Ratio and Proportions

- *Ratio:*

The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

In the ratio $a : b$, we call a as the first term or *antecedent* and b, the second term or *consequent*.

Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

- *Proportion:*

The equality of two ratios is called proportion.

If $a : b = c : d$, we write $a : b$ **::** $c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called *extremes*, while b and c are called *mean terms*.

Product of means = Product of extremes.

Thus, $a : b$ **::** $c : d \Leftrightarrow (b \times c) = (a \times d)$.

- *Fourth Proportional:*

If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b.

Mean Proportional:

Mean proportional between a and b is ab.

- *Comparison of Ratios:*

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

Compounded Ratio:

The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

- *Duplicate Ratios:*

Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

Sub-duplicate ratio of $(a : b)$ is $(a : b)$.

Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a+b}{a-b} = \dfrac{c+d}{c-d}$.   [componendo and dividendo]

- *Variations:*

We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

We say that x is inversely proportional to y, if $xy = k$ for some constant k and

we write, $x \propto \dfrac{1}{y}$.

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