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Subject — DSP | Subject code — ETEC 306
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Ans 1) $x_1 = [1, 6, 2, 7]$ $x_2 = [-1, 4, -3, 2]$

$$\begin{bmatrix} 1 & 7 & 2 & 6 \\ 6 & 1 & 7 & 2 \\ 2 & 6 & 1 & 7 \\ 7 & 2 & 6 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 33 \\ -19 \\ 33 \\ -15 \end{bmatrix}$$

$$x_1 * x_2 = [33, -19, 33, -15]$$

Ans 2) $x(n) \xleftrightarrow[N]{DFT} X(K)$

then $x^*(n) \xleftrightarrow[N]{DFT} X^*((-K)_N) = X^*(N-K)$

and $x^*(N-n) = x^*((-n)_N) \xleftrightarrow[N]{DFT} X^*(k)$

Proof

$$DFT[x^*(n)] = \sum_{n=0}^{N-1} x^*(n) e^{j2\pi/N kn}$$

$$= \left[\sum_{n=0}^{N-1} x(n) e^{j\frac{2\pi}{N} kn} \right]$$

which can be rewritten as,

$$\left[\sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}(-k)n}\right]^*$$

$$= \left[X((-k)_N)\right]^* = X^*((-k)_N) = X^*(N-k)$$

$$IDFT\left[X^*(k)\right] = \frac{1}{N}\sum_{k=0}^{N-1} X^*(k)\, e^{j\frac{2\pi}{N}kn} = \left[\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{-j\frac{2\pi}{N}kn}\right]^*$$

$$= \left[\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}k(-n)}\right]^*$$

$$= \left[x((-n)_N)\right]^* = x^*((-n)_N) = x^*(N-n)$$

Hence for real sequence, the DFT is always conjugate symmetric

when $x(n)$ is real,

$$x(n) = x^*(n)$$

then $DFT[x(n)] = DFT[x^*(n)]$

$$X(k) = X^*((-k)_N) = X^*(N-k)$$

$$X^*(k) = X((-k)_N) = X(N-k)$$

The DFT of a real signal is conjugate symmetric

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Ans 1) c

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ -1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 4-6j \\ 0 \\ 4+6j \end{bmatrix}$$

Ans 2 a

Overlap-save Method	Overlap add method
1. The first (M-1) samples of first section are set to 0	The last (M-1) samples of the first section are set to zero
2. In this method the size of input data block is L+M-1	In this method the size of input data block is L.
3. Each data block consist of last M-1 data point of previous data block followed by L new points	Each data block is L points and we append M-1 zeroes to compute N-point DFT.

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Overlap-save method	Overlap add method
4. To form the output sequence the first M-1 data points are discarded in each output block and remaining data are filtered together	To form the output sequence, the last M-1 points from each output block is added to first (m-1) points of succeeding block

Ans (4) (a)

Linear Convolution using circular convolution

Let $x(n)$ & $h(n)$ be of length l and m

length of o/p = $l+m-1$

$x(n) = [1, 2, 3, 1]$ $\qquad L = 4$

$h[n] = [1, 1, 1]$ $\qquad M = 3$

$x(n) = [1, 2, 3, 1, 0, 0]$

$h(n) = [1, 1, 1, 0, 0, 0]$

$L+M-1 = 6$

Using matrix method,

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+0+0 \\ 2+1 \\ 3+2+1 \\ 1+3+2 \\ 1+3 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 \\ 3 \\ 6 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

$y(n) = [1, 3, 6, 6, 4, 1]$ [Linear convolution]

Circular using linear convolution

$x[n] = [1\ 3\ 2\ 1]$ $\qquad$ $h(n) = [1, 1, -1, 1]$

$y_1(n) = x(n) * h(n)$

	1	3	2	1
1	1	3	2	1
1	1	3	2	1
−1	−1	−3	−2	1
1	1	3	2	1

$y_1(n) = \{1, 4, 4, 1, 2, 1, 1\}$

$y(n) = \{3, 5, 5, 1\}$ This is circular convolutio

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Ans (1) (d) $\alpha_p = 0.5dB$

$\alpha_s = 30dB$

$\Omega_p = 1200Hz$

$\Omega_s = 2400Hz$

For order we use,

$$N = \frac{\cosh^{-1}\left(\frac{10^{0.1\alpha_s(dB)}-1}{10^{0.1\alpha_p(dB)}-1}\right)^{1/2}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N = \frac{\cosh^{-1}\left(\frac{10^{0.1\times 30}-1}{10^{0.1\times 0.5}-1}\right)^{1/2}}{\cosh^{-1}\left(\frac{2400}{1200}\right)}$$

$$N = \frac{\cosh^{-1}\left(\frac{10^{3}-1}{10^{0.05}-1}\right)^{1/2}}{\cosh^{-1}(2)}$$

$$= \frac{\cosh^{-1}\left(\frac{999}{0.1220}\right)^{1/2}}{\cosh^{-1}(2)}$$

$$= \frac{\cosh^{-1}(8188\cancel{0}5.24)^{1/2}}{\cosh^{-1}(2)}$$

Ans (9) (b) Parseval theorem

If $DFT\{x_1(n)\} = X_1(k)$

& $DFT\{x_2(n)\} = X_2(k)$

then
$$\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k) \cdot X_2^*(k)$$

Proof -

$$x_2(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_2(k)\, e^{j\frac{2\pi}{N}kn}$$

$$x_2^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_2^*(k)\, e^{-j\frac{2\pi}{N}kn}$$

$$\sum_{n=0}^{N-1} x_1(n) \cdot x_2^*(n) = \sum_{n=0}^{N-1} x_1(n) \cdot \frac{1}{N}\sum_{k=0}^{N-1} X_2^*(k)\, e^{-j\frac{2\pi}{N}kn}$$

On rearranging

$$\frac{1}{N}\sum_{k=0}^{N-1} X_2^*(k) \cdot \sum_{n=0}^{N-1} x_1(n) \cdot e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_2^*(k) \cdot X_1(k)$$

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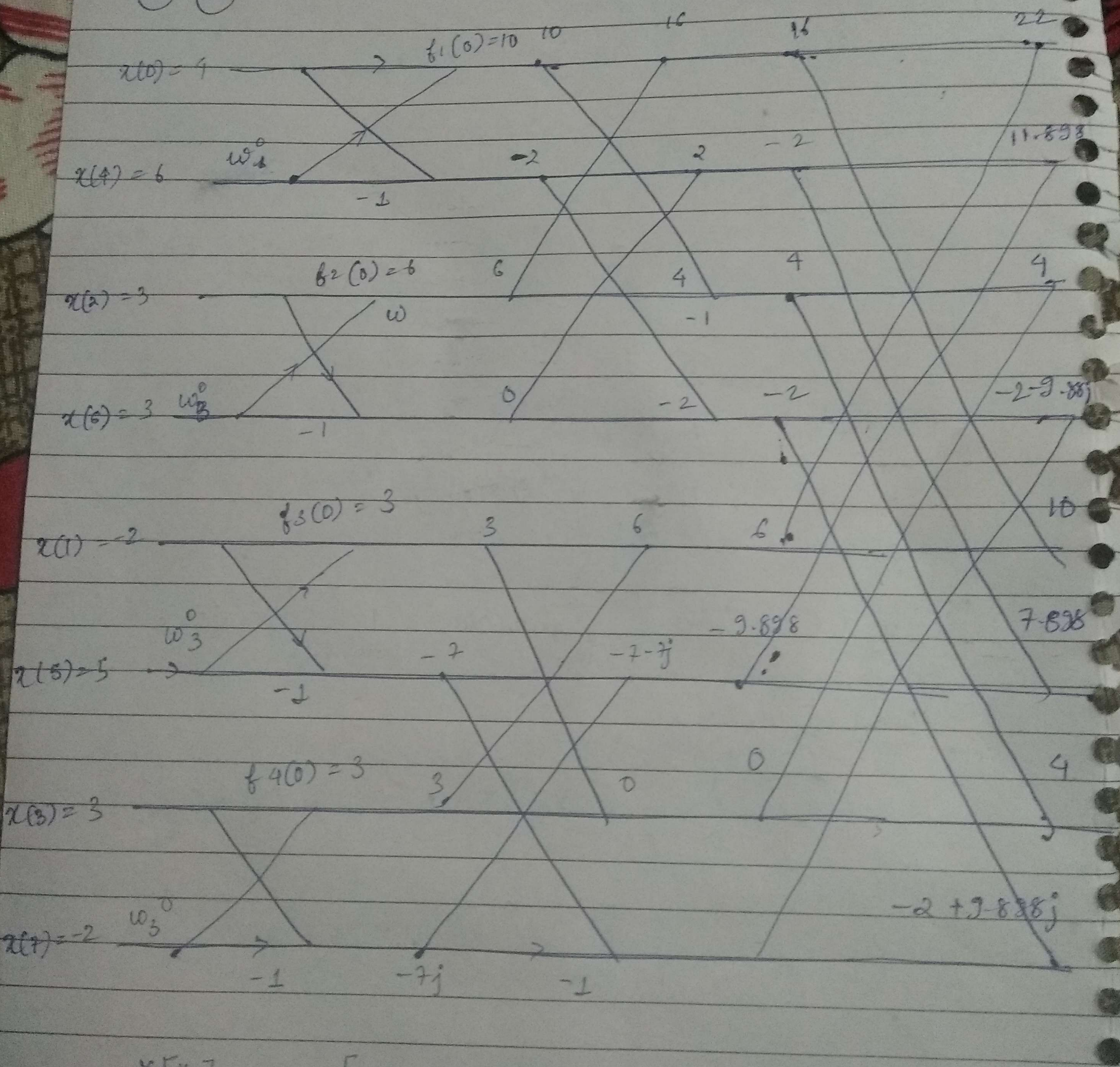
Ans (2) (b)



Answer

$X[k] = [22,\ 11.898,\ 4,\ -2-9.898j,\ 10,\ 7.898,\ 4,\ -2+9.898j]$