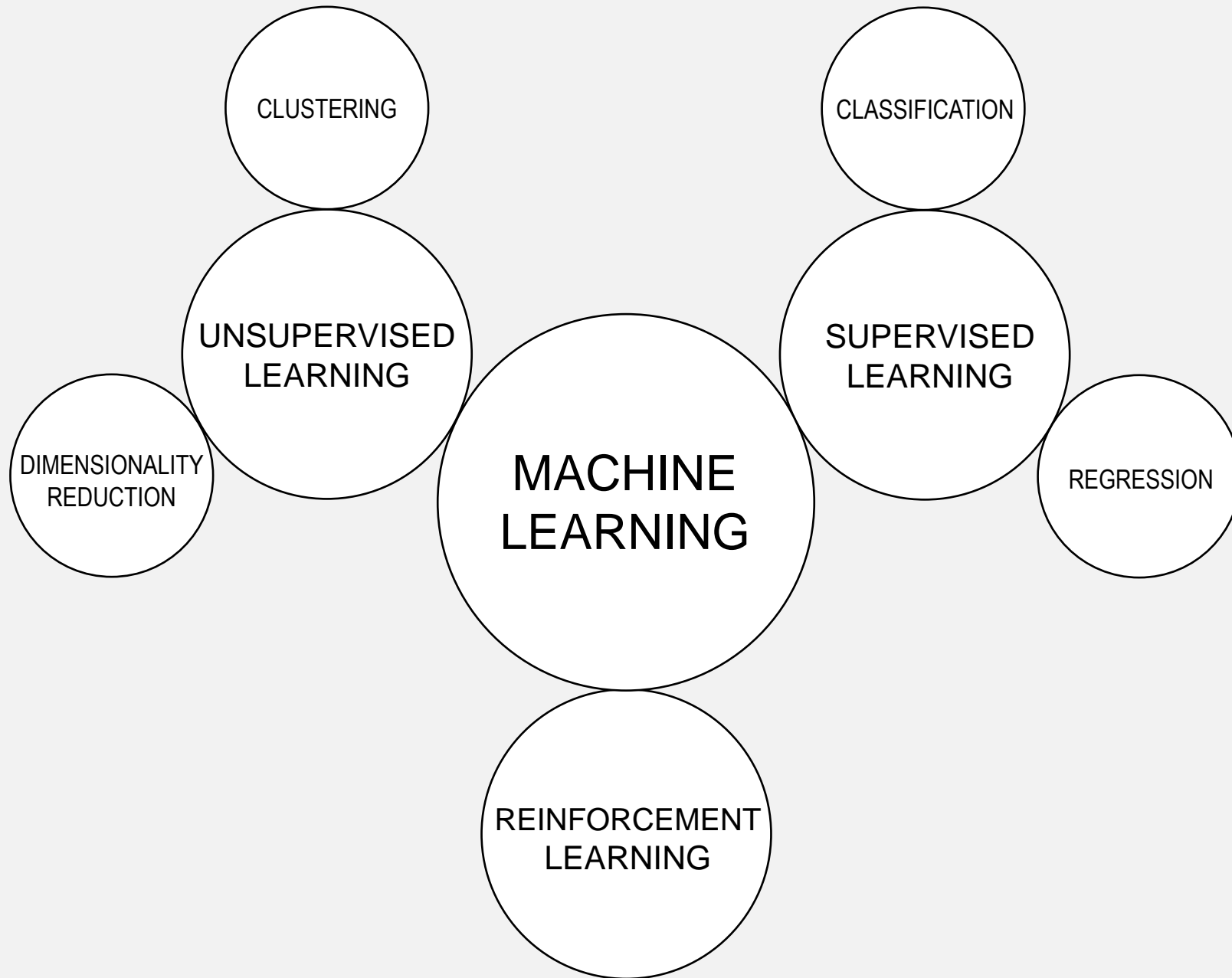


# CLUSTERING

Lecture 10  
MALI, 2024



# CLUSTERING

# CLUSTERING

- What is clustering?
- *k*-means clustering
- Agglomerative clustering
- DBSCAN
- Application

# WHAT IS CLUSTERING?

grouping data: unlabeled version of classification

↑  
most data  
in the world

# REVERSE IMAGE SEARCH

I want to know what this bird in my garden is



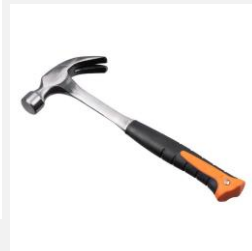
The corresponding websites  
tell me it's a common linnet

# REVERSE IMAGE SEARCH



All the images in the dataset ...

# REVERSE IMAGE SEARCH



... are **clustered** into groups.

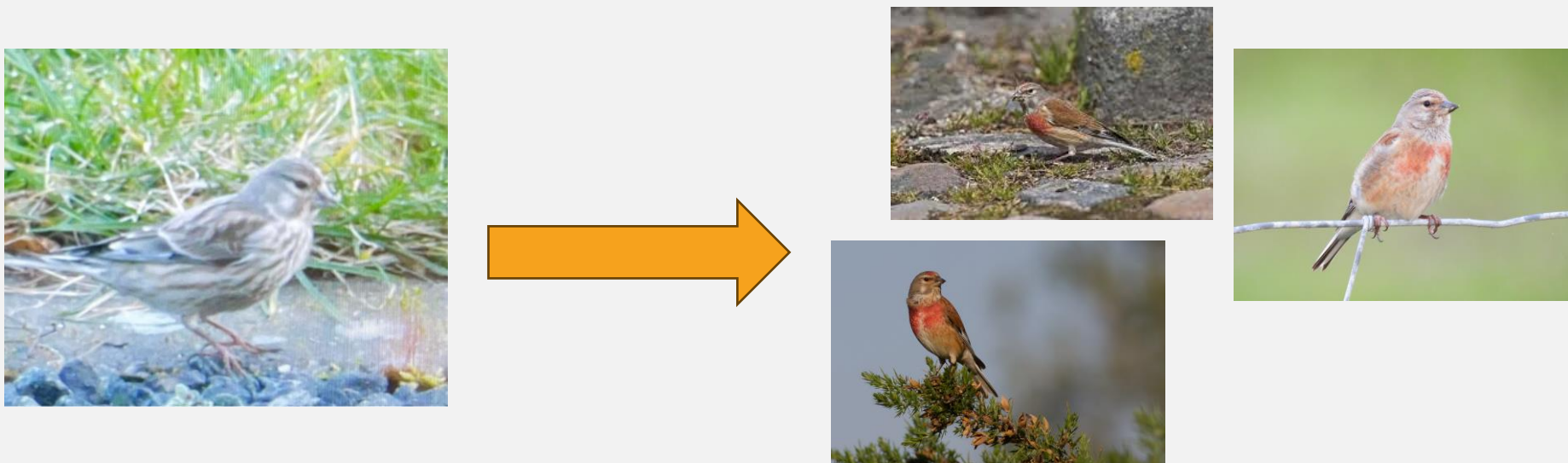


# REVERSE IMAGE SEARCH



The image we search with is assigned to a cluster ...

# REVERSE IMAGE SEARCH



... and the other images in the cluster are returned.

## DIFFERENCE FROM CLASSIFICATION?

At no point did we label  
the images.

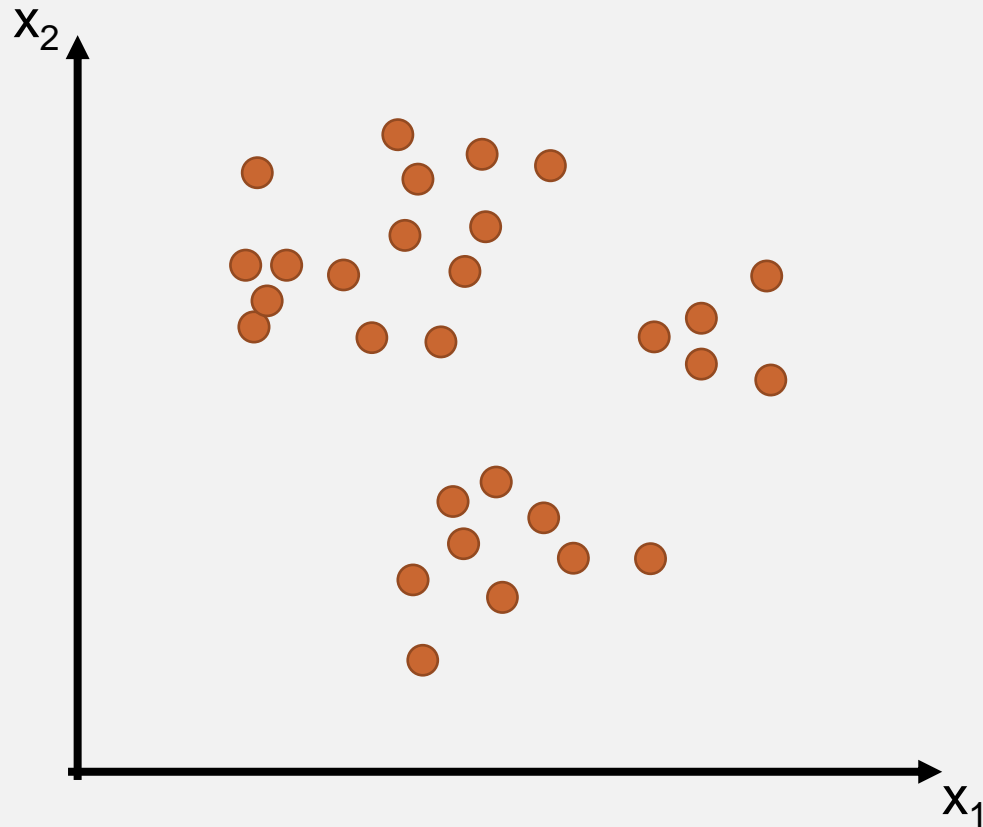
We just saw/figured out that  
some were similar

(measure of similarity)

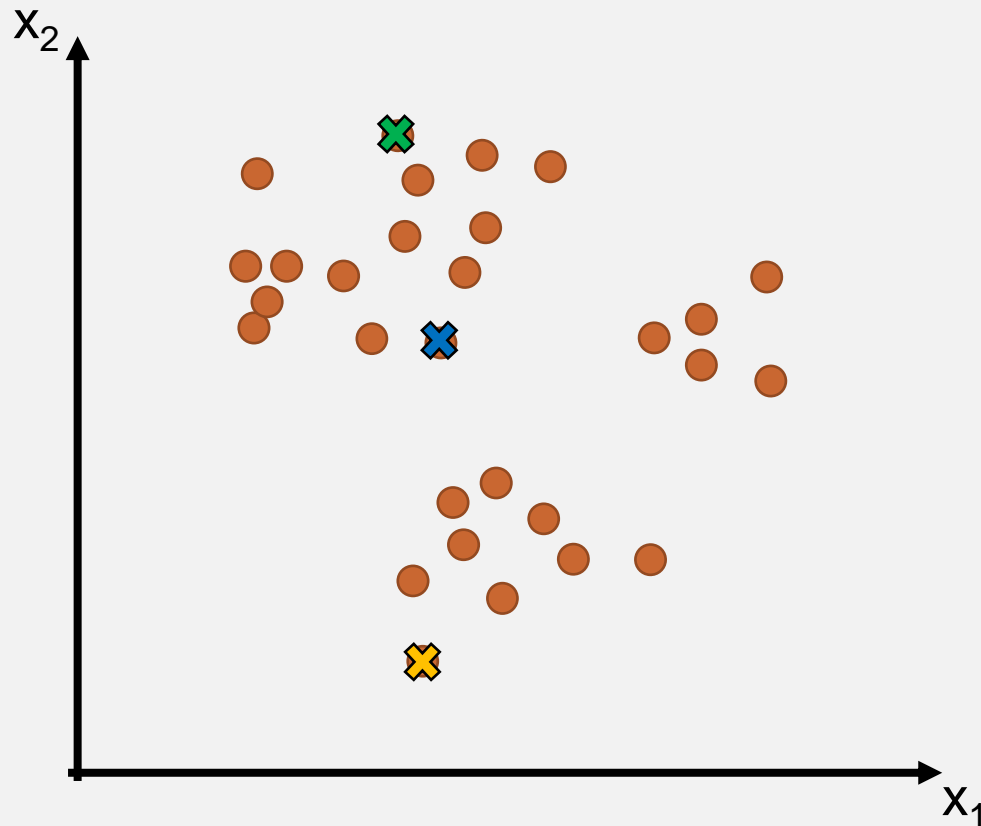
# CLUSTERING

- What is clustering?
- ***k*-means clustering**
- Agglomerative clustering
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- Application

# $k$ -MEANS CLUSTERING

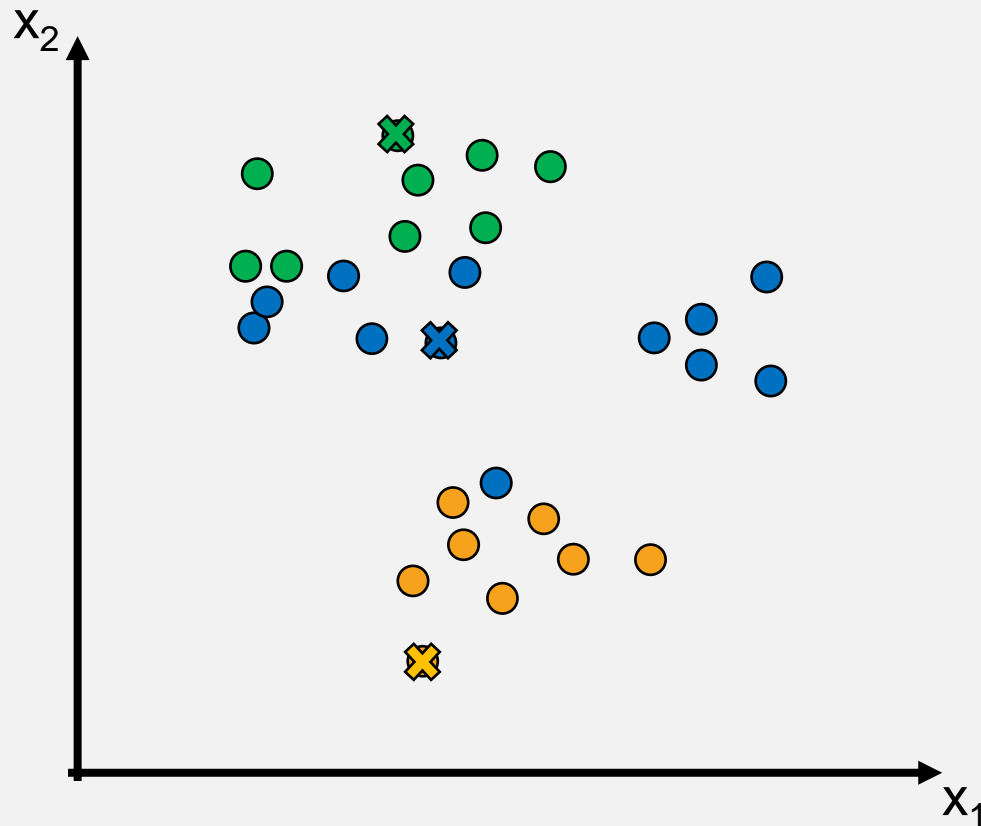


# $k$ -MEANS CLUSTERING



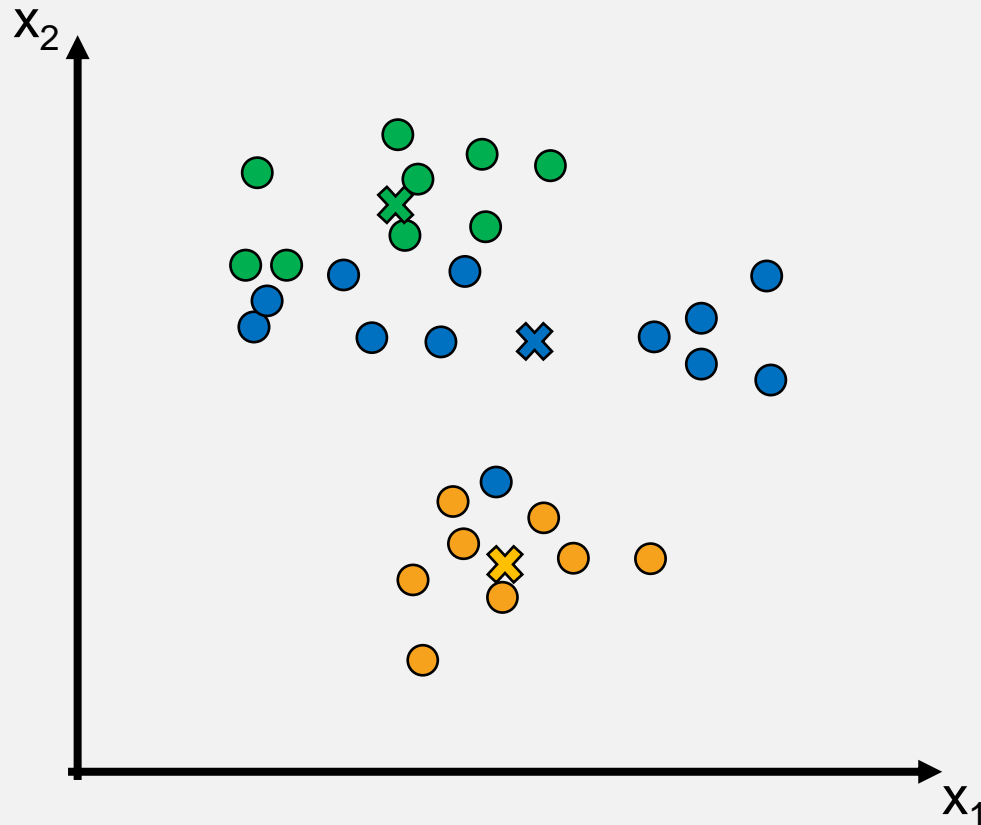
I. Assign  $k(=3)$  random points as **centroids**

# $k$ -MEANS CLUSTERING



1. Assign  $k(=3)$  random points as **centroids**
2. Group the data by their distance to the centroids

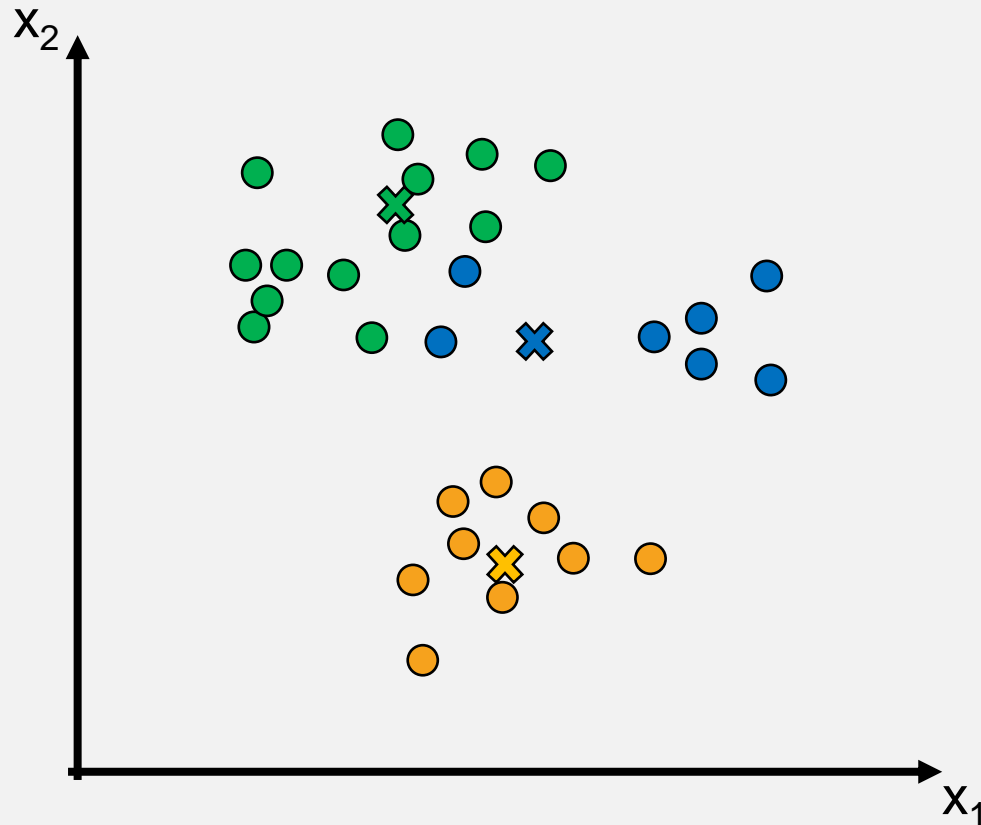
# $k$ -MEANS CLUSTERING



1. Assign  $k(=3)$  random points as **centroids**
2. Group the data by their distance to the centroids
3. Move the centroids to the cluster centers

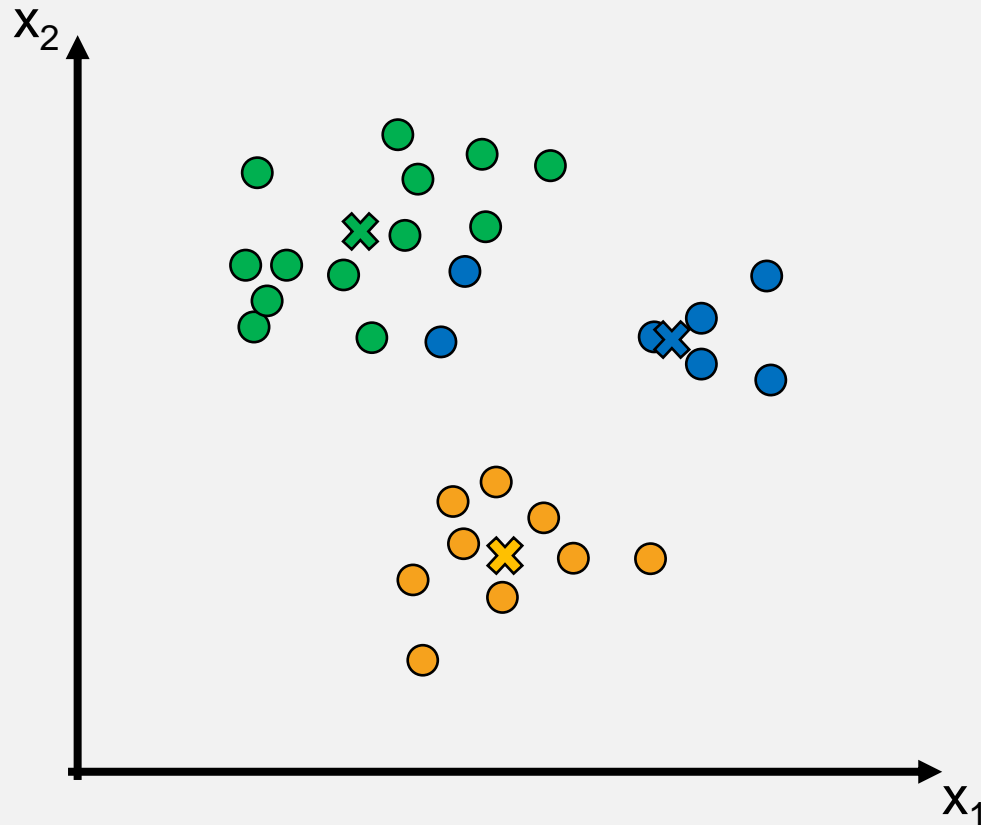


# $k$ -MEANS CLUSTERING



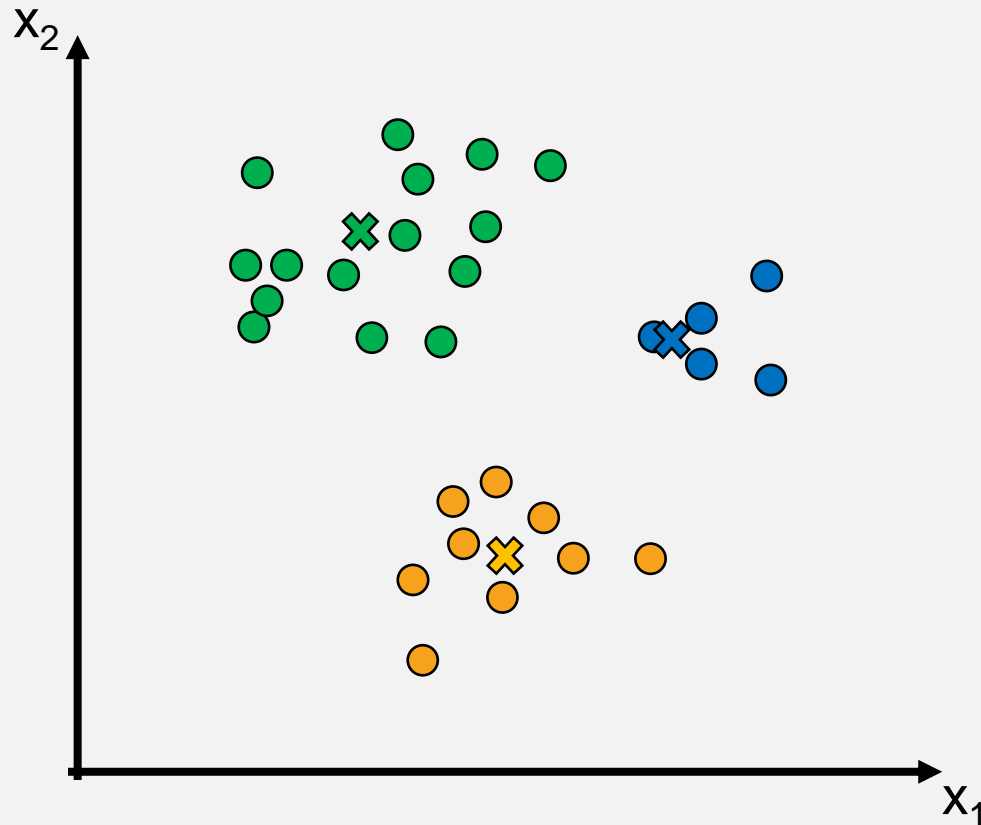
1. Assign  $k(=3)$  random points as **centroids**
2. Group the data by their distance to the centroids
3. Move the centroids to the cluster centers
4. Regroup the data

# $k$ -MEANS CLUSTERING



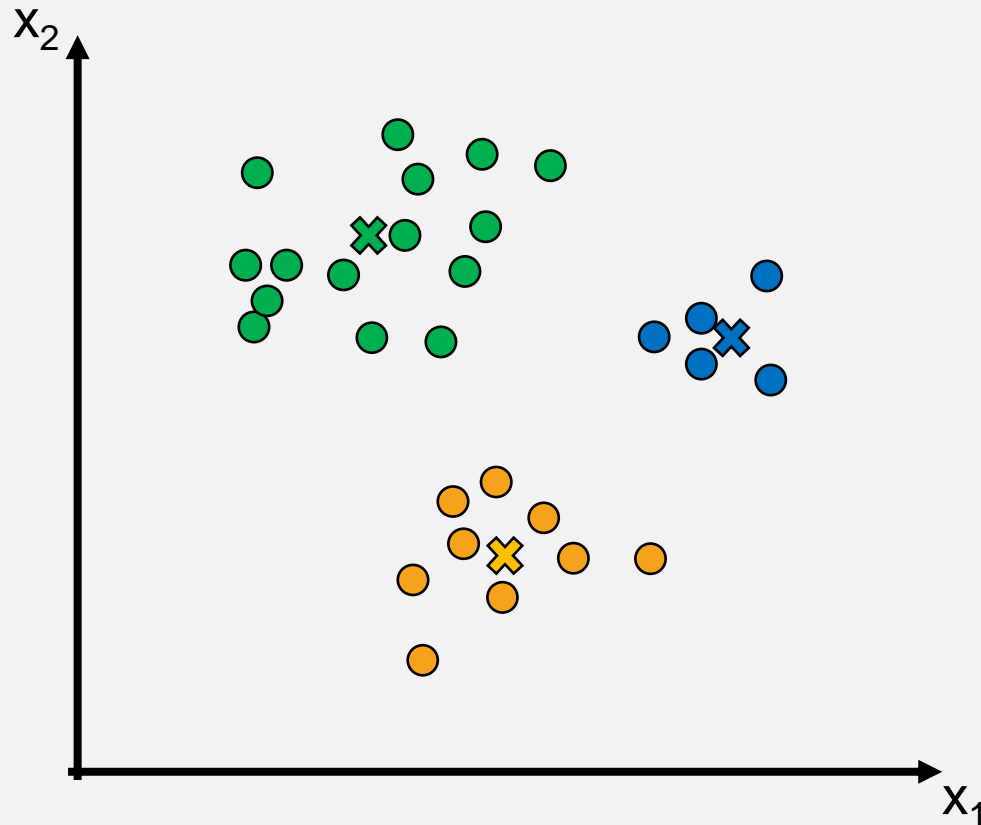
1. Assign  $k(=3)$  random points as **centroids**
2. Group the data by their distance to the centroids
3. Move the centroids to the cluster centers
4. Regroup the data
5. Repeat 3-4 until nothing changes

# $k$ -MEANS CLUSTERING



1. Assign  $k(=3)$  random points as **centroids**
2. Group the data by their distance to the centroids
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5. Repeat 3-4 until nothing changes

# $k$ -MEANS CLUSTERING



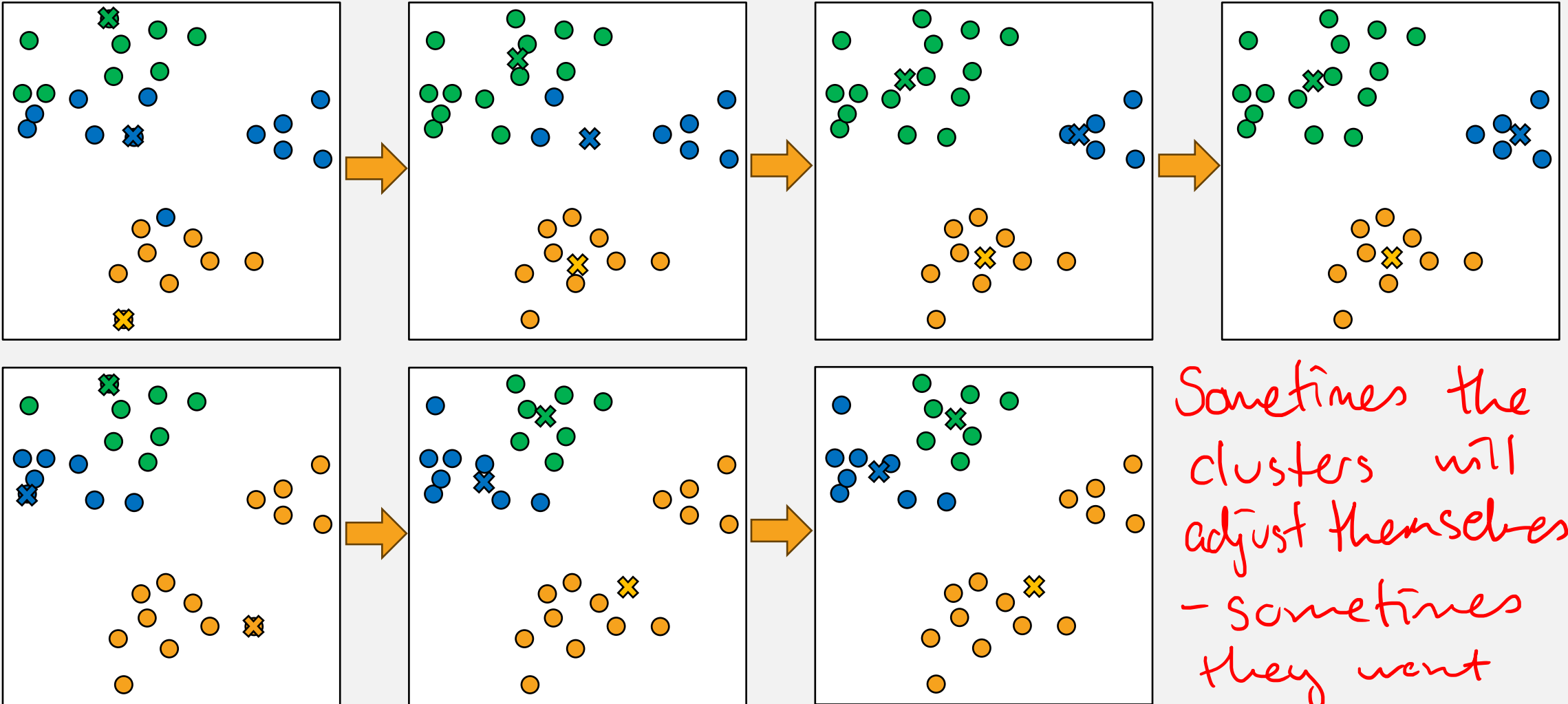
1. Assign  $k(=3)$  random points as **centroids**
2. Group the data by their distance to the centroids
3. Move the centroids to the cluster centers
4. Regroup the data
5. Repeat 3-4 until nothing changes

## A FEW THINGS WE HAVE TO DEAL WITH

The value of  $k$

The initial centroids

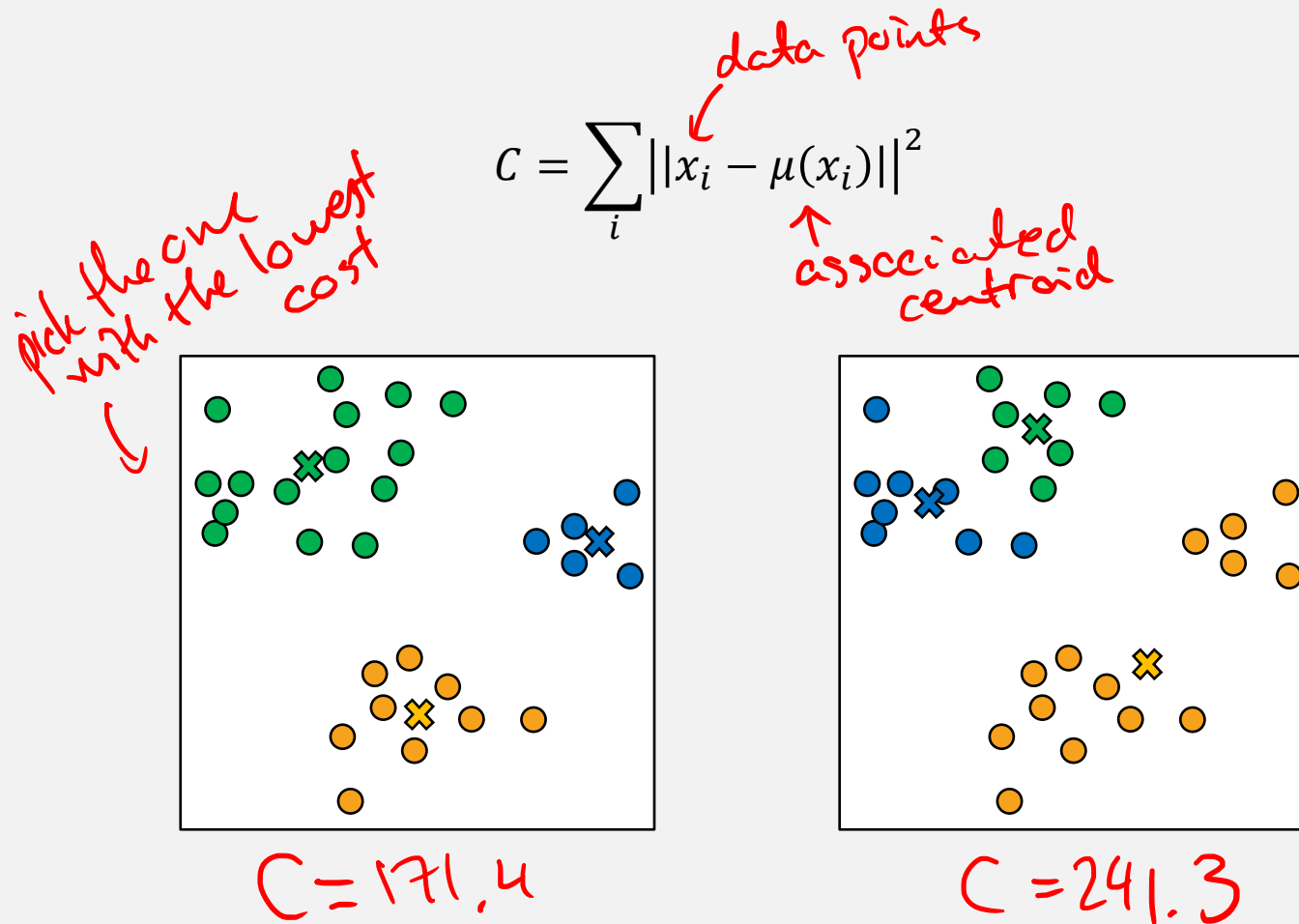
## THE INITIAL CENTROIDS



Sometimes the clusters will adjust themselves - sometimes they won't

# THE INITIAL CENTROIDS

Solution 1: Try different, randomized initializations and compare the **costs** of the final clusterings

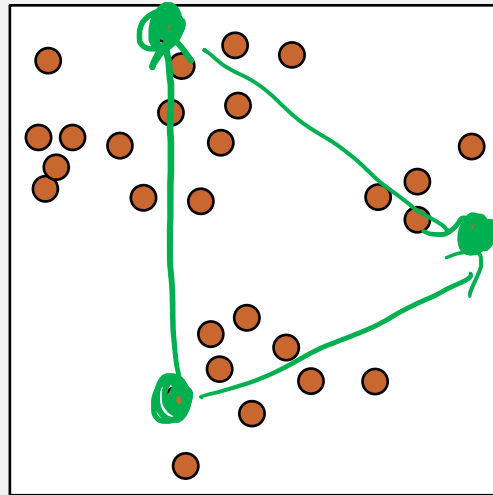


## THE INITIAL CENTROIDS

Solution 2: Choose the initial centroids based on the distance to the previous ones

Choose the point furthest away

⇒ likely to select different points from  
different cluster  $i$   
and outliers  $i$

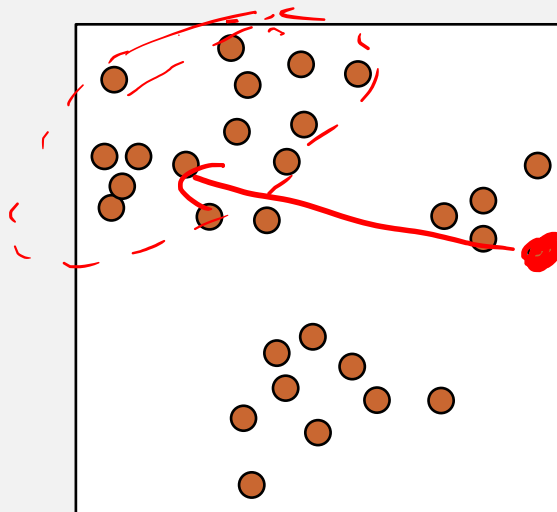




# THE INITIAL CENTROIDS

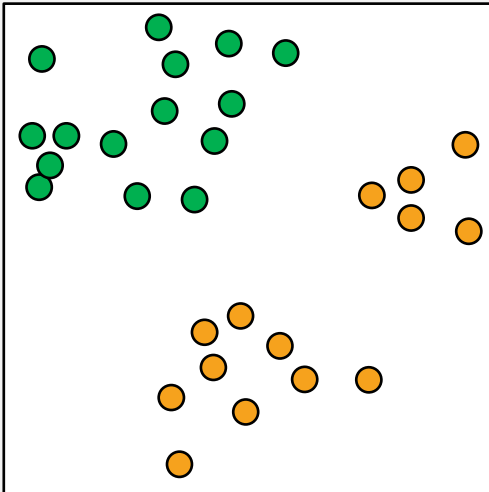
Solution 3: Choose "far away but random" points ("k-means++")

assign a distance-based probability  
of picking next point

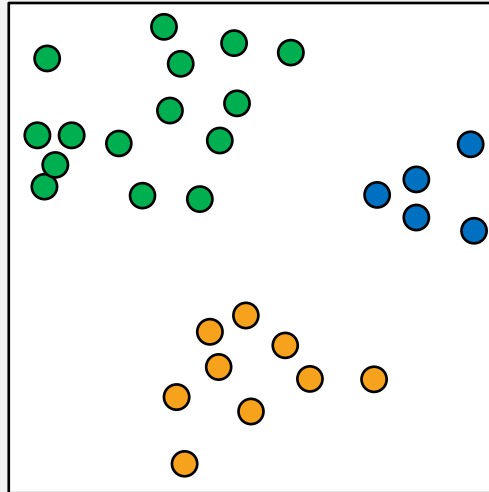


# THE NUMBER OF CLUSTERS ( $k$ )

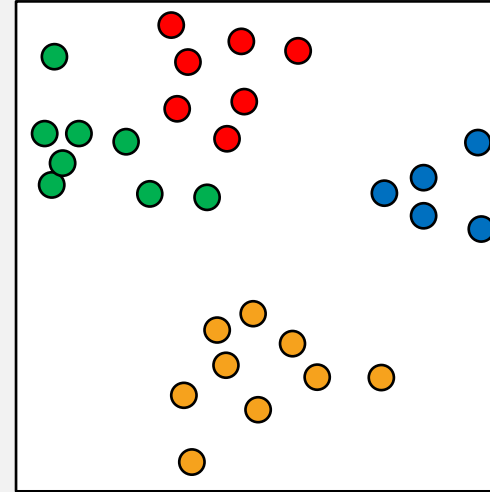
$k = 2$



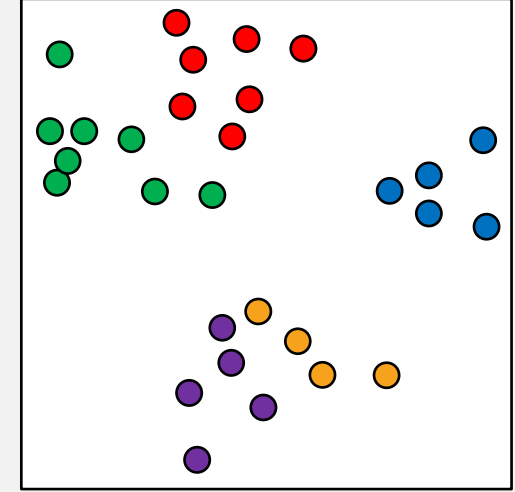
$k = 3$



$k = 4$



$k = 5$

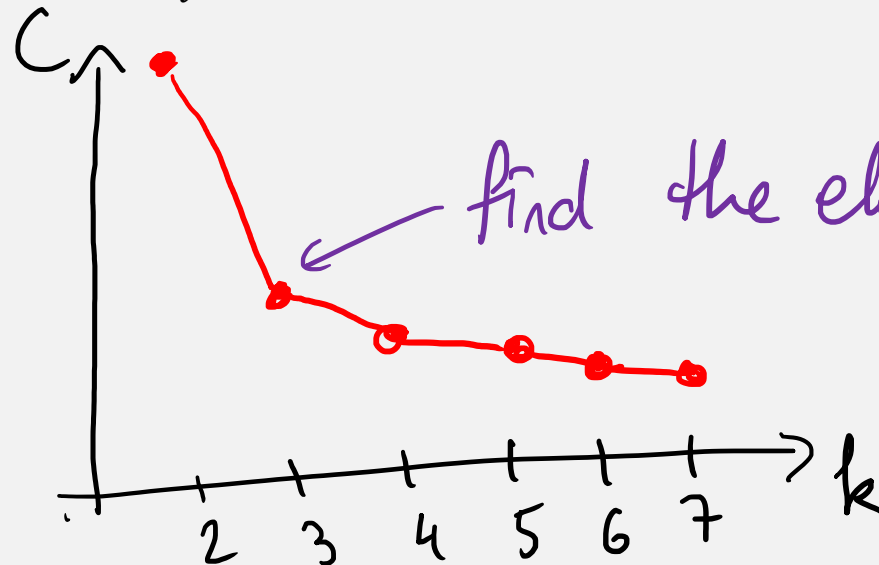


# THE NUMBER OF CLUSTERS ( $k$ )

The easy way: *You already know it (domain knowledge)*

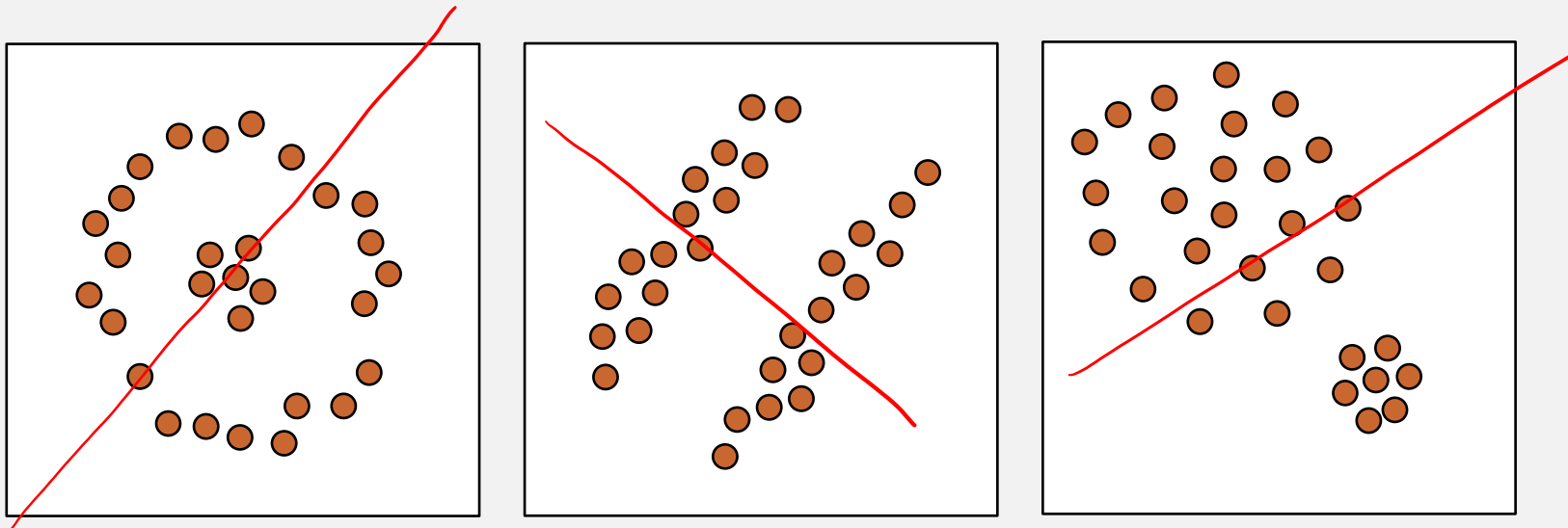
The hard way: *"The elbow method"*

*cost function*  $\rightarrow C = \sum_i ||x_i - \mu(x_i)||^2$  always decreases with  $k$

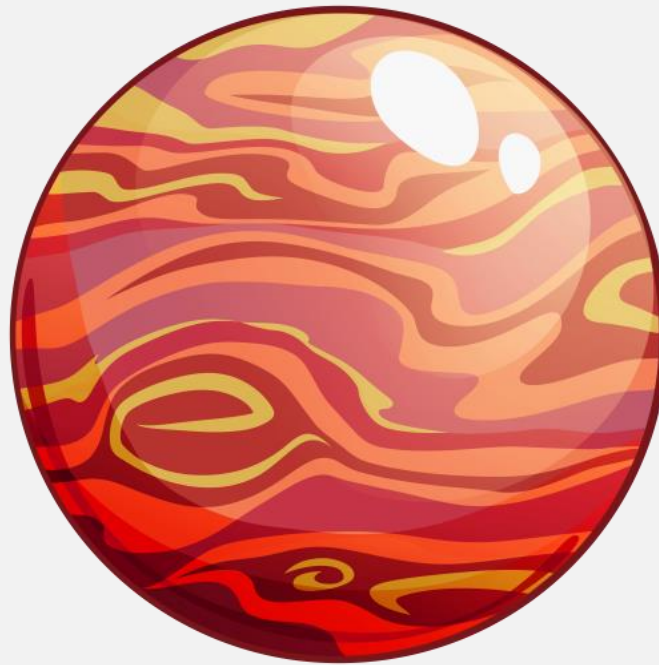


*find the elbow: new clusters don't contribute a lot*

## WHERE $k$ -MEANS FAILS



## CODE EXAMPLE



*Jupyter Notebook* **Clustering methods**

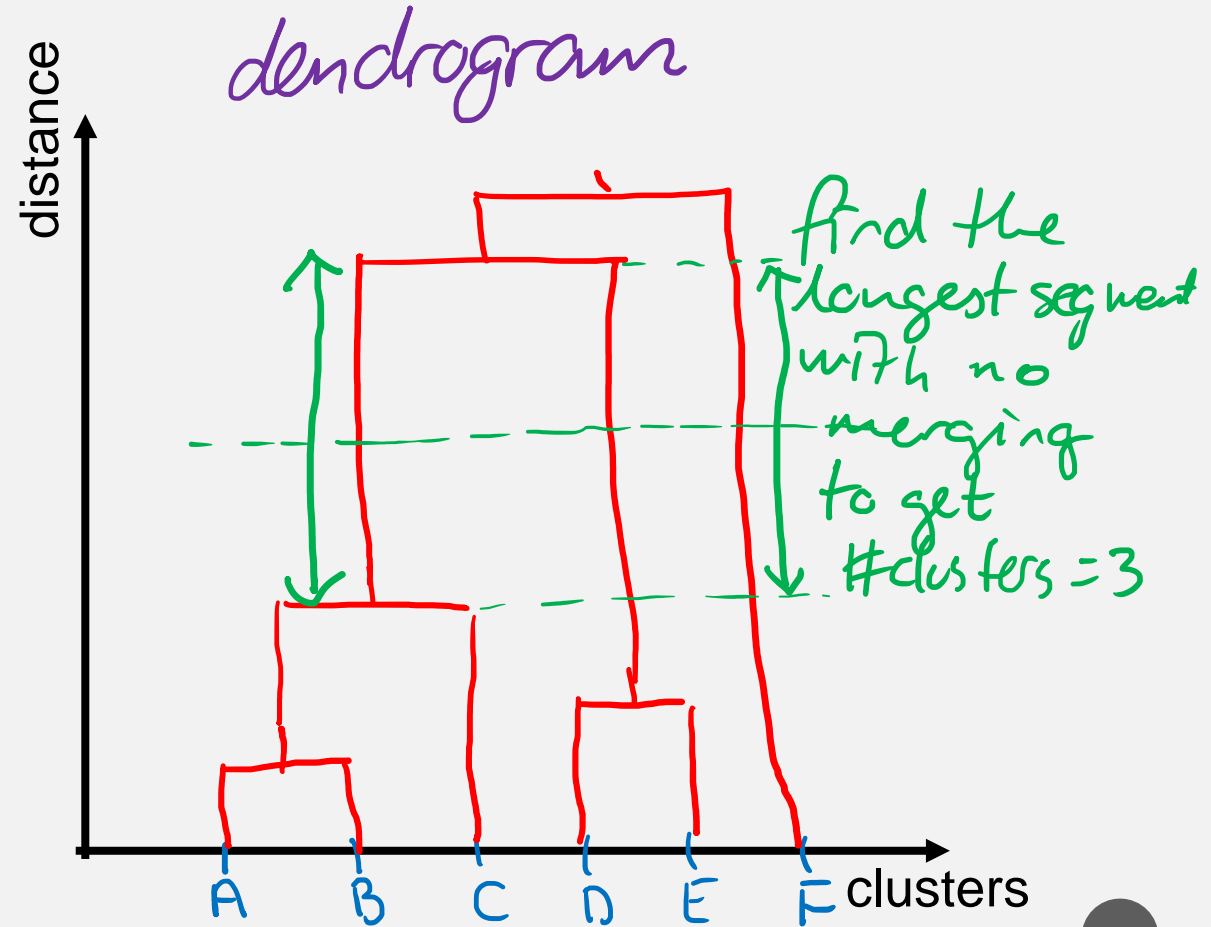
# CLUSTERING

- What is clustering?
- *k*-means clustering
- **Agglomerative clustering**
- DBSCAN
- Application

## AGGLOMERATIVE CLUSTERING

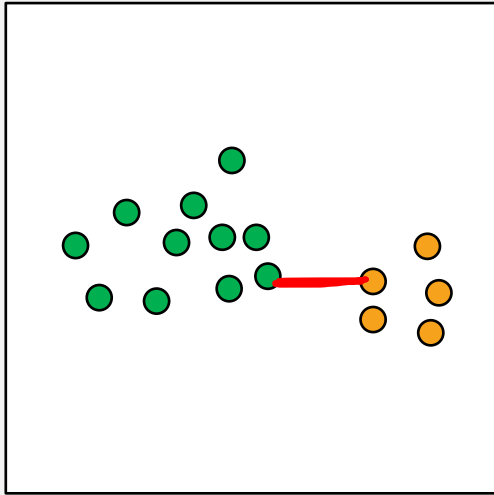
let each point be its own cluster  
while there are more than 1 cluster:  
merge the two closest clusters

# AGGLOMERATIVE CLUSTERING

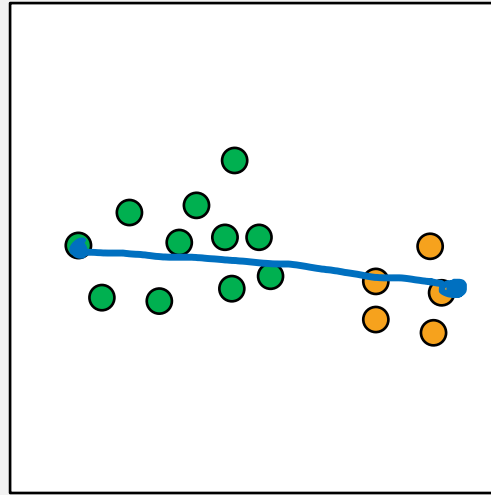




# THE DISTANCE BETWEEN CLUSTERS

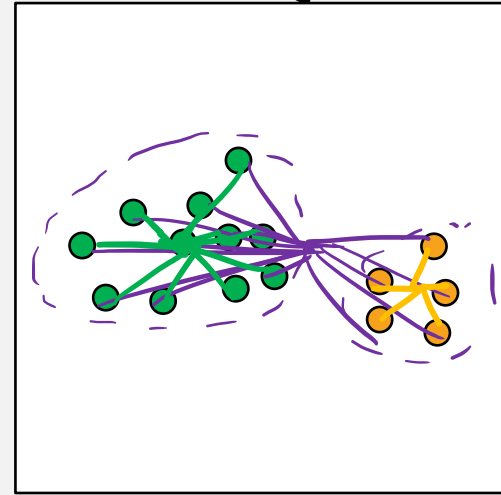


"single link"  
(min distance)  
→ sensitive to  
outliers/noise



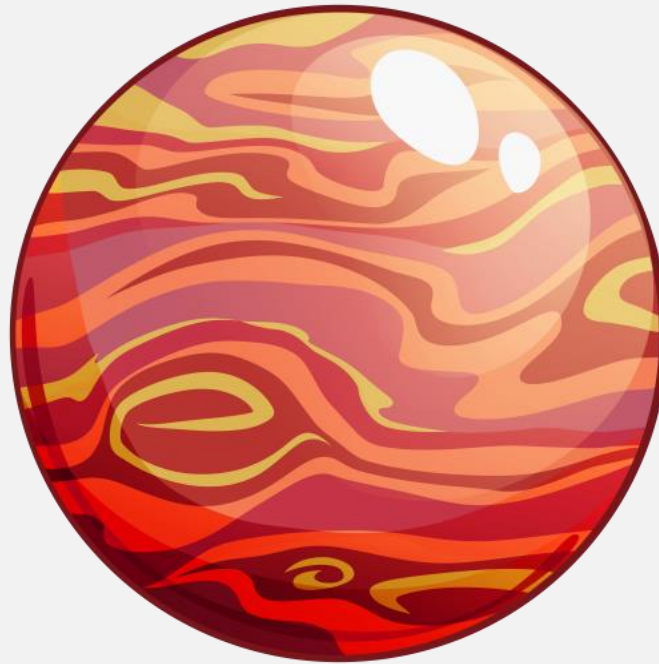
"complete link"  
(max distance)  
→ may break  
large clusters  
several others...

$$\text{purple star} - (\text{green star} + \text{yellow star})$$



"Ward's method"  
(change in cost function)  
→ difficulty with  
different sizes/  
weird shapes

## CODE EXAMPLE



*Jupyter Notebook* **Clustering methods**

# CLUSTERING

- What is clustering?
- *k*-means clustering
- Agglomerative clustering
- **DBSCAN**
- Application

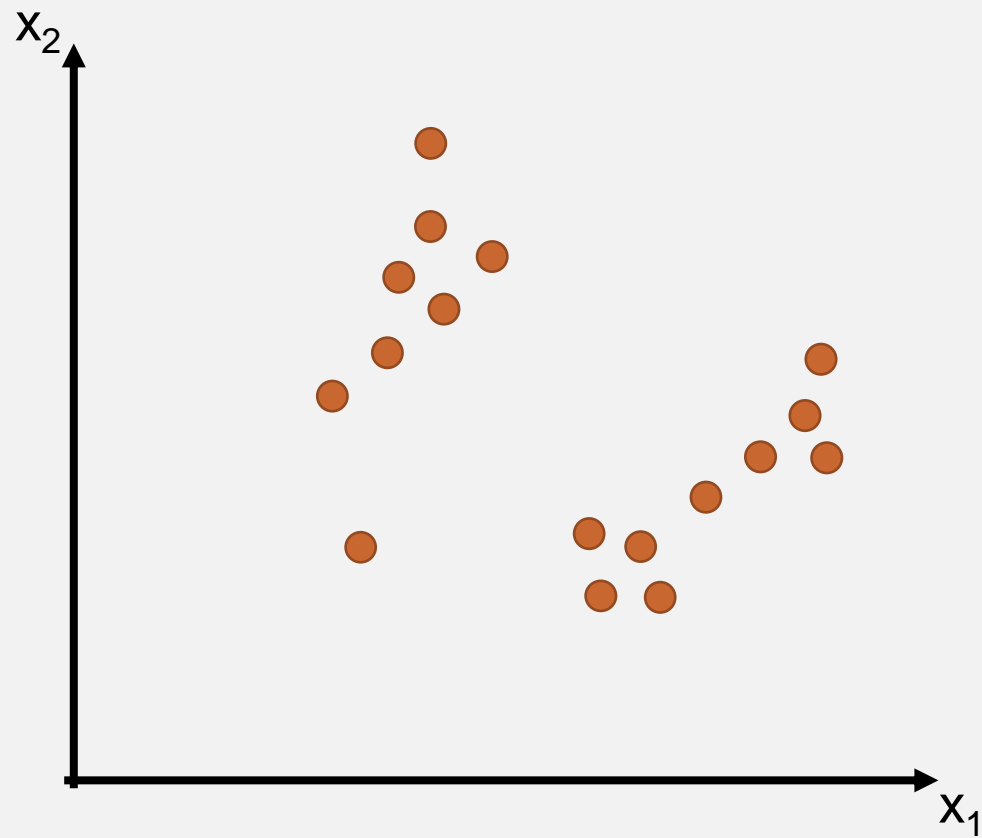
# DBSCAN

density-based spatial clustering of applications with noise

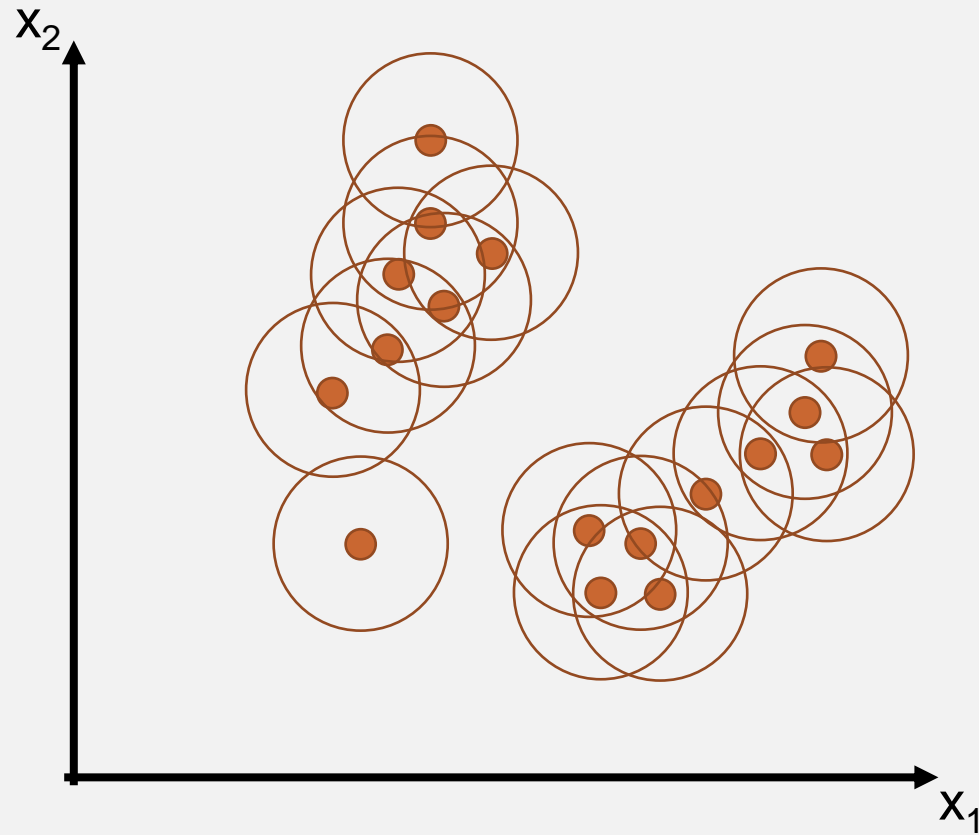
⇒ partition points into dense regions  
separated by not so dense regions

- How do we measure density?  
= number of points in a circle of radius  $\epsilon$
- What is a dense region?  
= density of at least  $n$  points

# DBSCAN

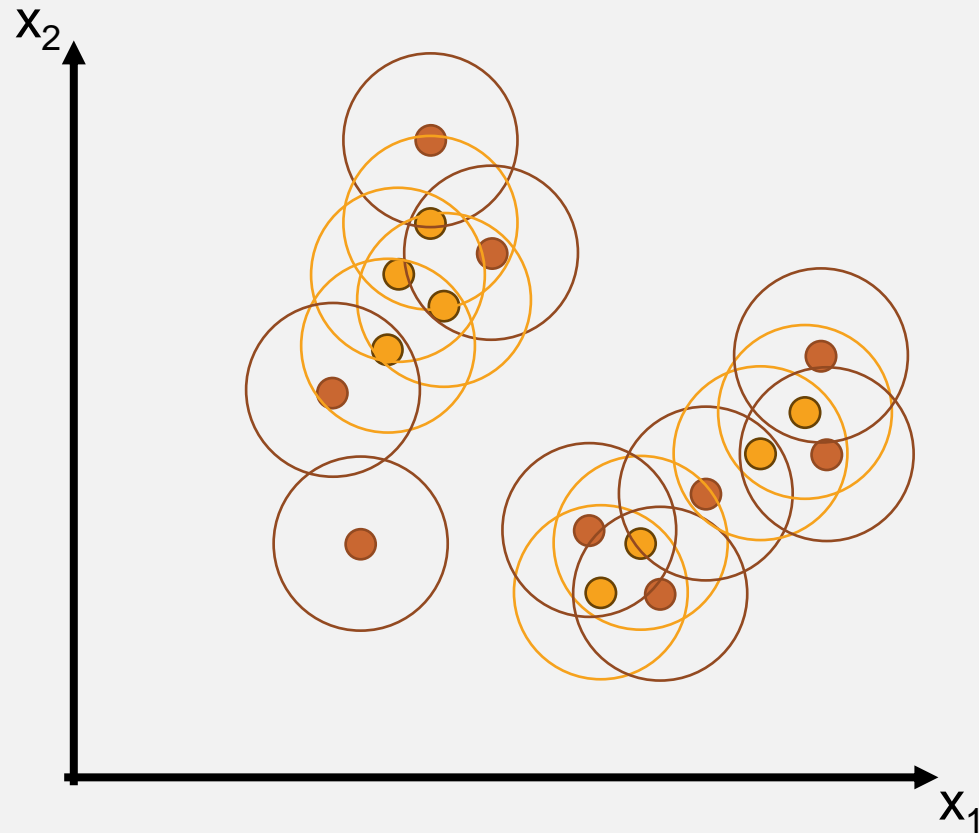


# DBSCAN



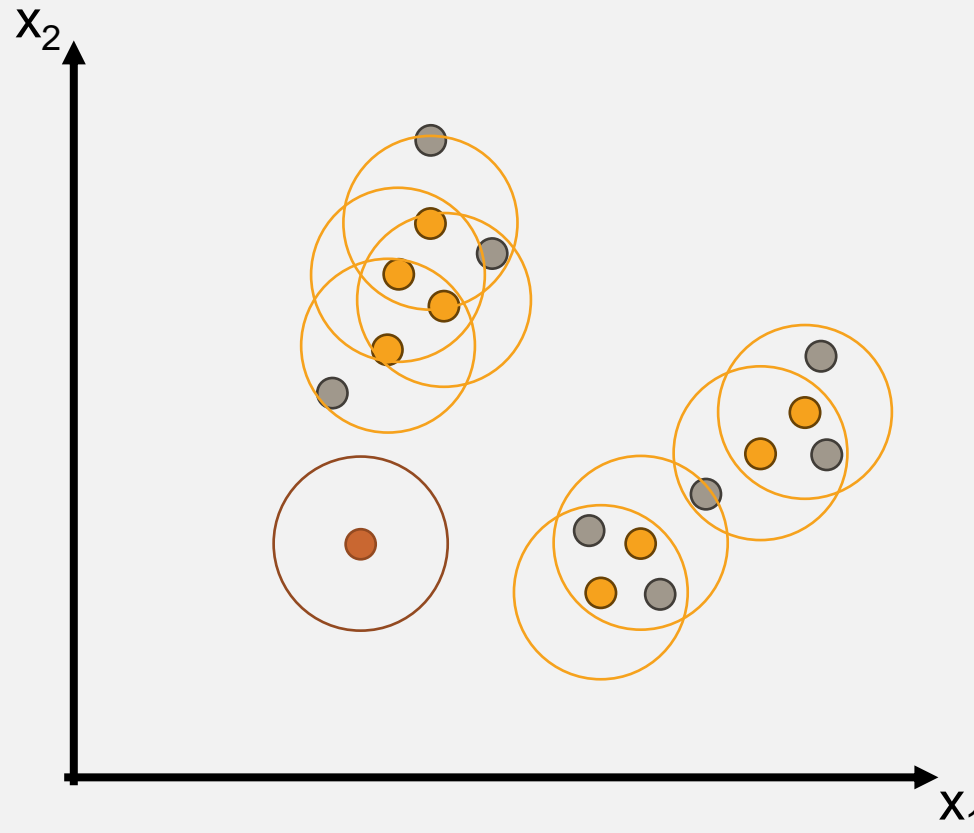
- I. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.

# DBSCAN



1. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.
2. If the  $\epsilon$ -neighbourhood contains at least  $n$  ( $=4$ ) points, we consider the point a **core** point ●.

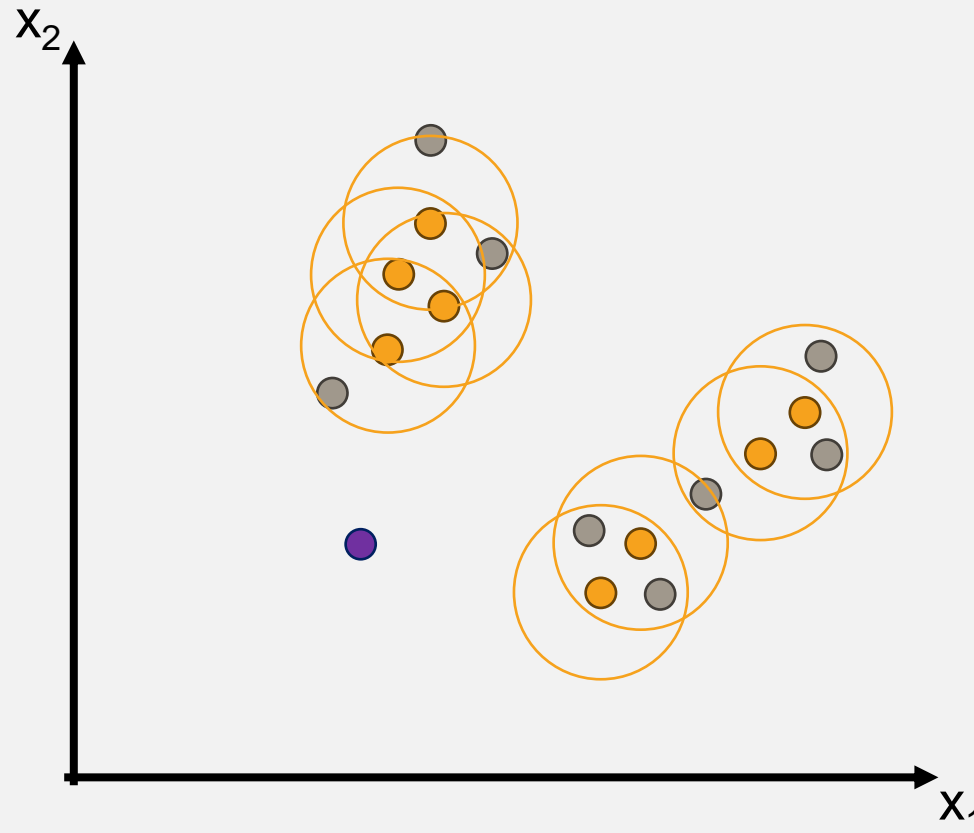
# DBSCAN



1. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.
2. If the  $\epsilon$ -neighbourhood contains at least  $n$  ( $=4$ ) points, we consider the point a **core** point ●.
3. If the point is not a core point, but is in the  $\epsilon$ -neighbourhood of one, it is a **border** point ●.

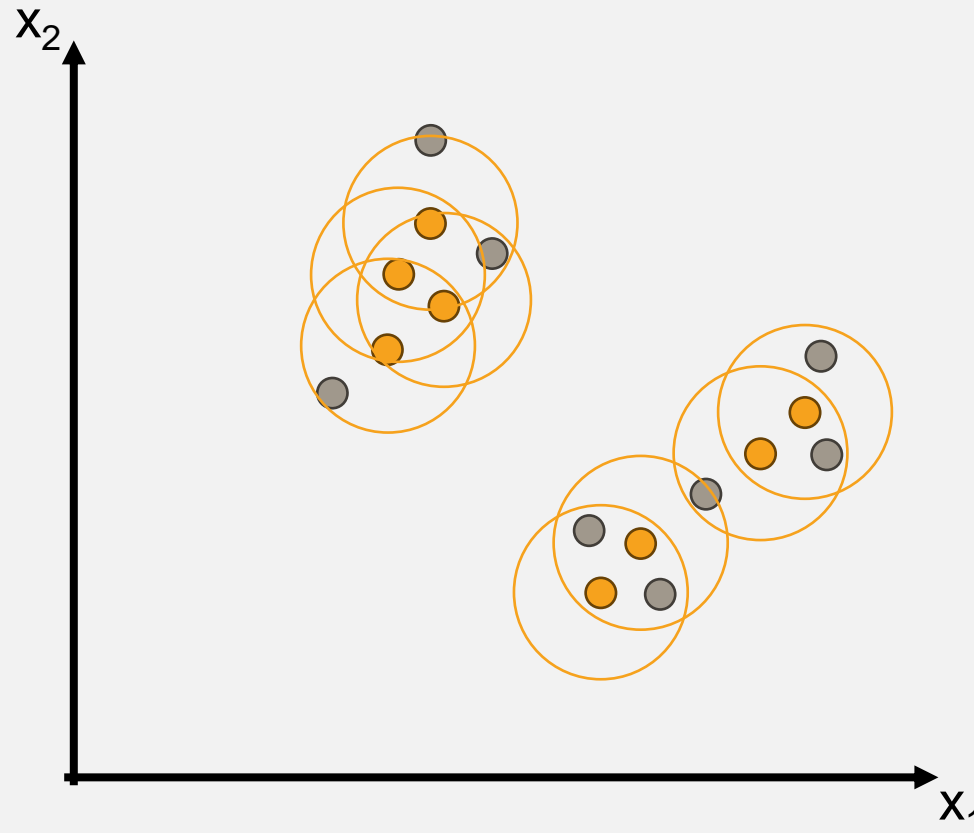


# DBSCAN



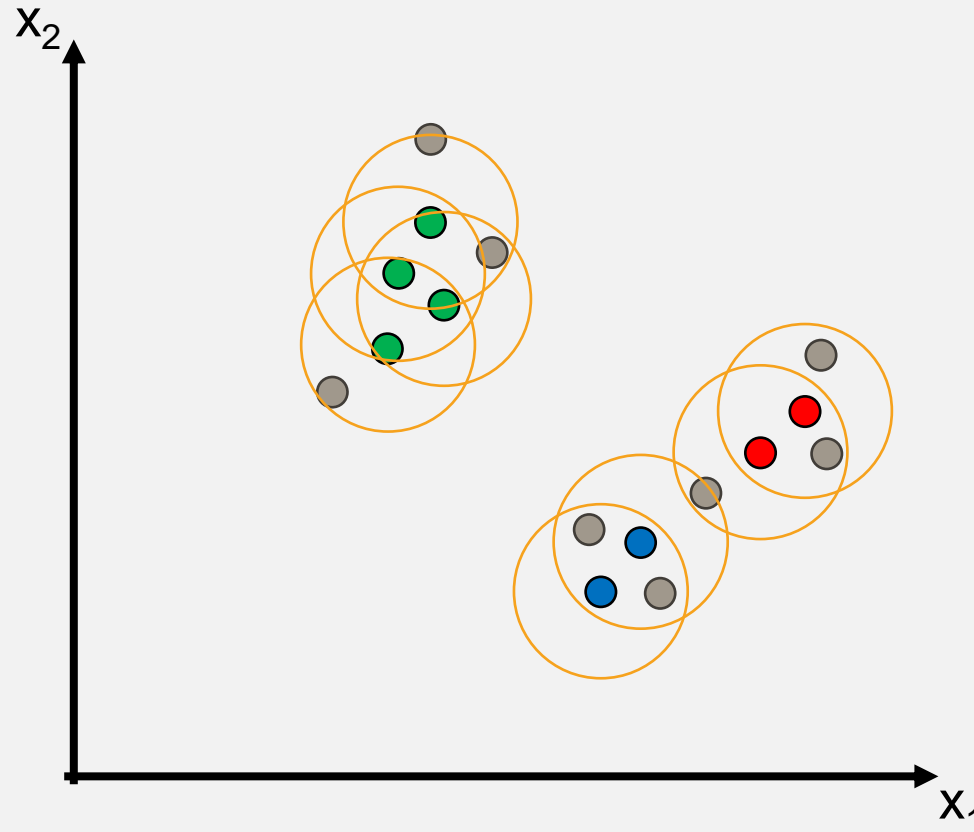
1. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.
2. If the  $\epsilon$ -neighbourhood contains at least  $n$  ( $=4$ ) points, we consider the point a **core** point ●.
3. If the point is not a core point, but is in the  $\epsilon$ -neighbourhood of one, it is a **border** point ●.
4. Otherwise, it is a **noise** point ●.

# DBSCAN



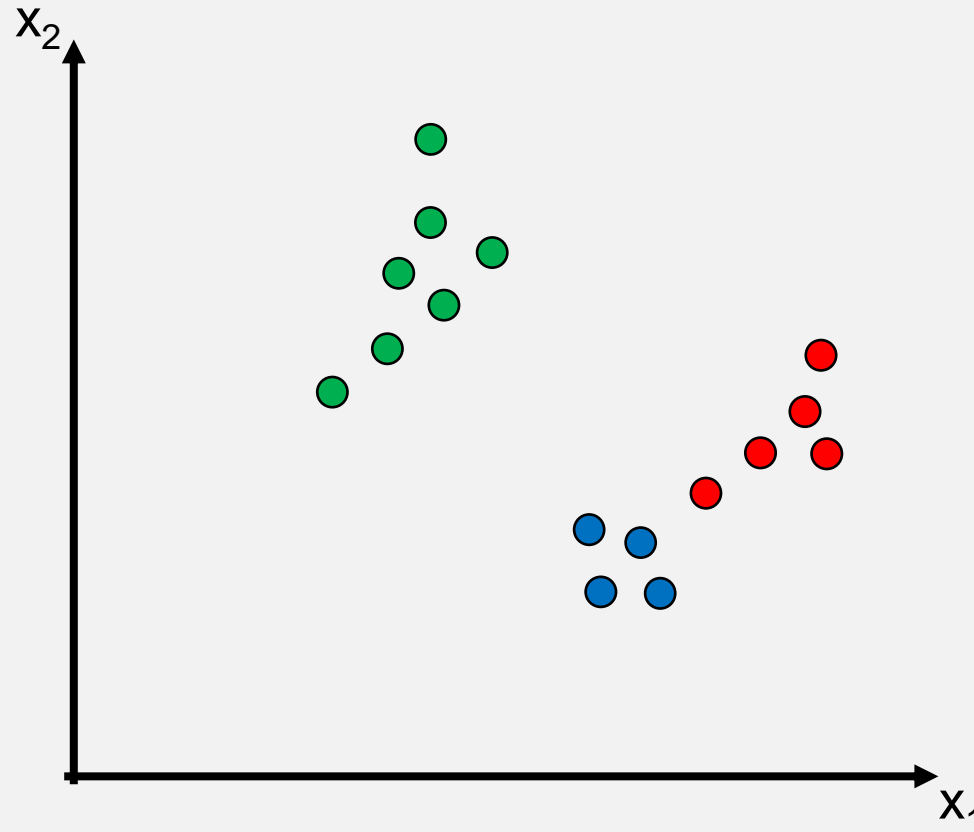
1. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.
2. If the  $\epsilon$ -neighbourhood contains at least  $n$  ( $=4$ ) points, we consider the point a **core** point ●.
3. If the point is not a core point, but is in the  $\epsilon$ -neighbourhood of one, it is a **border** point ●.
4. Otherwise, it is a **noise** point ●.
5. Get rid of **noise** points.




# DBSCAN



1. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.
2. If the  $\epsilon$ -neighbourhood contains at least  $n$  ( $=4$ ) points, we consider the point a **core** point (orange).
3. If the point is not a core point, but is in the  $\epsilon$ -neighbourhood of one, it is a **border** point (grey).
4. Otherwise, it is a **noise** point (blue).
5. Get rid of **noise** points.
6. All **core** points reachable through each other's  $\epsilon$ -neighbourhoods belong to the same cluster.

# DBSCAN



1. Draw a circle of radius  $\epsilon$  around every point.  
This region is the  $\epsilon$ -neighbourhood.
2. If the  $\epsilon$ -neighbourhood contains at least  $n$  ( $=4$ ) points, we consider the point a **core** point .
3. If the point is not a core point, but is in the  $\epsilon$ -neighbourhood of one, it is a **border** point .
4. Otherwise, it is a **noise** point .
5. Get rid of **noise** points.
6. All **core** points reachable through each other's  $\epsilon$ -neighbourhoods belong to the same cluster.
7. All **border** points are assigned to the cluster of closest core point.

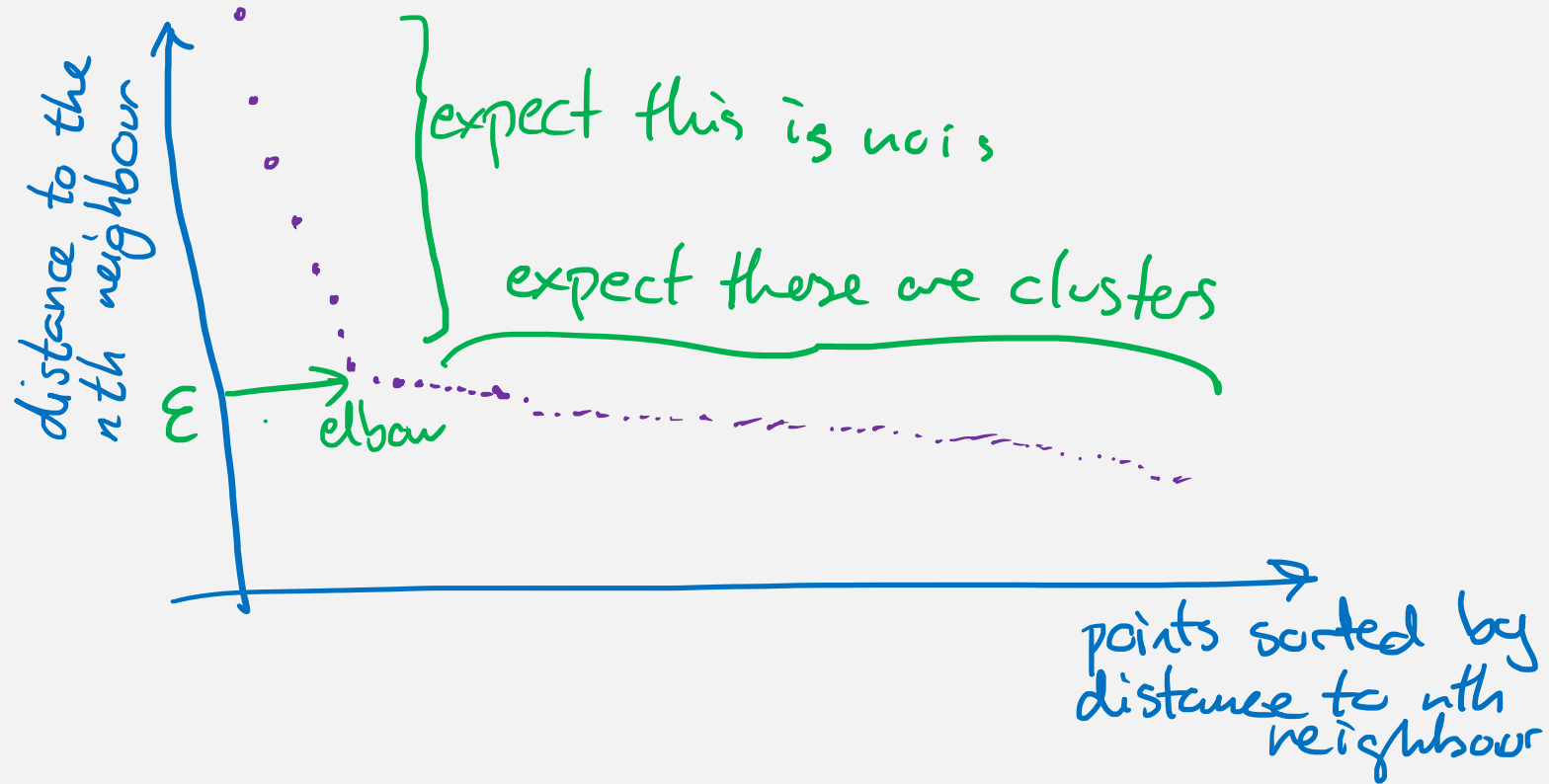
## DETERMINING $\epsilon$ AND $n$

(recommendations .....)

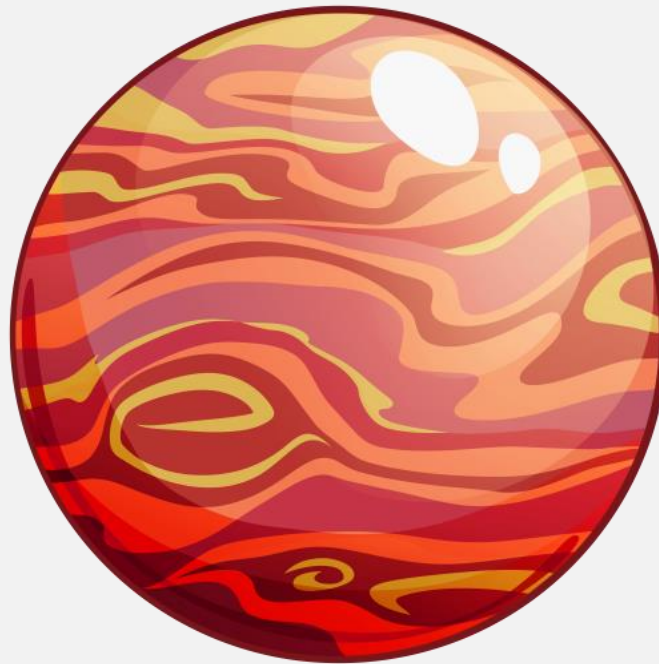
$$n \text{ (minPts)} = 2 \times d$$

dimensionality  
2D data  $\Rightarrow n=4$

③  $\epsilon$



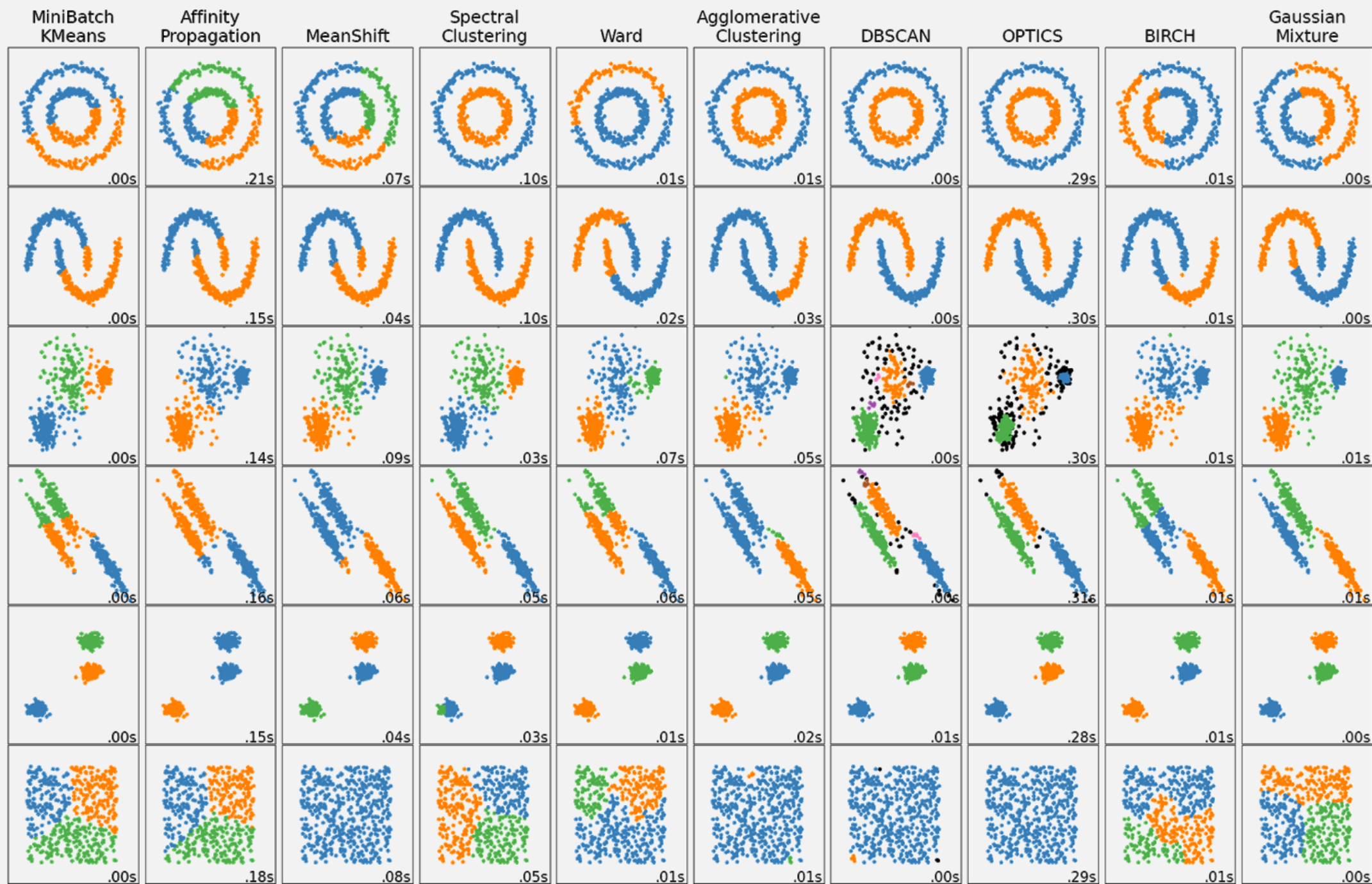
## CODE EXAMPLE



*Jupyter Notebook* **Clustering methods**

# COMPARING THE MODELS

	Pros	Cons
k-means clustering	Efficient	Can't handle noise/outliers Can't handle weird shapes Initialization User must provide $k$
Agglomerative clustering	No a priori knowledge of #clusters needed	Dendrograms can be ambiguous Computationally heavy Each distance metric has its own problems
DBSCAN	Weird shapes Handles outliers No a priori knowledge of #clusters needed	Trouble with different densities

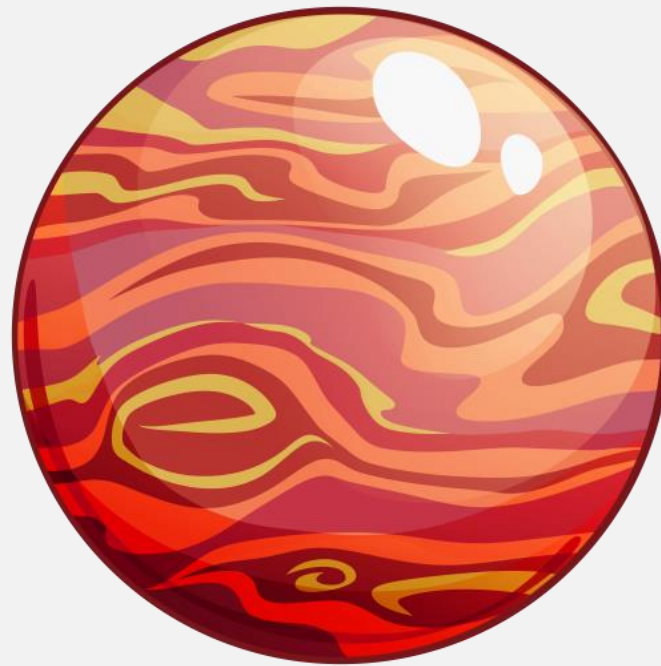




# CLUSTERING

- What is clustering?
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- Agglomerative clustering
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# APPLICATION: IMAGE SEGMENTATION



*Jupyter Notebook* **Image segmentation**

OUR ANALYSIS SHOWS THAT THERE ARE  
THREE KINDS OF PEOPLE IN THE WORLD:  
THOSE WHO USE K-MEANS CLUSTERING  
WITH  $K=3$ , AND TWO OTHER TYPES WHOSE  
QUALITATIVE INTERPRETATION IS UNCLEAR.

