# Maintenance Optimization

Understanding Optimization

# Life Data Analysis

Understanding Weibull Analysis

# An Overview of Basic Concepts

- In life data analysis (also called "Weibull analysis"), the practitioner attempts to make predictions about the life of all products in the population.
- The parameterized distribution for the data set can then be used to estimate important life characteristics of the product such as reliability or probability of failure at a specific time, the mean life and the failure rate.
- Life data analysis requires the practitioner to:
  - Gather life data for the product.
  - Select a lifetime distribution that will fit the data and model the life of the product.
  - Estimate the parameters that will fit the distribution to the data.
  - Generate plots and results that estimate the life characteristics of the product, such as the reliability or mean life.

# What’s the Weibull?- in English

- The “Weibull “ refers to the Weibull statistical distribution, named after its “rediscoverer” Waloddi Weibull in the late 1940’s.
- It provides a graphical solution to reliability questions even for small samples.
- The Weibull distribution is used extensively in reliability engineering because of its ability to describe failure distributions from Early Life(infant morality) to Useful Life(random failures) to Wear out.
- Examples include:
  - Early Life: Quality problems, assembly problems
  - Useful Life: foreign object damage, human error, quality or maintenance problems
  - Wear out: Low cycle fatigue, stress-rupture, corrosion

# Lifetime Distributions

- Statistical distributions have been formulated by statisticians, mathematicians and engineers to mathematically model or represent certain behaviour. The probability density function (*pdf*) is a mathematical function that describes the distribution.
- The equation below gives the *pdf* for the 3-parameter Weibull distribution.

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}$$

- Some distributions, such as the Weibull and lognormal, tend to better represent life data and are commonly called "lifetime distributions" or "life distributions.“
- Other commonly used life distributions include the exponential, lognormal and normal distributions. The analyst chooses the life distribution that is most appropriate to model each particular data set based on past experience and goodness-of-fit tests.

# Weibull Definitions

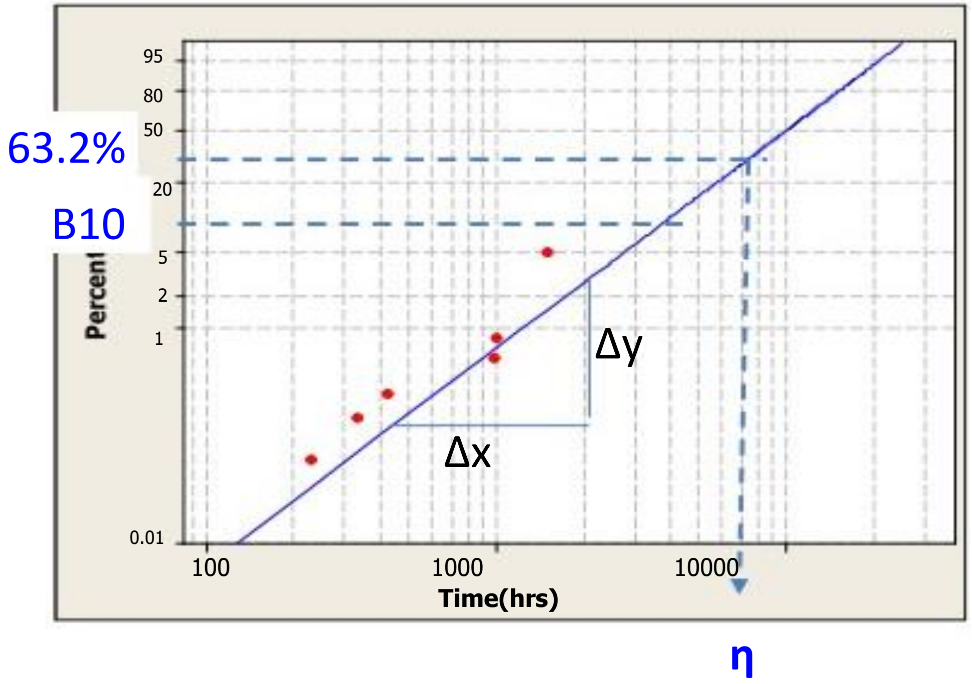
- η (Eta)= characteristic Life
  ≈ mean time to failure
- β (Beta) = “slope” of line
  ≈ failure mode type
- Weibull equation:

*Cumulative % failed* = $1-e^{-\left(\frac{t}{n}\right)^{\beta}}$

- If t=η:

*Cumulative % failed* (*at t*=η)=(1−$\frac{1}{e}$) *x*100%=63.2%

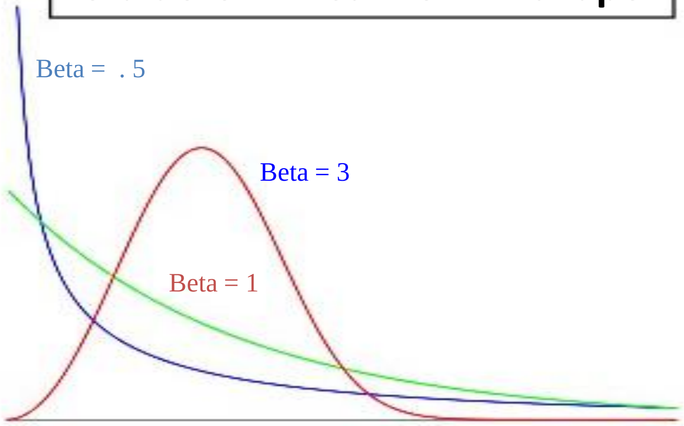
- Bxx life=age at which xx% of the fleet fail
- B1 life= age at which 1% of the fleet fail
- B10 life= age at which 10% of the fleet fail



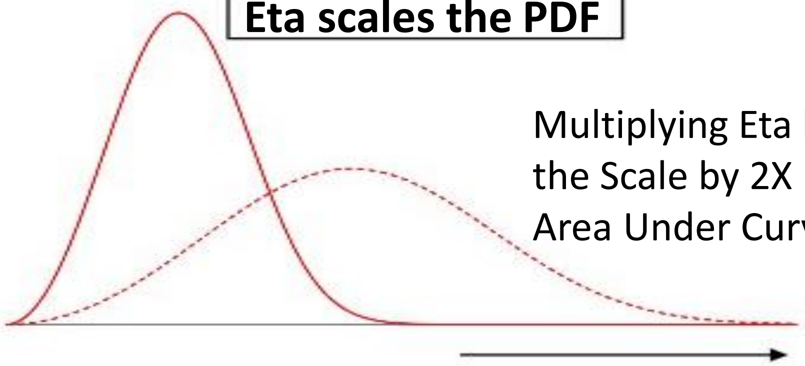
# β and η makes the Weibull work

The Weibull distribution is characterized by two parameters, a shape parameter we refer to as beta (β ) and a scale parameter we refer to as eta (η )

**Beta determines the PDF shape**




**Eta scales the PDF**



Multiplying Eta by 2 Stretches the Scale by 2X But Keeps Area Under Curve Equal to 1

Beta and Eta are calculated to best match the frequency, or density of data point occurrence along the x-axis

# Calculated Results and Plots

Once you have calculated the parameters to fit a life distribution to a particular data set, you can obtain a variety of plots and calculated results from the analysis, including:

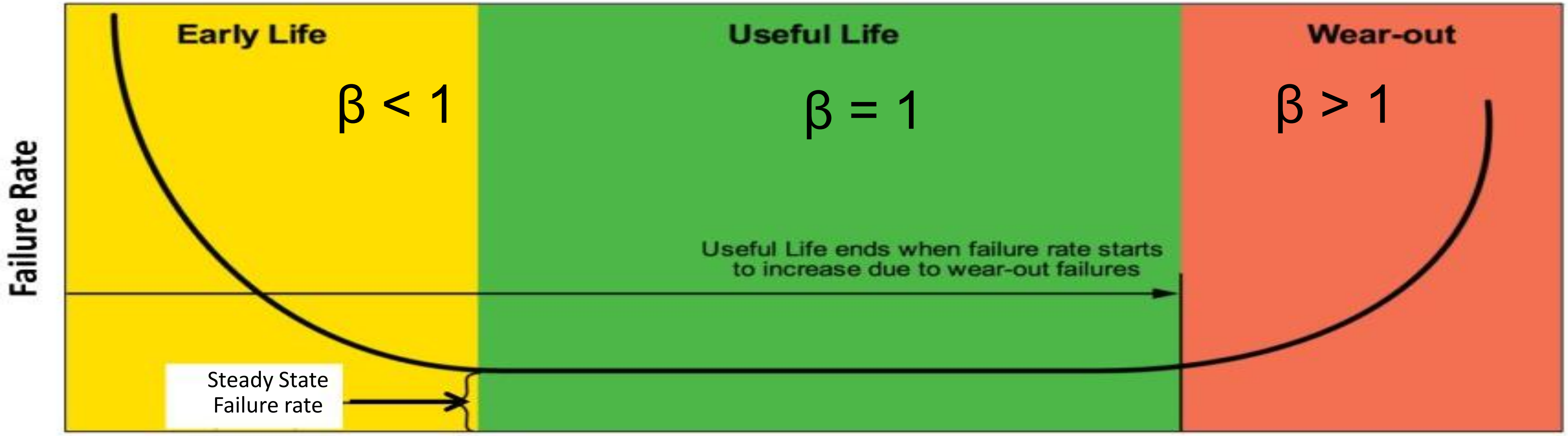
- ❑ **Reliability Given Time:** The probability that a unit will operate successfully at a particular point in time. For example, there is an 88% chance that the product will operate successfully after 3 years of operation.
- ❑ **Probability of Failure Given Time:** The probability that a unit will be failed at a particular point in time. Probability of failure is also known as "unreliability" and it is the reciprocal of the reliability. For example, there is a 12% chance that the unit will be failed after 3 years of operation (probability of failure or unreliability) and an 88% chance that it will operate successfully (reliability).
- ❑ **Mean Life:** The average time that the units in the population are expected to operate before failure. This metric is often referred to as "mean time to failure" (MTTF) or "mean time before failure" (MTBF).
- ❑ **Failure Rate:** The number of failures per unit time that can be expected to occur for the product.
- ❑**Reliable Life** (warranty time). The estimated time when the reliability will be equal to a specified goal. For example, the estimated time of operation is 4 years for a reliability of 90%.

# Calculated Results and Plots

- **B(X) Life:** The estimated time when the probability of failure will reach a specified point (X%). For example, if 10% of the products are expected to fail by 4 years of operation, then the B(10) life is 4 years. (Note that this is equivalent to a reliable life of 4 years for a 90% reliability.)
- **Probability Plot:** A plot of the probability of failure over time. (Note that probability plots are based on the linearization of a specific distribution. Consequently, the form of a probability plot for one distribution will be different than the form for another. For example, an exponential distribution probability plot has different axes than those of a normal distribution probability plot.)
- **Reliability vs. Time Plot:** A plot of the reliability over time.
- ***pdf* Plot:** A plot of the probability density function (*pdf*).
- **Failure Rate vs. Time Plot:** A plot of the failure rate over time.
- **Contour Plot:** A graphical representation of the possible solutions to the likelihood ratio equation. This is employed to make comparisons between two different data sets.

# Bathtub curve

The Weibull Distribution can describe each portion of the Bathtub curve




**Operating Time (hours, cycles, months, seconds)**

Typical failure modes:

* Inadequate burn-in
* Misassembly
* Some quality problems

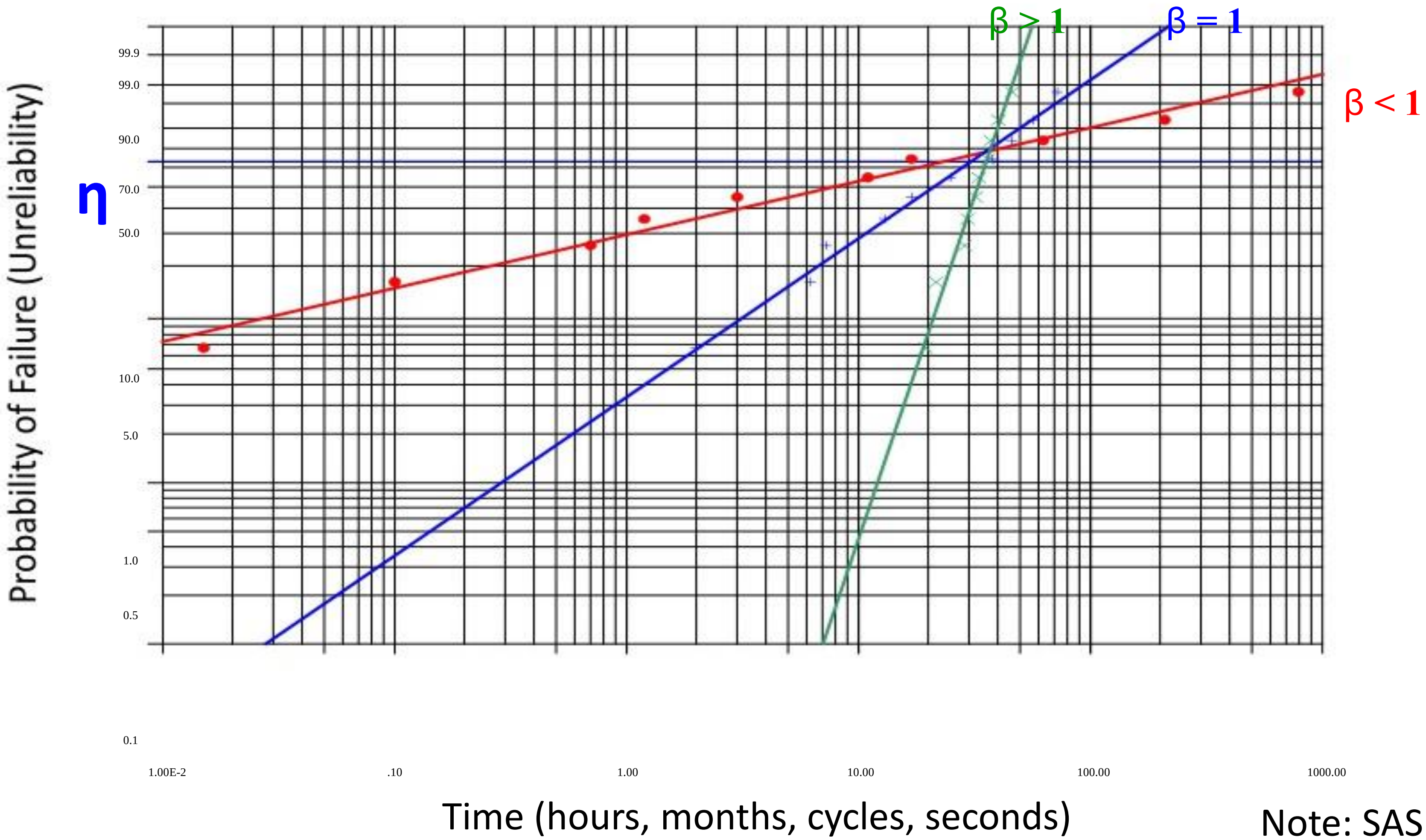
Typical failure modes

* Independent of time
* Maintenance errors
* Electronics
* Mixtures of problems

Typical failure modes:

* LCF
* TMF
* HCF
* Stress rupture
* Corrosion

# What Weibull β & η looks like on a Weibull Plot



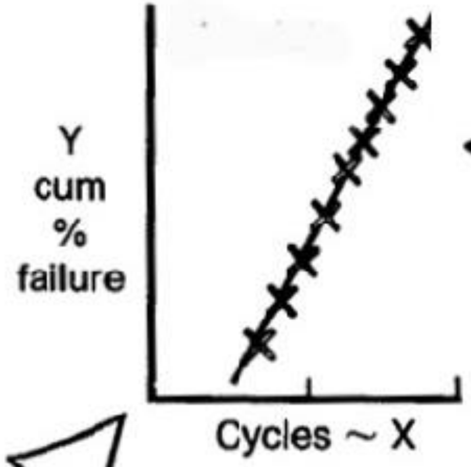

Note: SAS Weibull plot

# How is a Weibull Analysis done?

**Organize the Data**

| Failure Time-Cycles (X) | Order | Median Rank (Y) |
|---|---|---|
| 910 | 1 | 0.061 |
| 976 | 2 | 0.149 |
| 1125 | 3 | 0.237 |
| 1532 | 4 | 0.325 |
| ¦ | ¦ | ¦ |
| ¦ | ¦ | ¦ |
| 3680 | 11 | 0.939 |
| | N=11 | |

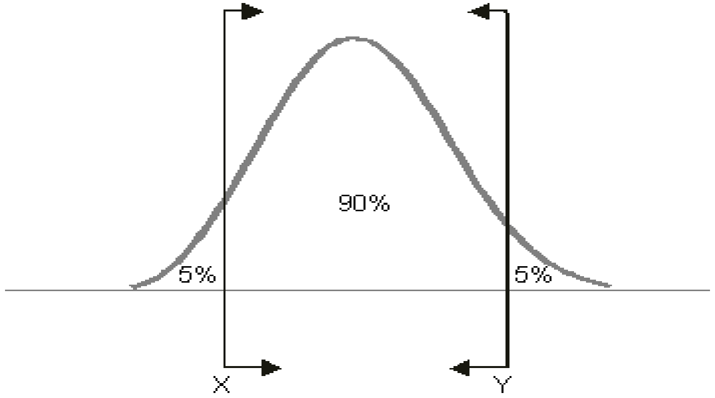
**Fit line to data**



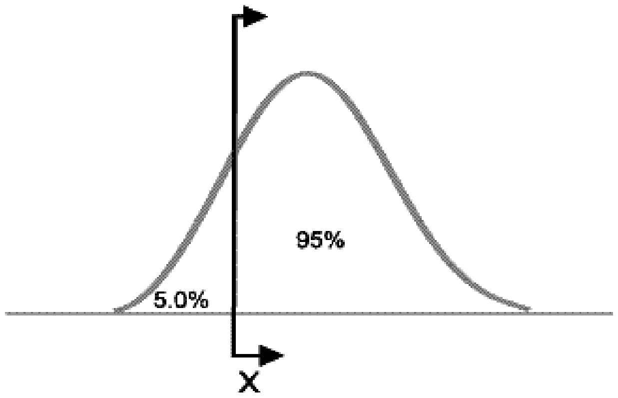

**Plot the Data**

# Confidence Bounds

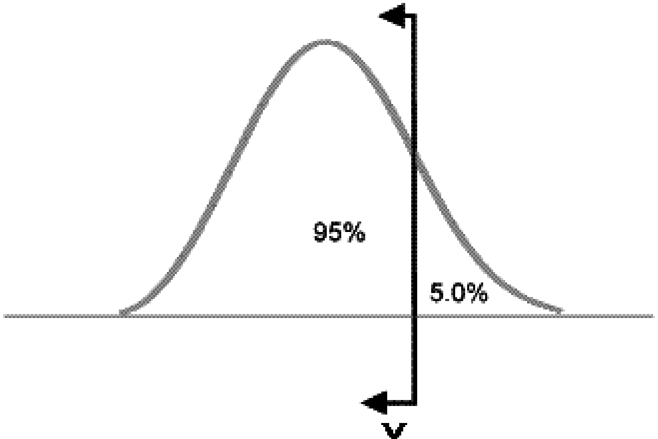
- ❑ Because life data analysis results are estimates based on the observed lifetimes of a sampling of units, there is uncertainty in the results due to the limited sample sizes.
- ❑ "Confidence bounds" (also called "confidence intervals") are used to quantify this uncertainty due to sampling error by expressing the confidence that a specific interval contains the quantity of interest.



**Two-Sided Confidence Bounds**



**Lower One-Sided Confidence Bounds**



**Upper One-Sided Confidence Bounds**

# Example Weibull Analysis

- We will now use an example to show how Weibull analysis can:
  - Indicate what type of failure mode is seen
  - Substantiate a design (or not)
  - Forecast failures
  - Evaluate corrective actions

# Examples

The following are examples of engineering problems solved with Weibull analysis:

• A project engineer reports three failures of a component in service operations during a three-month period. The Program Manager asks, "How many failures will we have in the next quarter, six months, and year?" What will it cost? What is the best corrective action to reduce the risk and losses?

• To order spare parts and schedule maintenance labor, how many units will be returned to depot for overhaul for each failure mode month-by-month next year? The program manager wants to be 95% confident that he will have enough spare parts and labor available to support the overall program.

• A state Air Resources Board requires a fleet recall when any part in the emissions system exceeds a 4% failure rate during the warranty period. Based on the warranty data, which parts will exceed the 4% rate and on what date?

• After an engineering change, how many units must be tested for how long, without any failures, to verify that the old failure mode is eliminated, or significantly improved with 90% confidence?

• An electric utility is plagued with outages from superheater tube failures. Based on inspection data forecast the life of the boiler based on plugging failed tubes. The boiler is replaced when 10% of the tubes have been plugged due to failure.

• The cost of an unplanned failure for a component, subject to a wear out failure mode, is twenty times the cost of a planned replacement. What is the optimal replacement interval?

# Example Weibull Analysis

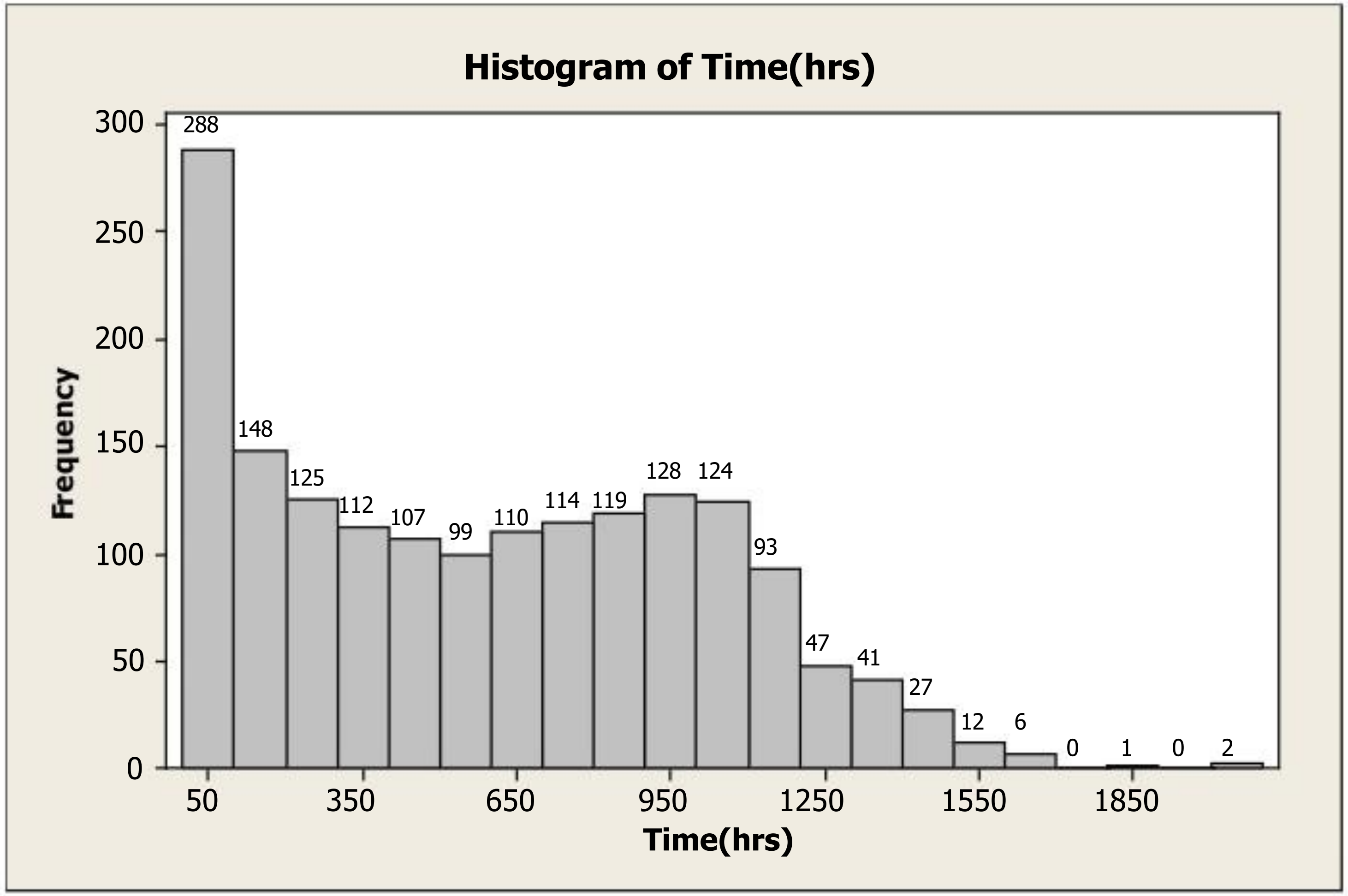
- ❑ Field failure of a bearing cage occurred at times: 230,334, 423, 990, 1009, & 1510 hours.*
- ❑ With an un failed population of over 1700 bearing cages:

- Is the demonstrated B10 life ≥ 8000 hours?
- If not:
  - How many failures will occur by 1000 hours? 4000 hours? (With no inspection)
  - How many failures will occur by 4000 hours if we initiated a 1000 hour inspection?
  - With a utilization rate of 25 hours per month, how many failures can you expect in the next year?
  - If we must redesign, how many bearing cages must I test for how long to be 90% confident I have a B10 life of 8000 hours?

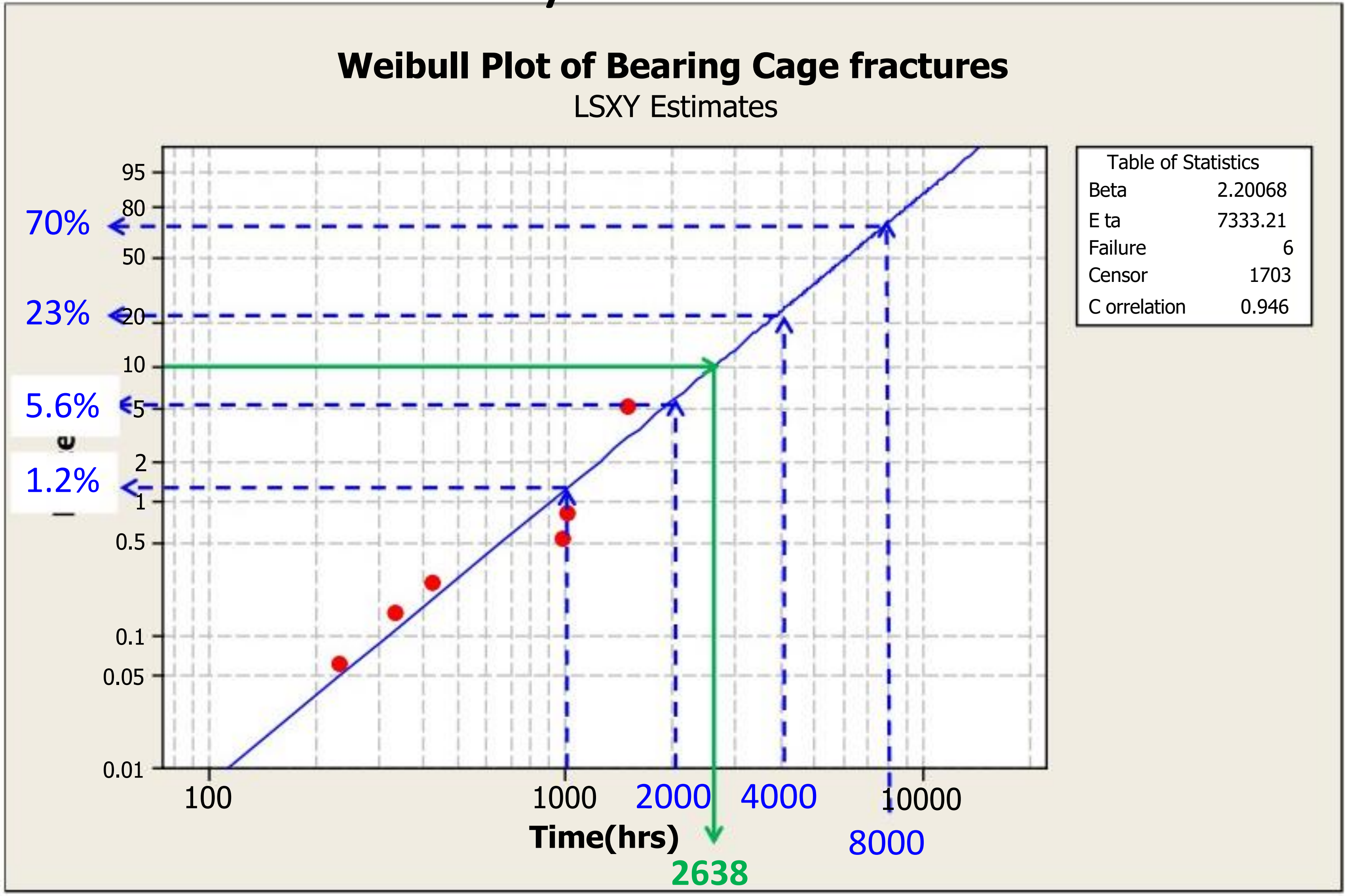
* Data taken from ***USAF Weibull Analysis Handbook***

# Example Weibull Analysis

- Bearing cage population:



# Example Weibull Analysis




Note: MINITAB Weibull plot

# Example Weibull Analysis-answers

- Is the demonstrated B10 life ≥ 8000 hours?

- Answer: Looking at the Weibull plot the answer is NO, the B10 life demonstrated is ~2638 hours.

- Then:

- How many failures will occur by 1000 hours? 4000 hours? (With no inspection)

- Answer: (Assuming failed units are not replaced)

- Number of failed units by 1000 hours=Prob of Failure by 1000 hours

  X number of units = .012 x 1703= 21

How many failures will occur by 4000 hours if we initiated a 1000 hour inspection?

- Answer: Each unit would cycle through 4 times,  so, by 4000 hours

  (.012+.012+.012+.012) x (1703)=82 failures are expected.

# Example Weibull Analysis-answers

- With a utilization rate of 25 hours per month, how many failures can you expect in the next year?
- Answer:

  (using EXCEL)

| Number of units (n) | Current time on each unit(t) | Time on each unit at end of year(t+300) | F(t) | F(t+300) | Single unit risk: (F(t+300)-F(t))/ (1-F(t)) | Total risk: n*(F(t+300)-F(t))/ (1-F(t)) |
|---|---|---|---|---|---|---|
| 289 | 50 | 350 | 0.0000 | 0.0012 | 0.0012 | 0.35 |
| 149 | 150 | 450 | 0.0002 | 0.0022 | 0.0020 | 0.29 |
| 125 | 250 | 550 | 0.0006 | 0.0033 | 0.0028 | 0.34 |
| 112 | 350 | 650 | 0.0012 | 0.0048 | 0.0036 | 0.40 |
| 107 | 450 | 750 | 0.0022 | 0.0066 | 0.0045 | 0.48 |
| 98 | 550 | 850 | 0.0033 | 0.0087 | 0.0054 | 0.53 |
| 110 | 650 | 950 | 0.0048 | 0.0111 | 0.0063 | 0.69 |
| 114 | 750 | 1050 | 0.0066 | 0.0138 | 0.0072 | 0.83 |
| 118 | 850 | 1150 | 0.0087 | 0.0168 | 0.0082 | 0.97 |
| 128 | 950 | 1250 | 0.0111 | 0.0202 | 0.0092 | 1.18 |
| 124 | 1050 | 1350 | 0.0138 | 0.0239 | 0.0102 | 1.27 |
| 93 | 1150 | 1450 | 0.0168 | 0.0279 | 0.0112 | 1.04 |
| 47 | 1250 | 1550 | 0.0202 | 0.0322 | 0.0123 | 0.58 |
| 41 | 1350 | 1650 | 0.0239 | 0.0369 | 0.0133 | 0.55 |
| 27 | 1450 | 1750 | 0.0279 | 0.0419 | 0.0144 | 0.39 |
| 12 | 1550 | 1850 | 0.0322 | 0.0472 | 0.0155 | 0.19 |
| 6 | 1650 | 1950 | 0.0369 | 0.0528 | 0.0165 | 0.10 |
| 0 | 1750 | 2050 | 0.0419 | 0.0588 | 0.0176 | 0.00 |
| 1 | 1850 | 2150 | 0.0472 | 0.0650 | 0.0188 | 0.02 |
| 0 | 1950 | 2250 | 0.0528 | 0.0716 | 0.0199 | 0.00 |
| 2 | 2050 | 2350 | 0.0588 | 0.0785 | 0.0210 | 0.04 |
| | | | | | | |
| | | | | | **Overall fleet risk=** | **10.23** |

defour analytics

# Example Weibull Analysis-answers

- If we must redesign, how many bearing cages must I test for how long to be 90% confident I have a B10 life of 8000 hours?

- Answer: based on a β=2.2, we can use the table on the next page to calculate the factor to multiply the characteristic life of the distribution we need to demonstrate with 90% confidence.
- Since we know B10 life and β, we can calculate the η of the desired Weibull:

$$.10=1-e^{-\left(\frac{8000}{\eta}\right)^{2.2}} \Rightarrow \eta=22,250\,hours$$

- Hence, we could choose any number of test units(depending on budget):
- For example, I chose 2,3,4,or 5 new design bearing cages:

| N | Multiplier | η | Test time |
|---|---|---|---|
| 2 | 1.066134 | 22250 | 23721.48 |
| 3 | 0.886687 | 22250 | 19728.78 |
| 4 | 0.778 | 22250 | 17310.51 |
| 5 | 0.702959 | 22250 | 15640.83 |

# Zero-failure test Plans

Confidence level= 0.9

| | Beta | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.5 | 1 | 1.5 | 2 | 2.2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
| N | Infant Mortality | Random | Early Wearout | | | | | | Old Age Rapid Wearout | | |
| 2 | 1.3255 | 1.1513 | 1.0985 | 1.0730 | 1.0661 | 1.0580 | 1.0481 | 1.0411 | 1.0358 | 1.0318 | 1.0286 |
| 3 | 0.5891 | 0.7675 | 0.8383 | 0.8761 | 0.8867 | 0.8996 | 0.9156 | 0.9272 | 0.9360 | 0.9429 | 0.9485 |
| 4 | 0.3314 | 0.5756 | 0.6920 | 0.7587 | 0.7780 | 0.8018 | 0.8319 | 0.8540 | 0.8710 | 0.8845 | 0.8954 |
| 5 | 0.2121 | 0.4605 | 0.5963 | 0.6786 | 0.7030 | 0.7333 | 0.7722 | 0.8013 | 0.8238 | 0.8417 | 0.8563 |
| 6 | 0.1473 | 0.3838 | 0.5281 | 0.6195 | 0.6471 | 0.6818 | 0.7267 | 0.7606 | 0.7871 | 0.8083 | 0.8257 |
| 7 | 0.1082 | 0.3289 | 0.4765 | 0.5735 | 0.6033 | 0.6410 | 0.6903 | 0.7278 | 0.7573 | 0.7811 | 0.8006 |
| 8 | 0.0828 | 0.2878 | 0.4359 | 0.5365 | 0.5677 | 0.6076 | 0.6603 | 0.7006 | 0.7325 | 0.7582 | 0.7795 |
| 9 | 0.0655 | 0.2558 | 0.4030 | 0.5058 | 0.5381 | 0.5797 | 0.6348 | 0.6774 | 0.7112 | 0.7386 | 0.7614 |
| 10 | 0.0530 | 0.2303 | 0.3757 | 0.4799 | 0.5130 | 0.5558 | 0.6129 | 0.6573 | 0.6927 | 0.7216 | 0.7455 |
| 11 | 0.0438 | 0.2093 | 0.3525 | 0.4575 | 0.4912 | 0.5350 | 0.5938 | 0.6397 | 0.6764 | 0.7064 | 0.7314 |
| 12 | 0.0368 | 0.1919 | 0.3327 | 0.4380 | 0.4722 | 0.5167 | 0.5768 | 0.6240 | 0.6618 | 0.6929 | 0.7188 |
| 13 | 0.0314 | 0.1771 | 0.3154 | 0.4209 | 0.4553 | 0.5004 | 0.5616 | 0.6098 | 0.6487 | 0.6807 | 0.7074 |
| 14 | 0.0271 | 0.1645 | 0.3002 | 0.4055 | 0.4402 | 0.4858 | 0.5479 | 0.5971 | 0.6368 | 0.6696 | 0.6970 |
| 15 | 0.0236 | 0.1535 | 0.2867 | 0.3918 | 0.4266 | 0.4726 | 0.5354 | 0.5854 | 0.6259 | 0.6594 | 0.6874 |
| 16 | 0.0207 | 0.1439 | 0.2746 | 0.3794 | 0.4143 | 0.4605 | 0.5240 | 0.5747 | 0.6159 | 0.6500 | 0.6786 |
| 17 | 0.0183 | 0.1354 | 0.2637 | 0.3680 | 0.4030 | 0.4495 | 0.5136 | 0.5649 | 0.6067 | 0.6413 | 0.6704 |
| 18 | 0.0164 | 0.1279 | 0.2539 | 0.3577 | 0.3927 | 0.4393 | 0.5039 | 0.5557 | 0.5980 | 0.6332 | 0.6628 |
| 19 | 0.0147 | 0.1212 | 0.2449 | 0.3481 | 0.3832 | 0.4299 | 0.4949 | 0.5472 | 0.5900 | 0.6256 | 0.6557 |
| 20 | 0.0133 | 0.1151 | 0.2367 | 0.3393 | 0.3743 | 0.4212 | 0.4865 | 0.5392 | 0.5825 | 0.6185 | 0.6490 |
| 21 | 0.0120 | 0.1096 | 0.2291 | 0.3311 | 0.3661 | 0.4130 | 0.4786 | 0.5318 | 0.5754 | 0.6119 | 0.6427 |
| 22 | 0.0110 | 0.1047 | 0.2221 | 0.3235 | 0.3585 | 0.4054 | 0.4713 | 0.5247 | 0.5688 | 0.6056 | 0.6367 |
| 23 | 0.0100 | 0.1001 | 0.2156 | 0.3164 | 0.3513 | 0.3983 | 0.4643 | 0.5181 | 0.5625 | 0.5996 | 0.6311 |
| 24 | 0.0092 | 0.0959 | 0.2096 | 0.3097 | 0.3446 | 0.3916 | 0.4578 | 0.5119 | 0.5565 | 0.5940 | 0.6258 |
| 25 | 0.0085 | 0.0921 | 0.2039 | 0.3035 | 0.3382 | 0.3852 | 0.4516 | 0.5059 | 0.5509 | 0.5886 | 0.6207 |
| 26 | 0.0078 | 0.0886 | 0.1987 | 0.2976 | 0.3323 | 0.3792 | 0.4457 | 0.5003 | 0.5455 | 0.5835 | 0.6158 |
| 27 | 0.0073 | 0.0853 | 0.1937 | 0.2920 | 0.3266 | 0.3735 | 0.4402 | 0.4949 | 0.5404 | 0.5786 | 0.6112 |
| 28 | 0.0068 | 0.0822 | 0.1891 | 0.2868 | 0.3213 | 0.3681 | 0.4349 | 0.4898 | 0.5355 | 0.5740 | 0.6068 |
| 29 | 0.0063 | 0.0794 | 0.1847 | 0.2818 | 0.3162 | 0.3630 | 0.4298 | 0.4849 | 0.5308 | 0.5695 | 0.6025 |
| 30 | 0.0059 | 0.0768 | 0.1806 | 0.2770 | 0.3113 | 0.3581 | 0.4250 | 0.4802 | 0.5263 | 0.5653 | 0.5984 |
| 40 | 0.0033 | 0.0576 | 0.1491 | 0.2399 | 0.2732 | 0.3192 | 0.3861 | 0.4423 | 0.4898 | 0.5302 | 0.5650 |
| 50 | 0.0021 | 0.0461 | 0.1285 | 0.2146 | 0.2468 | 0.2919 | 0.3584 | 0.4150 | 0.4632 | 0.5046 | 0.5403 |

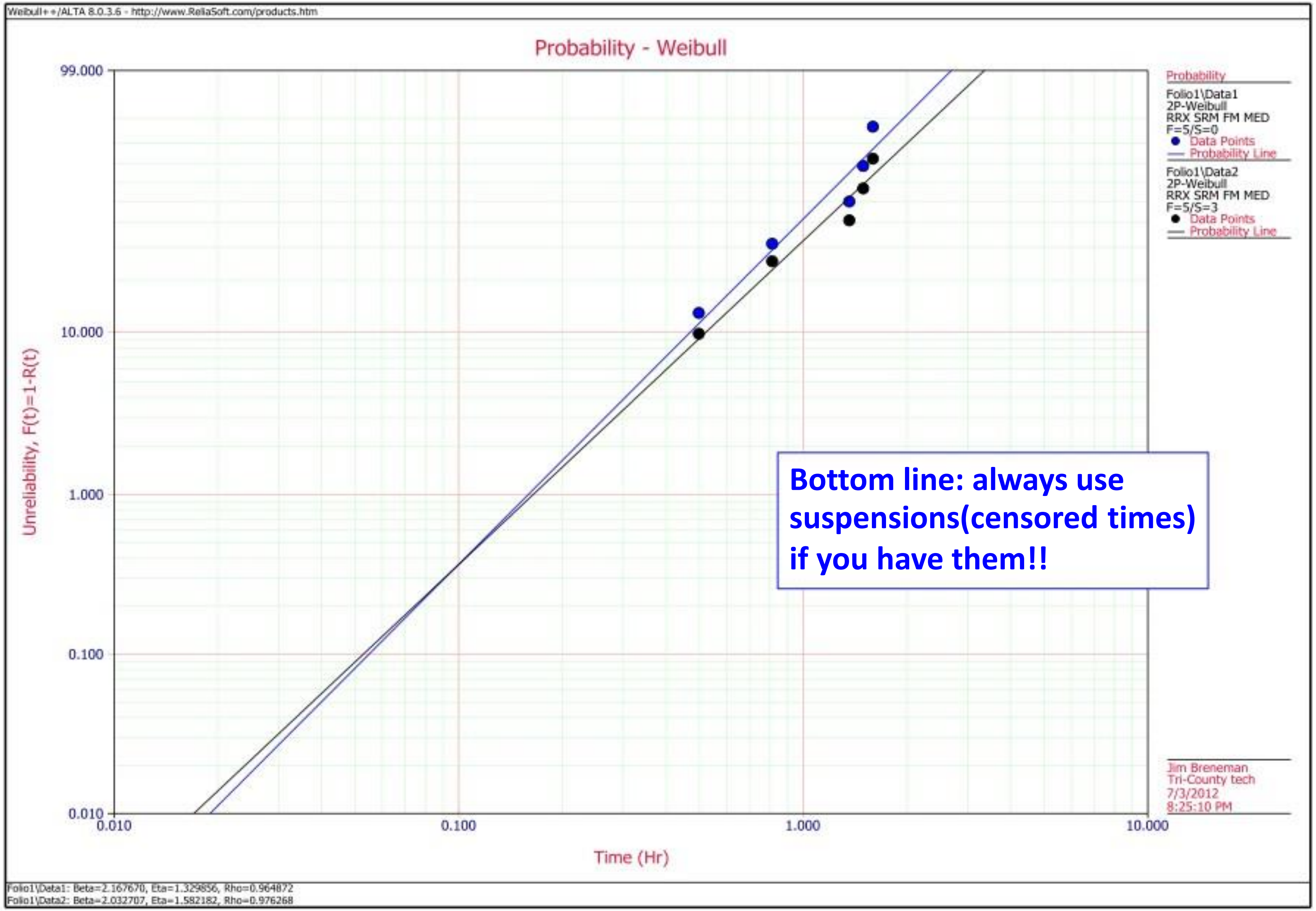
Generated by EXCELTM

# Weibull Analysis & the Reality of Data

- Failures only (perfect data…not the usual case)
- Censored (unfailed data along with failures)

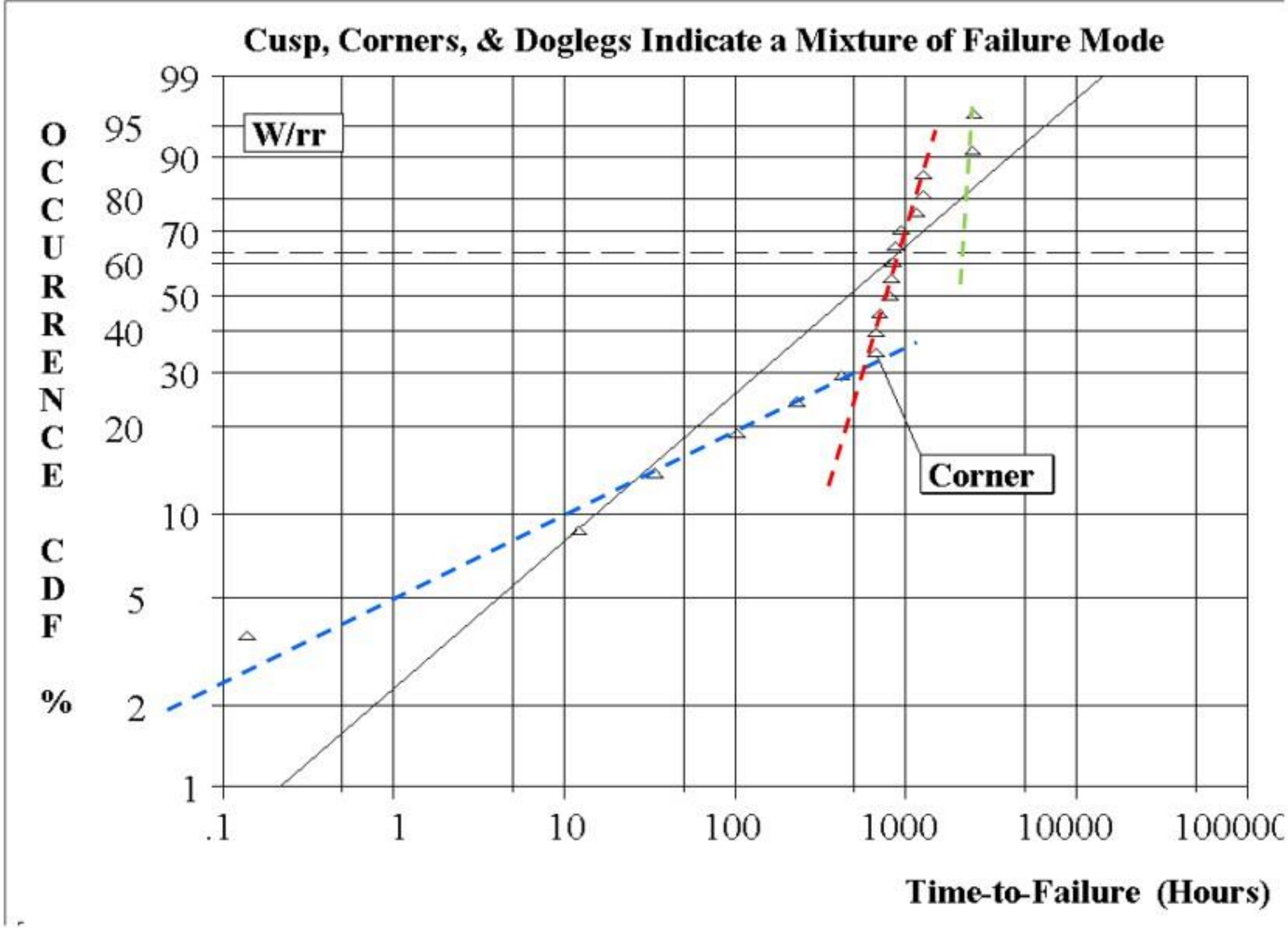
"Dirty" data:

- Mixtures of failure modes
- Curved data on Weibull plot
- Nonzero time origin
- No failure data
- Interval inspection data
- Failed units not identified
- Unknown ages for successful units
- Extremely small samples (as small as one failure)




**Bottom line: always use suspensions(censored times) if you have them!!**

Note: Weibull++8 Weibull plot




Note: Supersmith Weibull plot

# Is Weibull for you?

If you have lab or field failures that you need to analytically determine:

1. What type of failure mode is it?
2. Are there more than one failure mode for this part?
3. How many more will I have? Next year? Next 2 years…?
4. How many of the new design should I test? For how long?
5. What happens if I wait to retrofit a new design until the MOH? Force retrofit?
6. How many spare parts do I need?
7. How much is the warranty going to cost us?

**The answer is YES!**
**You can use the Weibull to help answer these questions!**

# Weibull in R

# References

1. "USAF Weibull Analysis Handbook", AFWAL-TR-83-2079, 1983.
2. "The New Weibull Handbook", Bob Abernethy, Fifth Ed.
3. "Weibull Analysis Primer", James McLinn, ASQ-RD
4. "A Statistical Distribution function of Wide Applicability," Waloddi Weibull, J. Appl. Mech, 18:293-297, 1951

# Design Thinking

Ask the participants what kind of problems related to their industry can be solved using this concept.

# Question & Answers

# THANK YOU