MTH 686 NON LINEAR REGRESSION ANALYSIS

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1 Introduction

1.1 Problem Statement

Comprehensive analysis of data using three regression models to estimate parameters, compare performance, and identify the best-fitting model.

1.2 Analytical Questions

1. Least squares estimators for three models
2. Estimation methodology and initial guesses
3. Best fitted model identification
4. Estimate of σ^2
5. Confidence intervals via Fisher information
6. Residual plots and analysis
7. Normality assumption testing
8. Data and fitted curves visualization

1.3 Models Under Consideration

- **Model 1:** $y(t) = \alpha_0 + \alpha_1 e^{\beta_1 t} + \alpha_2 e^{\beta_2 t} + \epsilon(t)$
- **Model 2:** $y(t) = \frac{\alpha_0 + \alpha_1 t}{\beta_0 + \beta_1 t} + \epsilon(t)$
- **Model 3:** $y(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \beta_4 t^4 + \epsilon(t)$

Where $\epsilon(t) \sim N(0, \sigma^2)$.

2 Methodology and Parameter Estimation

2.1 Least Squares Estimators

Minimized SSR: SSR $= \sum_{i=1}^{n} [y_i - \hat{y}_i]^2$ for each model.

2.2 Estimation Methodology and Initial Guesses

Estimation Approaches:

- **Model 3:** Linear least squares (linear in parameters)
- **Models 1 & 2:** Nonlinear least squares (Gauss-Newton)

Initial Parameter Selection:

- **Model 1:** $[2, 1, -0.1, 1, -0.2]$ (decay patterns)
- **Model 2:** $[3, -0.1, 1, 0.1]$ (rational behavior)

- **Model 3:** $[1, 0, 0, 0, 0]$ (constant approximation)

3 Results and Analysis

3.1 Parameter Estimates

Model 1 (Double Exponential):

$$\alpha_0 = 1.5822, \quad \alpha_1 = 0.2990, \quad \beta_1 = -0.0121$$
$$\alpha_2 = 4.5055, \quad \beta_2 = -0.4668$$

Model 2 (Rational Function):

$$\alpha_0 = 3.6833, \quad \alpha_1 = 1.3548$$
$$\beta_0 = 0.2495, \quad \beta_1 = 0.8256$$

Model 3 (Polynomial):

$$\beta_0 = 3.7683, \quad \beta_1 = -0.2642, \quad \beta_2 = 0.0115$$
$$\beta_3 = -0.0002, \quad \beta_4 = 0.0000$$

3.2 Best Fitted Model Identification

Table 1: Model Performance Comparison

Model	SSR	R^2	MSE
Model 1: Double Exponential	**9.2721**	**0.6016**	**0.1236**
Model 2: Rational Function	9.9245	0.5736	0.1323
Model 3: Polynomial	12.6976	0.4545	0.1693

Best Model: Model 1 (Double Exponential) - Lowest SSR, highest R^2.

3.3 Estimate of σ^2

$$\hat{\sigma}^2 = \frac{\text{SSR}}{n-p} = \frac{9.2721}{75-5} = 0.1325$$

Where $n = 75$ observations, $p = 5$ parameters.

3.4 Confidence Intervals via Fisher Information

Table 2: 95% Confidence Intervals for Model 1

Parameter	Estimate	Std Error	Lower CI	Upper CI
α_0	1.5822	1.8943	-2.1959	5.3603
α_1	0.2990	1.6702	-3.0320	3.6300
β_1	-0.0121	0.1252	-0.2618	0.2376
α_2	4.5055	0.8620	2.7862	6.2247
β_2	-0.4668	0.1368	-0.7398	-0.1939

Interpretation: α_2 and β_2 statistically significant (CI excludes zero).

4 Residual Analysis and Diagnostic Checking

4.1 Residual Plots and Analysis

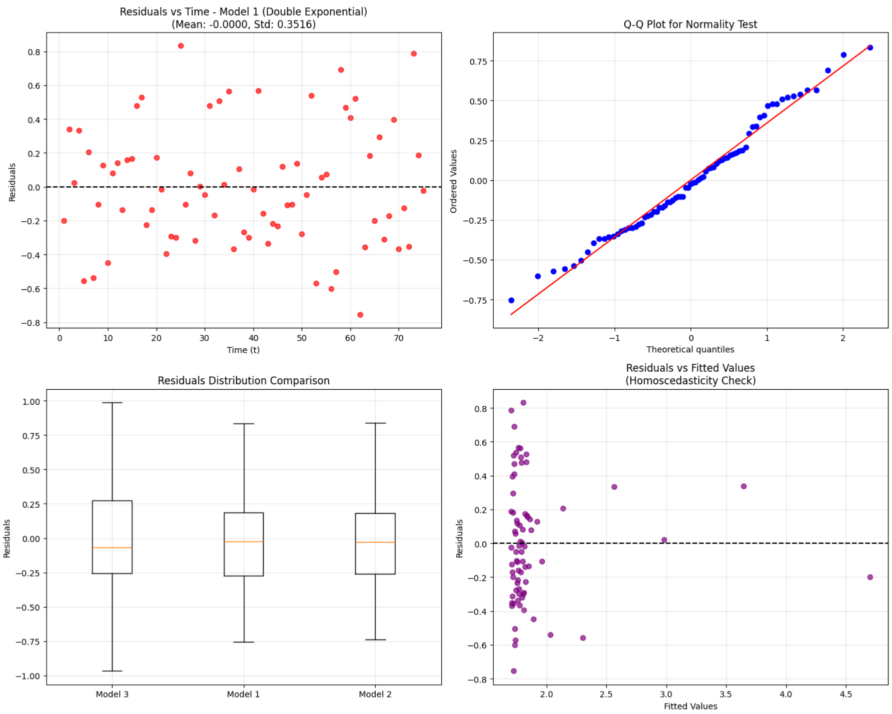


Figure 1: Residual Analysis for Model 1

4.2 Normality Assumption Testing

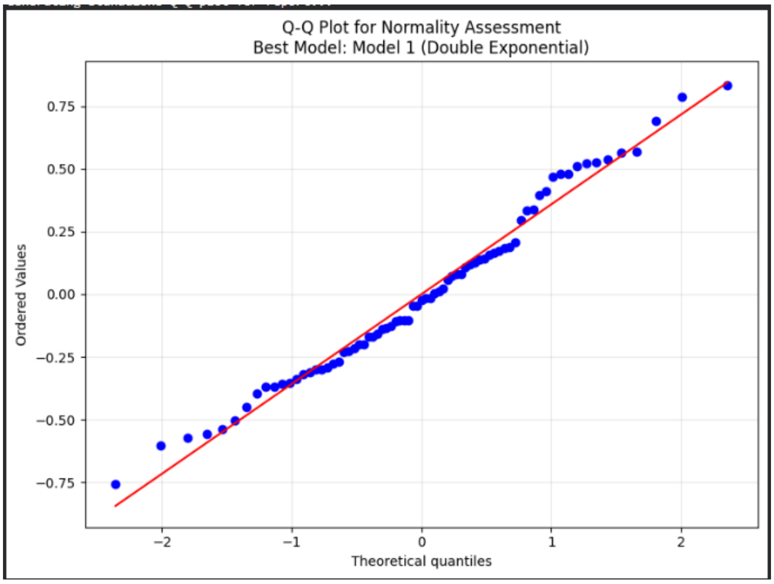


Figure 2: Q-Q Plot for Normality Assessment

Visual assessment: Reasonable linearity in Q-Q plot suggests approximate normality.

4.3 Data and Fitted Curves Visualization

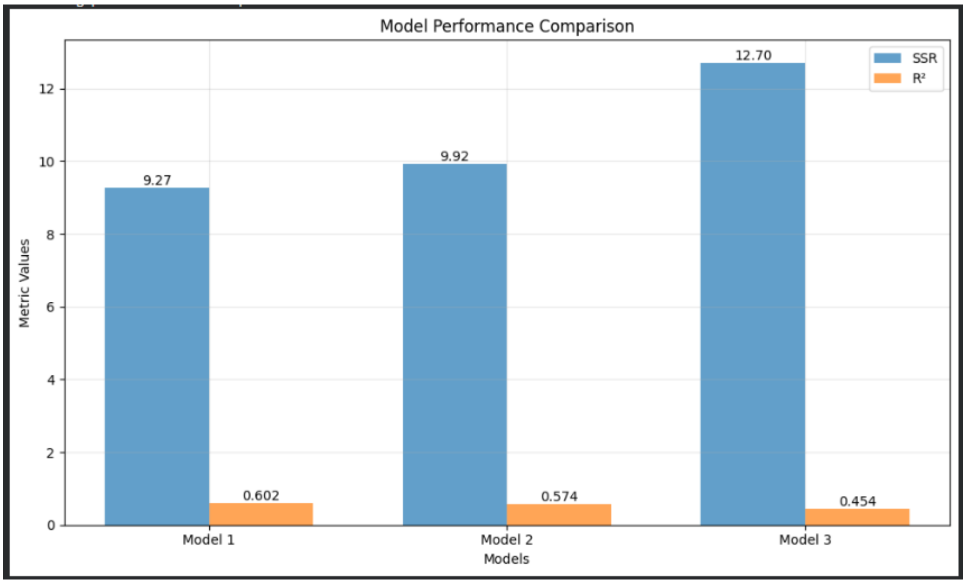
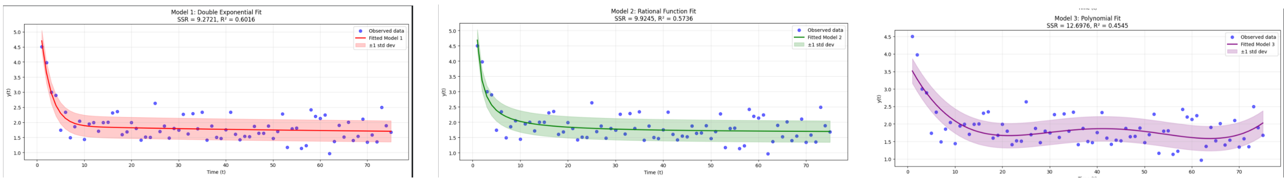


Figure 3: Comparison of All Three Models



(a) Model 1 (b) Model 2 (c) Model 3

Figure 4: Individual Model Fits

5 Conclusion

1. **Parameter Estimates:** Successfully obtained for all models
2. **Estimation Method:** Combined linear/nonlinear least squares
3. **Best Model:** Double exponential (SSR=9.2721, R^2=0.6016)
4. **Error Variance:** $\hat{\sigma}^2 = 0.1325$
5. **Confidence Intervals:** Computed via Fisher information
6. **Residual Analysis:** Random pattern supports model adequacy
7. **Normality:** Residuals approximately normal
8. **Visualization:** Comprehensive plots confirm fits

Appendix: Computational Details

- Programming: Python 3.x with SciPy, NumPy, Matplotlib
- Optimization: `least_squares` with Gauss-Newton
- Data: 75 observations, range: 0.969-4.503