Intraday Volatility Monitor Report

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1 Abstract

This work investigate a reliable combination of algorithms that work together to detect intraday volatility from minute by minute returns from high-frequency financial time series data. The algorithms implementation are cumulative sum (control chart) (CUSUM), Page-Hinkley (PH), Bayesian Online Change Point Estimation (BOCPE). Individually they are fed the same live data and make their decision if volatility has changed and the overall algorithm takes all decisions into consideration to eliminate bias and reduce false alarms.

2 Introduction

2.1 Motivation

Financial markets function in environments marked by rapid information flow, structural discontinuities, and sudden regime transitions. Within high-frequency trading contexts, volatility may adjust within minutes in response to macroeconomic announcements, liquidity disruptions, demand, economic demand or unforeseen events. The prompt identification of intraday volatility shifts is therefore critical for effective risk management, personal investment, portfolio hedging, and regular surveillance of prospective markets.

2.2 Statistical Challenges

Conventional volatility modeling techniques—such as rolling standard deviations, implied volatility from VIX or Bollinger Bands and are well suited for capturing persistence and clustering effects. However, these approaches are not primarily designed for the real-time identification of abrupt structural breaks. In high-frequency settings, the principal challenge extends beyond estimating volatility levels to determining precisely when the underlying return distribution has changed. This objective aligns naturally with the theory of sequential change-point detection.

Sequential monitoring procedures, including the CUSUM control chart, the PH test, and BOCPD, have been extensively examined in statistical process control and time-series analysis. Each method provides a systematic framework for detecting distributional shifts as observations arrive sequentially. CUSUM is particularly effective at identifying small but persistent deviations from a baseline, making it sensitive to gradual volatility regime transitions. The Page–Hinkley test, by contrast, is designed to react quickly to abrupt and pronounced changes, offering faster detection of sharp volatility spikes. Bayesian Online Change Point Estimation incorporates probabilistic updating and explicitly models uncertainty, producing a structured posterior assessment of regime changes at the expense of greater computational complexity. Because each algorithm exhibits distinct sensitivity characteristics and response dynamics, no single procedure is uniformly optimal across all volatility environments. Their integration therefore creates a complementary monitoring system that improves robustness and reduces the likelihood of both delayed detection and false alarms.

3 System Overview

3.1 Data Acquisition

Minute-level price data for the asset under study is obtained from historical market records. Each observation consists of a timestamp and the corresponding traded price P_t. Data are sorted in ascending time order and processed sequentially to simulate real-time monitoring conditions.

Missing values and non-trading intervals are removed prior to analysis to ensure consistency in the sampling frequency.

3.2 Return Construction

Monitoring algorithms operate on transformations of the price series rather than raw prices. Two return definitions are considered:

Arithmetic Returns

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}. \tag{1}$$

Log Returns

$$r_t = \log(P_t) - \log(P_{t-1}). \tag{2}$$

Arithmetic returns provide a direct measure of proportional price change, while log returns possess desirable additive properties over time. Individual monitoring algorithms may operate on either representation depending on their modeling assumptions.

3.3 Streaming Monitoring Architecture

The framework is designed for sequential operation. At each time step t, the newly observed return is supplied simultaneously to all monitoring algorithms. The general process for receiving minute by minute data for each detector follows the general principal:

Let $x_t^{(i)}$ denote the observation provided to detector i where i can be CUSUM, PH or BOCPE. Each detector updates its internal state recursively according to

$$S_t^{(i)} = g_i\big(S_{t-1}^{(i)}, x_t^{(i)}\big), \tag{3}$$

where $g_i(\cdot)$ represents the detector-specific update rule.

Each monitoring procedure produces a binary flag output

$$F_t^{(i)} \in \{0, 1\}, \tag{4}$$

where $F_t^{(i)} = 1$ indicates a detected volatility shift (flag) and $F_t^{(i)} = 0$ indicates no detection (no flag).

3.4 Detector Interface

All monitoring algorithms share a common operational interface. Upon receiving a new observation x_t, a detector:

1. Updates its internal state,
2. Evaluates a decision condition,
3. Outputs a Boolean alarm flag.

This standardized interface enables parallel execution of multiple detectors within a unified monitoring system.

3.5 Evaluation Event Definition

For performance assessment, proxy volatility events are defined using a magnitude threshold on returns. Let $\hat{\sigma}$ denote the empirical standard deviation of the selected return series. An event is recorded when

$$|x_t| > 2\hat{\sigma}. \tag{5}$$

This event definition is used solely for evaluating detection performance and does not influence the detector update rules.

3.6 System Output

The ensemble monitoring system collects the alarm signals $\{F_t^{(1)}, F_t^{(2)}, F_t^{(3)}\}$ generated by the individual detectors. These signals are subsequently combined through a decision rule described in Section 5 to produce a final volatility alarm.

4 Individual Return Monitors

4.1 Cumulative Sum (CUSUM) Monitor

4.1.1 Conceptual Overview

CUSUM detects small standard deviation, persistent shifts in a process mean over time. In the intraday volatility framework, CUSUM accumulates evidence of sustained directional movement in the return series. Instead of assessing large blocks of returns within a given time frame $t > 60s$ and creating a decision based on the mean value against standard deviations, CUSUM looks at small single-period threshold methods effectively integrating small deviations over time, making it particularly sensitive to gradual regime shifts.

4.1.2 Mathematical Formulation

Let x_t denote the return observation supplied to the CUSUM detector at time t. We implement a two-sided (split) CUSUM procedure defined by

$$S_t^+ = \max\left(0, S_{t-1}^+ + x_t - (\mu + k)\right), \tag{6}$$
$$S_t^- = \min\left(0, S_{t-1}^- + x_t + (\mu + k)\right), \tag{7}$$

with initialization

$$S_0^+ = S_0^- = 0. \tag{8}$$

Here:

- μ is the reference mean,
- k is a drift parameter controlling sensitivity,
- S_t^+ accumulates upward deviations,
- S_t^- accumulates downward deviations.

4.1.3 Alarm Condition

A volatility alarm is triggered at time t if either cumulative sum exceeds a pre-specified threshold h:

$$\text{Alarm if } S_t^+ > h \quad \text{or} \quad S_t^- < -h. \tag{9}$$

The detector output is defined as

$$F_t^{(\text{CUSUM})} = \begin{cases} 1, & \text{if alarm condition is satisfied,} \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

4.1.4 Algorithmic Implementation

At each time step:

1. Receive new return observation x_t.
2. Update S_t^+ and S_t^- using the recursive equations above.
3. Evaluate the threshold condition.
4. Output a Boolean flag.
5. Reset cumulative sums after a defined value of time t.

This is a recursive loop such that it enables efficient minute-by-minute monitoring.

4.1.5 Outputs

The CUSUM detector produces:

- Upward cumulative signal S_t^+,
- Downward cumulative signal S_t^-,
- Binary alarm flag $F_t^{(\text{CUSUM})}$.
- Control Chart 1

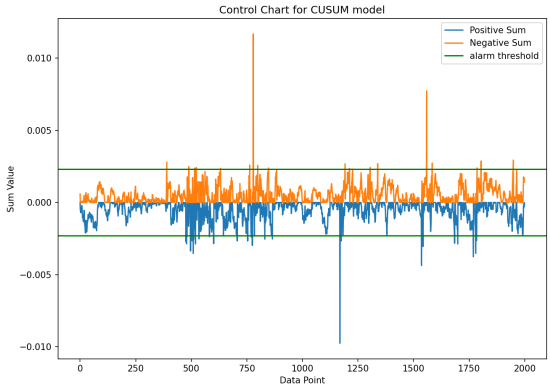

Figure 1: CUSUM Control Chart

These outputs provide both directional information and detection timing for intraday volatility shifts.

4.2 Page–Hinkley (PH) Monitor

4.2.1 Conceptual Overview

The Page–Hinkley (PH) test is a change-detection procedure that is well-suited to identifying abrupt shifts in the distribution of incoming data. In the IVM setting, PH is applied to a volatility computed from minute by minute intraday returns. The detector accumulates evidence in favor of a transition from a low-volatility regime to a high-volatility regime and triggers makes a flag once the accumulated evidence exceeds a threshold. This formulation follows the likelihood-ratio interpretation described in the document cited for explanation of PH [1].

4.2.2 Monitored Signal and Regime Models

Let $\{x_n\}_{n\geq 1}$ denote the sequential volatility proxy supplied to PH, where x_n is the rolling standard deviation (or other rolling dispersion measure) computed from intraday returns.

The PH implementation assumes two candidate regimes:

- **Low-volatility regime** with density $f_0(x)$,
- **High-volatility regime** with density $f_1(x)$,

so that each new observation x_n can be evaluated under both models. In the The volatility proxy is suggested to be approximately log-normal in each regime, motivating log-normal model choices for f_0 and f_1 (or empirically estimated densities) [1]

4.2.3 Log-Likelihood Ratio Increment

For each incoming observation x_n, define the log-likelihood ratio increment

$$X_n \;=\; \log\bigl(f_1(x_n)\bigr) \;-\; \log\bigl(f_0(x_n)\bigr). \tag{11}$$

When X_n is persistently positive, the observed volatility proxy is more consistent with the high-volatility regime than the low-volatility regime, and evidence accumulates toward a regime shift. This is the core statistic used in the implementation.

4.2.4 Cumulative Evidence and PH Statistic

Define the cumulative sum of log-likelihood ratio increments by

$$S_0 = 0, \qquad S_n \;=\; \sum_{k=1}^{n} X_k. \tag{12}$$

The Page–Hinkley statistic tracks how far the current cumulative evidence S_n has risen above its minimum historical value:

$$m_n \;=\; \min_{0 \le k \le n} S_k, \qquad g_n \;=\; S_n - m_n. \tag{13}$$

Equivalently, g_n measures the maximum upward excursion of the cumulative log-likelihood ratio relative to its past minimum.

4.2.5 Alarm Condition and Binary Output

Given a detection threshold $\delta > 0$, an alarm is triggered at time n when

$$g_n > \delta. \tag{14}$$

Define the PH binary flag as

$$F_n^{(\mathrm{PH})} = \begin{cases} 1, & g_n > \delta, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

In the IVM implementation, $F_n^{(\mathrm{PH})} = 1$ is interpreted as entry into a high-volatility regime.

4.2.6 Recursive Implementation

To support streaming operation, the quantities above can be updated recursively. Let m_{n-1} denote the running minimum of S_k up to time $n - 1$. Then:

$$S_n = S_{n-1} + X_n, \tag{16}$$

$$m_n = \min(m_{n-1}, S_n), \tag{17}$$

$$g_n = S_n - m_n. \tag{18}$$

4.2.7 Outputs

At each time step n, the PH monitor provides the following outputs:

- the log-likelihood ratio increment X_n,
- cumulative evidence S_n,
- running minimum m_n,
- PH statistic g_n,
- binary alarm flag $F_n^{(\mathrm{PH})}$.
- Graphical volatility flags 2

These outputs support both real-time volatility monitoring (via the flag) and diagnostic analysis (using X_n, S_n, and g_n).

Visually PH depicts exactly where and when it is flagging returns, see figure 2

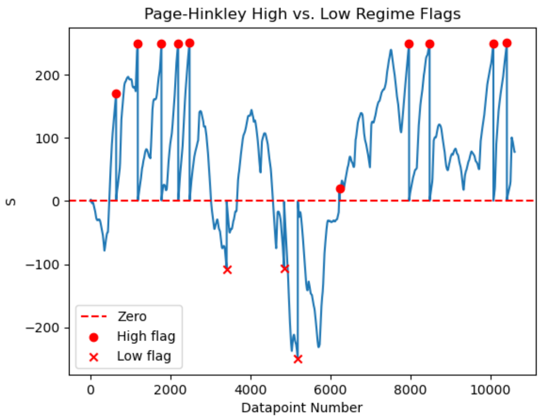

Figure 2: Page-Hinkley flagged returns graph from Oct 9th 2025-Nov 26th 2025

4.3 Bayesian Online Change Point Estimation (BOCPE)

4.3.1 Statistical Model

Bayesian Online Change Point Estimation (BOCPE) provides a probabilistic framework for detecting regime changes in a sequentially observed process. The method assumes that observations are generated from a sequence of piecewise stationary regimes separated by unknown change points.

Let $\{x_t\}_{t=1}^{T}$ denote the sequential observations supplied to the detector. Define the *run length* variable

$$r_t \in \{0, 1, 2, \dots\}, \tag{19}$$

where r_t represents the number of time steps elapsed since the most recent change point. A change point occurring at time t corresponds to $r_t = 0$.

4.3.2 Hazard Model

Regime transitions are governed by a constant hazard rate H, representing the probability that a change occurs at any time step:

$$\Pr(r_t = 0 \mid r_{t-1}) = H, \tag{20}$$
$$\Pr(r_t = r_{t-1} + 1 \mid r_{t-1}) = 1 - H. \tag{21}$$

This assumption implies that regime durations follow a geometric distribution.

4.3.3 Observation Model

Within each regime, observations are assumed conditionally Gaussian with unknown mean and known variance:

$$x_t \mid \mu \sim \mathcal{N}(\mu, \sigma_x^2), \tag{22}$$

where σ_x^2 denotes the observation variance.

A conjugate Normal prior is placed on the regime mean:

$$\mu \sim \mathcal{N}(\mu_0, \tau_0^{-1}), \tag{23}$$

where μ_0 is the prior mean and τ_0 is the prior precision.

4.3.4 Predictive Distribution

For each possible run length r, the posterior distribution of the regime mean after observing data up to time $t-1$ is

$$\mu \mid r_{t-1} = r, x_{1:t-1} \sim \mathcal{N}\big(m_{t-1}^{(r)}, (\tau_{t-1}^{(r)})^{-1}\big). \tag{24}$$

Integrating over the uncertainty in μ yields the predictive distribution

$$p(x_t \mid r_{t-1} = r, x_{1:t-1}) = \mathcal{N}\left(x_t \mid m_{t-1}^{(r)}, \sigma_x^2 + (\tau_{t-1}^{(r)})^{-1}\right). \tag{25}$$

4.3.5 Run-Length Posterior Update

Let

$$\pi_{t-1}(r) = \Pr(r_{t-1} = r \mid x_{1:t-1})$$

denote the posterior run-length distribution.

Two competing events are considered at each time step.

Growth (no change point)

$$\tilde{\pi}_t(r+1) = \pi_{t-1}(r)\, p(x_t \mid r)(1-H). \tag{26}$$

Change point

$$\tilde{\pi}_t(0) = \sum_{r \geq 0} \pi_{t-1}(r)\, p(x_t \mid r) H. \tag{27}$$

Normalization gives the posterior distribution

$$\pi_t(r) = \frac{\tilde{\pi}_t(r)}{\sum_j \tilde{\pi}_t(j)}. \tag{28}$$

4.3.6 Parameter Posterior Updates

Using Normal–Normal conjugacy, posterior parameters update recursively.

For a newly initiated regime ($r_t = 0$):

$$\tau_t^{(0)} = \tau_0 + \tau_x, \tag{29}$$

$$m_t^{(0)} = \frac{\tau_0 \mu_0 + \tau_x x_t}{\tau_0 + \tau_x}, \tag{30}$$

where $\tau_x = 1/\sigma_x^2$.

For continuing regimes:

$$\tau_t^{(r+1)} = \tau_{t-1}^{(r)} + \tau_x, \tag{31}$$

$$m_t^{(r+1)} = \frac{\tau_{t-1}^{(r)} m_{t-1}^{(r)} + \tau_x x_t}{\tau_{t-1}^{(r)} + \tau_x}. \tag{32}$$

4.3.7 Change-Point Decision Rule

The posterior probability of a change point occurring at time t is

$$\Pr(r_t = 0 \mid x_{1:t}) = \pi_t(0). \tag{33}$$

An alarm is generated when

$$F_t^{(\mathrm{BOCPE})} = \begin{cases} 1, & \pi_t(0) \geq \theta, \\ 0, & \text{otherwise}, \end{cases} \tag{34}$$

where θ is a predefined detection threshold.

Additionally, a change is inferred when the maximum a posteriori (MAP) run length decreases,

$$\hat{r}_t = \arg\max_r \pi_t(r), \qquad \hat{r}_t < \hat{r}_{t-1}. \tag{35}$$

4.3.8 Outputs

The BOCPE detector produces:

- posterior run-length distribution $\pi_t(r)$,
- estimated change-point probability $\pi_t(0)$,
- MAP run length estimate $\hat{r}_t$,
- binary volatility flag $F_t^{(\mathrm{BOCPE})}$.

These quantities provide probabilistic assessment of regime changes while maintaining fully online recursive computation.

5 Decision Rule

5.1 Overview

At each minute t, the monitoring system receives a new return observation and distributes it simultaneously to the three detectors: CUSUM, Page–Hinkley (PH), and BOCPE.

Each detector produces a binary flag

$$F_t^{(i)} \in \{0,1\}, \quad i \in \{\mathrm{CUSUM}, \mathrm{PH}, \mathrm{BOCPE}\},$$

where $F_t^{(i)} = 1$ indicates detection of a volatility shift and $F_t^{(i)} = 0$ indicates no detection.

The objective of the decision rule is to aggregate these individual flags into a single system-level alarm signal.

5.2 Two-Out-of-Three Voting Mechanism

Define the aggregate vote at time t as

$$V_t = \sum_{i=1}^{3} F_t^{(i)}. \tag{36}$$

The system alarm is triggered whenever at least two detectors simultaneously indicate a volatility shift:

$$A_t = \begin{cases} 1, & \text{if } V_t \geq 2, \\ 0, & \text{otherwise.} \end{cases} \tag{37}$$

Thus, a minimum consensus of two independent detection procedures is required to register an alarm. This majority rule reduces sensitivity to isolated false positives produced by any single algorithm while preserving responsiveness to genuine regime transitions.

5.3 Sequential Processing and Flag Persistence

The monitoring system operates in a streaming, minute-by-minute manner. At each time step:

1. A new return observation x_t is received.
2. Each detector updates its internal state and produces $F_t^{(i)}$.
3. The votes are aggregated to compute V_t.
4. If $V_t \geq 2$, a system alarm $A_t = 1$ is registered.

To stabilize detection behavior, detector flags remain active (i.e., $F_t^{(i)} = 1$) once triggered, until either:

- A system alarm is confirmed, or
- A predefined monitoring reset interval is reached.

In practice, volatility shifts may cause detectors to flag at slightly different minutes; maintaining active flags over a short window increases the probability of consensus.

5.4 Reset Mechanism

After a system alarm is generated, the monitoring framework enters a reset phase.

Let T_{alarm} denote the time at which $A_t = 1$. Following this event:

- Detector flags are cleared.
- Internal decision buffers are reset.
- Monitoring resumes on subsequent observations.

Formally, for $t > T_{\text{alarm}}$,

$$F_t^{(i)} \leftarrow 0 \quad \text{until new detector-specific conditions are met.} \tag{38}$$

This reset prevents repeated alarms from the same volatility event and allows the system to detect future regime transitions independently.

5.5 Final System Output

The final intraday volatility alarm process is therefore defined as

$$A_t = \mathbb{I}\left(F_t^{(\text{CUSUM})} + F_t^{(\text{PH})} + F_t^{(\text{BOCPE})} \geq 2\right),$$

where $\mathbb{I}(\cdot)$ denotes the indicator function.

The output sequence $\{A_t\}$ represents the final decision stream of the Intraday Volatility Monitor.

6 Conclusion

This work introduced the Intraday Volatility Monitor (IVM), a sequential monitoring framework designed to detect regime shifts in intraday financial volatility. The system integrates three distinct change-detection methodologies: CUSUM, Page–Hinkley (PH), and Bayesian Online Change Point Estimation (BOCPE). Each detector operates on streaming return data and produces a binary alarm signal indicating the presence or absence of a volatility shift.

The CUSUM monitor provides sensitivity to small but persistent deviations from a baseline mean by recursively accumulating directional evidence. The Page–Hinkley procedure evaluates log-likelihood ratio increments under competing volatility regime models, enabling rapid detection of abrupt structural changes. BOCPE extends the framework into a fully probabilistic domain, maintaining a posterior distribution over possible run lengths and quantifying uncertainty regarding the timing of change points.

By combining outputs through a unified decision, the IVM establishes reliable and relatively unbiased framework. This ensemble approach mitigates the limitations of any single algorithm's bias and false alarms while promoting robustness across many possible volatility environments in need of monitoring.

6.1 Future Directions

Future extensions of the IVM framework may include:

- Adaptive threshold selection based on market state,
- Multivariate volatility monitoring,
- Alternative observation models (e.g., heavy-tailed distributions),
- Integration with execution or hedging strategies.

The modular design of the current system ensures that such extensions can be incorporated without altering the core sequential monitoring structure.

References

[1] Matthew. *Page–Hinkley Notes and Pseudocode*. Feb. 8, 2026. URL: https://raw.githubusercontent.com/bhunter64/Intraday-Volatility-Monitor/Matthew/research/Page_Hinkley_Notes_Pseudocode.pdf (visited on 02/28/2026).