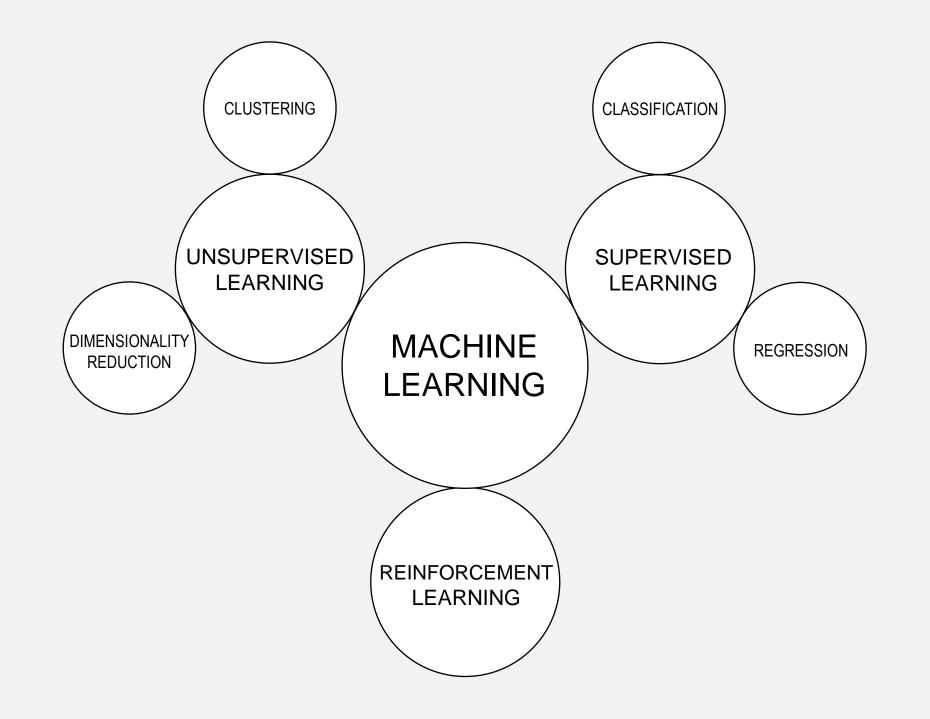
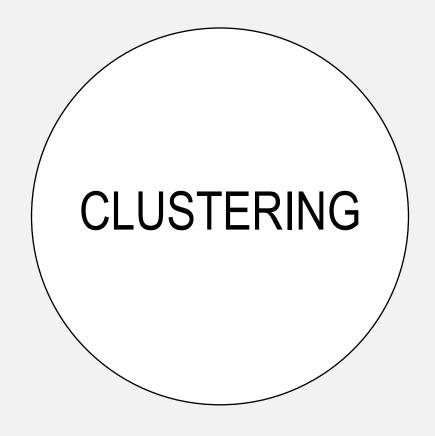
# **CLUSTERING**

Lecture 10

MALI, 2024





### WHAT IS CLUSTERING?

I want to know what this bird in my garden is











The corresponding websites tell me it's a common linnet













































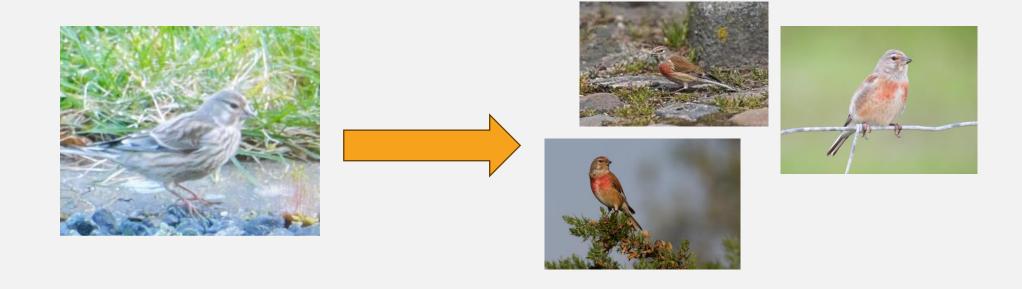




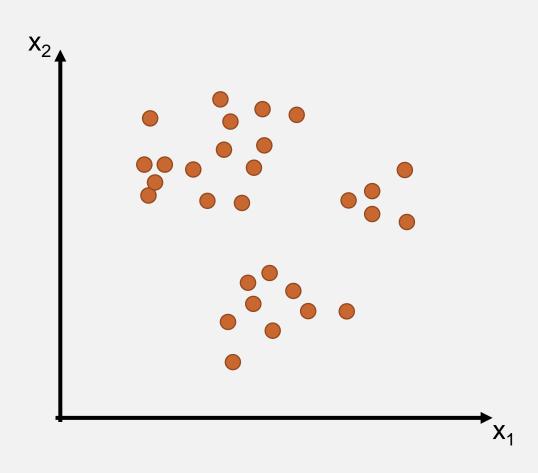


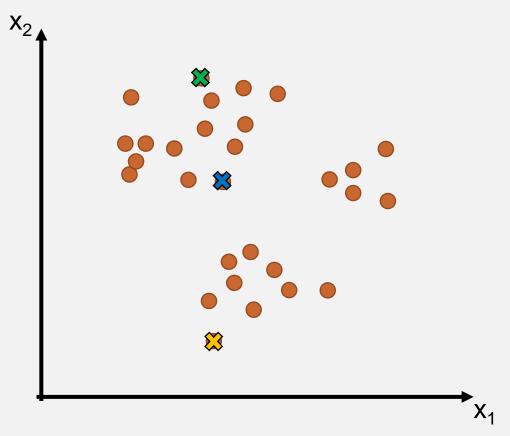




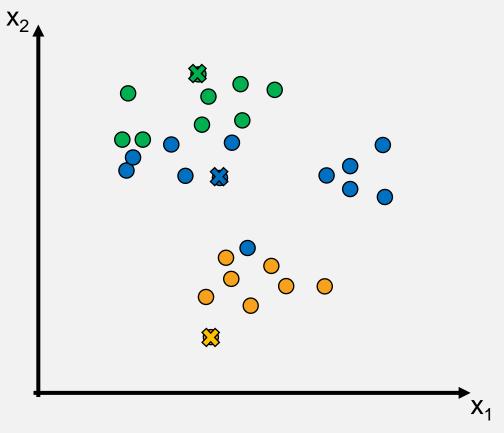


### DIFFERENCE FROM CLASSIFICATION?

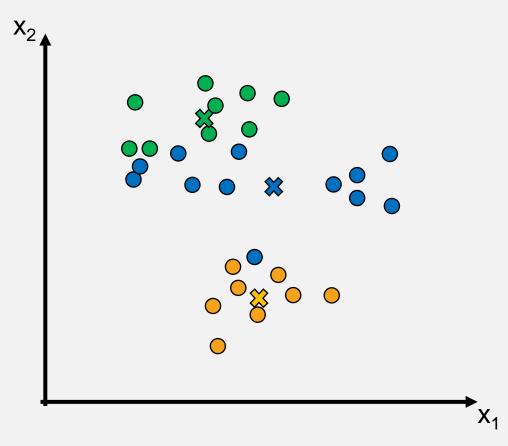




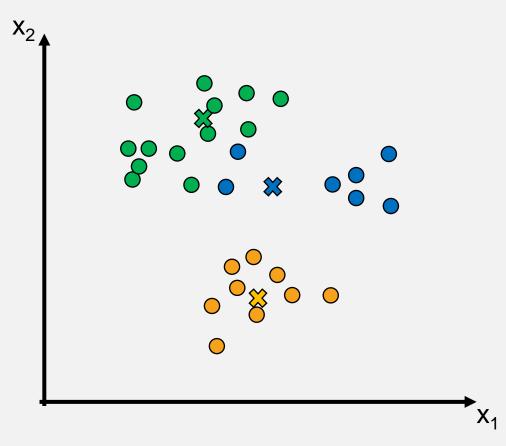
I. Assign k(=3) random points as centroids



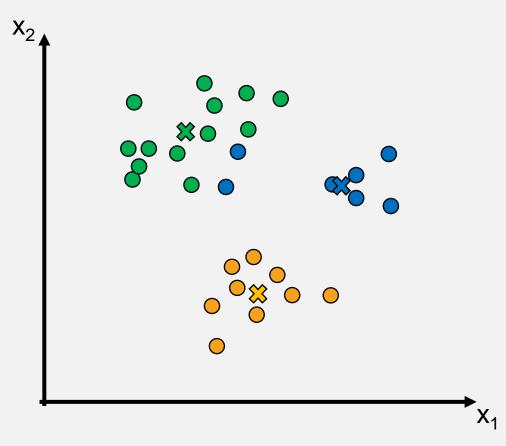
- I. Assign k(=3) random points as **centroids**
- 2. Group the data by their distance to the centroids



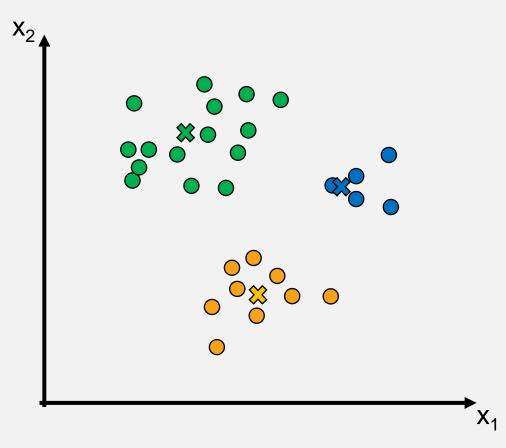
- I. Assign k(=3) random points as **centroids**
- 2. Group the data by their distance to the centroids
- 3. Move the centroids to the cluster centers



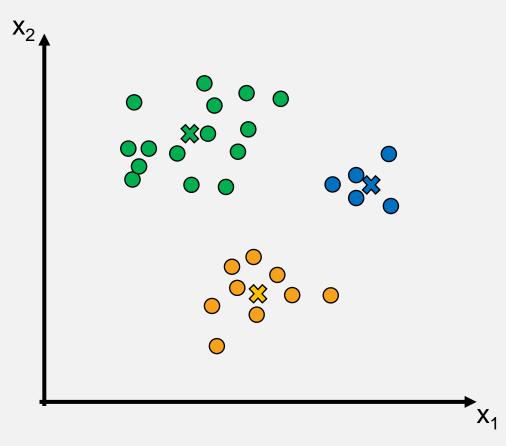
- I. Assign k(=3) random points as **centroids**
- 2. Group the data by their distance to the centroids
- 3. Move the centroids to the cluster centers
- 4. Regroup the data



- I. Assign k(=3) random points as **centroids**
- 2. Group the data by their distance to the centroids
- 3. Move the centroids to the cluster centers
- 4. Regroup the data
- 5. Repeat 3-4 until nothing changes

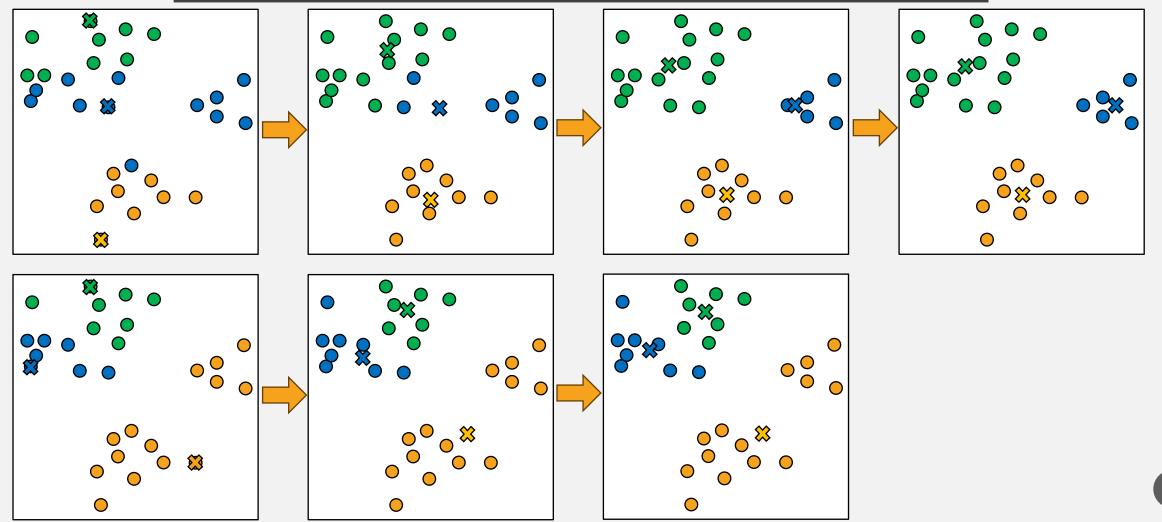


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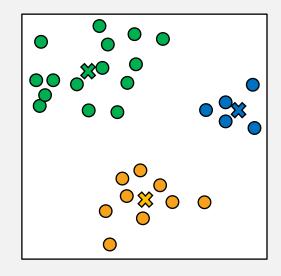


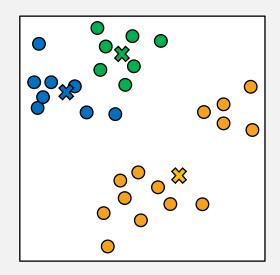
- I. Assign k(=3) random points as **centroids**
- 2. Group the data by their distance to the centroids
- 3. Move the centroids to the cluster centers
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- 5. Repeat 3-4 until nothing changes

## A FEW THINGS WE HAVE TO DEAL WITH

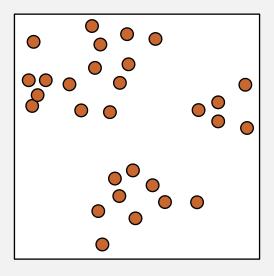


Solution 1: Try different, randomized initializations and compare the **costs** of the final clusterings

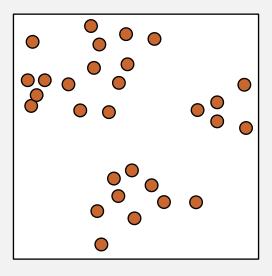




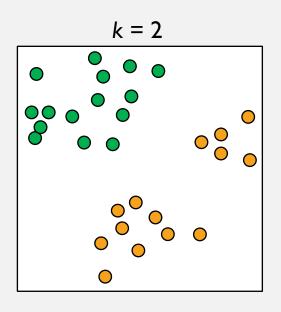
Solution 2: Choose the initial centroids based on the distance to the previous ones

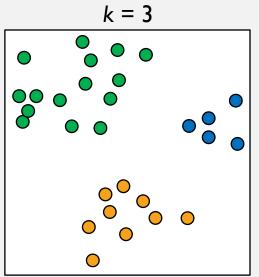


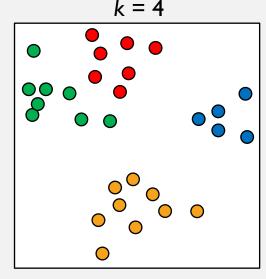
Solution 3: Choose "far away but random" points ("k-means++")

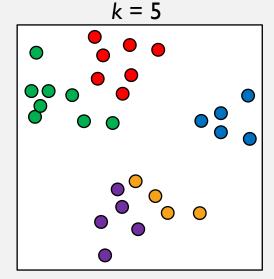


## THE NUMBER OF CLUSTERS (k)









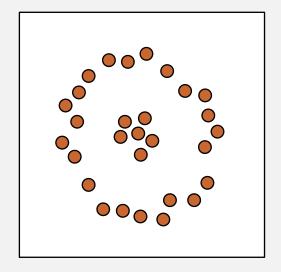
## THE NUMBER OF CLUSTERS (k)

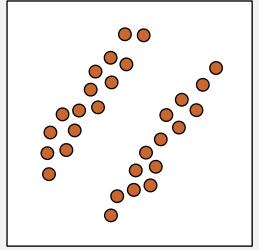
The easy way:

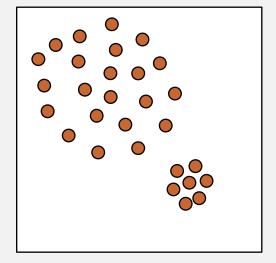
The hard way:

$$C = \sum_{i} \left| |x_i - \mu(x_i)| \right|^2$$

#### WHERE k-MEANS FAILS







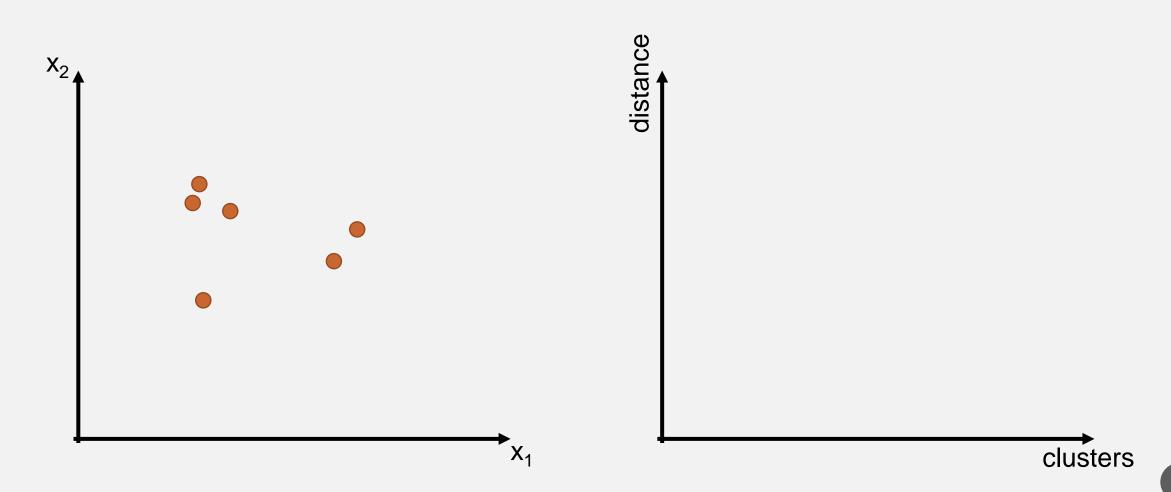
### CODE EXAMPLE



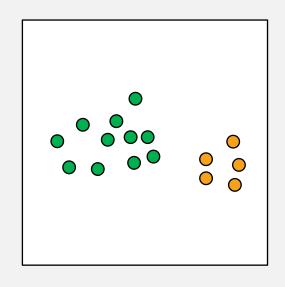
Jupyter Notebook Clustering methods

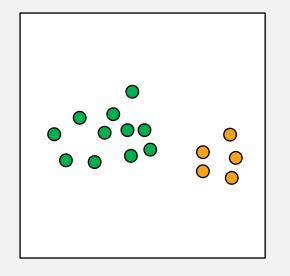
## AGGLOMERATIVE CLUSTERING

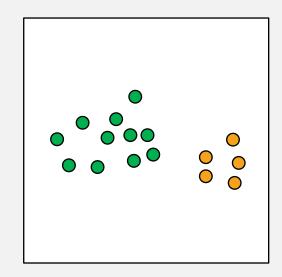
### AGGLOMERATIVE CLUSTERING



#### THE DISTANCE BETWEEN CLUSTERS





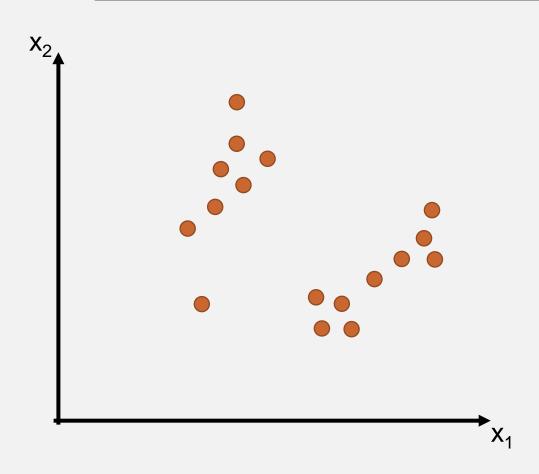


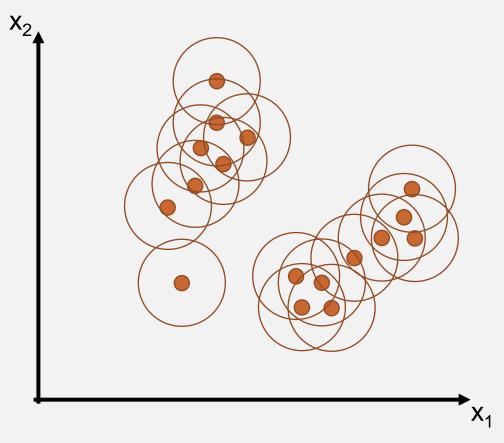
### CODE EXAMPLE



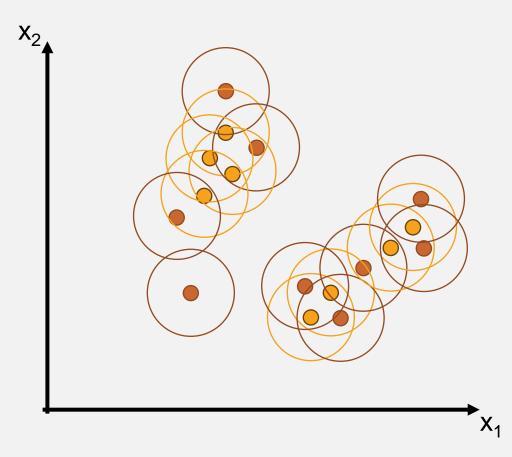
Jupyter Notebook Clustering methods

- How do we measure density?
- What is a dense region?

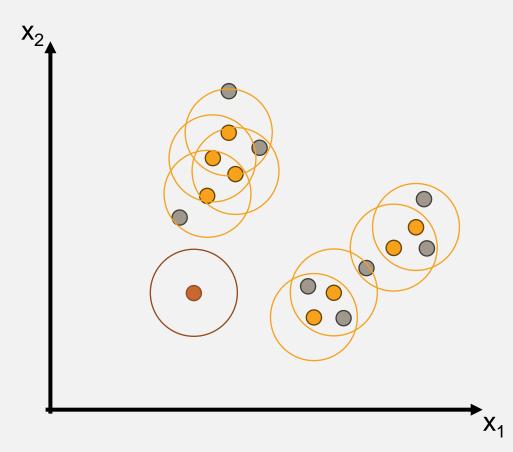




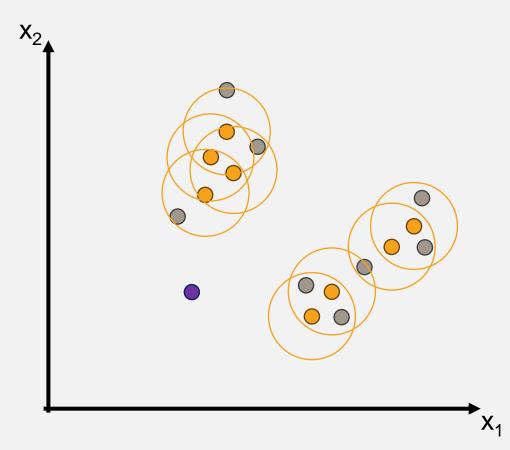
I. Draw a circle of radius  $\epsilon$  around every point. This region is the  $\epsilon$ -neighbourhood.



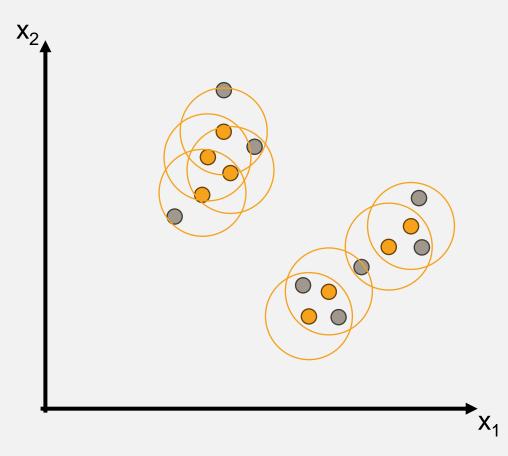
- I. Draw a circle of radius  $\epsilon$  around every point. This region is the  $\epsilon$ -neighbourhood.
- 2. If the ε-neighbourhood contains at least n (=4) points, we consider the point a **core** point •.



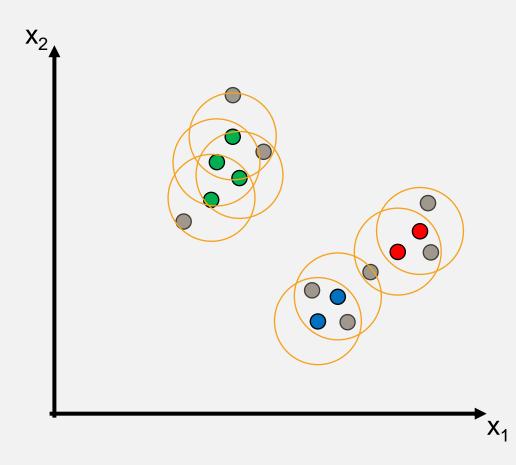
- Draw a circle of radius ε around every point.
  This region is the ε-neighbourhood.
- 2. If the ε-neighbourhood contains at least n (=4) points, we consider the point a **core** point •.
- 3. If the point is not a core point, but is in the ε-neighbourhood of one, it is a **border** point ...



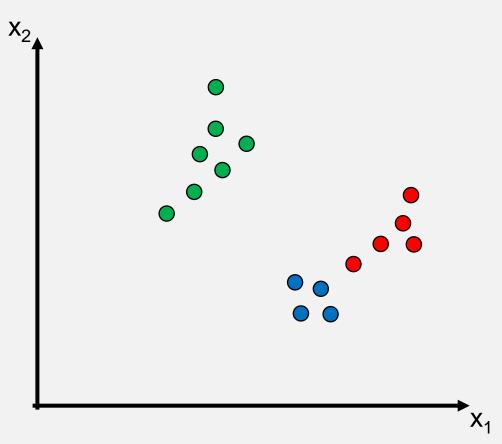
- Draw a circle of radius ε around every point.
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- 3. If the point is not a core point, but is in the ε-neighbourhood of one, it is a **border** point ...
- 4. Otherwise, it is a **noise** point **.**



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- 4. Otherwise, it is a **noise** point **.**
- 5. Get rid of **noise** points.



- Draw a circle of radius ε around every point.
  This region is the ε-neighbourhood.
- 2. If the ε-neighbourhood contains at least n (=4) points, we consider the point a **core** point •.
- 3. If the point is not a core point, but is in the ε-neighbourhood of one, it is a **border** point ...
- 4. Otherwise, it is a **noise** point **.**
- 5. Get rid of **noise** points.
- 6. All **core** points reachable through each other's ε-neighbourhoods belong to the same cluster.



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  This region is the ε-neighbourhood.
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- 3. If the point is not a core point, but is in the ε-neighbourhood of one, it is a **border** point ...
- 4. Otherwise, it is a **noise** point **.**
- 5. Get rid of **noise** points.
- 6. All **core** points reachable through each other's ε-neighbourhoods belong to the same cluster.
- 7. All **border** points are assigned to the cluster of closest core point.

## DETERMINING ε AND n

### CODE EXAMPLE



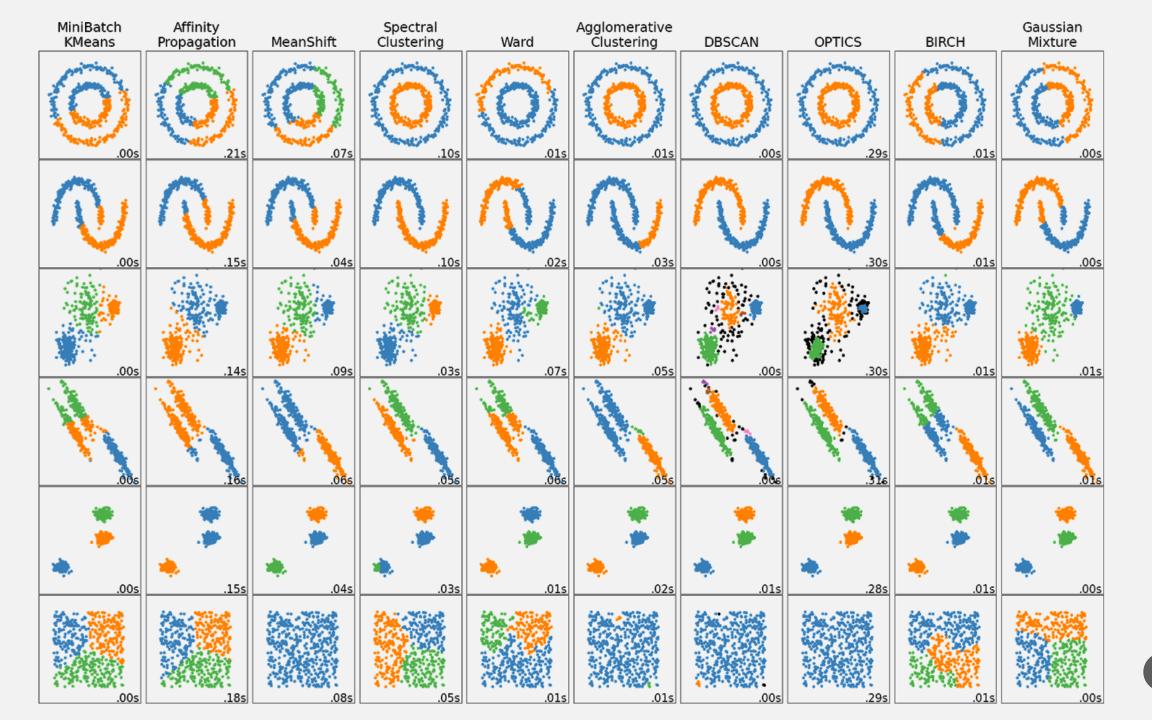
Jupyter Notebook Clustering methods

#### COMPARING THE MODELS

Pros Cons

k-means clustering

**Agglomerative clustering** 



## APPLICATION: IMAGE SEGMENTATION



Jupyter Notebook Image segmentation

OUR ANALYSIS SHOWS THAT THERE ARE THREE KINDS OF PEOPLE IN THE WORLD: THOSE WHO USE K-MEANS CLUSTERING WITH K=3, AND TWO OTHER TYPES WHOSE QUALITATIVE INTERPRETATION 15 UNCLEAR.