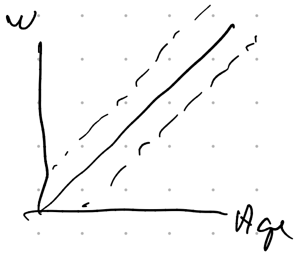
Statistical Intervals

Point estimate vs. Interval estimate

Types of interval estimates:

* Confidence intervals (8.1 – 8.6) + Ch. 10
* Prediction intervals
* Tolerance interval

(8.7)



Confidence Interval:

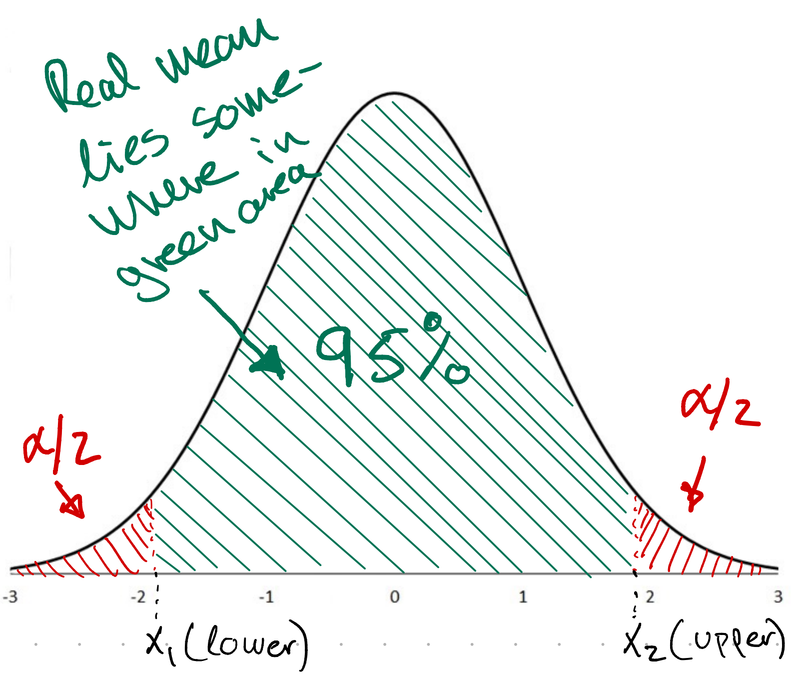
An interval within which the true parameter is believed to lie. Our belief is associated with a degree of belief

The interval depends on:

1) The parameter of interest (μ, σ, p)
2) What is known (e.g. σ)

1. CI for mean, true σ is known, Data Normal

$$\text{C.I.}: \ \bar{X} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$



α = error

confidence level $= 1 - \alpha$

upper:

$$\bar{X} + Z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

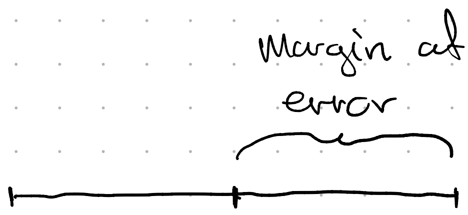
lower:

$$\bar{X} - Z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Z-scores for different C.I levels:

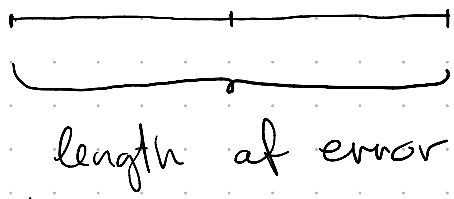
99% :	$Z_{1-\alpha/2} = 2.58$
95% :	$Z_{1-\alpha/2} = 1.96$
90% :	$Z_{1-\alpha/2} = 1.64$

$Z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$: margin of error



$\frac{\sigma}{\sqrt{n}}$; standard error

$2 \cdot \left[Z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right]$: length of error



example: Find 95% and 99% C.I:

$n = 25$, $\sigma = 20$, $\bar{x} = 122$

95% C.I :

$122 \pm 1.96 \cdot \frac{20}{\sqrt{25}}$

$122 \pm 1.96 \cdot 4$: $[114.16 ; 129.84]$

99% C.I :

$122 \pm 2.58 \cdot 4$: $[111.70 ; 132.30]$

Finding n:

Assume you need to find a sample size for some test/research:

a) Determine a confidence level ($Z_{\alpha/2}$)

b) Determine an allowed margin of error E

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

One-Sided Confidence Bounds:

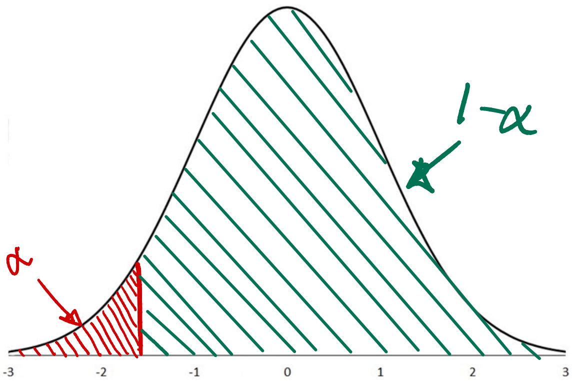
A $100(1-\alpha)\%$ upper-confidence bound for μ is

$$\mu \le \bar{x} + z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}}$$

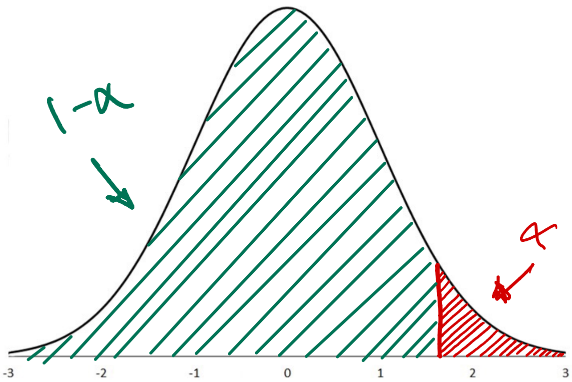
A $100(1-\alpha)\%$ lower-confidence bound for μ is

$$\bar{x} - z_{1-\alpha} \cdot \frac{\sigma}{\sqrt{n}} \le \mu$$

Lower



Upper



2. Large Sample C.I ($n \ge 30$ or $n \ge 40$)

$$\bar{x} - z_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}} \le \mu \le \bar{x} + z_{1-\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

Similar considerations apply for one-sided intervals.

3. C.I on mean, σ unknown, data normal, n small

Here we use a more conservative normal distribution called the t-distribution.

↳ standard deviation calculated from sample:

$$\bar{x} \pm t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}$$

```python
stats.t.cdf(1.96, 20)
```

0.9679608734982115

```python
stats.t.ppf(0.95, 19)
```

1.729132811521367

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%

Similar considerations apply for one-sided intervals.

Degree of freedom:

Since $\bar{x}$ was estimated and is used to estimate s, we have $n-1$ d.o.f.

Note: the sample S.D:

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

But Numpy does:

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

Must pass ddof=1

Statistics module uses n−1:

```
[14] from statistics import *
```

```
[15] stdev([1.5, 2.5, 2.5, 2.75, 3.25, 4.75])

1.0810874155219827
```

and so does scipy:

```
stats.tstd(data)

1.0810874155219827
```

Has a trimmed std.

4. CI for proportions:

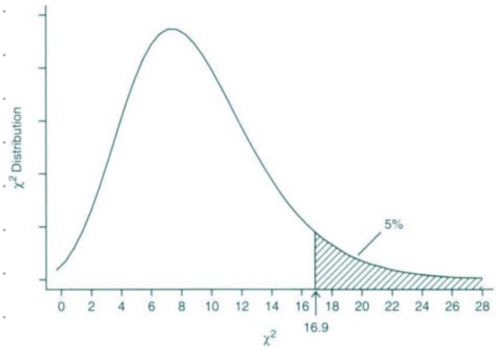
$$\hat{p} \pm Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$$

$n \cdot p \geq 5$ or $n(1-p) > 5$

Similar considerations of sample size and one-sided intervals.

5. CI for Variance / St.Dev:

We use a completely different distribution for this called the Chi-squared distribution



Not symmetric

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,\,n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,\,n-1}}$$

larger value

smaller value

Problem: A researcher is studying the amount of time it takes for customers to complete transactions at a local store. They collect a random sample of 25 transaction times (in minutes) and record the following data:

```
[27] data = [4, 5, 6, 5, 7, 5, 4, 4, 6, 5, 7, 6, 5, 4, 6, 7, 5, 6, 6, 4, 5, 6, 6, 7, 8]
```

```
[37] from scipy.stats import chi2
     from statistics import variance

     # Find variance and sample size
     var = variance(data)
     n = len(data)

     # Determine the chi-squared values
     alpha = 0.05
     chi2_low = chi2.ppf(alpha / 2, n - 1)
     chi2_high = chi2.ppf(1 - alpha / 2, n - 1)

     # Find the CI
     lower = (n - 1) * var / chi2_high
     upper = (n - 1) * var / chi2_low

     # return interval
     print("95% Confidence Interval for the Variance: [{:.4f}; {:.4f}]".format(lower, upper))
```

```
95% Confidence Interval for the Variance: [0.7662; 2.4320]
```

Note: This is a C.I for Variance
If you want for St.Dev, you need to take the square root.