Recap

PMF:

1) $f(x_i) \geq 0$
2) $\sum f(x_i) = 1$
3) $f(x_i) = P(X = x_i)$

CDF:

$F(x_i) = P(X \leq x_i)$

$0 \leq F(x) \leq 1$

if $x \leq y \rightarrow F(x) \leq F(y)$

$\frac{\sum x_i}{n}$

Expected Value:

$$E(x) = E[X] = EX = \mu_x = \sum_{i=1}^{n} x_i \cdot f(x_i)$$

Variance:

$$\sigma^2 = V(x) = Var(x)$$
$$= \sum x_i^2 \cdot f(x_i) - (E(x))^2$$
$$= E[x^2] - (E[X])^2$$

Standard Deviaton:

$$\sqrt{\sigma^2} = \sigma = \sqrt{E[x^2] - (E[X])^2}$$

Variance is not linear:

$$Var(aX+b) = a^2 \cdot Var(X)$$

Expectation is linear:

$$E(aX+b) = a \cdot E(X) + b$$

CDF vs. SF

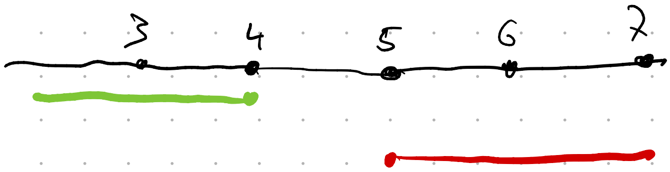
$$P(X \le 4) = P(X=0) + P(X=1) + \dots P(X=4)$$

$$P(X \ge 4) = 1 - P(X \le 3)$$

$cdf(4) + sf(4) = 1$

$P(X \le 4) + 1 - P(X \le 4)$

$$Sf + CDF = 1$$

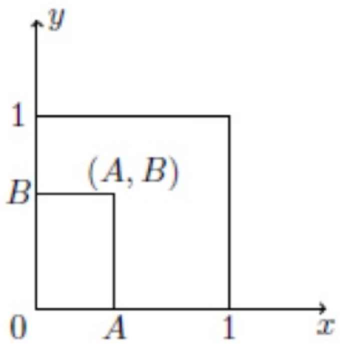


CDF includes x

SF does not include x.

Exercise 7

You choose a point (A, B) uniformly at random in the unit square $\{(x, y) : 0 \le x, y \le 1\}$.



What is the probability that the equation

$$AX^2 + X + B = 0$$

has real solutions?

Real solution $\rightarrow$ $b^2 - 4 \cdot ac \geq 0$

$\hookrightarrow ax^2 + bx + c = 0$

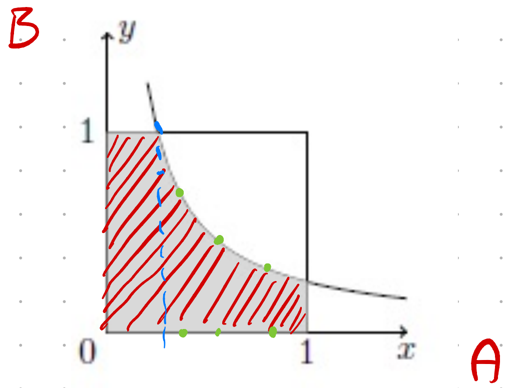
$\hookrightarrow a = A, b = 1, c = B$

$1^2 - 4 \cdot A \cdot B \geq 0$

$4AB \leq 1$

$AB \leq 1/4$

$\hookrightarrow B \leq \frac{1}{4A}$



So the question is:

How much is shaded out of the total?

$(ax^n)' = anx^{n-1}$

$\int ax^n = \frac{a}{n+1} x^{n+1}$

$X: [0; 1/4] \rightarrow y = 1$ so $1/4$

$x: [1/4, 1] \rightarrow y = \frac{1}{4x}$

$f(x) = 2x^6 + 9x^2 + 7$

$= 6 \cdot 2x^5 + 2 \cdot 9x^1 + 0 \cdot 7x^{-1}$

$$\text{Area} = \frac{1}{4} + \int_{1/4}^{1} \frac{1}{4x}\,dx = \frac{1}{4} + \frac{1}{4}\left[\ln x\right]_{1/4}^{1}$$

$$= \frac{1}{4} + \frac{1}{4}\left[0 - \ln 1/4\right] = \frac{1}{4} - \frac{1}{4} \cdot \ln 4^{-1}$$

$$= \frac{1}{4}(1 + \ln 4)$$

$P(X \geq 4) = 0.512$

Let Y denote the event that $X \geq 4$

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - \left[P(X=0) + P(X=1) \right]$$

$$= 1 - \left[\frac{\lambda^0 \cdot e^{-\lambda}}{0!} + \frac{\lambda^1 \cdot e^{-\lambda}}{1!} \right]$$

$$= 1 - \left[\frac{1^0 \cdot e^{-1}}{0!} + \frac{1^1 \cdot e^{-1}}{1!} \right]$$

m_1 m_2 ----- m_{10}

[1 1 0 1 1 0 1 1 7]

$$P(X = x_i) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

$$P(X \leq x_i) = 0.95$$