Q2

v =

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$$

B =

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 0 \\ 6 & 5 & 0 \end{bmatrix}$$

A =

$$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

- 2.1.a) $v + 1$ (1 point)
- 2.1.b) $v + v^T$ (1 point)
- 2.1.c) $v \cdot v^T$ (dot product) (2 points)
- 2.1.d) $v \times B$ (cross product). Consider that each column in B represents an individual vector. You can follow that same consideration when writing your answer (i.e., as a matrix) (4 points)
- 2.1.e) $v \bullet B$ (matrix multiplication) (2 points)
- 2.1.f) Calculate the eigenvalues of A (10 points)
- 2.1.g) Give one example of a property of the data that eigenvalues/eigenvectors represent (i.e., why is calculating the eigenvalues/eigenvectors useful)? (3 marks)

a) $v+1$ is not possible since they're different sizes

b) $v + v^T = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Different sizes so not mathematically possible.

numpy can do it using broadcasting

c) $v \cdot v^T = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = 3^2 + 2^2 + 1^2 = 14$

d) $v \times B = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 0 \\ 6 & 5 & 0 \end{bmatrix}$

$$\begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \end{array} \times \begin{array}{c} i \\ j \\ k \end{array} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}^T = 12k - 18j - 4k + 12i + 2j - 4i = \langle 8, -16, 8 \rangle$$

$$\begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \end{array} \times \begin{array}{c} i \\ j \\ k \end{array} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}^T = 9k - 15j - 2k + 10i + 1j - 3i = \langle 7, -14, 7 \rangle$$

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T = 0 = \langle 0, 0, 0 \rangle$$

$$\begin{bmatrix} 8 & 7 & 0 \\ -16 & -14 & 0 \\ 8 & 7 & 0 \end{bmatrix}$$

e) $\underset{\text{mult}}{v \cdot B} = \underset{1\times 3}{\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 0 \\ 6 & 5 & 0 \end{bmatrix}} = \begin{bmatrix} 20 & 14 & 0 \end{bmatrix}$

same, so allowed

f) $A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ $AX = \lambda X$ $\det(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - A = \begin{bmatrix} \lambda & -2 & -3 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda+1 \end{bmatrix}$$

$$\det \begin{bmatrix} \lambda & -2 & -3 \\ -1 & \lambda-2 & -1 \\ -1 & 0 & \lambda+1 \end{bmatrix} = -1 \cdot \det \begin{vmatrix} -2 & -3 \\ \lambda-2 & -1 \end{vmatrix} + 0 + (\lambda+1) \cdot \det \begin{vmatrix} \lambda & -2 \\ -1 & \lambda-2 \end{vmatrix}$$

$$= -1 \cdot (2 - (-3(\lambda-2))) + (\lambda+1)(\lambda(\lambda-2) - (2))$$

$$= -1(2 - (-3\lambda+6)) + (\lambda+1)(\lambda^2 - 2\lambda - 2)$$

$$= -3\lambda + 4 + \lambda^3 - 2\lambda^2 - 2\lambda + \lambda^2 - 2\lambda - 2$$

$$= \lambda^3 - \lambda^2 - 7\lambda + 2$$

$$\lambda = 3.07,\ 0.2778,\ -2.346$$

e)