

# CUSUM documentation

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## 1 What Is CUSUM?

CUSUM stands for cumulative sum control chart and is a statistical method which is used in change detection which is the process of detecting when the probability distribution function of a stochastic(random) process changes. It can be used for detecting regime changes and volatility shifts and is particularly sensitive to slow changes which accumulate over time. In the most standard form, the CUSUM metric is defined as follows:

$$S_0 = 0 \quad , \quad S_t = \sum_{i=0}^t (x_i - \mu_0) \quad n > 0$$

Where  $\mu_0$  is the reference mean and  $x_i$  is the  $i$ th data point collected. The control chart refers to a plot of  $S_t$  vs  $t$ . If  $S_t$  starts increasing over time for instance, this means  $x_i$  has been trending above the mean in recent time steps. When  $S_n$  becomes greater in absolute value than a pre-set threshold value  $h$ , a flag is put up that a change has occurred.

An equivalent definition is as follows:

$$S_0 = 0 \quad , \quad S_t = S_{t-1} + x_t - \mu_0$$

The above definition can be more efficient for implementation because only the previous term is needed to compute the next one, as opposed to the whole list of  $x_i$ 's. Another form of the metric is by using two different sums, one for tracking a trend in each direction and given by:

$$\begin{aligned} S_0^+ &= S_0^- = 0 \\ S_{t+1}^+ &= \max(0, S_t^+ + x_{t+1} - (\mu_0 + k)) \\ S_{t+1}^- &= \min(0, S_t^- + x_{t+1} - (\mu_0 - k)) \end{aligned}$$

Here  $k$  is a bias term and can be set to 0, but simply represents a threshold for how big a change should warrant a non-zero response from the cumulative sum.

The idea is in order to increase  $S^+$  from 0 to something positive,  $x_t$  must be greater than  $(\mu_0 + k)$ . Note by swapping the max for a min in the definition of  $S^-$ , we effectively reverse the direction of change we are tracking, only lowering  $S^-$  to negative numbers when  $x_n$  is sufficiently lower than the mean.

## 2 Pseudo code

The following function definitions are for each of the implementations outlined above.

```
def CUSUM_std (t,  $\mu$ , X, h):
    S = 0
    for i = 0 to t:
        S += X[i] -  $\mu$ 
    if |S| > h:
        print("flag: significant change")
    return S

def CUSUM_split(t,  $\mu$ , X, k, h):
    S_plus = 0
    S_minus = 0
    for i = 0 to t:
        S_plus = max(0, S_plus + X[i] - ( $\mu$  + k))
        S_minus = min(0, S_minus + X[i] - ( $\mu$  - k))
    if S_plus > h:
        print("flag: increase of sum beyond threshold")
    if S_minus < -h:
        print("flag: decrease of sum beyond threshold")
    return S_plus, S_minus
```