

Programming in Prolog

CSE 505 – Computing with Logic

Stony Brook University

<http://www.cs.stonybrook.edu/~cse505>

Relations/Predicates

- Predicates are building-blocks in predicate calculus: $p(a_1, a_2, \dots, a_k)$

- **parent(X, Y)** : X is a parent of Y.

parent(pam, bob) . parent(bob, ann) .

parent(tom, bob) . parent(bob, pat) .

parent(tom, liz) . parent(pat, jim) .

- **male(X)** : X is a male.

male(tom) .

male(bob) .

male(jim) .

we attach meaning to them, but within the logical system they are simply structural building blocks, with no meaning beyond that provided by explicitly-stated interrelationships

Relations

- **female (X) :** X is a female.

female (pam) .

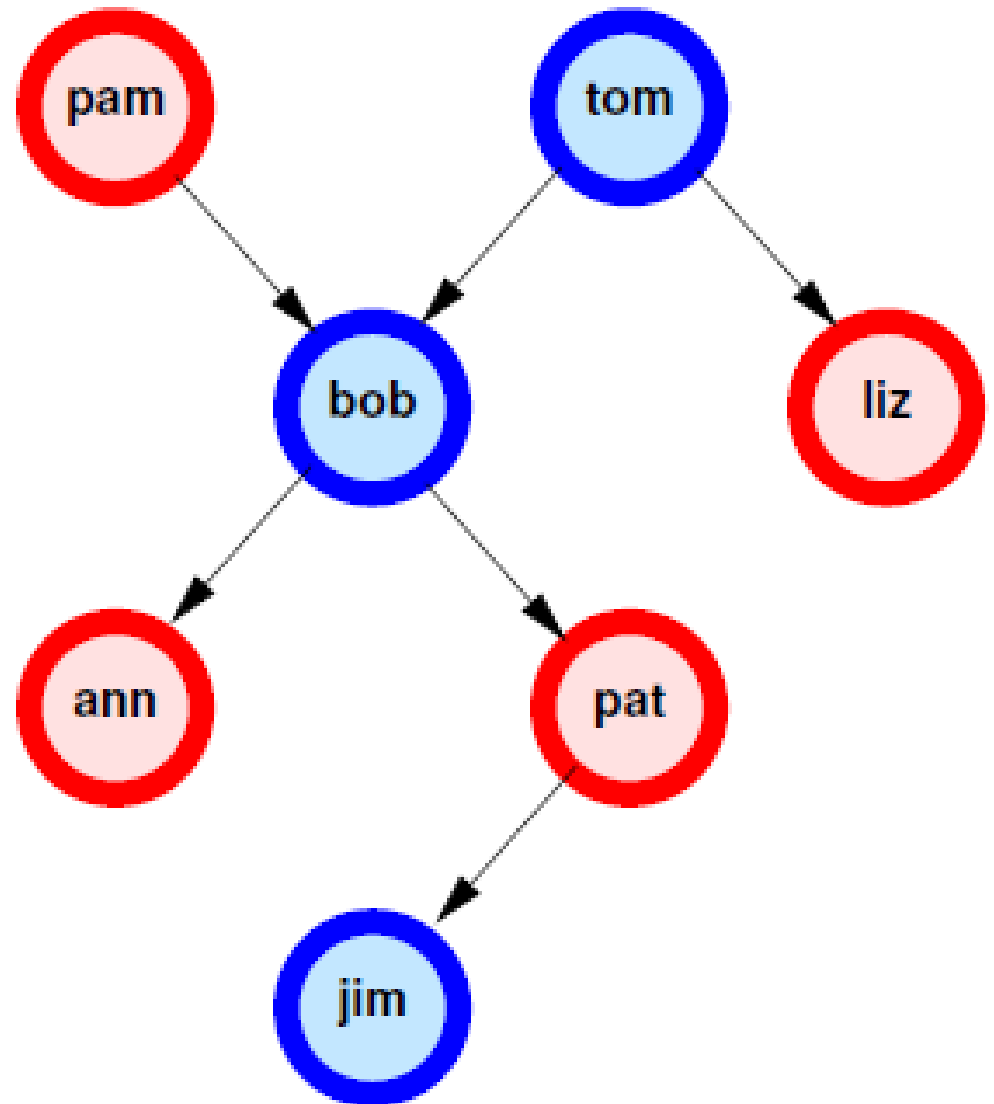
female (pat) .

female (ann) .

female (liz) .

Relations

`parent(pam, bob) .`
`parent(tom, bob) .`
`parent(tom, liz) .`
`parent(bob, ann) .`
`parent(bob, pat) .`
`parent(pat, jim) .`
`female(pam) .`
`female(pat) .`
`female(ann) .`
`female(liz) .`
`male(tom) .`
`male(bob) .`
`male(jim) .`



Relations

- Rules:

- **mother (X, Y)** : X is the mother of Y.

-FOL:

$$\forall X, Y \text{ (parent}(X, Y) \wedge \text{female}(X) \Rightarrow \text{mother}(X, Y))$$

-In Prolog:

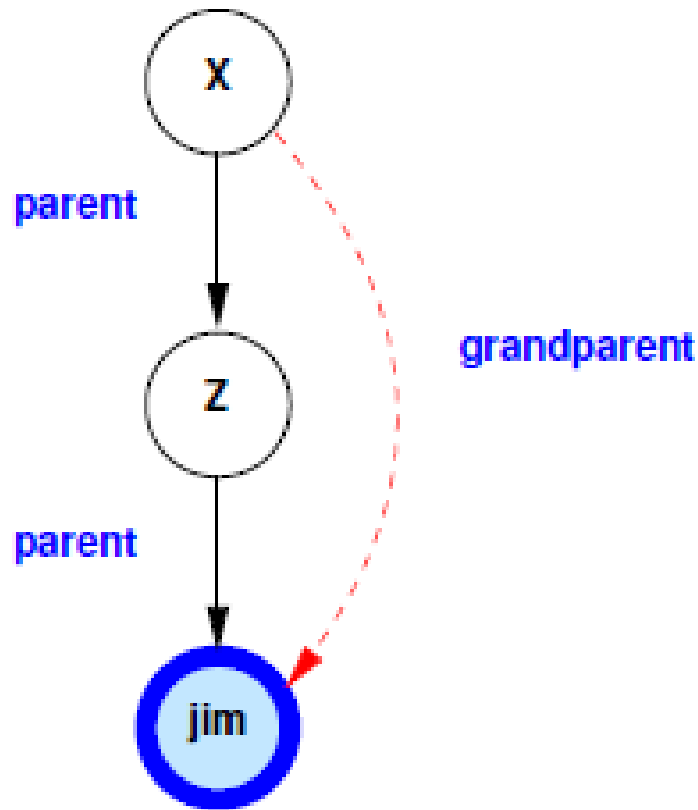
```
mother (X, Y) :-  
    parent (X, Y) ,  
    female (X) .
```

- “,” means *and* (conjunction), “:-” means *if* (implication) and “;” means *or* (disjunction).

Relations

- More Relations:

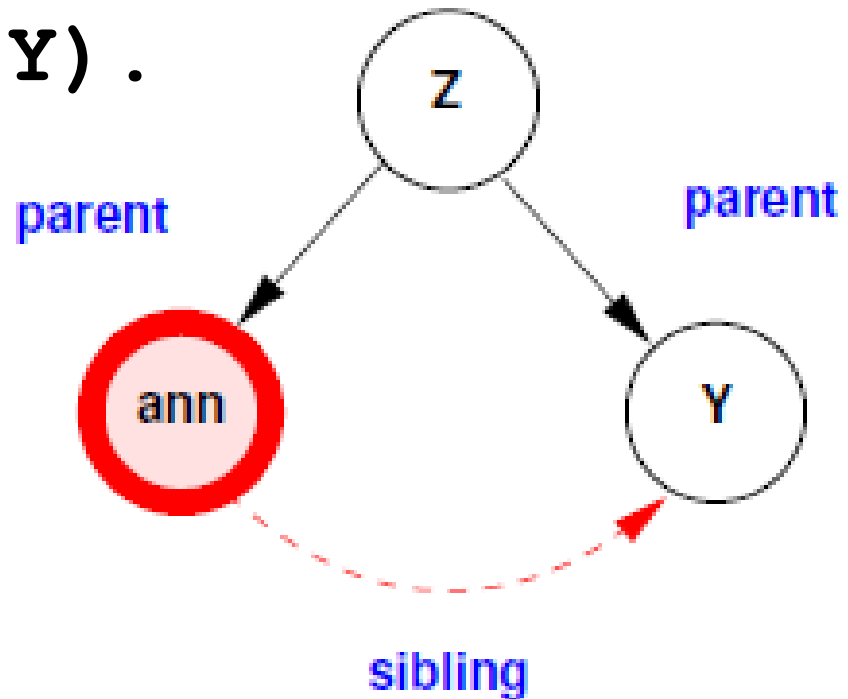
grandparent(X, Y) :- **parent**(X, Z) , **parent**(Z, Y) .



Relations

```
sibling(X,Y) :- parent(Z,X),  
                parent(Z,Y), X \= Y.
```

```
?- sibling(ann,Y).
```



Relations

- More Relations:

cousin(X,Y) :- ...

greatgrandparent(X,Y) :- ...

greatgreatgrandparent(X,Y) :- ...

Recursion

```
ancestor(X,Y) :-  
    parent(X,Y) .
```

```
ancestor(X,Y) :-  
    parent(X,Z) ,  
    ancestor(Z,Y) .
```

```
?- ancestor(X,jim) .
```

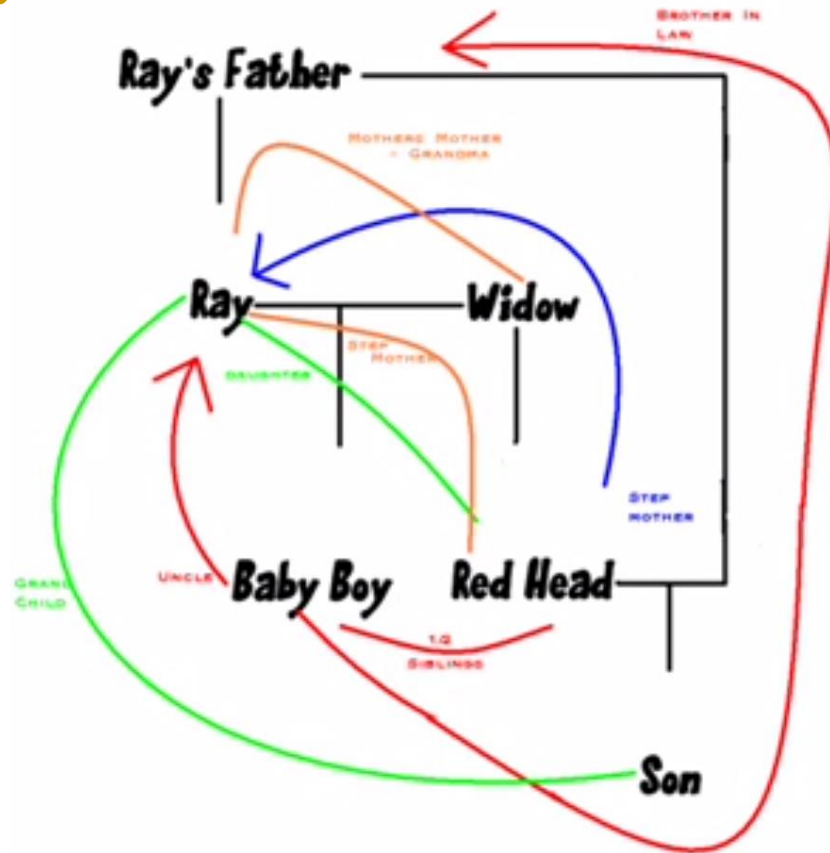
```
?- ancestor(pam,X) .
```

```
?- ancestor(X,Y) .
```

Relations

- How to implement “I’m My Own Grandpa”?

<https://www.youtube.com/watch?v=eYlJH81dSiw>



Recursion

- What about:

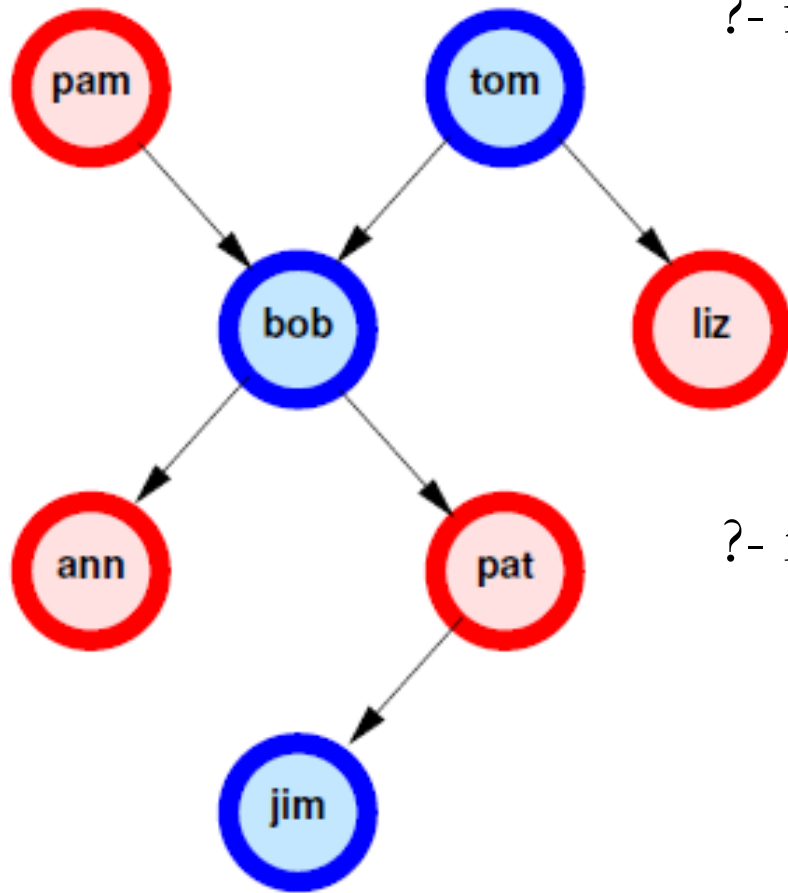
```
ancestor(X,Y) :-  
    ancestor(X,Z) ,  
    parent(Z,Y) .
```

```
ancestor(X,Y) :-  
    parent(X,Y) .
```

```
?- ancestor(X,Y) .
```

INFINITE LOOP

Computations in Prolog



?- mother(M, bob).

?- parent(M, bob), female(M).

?- M=pam, female(pam).

M = pam true

?- father(M, bob).

?- parent(M, bob), male(M)

(i) ?- M=pam, male(pam).

fail

(ii) ?- M=tom, male(tom).

M = tom true

Prolog Execution

- Call: Call a predicate (invocation)
- Exit: Return an answer to the caller
- Fail: Return to caller with no answer
- Redo: Try next path to find an answer

The XSB Prolog System

- <http://xsb.sourceforge.net>
 - Developed at Stony Brook by David Warren and many contributors.
- Overview of Installation:
 - Unzip/untar; this will create a subdirectory XSB
 - Windows: you are done
 - Linux:

```
cd XSB/build  
./configure  
./makexsb
```

That's it!
 - Cygwin under Windows: same as in Linux

Use of XSB

- Put your ruleset *and* data in a file with extension .P (or .pl)

```
p(X) :- q(X,_) .  
q(1,a) .  
q(2,a) .  
q(b,c) .  
?- p(X) .
```
- Don't forget: all rules and facts end with a period (.)
- Comments: /*...*/ or %.... (% acts like // in Java/C++)

- Type

.../XSB/bin/xsb

(Linux/Cygwin)

...\XSB\config\x86-pc-windows\bin\xsb (Windows)

where ... is the path to the directory where you downloaded XSB

- You will see a prompt

| ?-

and are now ready to type queries

Use of XSB

- Loading your program, myprog.P

?- [myprog] .

XSB will compile myprog.P (if necessary) and load it. Now you can type further queries, e.g.

?- p(X) .

?- p(1) .

- Some Useful Built-ins:

- **write(X)** – write whatever X is bound to
- **writeln(X)** – write then put newline
- **nl** – output newline
- Equality: **=**
- Inequality: **\=**

<http://xsb.sourceforge.net/manual1/index.html> (Volume 1)

<http://xsb.sourceforge.net/manual2/index.html> (Volume 2)

Use of XSB

- Some Useful Tricks:
 - XSB returns only the first answer to the query. To get the next, type **;** **<Return>**. For instance:

```
| ?- q(X).
```

```
X = 2;
```

```
X = 4
```

```
yes
```

- Usually, typing the **;**'s is tedious. To do this programmatically, use this idiom:

```
| ?- (q(_X), write('X='), writeln(_X), fail ; true).
```

X here tells XSB to not print its own answers, since we are printing them by ourselves. (XSB won't print answers for variables that are prefixed with a ****.)

Syntax of Prolog Programs

- A *program* is a sequence of clauses.
- Each *clause* is of the form **head :- body**.
- Head is one *term*.
- Body is a comma-separated list of terms.
- A clause with an empty body is called a *fact*.
- A clause is also sometimes called a *rule*.

Logic Programming Concepts

- Operators:
 - conjunction, disjunction, negation, implication
- Universal and existential quantifiers
- Statements
 - sometimes true, sometimes false, sometimes unknown
 - axioms - assumed true
 - theorems - provably true
 - goals - things we'd like to prove true

Logic Programming Concepts

- Most statements can be written many ways
- That's great for people but a nuisance for computers
 - It turns out that if you make certain restrictions on the format of statements you can prove theorems mechanically
- That's what logic programming systems do

Logic Programming Concepts

- In Prolog all statements are in the form of HORN CLAUSES consisting of a HEAD and a BODY:

HEAD :- BODY.

- The HEAD is a single term
- The BODY is a list (conjunction) of terms
- A term can be a constant, variable, or **STRUCTURE** consisting of a **FUNCTOR** and a parenthesized list of arguments

Logic Programming Concepts

- A structure can play the role of a data structure or a predicate
- A constant is either an ATOM or a NUMBER
 - An atom is either what looks like an identifier beginning with a lowercase letter, or a single quoted character string
 - A number looks like an integer or real from some more ordinary language
- A variable looks like an identifier beginning with an upper-case letter
- There are NO declarations (vars, terms, predicates)
- All types are discovered implicitly

Logic Programming Concepts

- The meaning of a rule is that *the conjunction of the structures in the body implies the head*.
 - A clause with an empty body is called a FACT:
raining(ny).
 - A clause with both sides is a RULE:
wet(X) :- raining(X).
Single-assignment variables: X must have the same value on both sides
 - A clause with an empty head is a QUERY, or top-level GOAL:
?- wet(X).
- The Prolog interpreter has a collection of facts and rules in its DATABASE.
 - Facts are axioms - things the interpreter assumes to be true.
 - Prolog provides an automatic way to deduce true results from facts and rules.

Logic Programming Concepts

- So, rules are theorems that allow the interpreter to infer things

- To be interesting, rules generally contain variables

employed(X) :- employs(Y,X) .

can be read:

"for all X, X is employed if there exists a Y such that Y employs X"

- Note the direction of the implication
- Also, the example does NOT say that X is employed ONLY IF there is a Y that employs X

Logic Programming Concepts

- The scope of a variable is the clause in which it appears:
 - Variables whose first appearance is on the **left hand side of the clause (the head)** have implicit **universal** quantifiers
 - Variables whose **first appearance is in the body** of the clause have implicit **existential** quantifiers in that body

Logic Programming Concepts

```
grandmother (A, C) :-  
    mother (A, B) ,  
    mother (B, C) .
```

can be read:

"for all A, C [A is the grandmother of C if there exists a B such that A is the mother of B and B is the mother of C]"

- We probably want another rule that says:

```
grandmother (A, C) :-  
    mother (A, B) ,  
    father (B, C) .
```

Recursion

- Transitive closure:
 - Example: a graph declared with facts (true statements)

edge (1 , 2) .

edge (2 , 3) .

edge (2 , 4) .

1) if there's an edge from X to Y, we can reach Y from X:

reach (X , Y) :- edge (X , Y) .

2) if there's an edge from X to Z, and we can reach Y from Z, then we can reach Y from X:

**reach (X , Y) :-
edge (X , Z) ,
reach (Z , Y) .**

?- reach (X, Y) .

X = 1

Y = 2 ;

← Type a semi-colon repeatedly for
more answers

X = 2

Y = 3 ;

X = 2

Y = 4 ;

X = 1

Y = 3 ;

X = 1

Y = 4 ;

no



reach (X, Y) :- edge (X, Y) .

reach (X, Y) :-

edge (X, Z) ,

reach (Z, Y) .

Prolog Programs

- We will now explore Prolog programs in more detail:
 - Syntax of Prolog Programs
 - A *program* is a sequence of clauses (Horn rules).
 - Each *clause* is of the form **head :- body.**
 - Head is one *term*.
 - Body is a comma-separated list of terms.
 - A clause with an empty body is called a *fact*.
 - A clause is also sometimes called a *rule*.

Terms

- Atomic data
- Variables
- Structures

Atomic Data

- *Numeric constants*: Integers, floating point numbers (e.g. **1024**, **-42**, **3.1415**, **6.023e23**,...)
- *Atoms*:
 - Identifiers: sequence of letters, digits, underscore, beginning with a lower case letter (e.g. **paul**, **r2d2**, **one_element**).
 - Strings of characters enclosed in single quotes (e.g. **'Stony Brook'**)

Variables

- Variables are denoted by identifiers beginning with an Uppercase letter or underscore (e.g. **X**, **Index**, **_param**).
- *These are Single-Assignment Logical variables:*
 - Variables can be assigned only once
 - Different occurrences of the same variable in a clause denote the same data.
 - Variables are implicitly declared upon first use
 - Variables are not typed
 - All types are discovered implicitly (no declarations in LP)
 - If the variable does not start with underscore, it is assumed that it appears multiple times in the rule.
 - If it does not appear multiple times, then a warning is produced: "*Singleton variable*"
 - You can use variables preceded with underscore to eliminate this warning

Variables

- *Anonymous variables* (also called *Don't care variables*): variables beginning with "_"
- Underscore, by itself (i.e., `_`), represents a variable
 - Each occurrence of `_` corresponds to a different variable; even within a clause, `_` does not stand for one and the same object.
- A variable with a name beginning with "_", but has more characters. E.g.: **`_radius`**, **`_Size`**
 - we want to give it a descriptive name
 - sometimes it is used to **create relationships within a clause (and must therefore be used more than once)**: a warning is produced: *"Singleton-marked variable appears more than once"*

Variables

- Warnings are used to identify bugs (most because of copy-paste errors)
 - Instead of declarations and type checking
 - Fix all the warnings in a program, so you know that you don't miss any logical error

Variables

- Variables can be assigned only once, but that value can be further refined:

?- $\mathbf{X=f(Y)}$,

$\mathbf{Y=g(Z)}$,

$\mathbf{Z=2}$.

Therefore, $\mathbf{X = f(g(2))}$

- The order also does not matter:

?- $\mathbf{Z=2}$,

$\mathbf{Y=g(Z)}$,

$\mathbf{X=f(Y)}$.

Therefore, $\mathbf{X = f(g(2))}$

- Even infinite structures:

?- $\mathbf{X=f(X)}$.

$\mathbf{X=f(f(f(f(f(f(f(f(f(\dots)))))}))}$

Logic Programming Queries

- To run a Prolog program, one asks the interpreter a question
 - This is done by asking a query which the interpreter tries to prove:
 - If it can, it says yes
 - If it can't, it says no
 - If your predicate contained variables, the interpreter prints the values it had to give them to make the predicate true.

?- wet(ny) . ?- reach(a, d) . ?- reach(d, a) .

Yes

Yes

No

?- wet(X) . ?- reach(X, d) . ?- reach(X, Y) .

X = ny ;

X=a

X=a, Y=d

X = seattle ;

no

Meaning of Logic Programs

- **Declarative Meaning:** What are the *logical consequences* of a program?
- **Procedural Meaning:** For what values of the variables in the query can I *prove* the query?
- The user gives the system a goal:
 - The system attempts to find axioms + inference steps to prove goal.
 - If goal contains variables, then also gives the values for those variables.

Declarative Meaning

```
brown(bear) .           big(bear) .
gray(elephant) .       big(elephant) .
black(cat) .           small(cat) .
dark(Z) :- black(Z) .
dark(Z) :- brown(Z) .
dangerous(X) :- dark(X) , big(X) .
```

- *Logical consequence of a program* L is the smallest set such that
 - All facts of the program are in L,
 - If $H :- B_1, B_2, \dots, B_n$ is an instance of a clause in the program such that B_1, B_2, \dots, B_n are all in L, then H is also in L.
- For the above program we get **dark(cat)** and **dark(bear)** and consequently **dangerous(bear)**.

Procedural Meaning of Prolog

```
brown(bear) .           big(bear) .
gray(elephant) .       big(elephant) .
black(cat) .           small(cat) .
dark(Z) :- black(Z) .
dark(Z) :- brown(Z) .
dangerous(X) :- dark(X) , big(X) .
```

- A *query* is, in general, a conjunction of goals: G_1, G_2, \dots, G_n
- To *prove* G_1, G_2, \dots, G_n :
 - Find a clause $H :- B_1, B_2, \dots, B_k$ such that G_1 and H match.
 - Under the substitution for variables, prove $B_1, B_2, \dots, B_k, G_2, \dots, G_n$

If nothing is left to prove then the proof succeeds!

If there are no more clauses to match, the proof fails!

Procedural Meaning of Prolog

```
brown(bear) .           big(bear) .
gray(elephant) .       big(elephant) .
black(cat) .           small(cat) .
dark(Z) :- black(Z) .
dark(Z) :- brown(Z) .
dangerous(X) :- dark(X) , big(X) .
```

- To prove: **?- dangerous(Q) .**

1. Select **dangerous(X) :- dark(X) , big(X)** and prove **dark(Q) , big(Q)** .
2. To prove **dark(Q)** select the first clause of dark, i.e. **dark(Z) :- black(Z)** , and prove **black(Q) , big(Q)** .
3. Now select the fact **black(cat)** and prove **big(cat)** .
4. Go back to step 2, and select the second clause of dark, i.e. **dark(Z) :- brown(Z)** , and prove **brown(Q) , big(Q)** .

This proof fails!

Procedural Meaning of Prolog

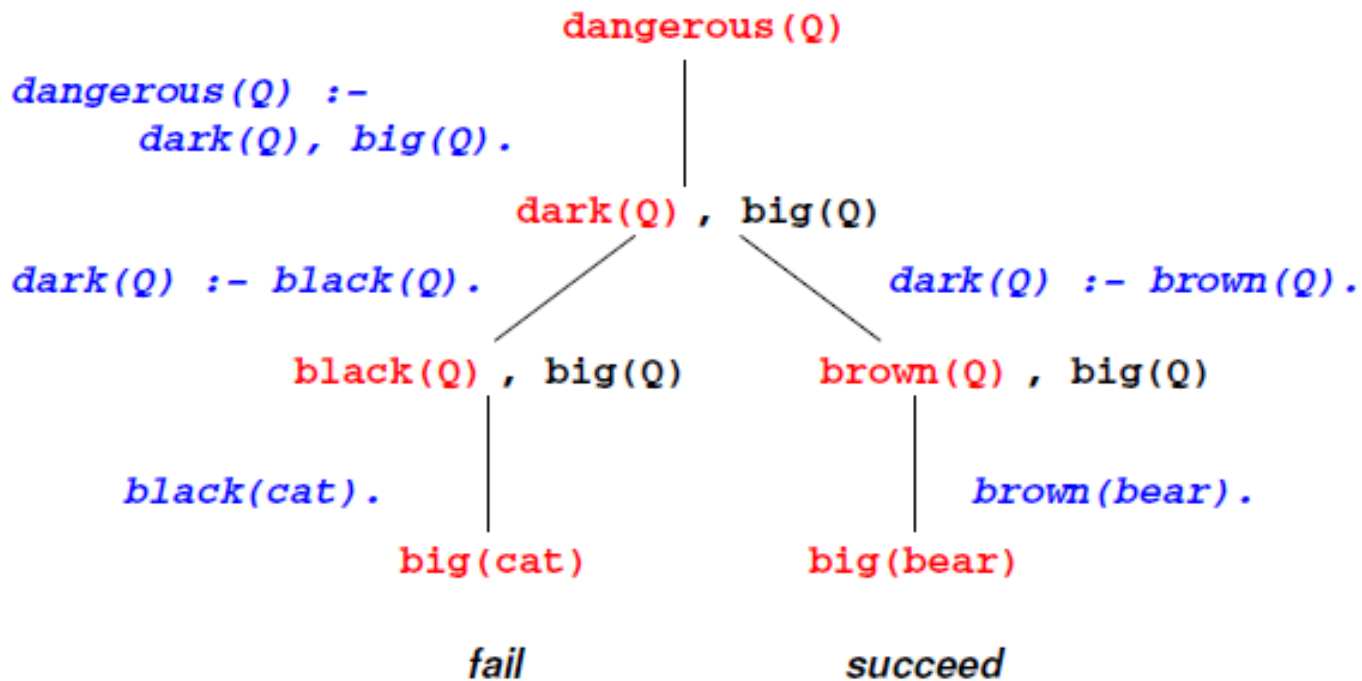
```
brown(bear) .           big(bear) .  
gray(elephant) .       big(elephant) .  
black(cat) .           small(cat) .  
dark(Z) :- black(Z) .  
dark(Z) :- brown(Z) .  
dangerous(X) :- dark(X) , big(X) .
```

- To prove: **?- dangerous(Q) .**
 5. Now select **brown(bear)** and prove **big(bear)** .
 6. Select the fact **big(bear)** .

There is nothing left to prove, so the proof succeeds

Procedural Meaning of Prolog

```
brown(bear) .           big(bear) .  
gray(elephant) .       big(elephant) .  
black(cat) .           small(cat) .  
dark(Z) :- black(Z) .  
dark(Z) :- brown(Z) .  
dangerous(X) :- dark(X) , big(X) .
```



Procedural Meaning of Prolog

- The Prolog interpreter works by what is called BACKWARD CHAINING (top-down, goal directed)
 - It begins with the thing it is trying to prove and works backwards looking for things that would imply it, until it gets to facts.
- It is also possible in theory to work forward from the facts trying to see if any of the things you can prove from them are what you were looking for (bottom-up resolution) - that can be very time-consuming
 - Example: Answer set programming, DLV, Potassco (the Potsdam Answer Set Solving Collection), OntoBroker
 - Fancier logic languages use both kinds of chaining, with special smarts or hints from the user to bound the searches

Procedural Meaning of Prolog

- The interpreter starts at the beginning of your database (this ordering is part of Prolog, NOT of logic programming in general) and looks for something with which to unify the current goal
 - If it finds a fact, great; it succeeds,
 - If it finds a rule, it attempts to satisfy the terms in the body of the rule depth first.
- This process is motivated by the RESOLUTION PRINCIPLE, due to Robinson, 1965:
 - It says that if $C1$ and $C2$ are Horn clauses, where $C2$ represents a true statement and the head of $C2$ unifies with one of the terms in the body of $C1$, then we can replace the term in $C1$ with the body of $C2$ to obtain another statement that is true if and only if $C1$ is true

Procedural Meaning of Prolog

- When it attempts resolution, the Prolog interpreter pushes the current goal onto a stack, makes the first term in the body the current goal, and goes back to the beginning of the database and starts looking again.
- If it gets through the first goal of a body successfully, the interpreter continues with the next one.
- If it gets all the way through the body, the goal is satisfied and it backs up a level and proceeds.

Procedural Meaning of Prolog

- If it fails to satisfy the terms in the body of a rule, the interpreter undoes the unification of the left hand side (this includes uninstantiating any variables that were given values as a result of the unification) **and keeps looking through the database for something else with which to unify** (BACKTRACKING).
- If the interpreter gets to the end of database without succeeding, it backs out a level (that's how it might fail to satisfy something in a body) and continues from there.

Procedural Meaning of Prolog

- PROLOG IS NOT PURELY DECLARATIVE:
 - The ordering of the database and the left-to-right pursuit of sub-goals gives a deterministic imperative semantics to searching and backtracking
 - Changing the order of statements in the database can give you different results:
 - It can lead to infinite loops,
 - It can certainly result in inefficiency.

Procedural Meaning of Prolog

- Transitive closure with left recursion in Prolog will run into an infinite loop:

```
reach(X,Y) :-  
    reach(X,Z) ,  
    edge(Z, Y) .
```

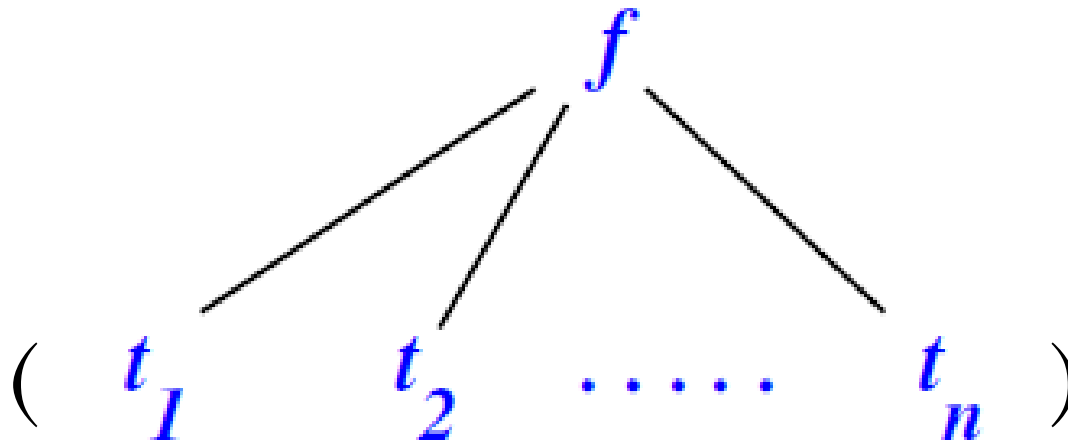
```
reach(X,Y) :-  
    edge(X,Y) .
```

```
?- reach(A,B) .
```

Infinite loop

Structures

- If f is an identifier and t_1, t_2, \dots, t_n are terms, then $f(t_1, t_2, \dots, t_n)$ is a term.

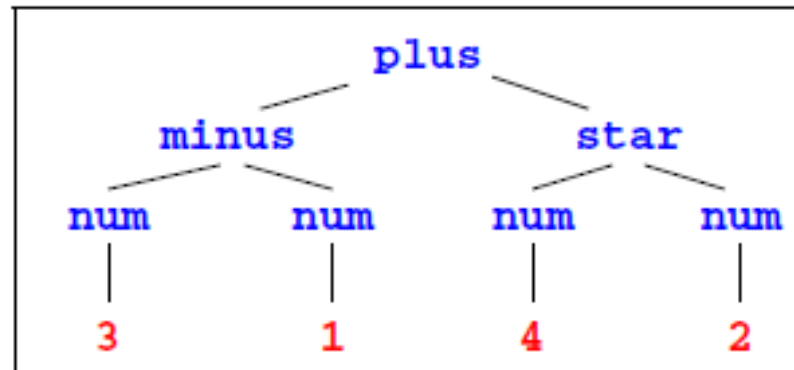


- In the above, f is called a *functor* and t_i is an *argument*.
- Structures are used to group related data items together (in some ways similar to struct in C and objects in Java).
- Structures are used to construct trees (and, as a special case, lists).

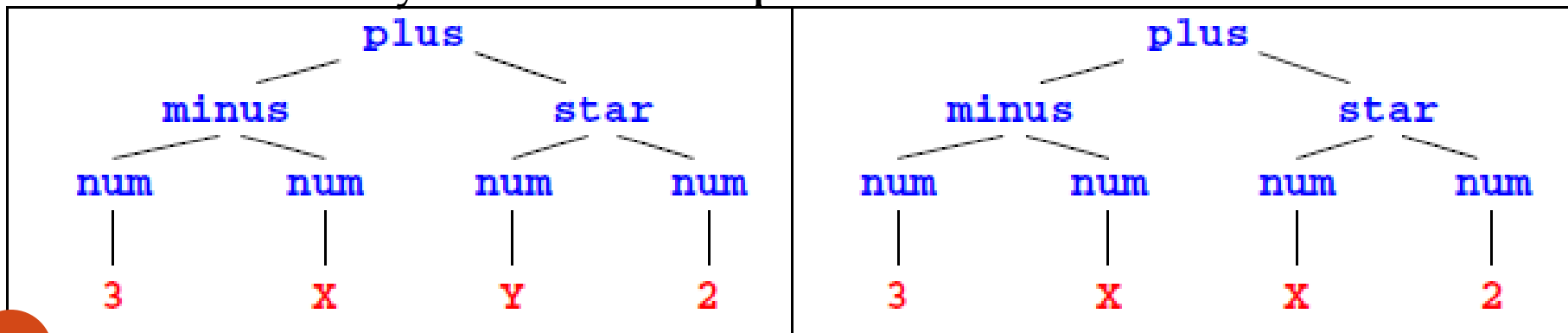
Trees

- Example: expression trees:

`plus (minus (num (3) , num (1)) , star (num (4) , num (2)))`

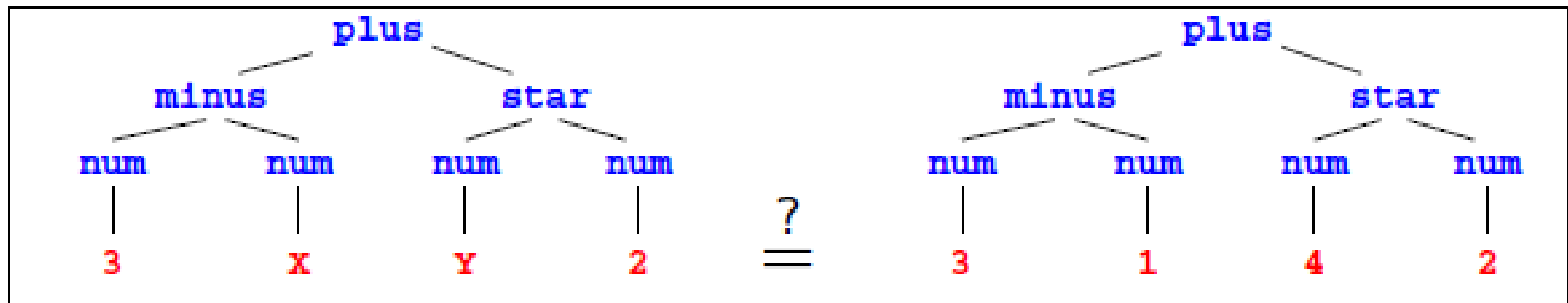


- Data structures may have variables. And the same variable may occur multiple times in a data structure.



Matching

- (We'll later introduce *unification*, a related operation that has logical semantics).
- $t1 = t2$: find substitutions for variables in $t1$ and $t2$ that make the two terms



Yes, with $X = 1$, $Y = 4$.

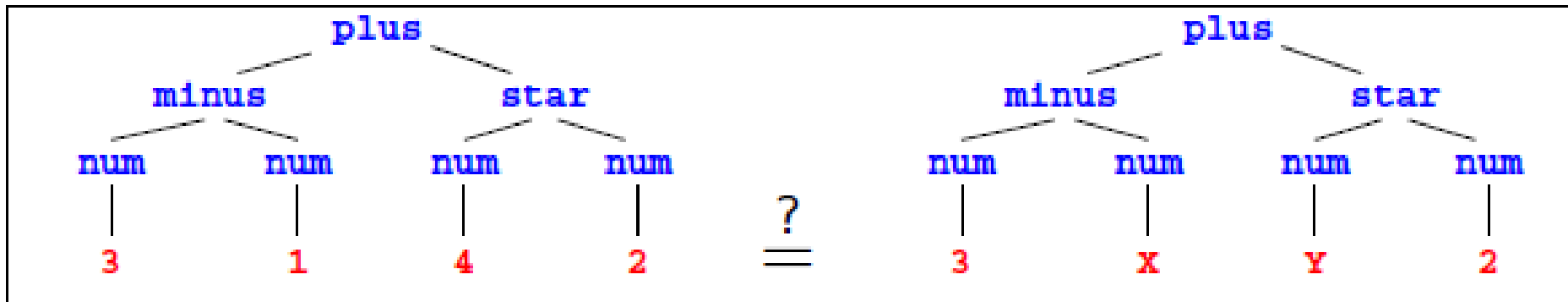
Matching

- Matching: given two terms, we can ask if they "*match*" each other. Rules:
 - A constant matches with itself: **42** unifies with **42**.
 - A variable matches with anything:
 - if it matches with something other than a variable, then it instantiates,
 - if it matches with a variable, then the two variables become associated.
 - **A=35, A=B** \rightarrow **B** becomes **35**.
 - **A=B, A=35** \rightarrow **B** becomes **35**.
- Two structures match if they:
 - Have the same functor,
 - Have the same arity,
 - Match recursively.
- **foo(g(42), 37)** matches with **foo(A, 37)**,
foo(g(A), B), etc.

Matching

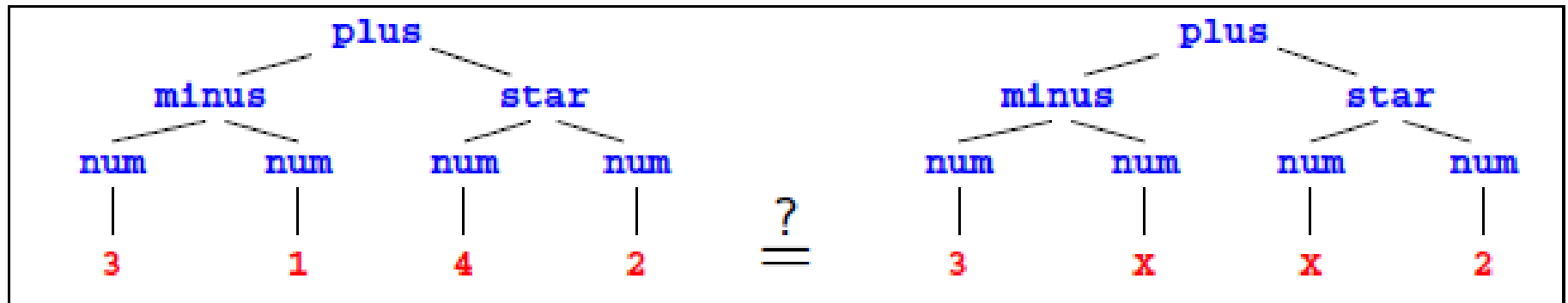
- General Rule to decide whether two terms, **S** and **T** *match* are as follows:
 - If **S** and **T** are constants, **S=T** if both are same object
 - If **S** is a variable and **T** is anything, **T=S**
 - If **T** is variable and **S** is anything, **S=T**
 - If **S** and **T** are structures, **S=T** if
 - **S** and **T** have same functor and same arity
 - All their corresponding arguments components have to match

Matching



Yes, with $X = 1$, $Y = 4$.

Matching



No! X cannot be 1 and 4 at the same time.

Matching

- Which of these match?
 - **A**
 - **100**
 - **func (B)**
 - **func (100)**
 - **func (C, D)**
 - **func (+ (99, 1))**

Matching

- Which of these match?
 - **A**
 - **100**
 - **func (B)**
 - **func (100)**
 - **func (C, D)**
 - **func (+ (99, 1))**
- **A** matches with **100**, **func (B)**, **func (100)**, **func (C,D)**, **func (+ (99, 1))**.
- **100** matches only with **A**.
- **func (B)** matches with **A**, **func (100)**, **func (+ (99, 1))**
- **func (C, D)** matches with **A**.
- **func (+ (99, 1))** matches with **A** and **func (B)**.

Accessing arguments of a structure

- Matching is the predominant means for accessing a structures arguments.
- Let `date('Sep', 1, 2015)` be a structure used to represent dates, with the month, day and year as the three arguments (in that order!).

Then `date(M,D,Y) = date('Sep',1,2020)` makes

`M = 'Sep', D = 1, Y = 2015.`

- If we want to get only the day, we can write

`date(_, D, _) = date('Sep', 1, 2015) .`

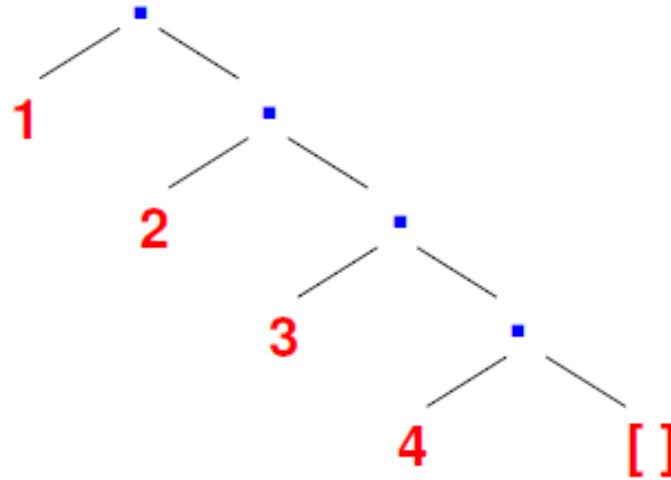
Then we only get: `D = 1.`

Lists

- Prolog uses a special syntax to represent and manipulate lists (syntactic sugar = internally, it uses structures):
 - $[1, 2, 3, 4]$: represents a list with **1**, **2**, **3** and **4**, respectively.
 - This can also be written as $[1 | [2, 3, 4]]$: a list with **1** as the *head* (first element) and $[2, 3, 4]$ as its *tail* (the list of remaining elements).
 - If $X = 1$ and $Y = [2, 3, 4]$ then $[X | Y]$ is same as $[1, 2, 3, 4]$.
 - The empty list is represented by $[]$ or **nil**.
 - The symbol " $|$ " (*pipe*) and is used to separate the beginning elements of a list from its tail.
 - For example: $[1, 2, 3, 4] = [1 | [2, 3, 4]] = [1 | [2 | [3, 4]]] = [1, 2 | [3, 4]] = [1, 2, 3 | [4]] = [1 | [2 | [3 | [4 | []]]]]$

Lists

- Lists are special cases of trees (i.e., (syntactic sugar = internally, it uses structures).
- For instance, the list **[1, 2, 3, 4]** is represented by the following structure:



- where the function symbol **. / 2** is the list constructor.
[1, 2, 3, 4] is same as **. (1, . (2, . (3, . (4, []))))**

Lists

- *Strings*: A sequence of characters surrounded by double quotes is equivalent to a list of (numeric) character codes: **"abc"**,
"John Smith",
"to be, or not to be".
 - ?- **X="abc"** .
X = [97,98,99]

Programming with Lists

- First example: **member**/2, to find if a given element occurs in a list:
- The program:

```
member (X, [X|_]) .  
member (X, [_|Ys]) :-  
    member (X, Ys) .
```

- Example queries:

```
?- member (2, [1,2,3]) .
```

```
?- member (X, [1,i,s,t]) .
```

```
?- member (f(X), [f(1),g(2),f(3),h(4)]) .
```

Programming with Lists

- **append/3**: concatenate two lists to form the third list:
- The program:

- Empty list append A is A.

append([], L, L) .

- Otherwise, break the first list up into a head X, tail L: if L append M is N, then X|N append M is X|N:

**append([X|L], M, [X|N]) :-
append(L, M, N) .**

- Example queries:

?- append([1,2], [3,4], X) .

?- append(X, Y, [1,2,3,4]) .

?- append(X, [3,4], [1,2,3,4]) .

Programming with Lists

- Is the predicate a function?
 - No. We are not applying arguments to get a result. Instead, we are proving that a theorem holds. Therefore, we can leave other variables unbound.

```
?- append(L, [2, 3], [1, 2, 3]).
```

```
    L = [ 1 ]
```

```
?- append([ 1 ], L, [1, 2, 3]).
```

```
    L = [2, 3]
```

```
?- append(L1, L2, [1, 2, 3]).
```

```
    L1 = [],           L2 = [1, 2, 3];
```

```
    L1 = [1],          L2 = [2, 3];
```

```
    L1 = [1, 2],        L2 = [3] ;
```

```
    L1 = [1, 2, 3],     L2 = [];
```

```
no
```


Append example trace

`append([], L, L) .`

`append([X|L], M, [X|N]) :- append(L, M, N) .`

`append([1,2], [3,4], X) ?`

Append example trace

`append([], L, L) .`

`append([X|L], M, [X|N]) :- append(L, M, N) .`



<code>append([1, 2], [3, 4], A) ?</code>	<code>X=1, L=[2], M=[3, 4], A=[X N]</code>
--	--

Append example trace

`append([], L, L) .`

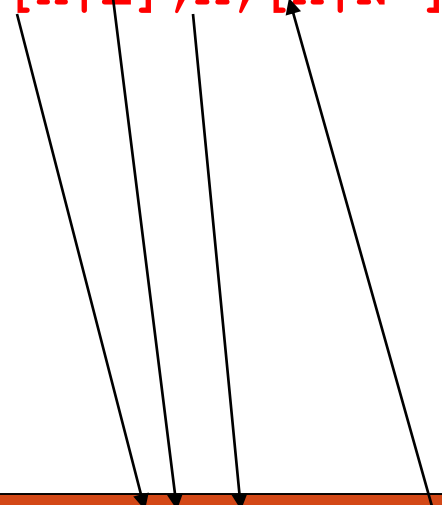
`append([X|L], M, [X|N]) :- append(L, M, N) .`

<code>append([2], [3, 4], N) ?</code>	
<code>append([1, 2], [3, 4], A) ?</code>	<code>X=1, L=[2], M=[3, 4], A=[X N]</code>

Append example trace

`append([], L, L) .`

`append([X|L], M, [X|N']) :- append(L, M, N') .`



<code>append([2], [3, 4], N) ?</code>	<code>X=2, L=[], M=[3, 4], N=[2 N']</code>
<code>append([1, 2], [3, 4], A) ?</code>	<code>X=1, L=[2], M=[3, 4], A=[1 N]</code>

Append example trace

append(**[]**,**L**,**L**) .

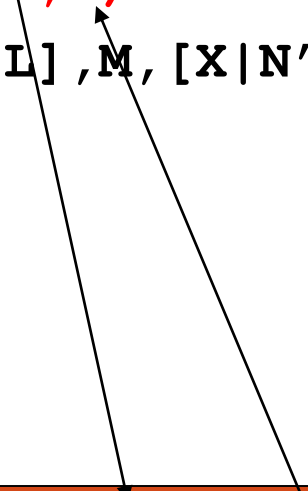
append(**[X|L]**,**M**,**[X|N']**) :- **append**(**L**,**M**,**N'**) .

append ([] , [3,4] , N') ?	
append ([2] , [3,4] , N) ?	X=2 , L=[] , M=[3,4] , N=[2 N']
append ([1,2] , [3,4] , A) ?	X=1 , L=[2] , M=[3,4] , A=[1 N]

Append example trace

append([], L, L) .

append([X|L], M, [X|N']) :- append(L, M, N') .



append([], [3, 4], N') ?	L = [3, 4], N' = L
append([2], [3, 4], N) ?	X=2, L=[], M=[3, 4], N=[2 N']
append([1, 2], [3, 4], A) ?	X=1, L=[2], M=[3, 4], A=[1 N]

Append example trace

`append([], L, L) .`

`append([X|L], M, [X|N']) :- append(L, M, N') .`

`A = [1|N]`

`N = [2|N']`

`N' = L`

`L = [3,4]`

Answer: `A = [1,2,3,4]`

<code>append([], [3,4], N') ?</code>	<code>L = [3,4], N' = L</code>
<code>append([2], [3,4], N) ?</code>	<code>X=2, L=[], M=[3,4], N=[2 N']</code>
<code>append([1,2], [3,4], A) ?</code>	<code>X=1, L=[2], M=[3,4], A=[1 N]</code>

Programming with Lists

- **len**/2 finds the length of a list (first argument).:

- The program:

```
len([], 0) .
```

```
len([_|Xs], N+1) :-  
    len(Xs, N) .
```

- Example queries:

```
?- len([], X) .  
    X = 0
```

```
?- len([l,i,s,t], 4) .  
    false
```

```
?- len([l,i,s,t], X) .  
    X = 0+1+1+1+1
```


Arithmetic

?- $1+2 = 3$.

false

- In Predicate logic, the basis for Prolog, the only symbols that have a meaning are the predicates themselves.
- In particular, function symbols are **uninterpreted**: have no special meaning and can only be used to construct data structures.

Arithmetic

- Meaning for arithmetic expressions is given by the built-in predicate **"is"**:

?- X is 1 + 2.

succeeds, binding X = 3.

?- 3 is 1 + 2.

succeeds.

- General form: **R is E** where **E** is an expression to be evaluated and **R** is matched with the expression's value.
- **Y is X + 1**, where **X** is a free variable, will give an error because **X** does not (yet) have a value, so, **X + 1** cannot be evaluated.

The list length example revisited

- **length**/2 finds the length of a list (first argument).:
- The program:

```
length([], 0).  
length([_|Xs], M) :-  
    length(Xs, N),  
    M is N+1.
```

- Example queries:

```
?- length([], X).  
?- length([l,i,s,t], 4).  
?- length([l,i,s,t], X).  
    X = 4  
?- length(List, 4).  
    List = [_1, _2, _3, _4]
```

Conditional Evaluation

- Conditional operator: the if-then-else construct in Prolog:
 - *if A then B else C* is written as (**A -> B ; C**)
 - To Prolog this means: try A. If you can prove it, go on to prove B and ignore C. If A fails, however, go on to prove C ignoring B.

```
max (X, Y, Z) :-  
    ( X =< Y  
    -> Z = Y  
    ; Z = X  
    ) .
```

```
?- max (1, 2, X) .  
X = 2 .
```

Conditional Evaluation

- Consider the computation of $n!$ (i.e. the factorial of n)

factorial(N, F) :- ...

- N is the input parameter; and F is the output parameter!
- The body of the rule specifies how the output is related to the input.
 - For factorial, there are two cases: $N \leq 0$ and $N > 0$.
 - if $N \leq 0$, then $F = 1$
 - if $N > 0$, then $F = N * \text{factorial}(N - 1)$

factorial(N, F) :-

(N > 0

-> N1 is N-1,

factorial(N1, F1),

F is N*F1

; F = 1

).

**?- factorial(12,X) .
X = 479001600**

Imperative features

- Other imperative features: we can think of prolog rules as imperative programs w/ backtracking.

```
program :-
```

```
    member(X, [1, 2, 3, 4]),  
    write(X),  
    nl,  
    fail.
```

```
program.
```

```
?- program. % prints all solutions
```

- **fail**: always fails, causes backtracking.
- **!** is the cut operator: prevents other rules from matching (we will see it later).

Therefore, Prolog Syntax:

- Assignments with arithmetic expressions is done using the keyword "**is**".

- If-then-else is written as

(cond -> then-part ; else-part)

- Arithmetic expressions are not directly used as arguments when calling a predicate; they are first evaluated, and then passed to the called predicate.
- If more than one action needs to be performed in a rule, they are written one after another, separated by a comma.

Arithmetic Operators

- Integer/Floating Point operators: $+$, $-$, $*$, $/$
 - Automatic detection of Integer/Floating Point
- Integer operators: mod , $//$ (integer division)
- Comparison operators: $<$, $>$, $=<$, $>=$,

$Expr1 ::= Expr2$ (succeeds if expression
 $Expr1$ evaluates to a number equal to $Expr2$),

$Expr1 \neq Expr2$ (succeeds if expression
 $Expr1$ evaluates to a number non-equal to $Expr2$)

Programming with Lists

- We want to define **delete**/3, to remove a given element from a list (called **select**/3 in XSB's basics library)
- Examples:
 - **delete**([1,2,3], 2, **x**) should succeed with **x** = [1,3].
 - **delete**([1,2,3], **x**, [1,3]) should succeed with **x** = 2.
 - **delete**(**x**, 2, [1,3]) should succeed with **x**=[2,1,3]; **x**=[1,2,3]; **x**=[1,3,2]; **fail**

Programming with Lists

- **Algorithm:**

- When X is selected from $[X \mid Ys]$, Ys results.
- When X is selected from the tail of $[H \mid Ys]$, $[H \mid Zs]$ results, where Zs is the result of taking X out of Ys .

Programming with Lists

- The program:

```
delete([],X,_) :- fail.  
delete([X|Ys], X, Ys).  
delete([Y|Ys], X, [Y|Zs]) :-  
    delete(Ys, X, Zs).
```

- Example queries:

```
?- delete([l,i,s,t], s, X).  
    X = [l, i, t]  
?- delete([l,i,s,t], X, Y).  
?- delete(X, s, [l,i,t]).  
?- delete(X, Y, [l,i,s,t]).
```

Permutations

- Define `permute/2`, to find a permutation of a given list.
 - E.g. `permute([1,2,3], X)` should return `X=[1,2,3]` and upon backtracking, `X=[1,3,2]`, `X=[2,1,3]`, `X=[2,3,1]`, `X=[3,1,2]`, and `X=[3,2,1]`.
 - Hint: What is the relationship between the permutations of `[1,2,3]` and the permutations of `[2,3]`?

<code>permute([2,3], Y)</code>	<code>permute([1,2,3], Y)</code>
<code>[2,3]</code>	<code>[1,2,3]</code>
	<code>[2,1,3]</code>
	<code>[2,3,1]</code>
<code>[3,2]</code>	<code>[1,3,2]</code>
	<code>[3,1,2]</code>
	<code>[3,2,1]</code>

Programming with Lists

- The program:

```
permute([], []).
```

```
permute([X|Xs], Ys) :-
```

```
    permute(Xs, Zs),
```

```
    delete(Ys, X, Zs).
```

- Example query:

```
?- permute([1,2,3], X).
```

The Issue of Efficiency

- Define a predicate, `rev/2` that finds the reverse of a given list.
 - E.g. `rev([1,2,3], X)` should succeed with $X = [3,2,1]$.
 - Hint: what is the relationship between the reverse of `[1,2,3]` and the reverse of `[2,3]`?

`rev([], []).`

**`rev([X|Xs], Ys) :- rev(Xs, Zs),
append(Zs, [X], Ys).`**

- How long does it take to evaluate `rev([1, 2, ..., n], X)`?
 - $T(n) = T(n - 1) + \text{time to add 1 element to the end of an } n - 1 \text{ element list}$
 $= T(n - 1) + n - 1 = T(n - 2) + n - 2 + n - 1 = \dots$
 - $\rightarrow T(n) = O(n^2)$

Making rev/2 faster

- Keep an accumulator: a stack all elements seen so far.
 - i.e. a list, with elements seen so far in reverse order.

- The program:

```
rev(L1, L2) :- rev(L1, [], L2).
```

```
rev([X|Xs], AccBefore, AccAfter) :-  
    rev(Xs, [X|AccBefore], AccAfter).
```

```
rev([], Acc, Acc). % Base case
```

- Example query:

```
?- rev([1,2,3], [], X).
```

```
which calls rev([2,3], [1], X)
```

```
which calls rev([3], [2,1], X)
```

```
which calls rev([], [3,2,1], X)
```

Tree Traversal

- Assume you have a binary tree, represented by
 - node/ 3 facts: for internal nodes: `node(a,b,c)` means that a has b and c as children.
 - leaf/ 1 facts: for leaves: `leaf(a)` means that a is a leaf.
 - Example:
`node(5, 3, 6). node(3, 1, 4). leaf(1). leaf(4). leaf(6).`
- Write a predicate `preorder/ 2` that traverses the tree (starting from a given node) and returns the list of nodes in pre-order

Tree Traversal

```
preorder(Root, [Root]) :-  
    leaf(Root).
```

```
preorder(Root, [Root|L]) :-  
    node(Root, Child1, Child2),  
    preorder(Child1, L1),  
    preorder(Child2, L2),  
    append(L1, L2, L).
```

- The program takes $O(n^2)$ time to traverse a tree with n nodes.

Difference Lists

- The lists in Prolog are singly-linked; hence we can access the first element in constant time, but need to scan the entire list to get the last element.
- However, unlike functional languages like Lisp or SML, we can use variables in data structures:
 - We can exploit this to make lists “*open tailed*”

Difference Lists

- When $X = [1, 2, 3 \mid Y]$, X is a list with 1, 2, 3 as its first three elements, followed by Y .
 - Now if $Y = [4 \mid Z]$ then $X = [1, 2, 3, 4 \mid Z]$.
 - We can think of Z as “pointing to” the end of X .
 - **We can now add an element to the end of X in constant time!!**
 - (e.g. $Z = [5 \mid W]$)
- Open-tailed lists are also called *difference lists* in Prolog.

Tree Traversal, Revisited

```
preorder1(Node, List, Tail) :-  
    node(Node, Child1, Child2),  
    List = [Node|List1],  
    preorder1(Child1, List1, Tail1),  
    preorder1(Child2, Tail1, Tail).  
preorder1(Node, [Node|Tail], Tail) :-  
    leaf(Node).  
preorder(Node, List) :-  
    preorder1(Node, List, []).
```

- The program takes $O(n)$ time to traverse a tree with n nodes.

Difference Lists: Conventions

- An difference list is represented by two variables: one referring to the entire list, and another to its (uninstantiated) tail.
 - e.g. $X = [1, 2, 3 \mid Z]$.
- Most Prolog programmers use the notation List - Tail to denote a list List with tail Tail.
- Note that “-” is used as a data structure symbol (not used here for arithmetic).

Difference Lists: Conventions

- The preorder traversal program may be written as:

```
preorder1 (Node, [Node|L]-T) :-  
    node (Node, Child1, Child2),  
    preorder1 (Child1, L-T1),  
    preorder1 (Child2, T1-T).  
preorder1 (Node, [Node|T]-T).
```

Graphs in Prolog

- There are several ways to represent graphs in Prolog:
 - represent each edge separately as one clause (fact):
edge (a , b) .
edge (b , c) .
 - isolated nodes cannot be represented, unless we have also **node** / 1 facts
 - the whole graph as one data object: as a pair of two sets (nodes and edges): **graph ([a , b , c , d , f , g] , [e (a , b) , e (b , c) , e (b , f)])**
 - list of arcs: [a-b, b-c, b-f]
 - adjacency-list: **[n (a , [b]) , n (b , [c , f]) , n (d , [])]**

Graphs in Prolog

- Path from one node to another one:
 - a predicate **path (G, A, B, P)** to find an acyclic path P from node A to node B in the graph G.
 - The predicate should return all paths via backtracking.
 - We will solve it using the graph as a data object, like in **graph ([a, b, c, d, f, g] , [e (a, b) , e (b, c) , e (b, f)]**

Graphs in Prolog

- Path from one node to another one:

```
path (G,A,B,P) :-
```

```
    path1 (G,A,[B],P) .
```

```
path1 (_,A,[A|P1],[A|P1]) . %Base
```

```
path1 (G,A,[Y|P1],P) :-
```

```
    adjacent (X,Y,G) ,
```

```
    \+ member (X,[Y|P1]) ,
```

```
    path1 (G,A,[X,Y|P1],P) .
```

Graphs in Prolog

- Acyclic graph path:

```
adjacent (X, Y, graph (_, Es) ) :-  
    member (e (X, Y) , Es) .
```

```
adjacent (X, Y, graph (_, Es) ) :-  
    member (e (Y, X) , Es) .
```

Graphs in Prolog

- Cycle from a given node:
 - a predicate **cycle (G,A,P)** to find a closed path (cycle) P starting at a given node A in the graph G.
 - The predicate should return all cycles via backtracking.

```
cycle (G,A,P) :-  
    adjacent (B,A,G) ,  
    path (G,A,B,P1) ,  
    length (P1,L) ,  
    L > 2 ,  
    append (P1,[A],P) .
```

Aggregates in XSB

- `setof(?Template, +Goal, ?Set)` : ?Set is the set of all instances of Template such that Goal is provable.
- `bagof(?Template, +Goal, ?Bag)` has the same semantics as `setof/3` except that the third argument returns an unsorted list that may contain duplicates.
- `findall(?Template, +Goal, ?List)` is similar to predicate `bagof/3`, except that variables in Goal that do not occur in Template are treated as existential, and alternative lists are not returned for different bindings of such variables.
- `tfindall(?Template, +Goal, ?List)` is similar to predicate `findall/3`, but the Goal must be a call to a single tabled predicate.

XSB Prolog

- Negation: *not* ($\backslash +$): negation-as-failure
- Another negation called *tnot* (*TABLING* = *memoization*)
 - Use: ... :- ..., *tnot*(foobar(X)).
 - All variables under the scope of *tnot* must also occur to the left of that scope in the body of the rule in other positive relations:
 - Ok: ... :- p(X,Y), *tnot*(foobar(X,Y)), ...
 - Not ok: ... :- p(X,Z), *tnot*(foobar(X,**Y**)), ...
- XSB also supports Datalog:
 - :- auto_table.at the top of the program file

XSB Prolog

- Read/write from and to files:
 - Edinburgh style:
 - ?- see('a.txt'), read(X), seen.
 - ?- tell('a.txt'),
write('Hello, World!'), told.

XSB Prolog

- Read/write from and to files:
 - ISO style:
?- open('a.txt', write, X),
write(X, 'Hello, World!'),
close(X).

Cut (logic programming)

- Cut (! in Prolog) is a goal which always succeeds, **but cannot be backtracked past.**

- **Green cut**

`gamble(X) :- gotmoney(X), !.`

`gamble(X) :- gotcredit(X), \+ gotmoney(X).`

- **cut** says “stop looking for alternatives”
- by explicitly writing `\+ gotmoney(X)`, it guarantees that the second rule will always work even if the first one is removed by accident or changed

- **Red cut**

`gamble(X) :- gotmoney(X), !.`

`gamble(X) :- gotcredit(X).`

Cut (logic programming)

- Consider:

$p(a). p(b).$

$q(a). q(b). q(c).$

$?- p(X), !.$

$X=a ;$

no

$?- p(X), !, q(Y).$

$X=a, Y=a ;$

$X=a, Y=b ;$

$X=a, Y=c ;$

Testing types

- **atom(X)**

Tests whether X is bound to a symbolic atom.

?- atom(a).

yes

?- atom(3).

no

- **integer(X)**

Tests whether X is bound to an integer.

- **real(X)**

Tests whether X is bound to a real number.

Testing for variables

- **is_list(L)**

Tests whether L is bound to a list.

- **ground(G)**

Tests whether G has unbound logical variables.

- **var(X)**

Tests whether X is bound to a Prolog variable.

Control / Meta-predicates

- **call(P)**

Force P to be a goal; succeed if P does, else fail.

Assert and retract

- **asserta(C)**

Assert clause C into database above other clauses with the same key predicate. The key predicate of a clause is the first predicate encountered when the clause is read from left to right.

- **assertz(C), assert(C)**

Assert clause C into database below other clauses with the same key predicate.

- **retract(C)**

Retract C from the database. C must be sufficiently instantiated to determine the predicate key.

Prolog terms and clauses

- **clause(H,B)**

Retrieves clauses in memory whose head matches H and body matches B. H must be sufficiently instantiated to determine the main predicate of the head.

- **functor(E,F,N)**

E must be bound to a functor expression of the form 'f(...)'. F will be bound to 'f', and N will be bound to the number of arguments that f has.

- **arg(N,E,A)**

E must be bound to a functor expression, N is a whole number, and A will be bound to the Nth argument of E

Prolog terms and clauses

- `=..`

converts between term and list. For example,

?- `parent(a,X) = .. L.`

`L = [parent, a, _X001]`