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Ans (1) (a) Advantages of LZW over Huffman

(i) LZW requires no prior information about input data stream
(ii) LZW can compress the input stream in one single pass and LZW is more simpler than huffman

Ans (1) (b) Entropy is the measure of average amount of information needed to represent an event drawn from a probability distribution for random variable.
It is average information content per source symbol.
Properties of Entropy -

- Entropy $H(x)$ of source is bounded as follows

$$0 \leq H(x) \leq \log M$$
$$H(x) = 0$$

- If and only if probability $P_K = 1$ for some k, the remaining all probabilities are zero; this lower bound an entropy corresponds to no uncertainty

$$H(x) = \log M$$

- If and only if $p_k = \frac{1}{M}$ for all k (i.e all message are equiprobable), this upper bound on entropy responds to maximum uncertainity

Ans (1) (c) Information rate is average number of bits of information per second.

$$R = rH$$

R is information rate, H is entropy and r is rate at which messages are generated.

$$R = (r \text{ in message/second}) \times (H \text{ in bits/message})$$

$$= \text{bits/second}$$

Ans (1) (d) A source from which the data is being emitted at successive intervals, which is ~~intervals~~ independent of previous values, can be termed as discrete memoryless source.

This source is discrete as it is not considered for a continuous time interval, but at discrete time interval. This source is memoryless as it is fresh at each instant of time, without considering previous value.

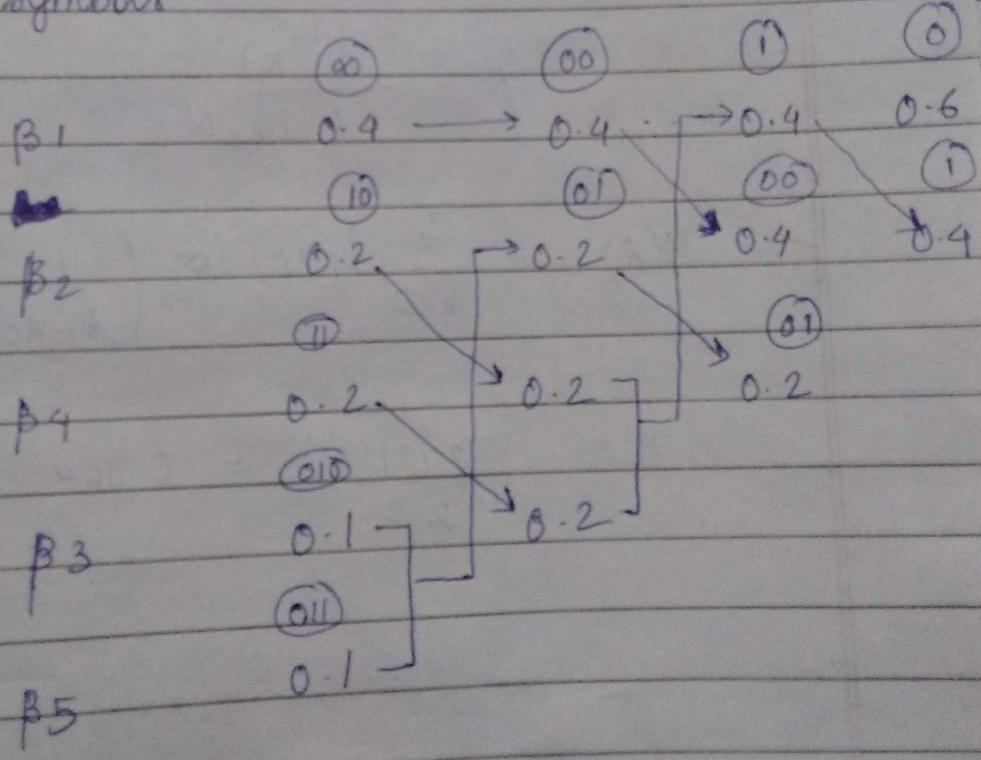
Ans (1) (e)

Kraft - McMillian inequality gives a necessary and sufficient condition for existence of prefix code.

For any instantaneous code over an alphabet of size D, the codeword lengths $l_1, l_2 \ldots l_m$ must satisfy

$$\sum_{i=1}^{m} D^{-l} \leq 1$$

Ans (2) (a) Symbols



(i) Binary codes

$\beta_1 = 00$ $\beta_4 = 11$

$\beta_2 = 10$ $\beta_5 = 011$

$\beta_3 = 010$

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(ii) average codeword length

$$L = \sum_{i=0}^{4} P_i N_i$$ where N = code length of symbol

$$= (0.4 \times 2) + (0.2 \times 2) + (0.1 \times 3) + (0.2 \times 2) + (0.1 \times 3)$$

$$= 0.8 + 0.4 + 0.3 + 0.4 + 0.3$$

$L = 2.2$ bits/symbol

(iii) Effeciency

$$\eta = \frac{H(X)}{L \log_2 M}$$

$$H(X) = \sum_{i=0}^{4} P(x_i) \log_2 \left(\frac{1}{P(x_i)} \right) = 2.12 \text{ bits/message}$$

$$\eta = \frac{2.121}{2.2} = 0.964 = 96.4\%$$

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Ans (2) (b) Proof $I(X;Y) = H(X) - H(X/Y)$

$$= \sum_{j=1}^{m} \sum_{k=1}^{n} P(x_j, y_k)\, I(x_j, y_k)$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{n} P(x_j, y_k) \log \frac{P(x_j / y_k)}{P(x_j)}$$

$$= \sum_{j=1}^{m} \sum_{k=1}^{m} P(x_j, y_k) \log (P(x_j))$$

$$= -\left[-\sum_{j=i}^{m} \sum_{k=1}^{n} P(x_j\, y_k) \log P(x_j / y_k)) \right] + H(X)$$

~~Hence~~ $H(X) - H(X/Y) = I(X:Y)$

proved,

Ans (4) (a) (i) Lossless channel

For a lossless channel no source information is lost in transmission. It has only one non zero element in each column.

Example —

$$P(Y/X) = \begin{bmatrix} 3/4 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In case of lossless channel $p(x/y) = 0/1$ as the probability of x given that y has occurred is $0/1$

This channel has $H(X/Y) = 0$ and $I(X,Y) = H(X)$

(ii) Deteerministic channel

This channel has only one non-zero element in each row of $P(Y/X)$. This has $H(Y/X) = 0$ and $I(X,Y) = H(Y)$

$$P(Y/X) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) Noiseless channel

This channel has $n = m$ and $P(Y/X)$ is an identity matrix $I(X,Y) = H(Y/X)$ & $H(Y/X) = H(X/Y)$

$$P(Y/X) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ans (4) (b) Capacity of a noiseless channel

$$x_1 \longrightarrow y_1$$
$$x_2 \longrightarrow y_2$$

$$P(y_j/x_i) = \begin{matrix} 1 & i = j \\ 0 & i \neq j \end{matrix}$$

$$P(x_i y_j) = P(y_j/x_i) \cdot P(x_i)$$

$$= \begin{matrix} P(x_i) & i = j \\ 0 & i \neq j \end{matrix}$$

Now,

$$P(y_j) = \sum_{i=1}^{m} P(x_i y_j)$$

But $P(x_i y_j) = P(x_i)$

for $i = j$

$$P(y_j) = P(x_i)$$

Hence $P(y_j) \log_2 P(y_j) = -P(x_i) \log P(x_i)$

$$H(Y) = H(X)$$

Also,

$$H(Y/X) = -\sum_i \sum_j P(x_i y_j) \log_2 P(y_j/x_i)$$

from above we get,

$$= -\sum_i P(x_i) \sum_j \log P(y_j/x_i)$$

$$= 1 \times \log 1 = 0$$

Capacity of noiseless channel,

$$C_5 = \max I(X;Y)$$

$$= H(X) - H(X/Y)$$

$$= H(Y) - H(Y/X)$$

$$H(X/Y) = -\sum_i \sum_j P(x_i y_j) \log_2 P(x_i/y_j)$$

$$= \sum_j P(y_j) \sum_i \log_2 P(x_i/y_j)$$

$$H(Y/X) = 0$$

$$C_5 = H(X) = H(Y)$$

$$C_5 = \log_2 m = \log_2 n$$