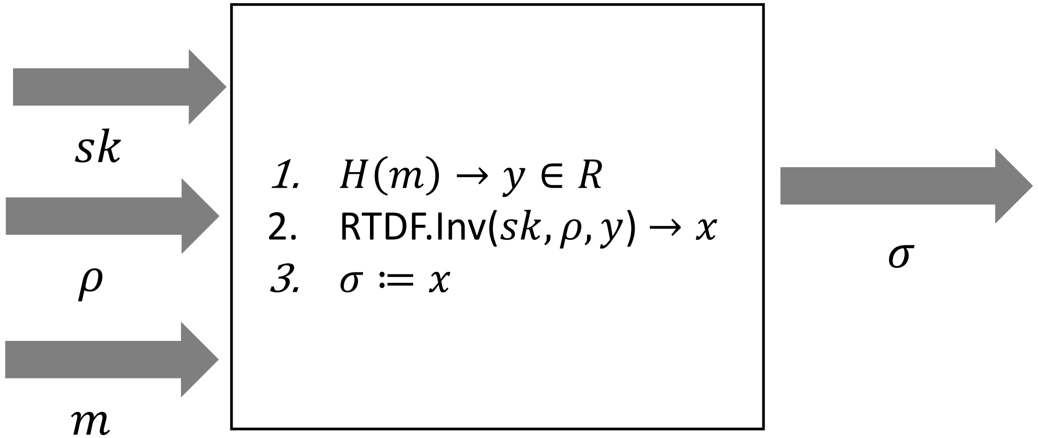

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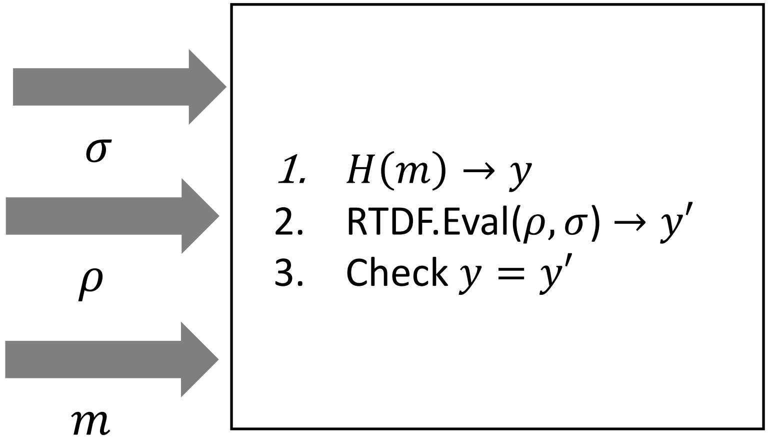

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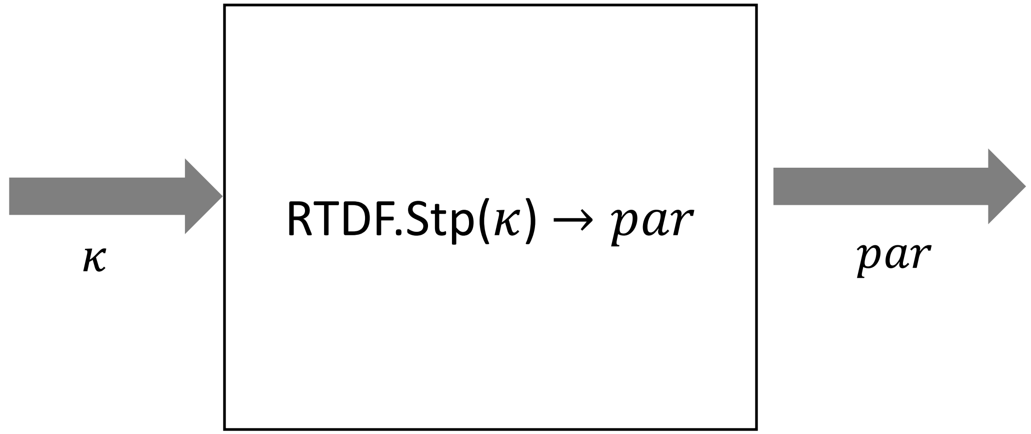

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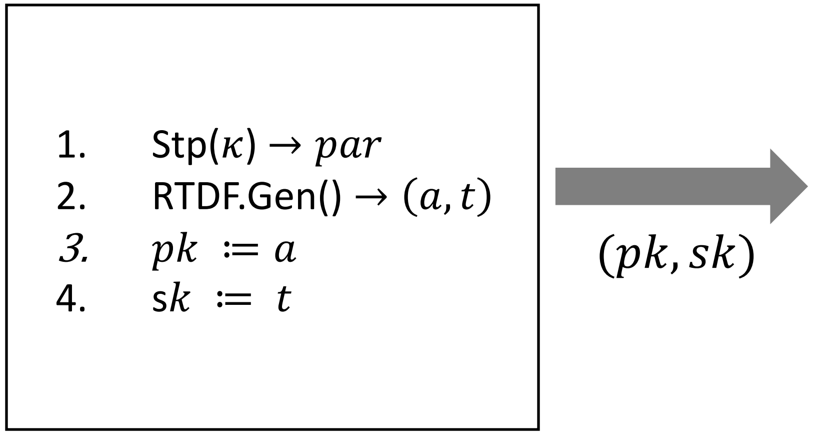

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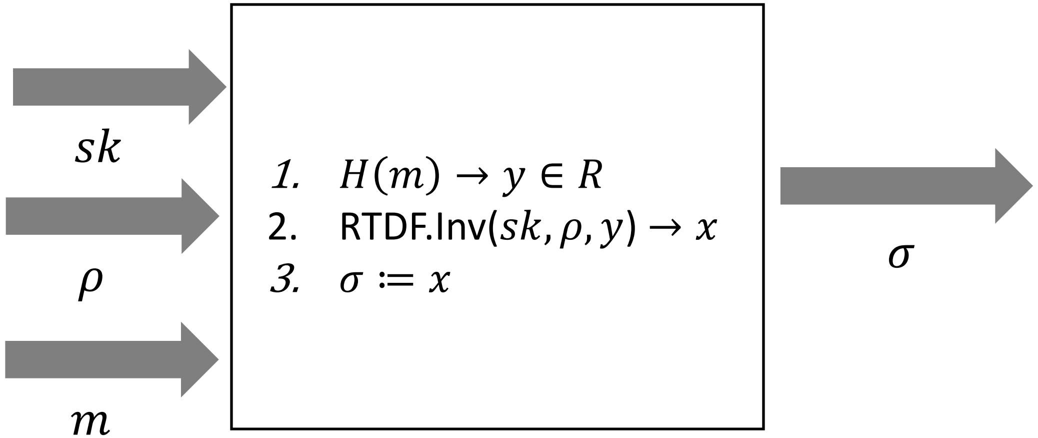

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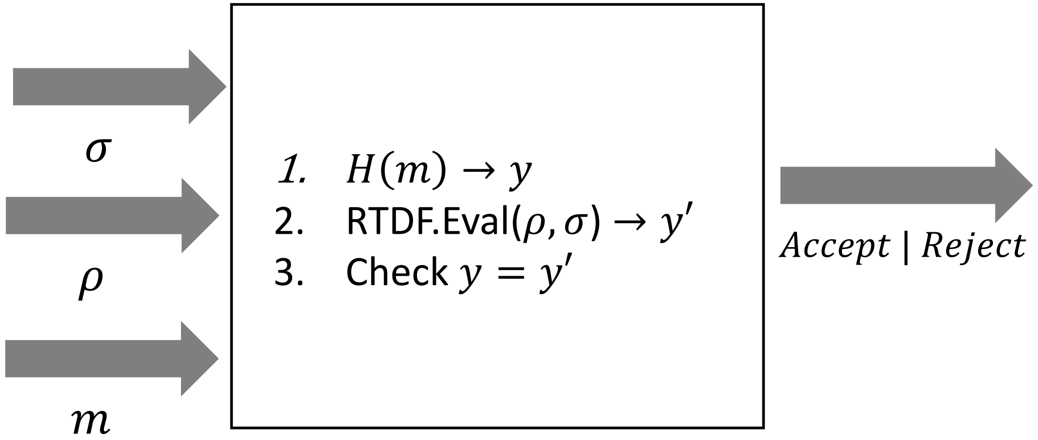

Sign

Summary

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Summary

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- RTDF allows improvements (e.g., scaling ring size) at the primitive level, propagating to ring signatures without redesign.
- RTDF serves as a building block for other anonymity-preserving primitives, promoting primitive reusability in cryptographic design.