

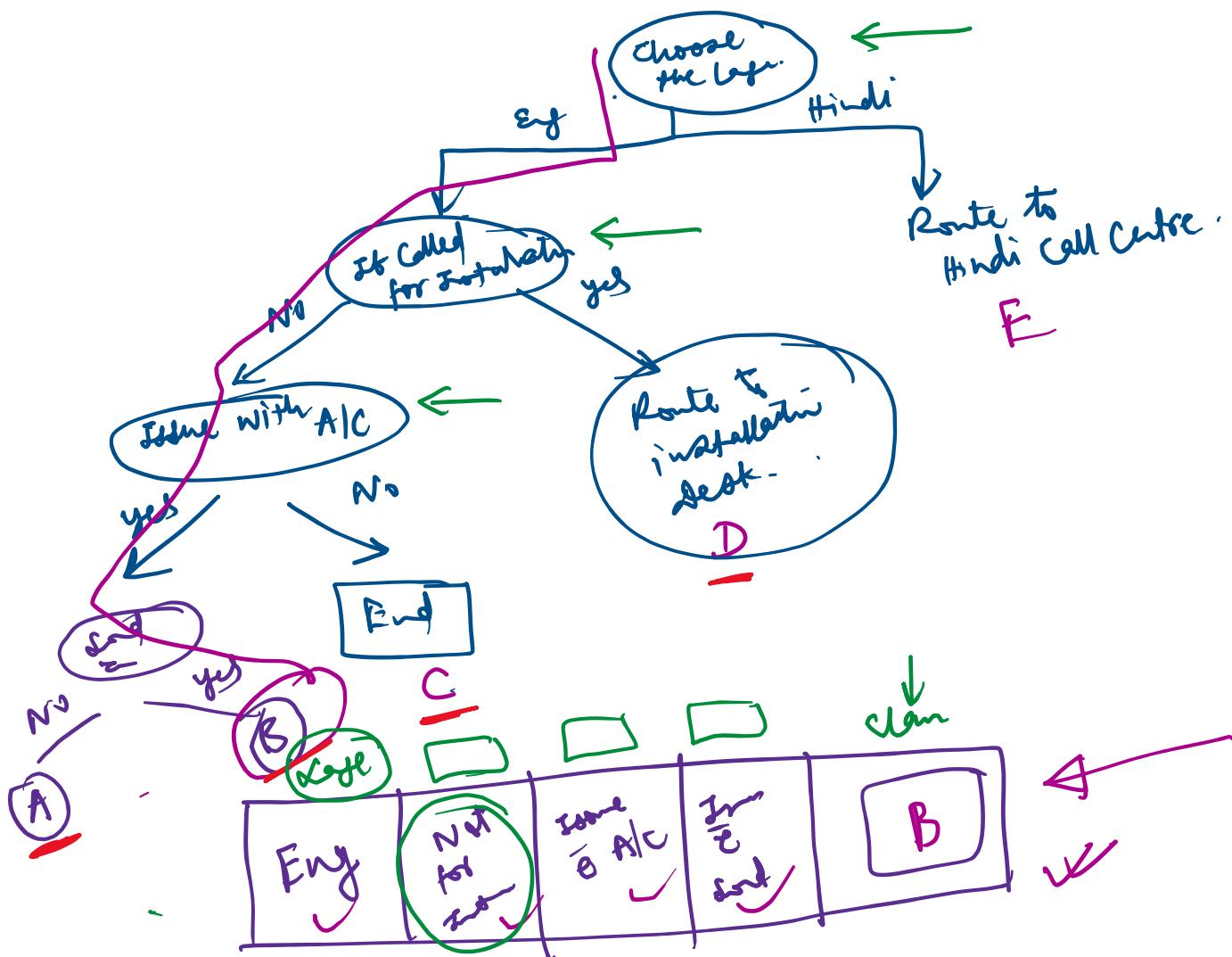
## Decision Tree

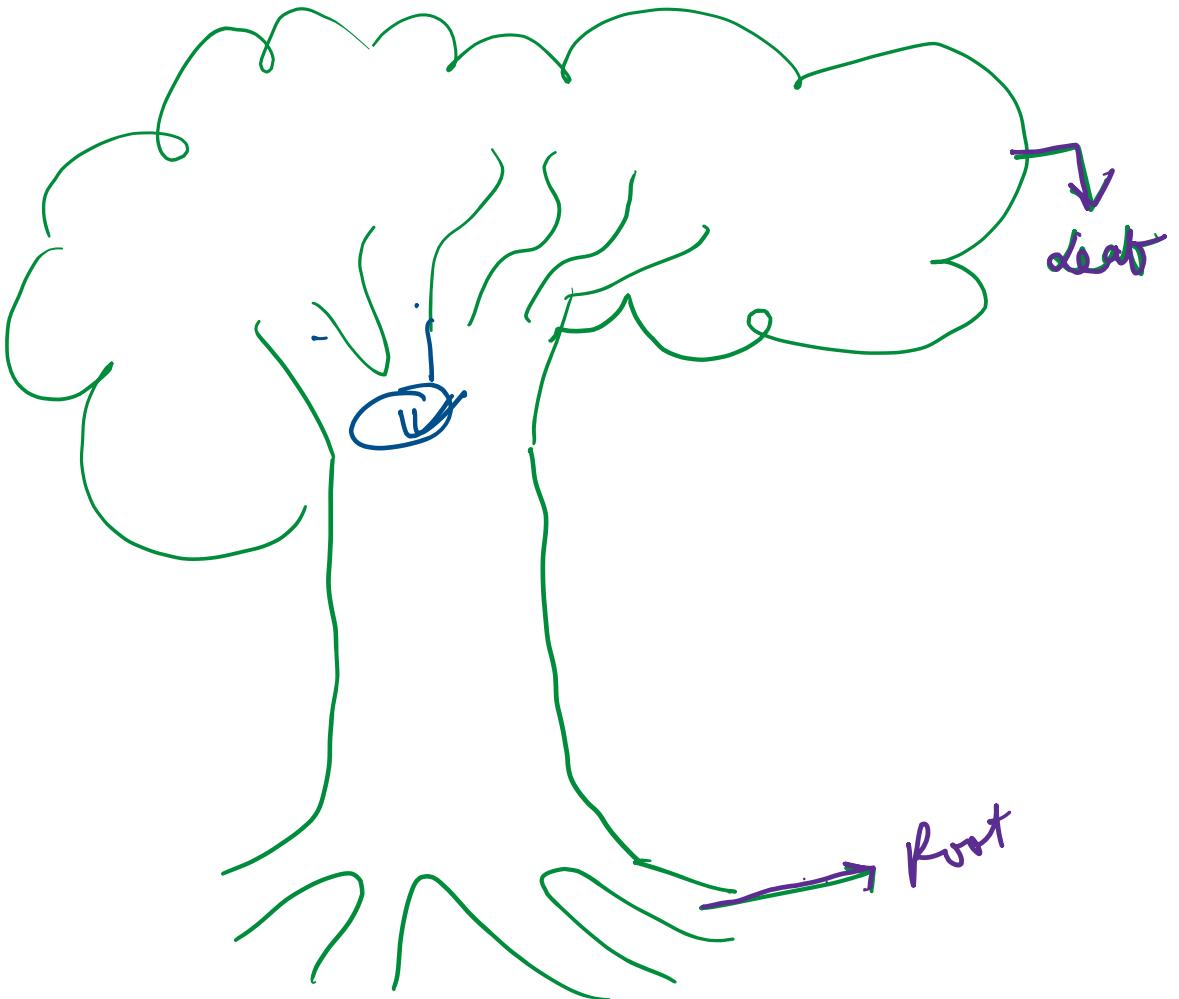
→ supervised ML Algo.

Mostly used for classification but can be used in Regression.

→ Tree based Structure -  
Based on decisions.

## A/C Service

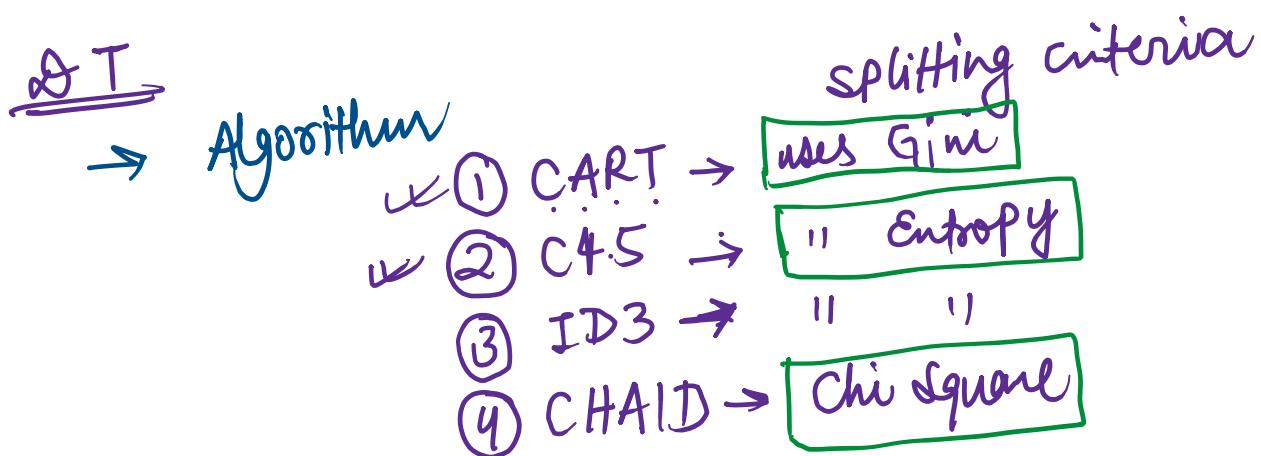




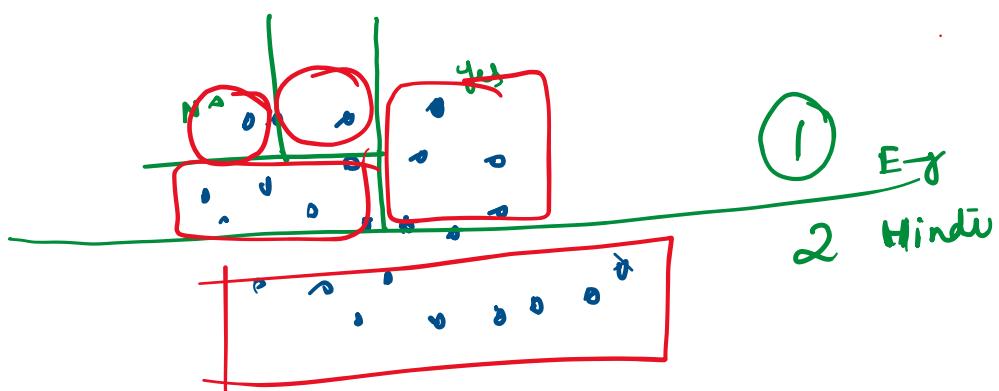
## Terminologies

- ① Root node : Main Node (splitting starts here).
- ② splitting : Process of dividing Nodes into Subnodes.
- ③ Decision Node : Condition node, at the node where  
(decision is made / nodes  
↓  
subnodes)
- ④ Parent/  
child nodes : Node which is divided into sub nodes  
are called Parent Node or Sub-nodes  
—o—  
these sub nodes are the child of "Parent node".

- ⑤ Subnode = Child node.
- ⑥ Terminal Node / Leaf Node : - Last Node  
(No further split).
- ⑦ Pruning : Process of Removing Subnodes  
↳ (reverse of splitting)

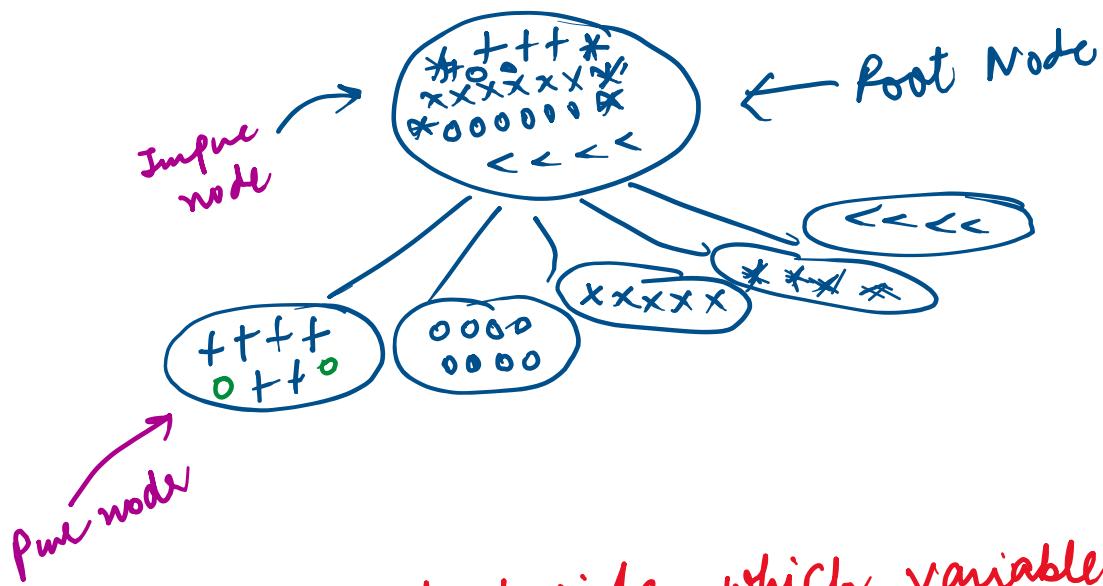


- Based on Recursive Partitioning
- Hierarchical
- Based on different splitting criteria.



## splitting criteria

The purpose is to get more pure nodes further.



Q. In D.T. how to decide which variable is used for splitting?

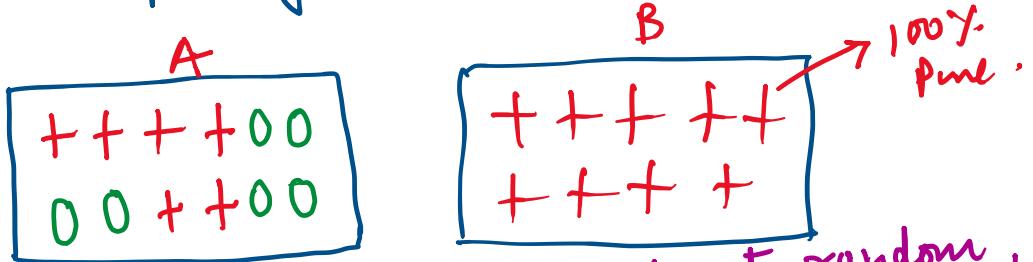
→ Based on Splitting criteria

- ① gini
- ② Entropy
- ③ Chi-Sq.
- ④ Red. in variance

Central Idea: To get more/better pure Node.

## ① Gini Impurity

$$\text{Gini Impurity} = 1 - \text{Gini}$$



→ If we select 2 items from a node at random, they must be from the same class.

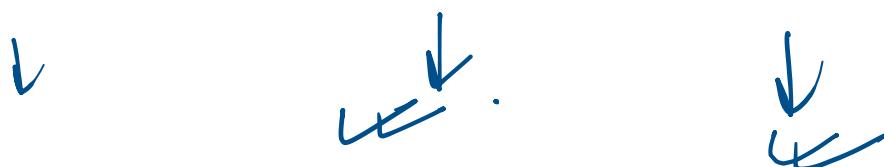
→ If it's 100% pure — Prob. of picking both items belonging to same class = 1 or 100%.

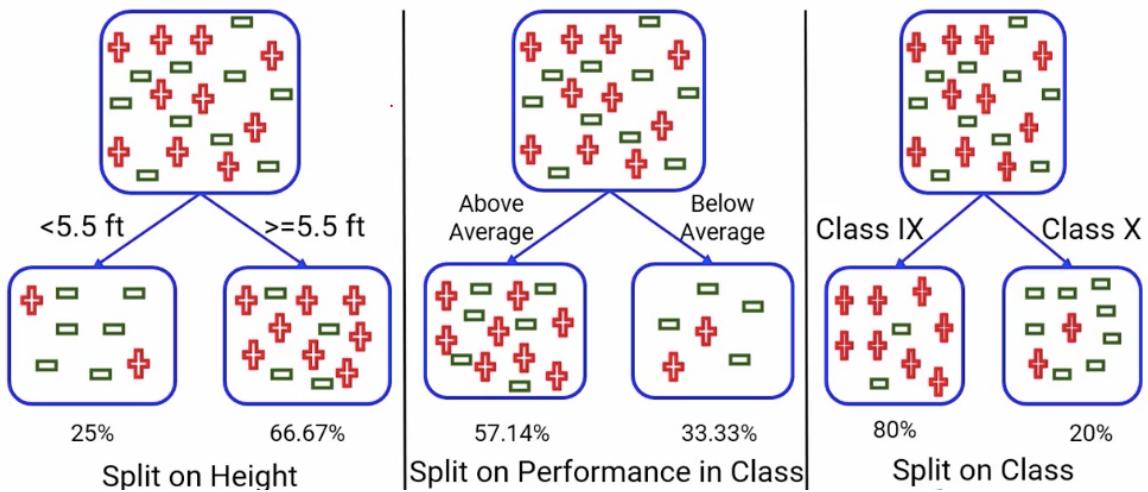
### Steps:

① Gini Impurity of subnodes

$$② \text{Gini} = (P_1^2 + P_2^2 + \dots + P_n^2)$$

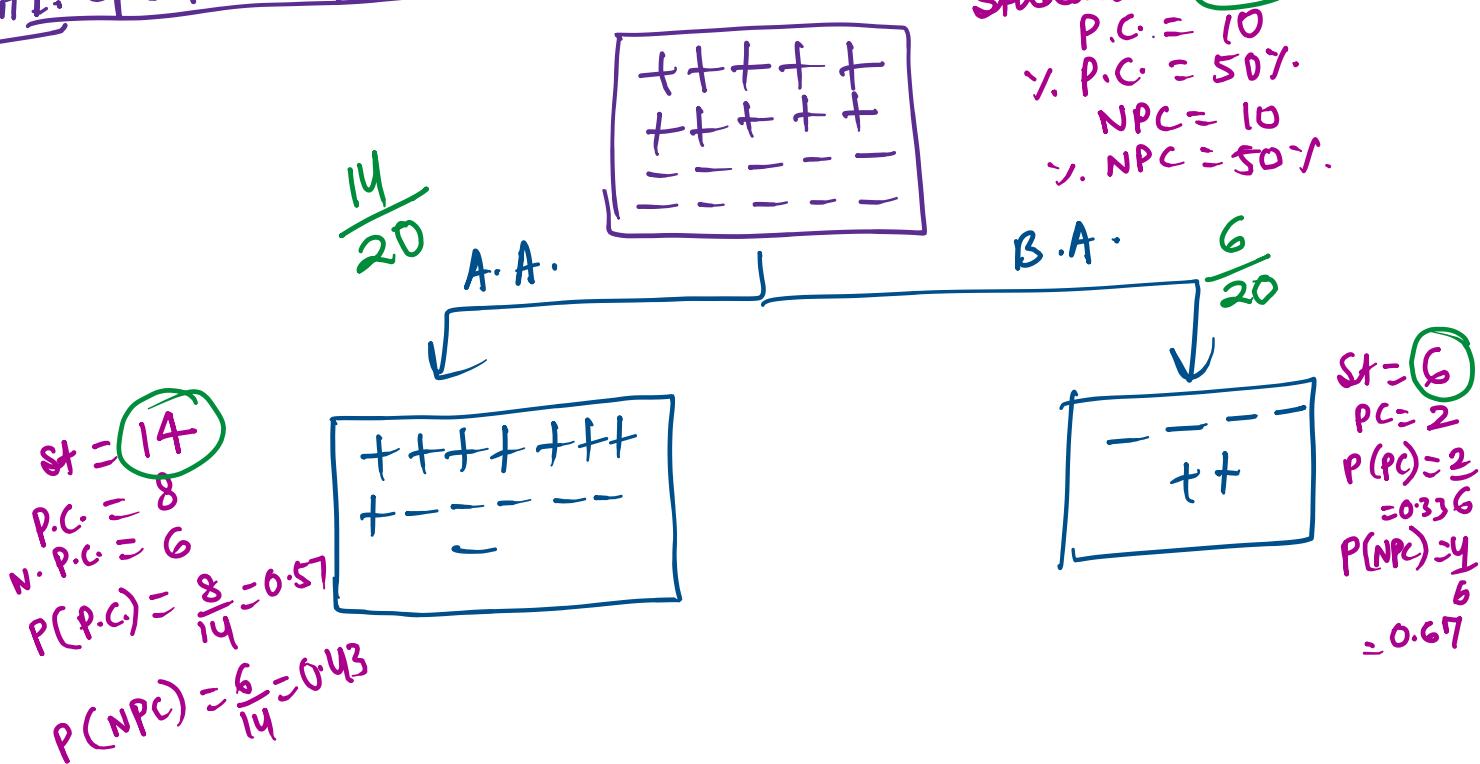
③ Gini Impurity of split  
→ taking the weighted gini impurity of subnodes or split.





height	Performance in Cls	Class	$y = \text{play} / N \cdot P.C.$
$< 5.5$	Above	X	+
$\geq 5.5$	Below	X	-

split1: Split on Performance



① G.I. of subnode A.A.

$$= 1 - ((0.57)^2 + (0.43)^2) = \boxed{0.49}$$

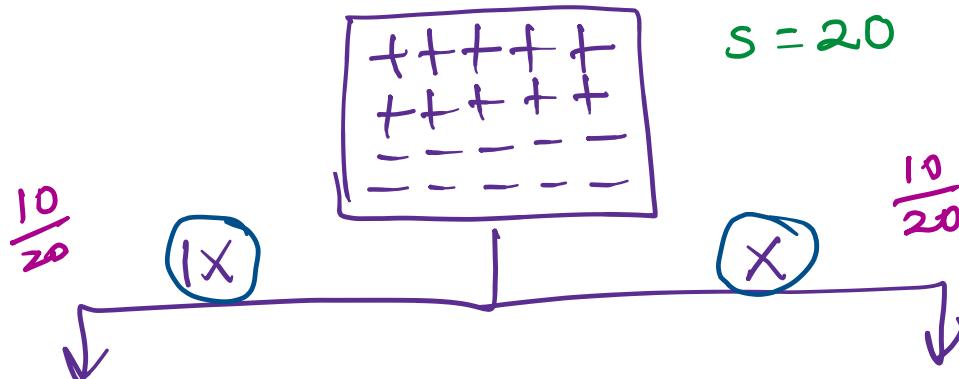
② G.I. of subnode B.A.

$$= 1 - ((0.33)^2 + (0.67)^2) = \boxed{0.44}$$

③ weighted G.I. of split

$$= \frac{14}{20} \times (0.49) + \frac{6}{20} \times (0.44)$$
$$= \boxed{0.475}$$

split 2: split on class



$S = 10$   
 $P_{LC} = 8$   
 $P(PC) = 0.8$   
 $P(NPC) = 0.2$

$S = 10$   
 $P_{LC} = 2$   
 $P(PC) = 0.2$   
 $P(NPC) = 0.8$

① G.I. of subnode ~~X~~:

$$= 1 - [(0.8)^2 + (0.2)^2] = 0.32$$

② G.I. of subnode X

$$= 1 - [(0.2)^2 + (0.8)^2] = 0.32$$

③ weighted G.I. of split by class

$$= \frac{10}{20} \times 0.32 + \frac{10}{20} \times 0.32 \\ = 0.32$$

decision

split	weighted G.I.
Part	0.475
class	0.32



since the G.I. is least when we split by class  
hence

1st split will happen by class.

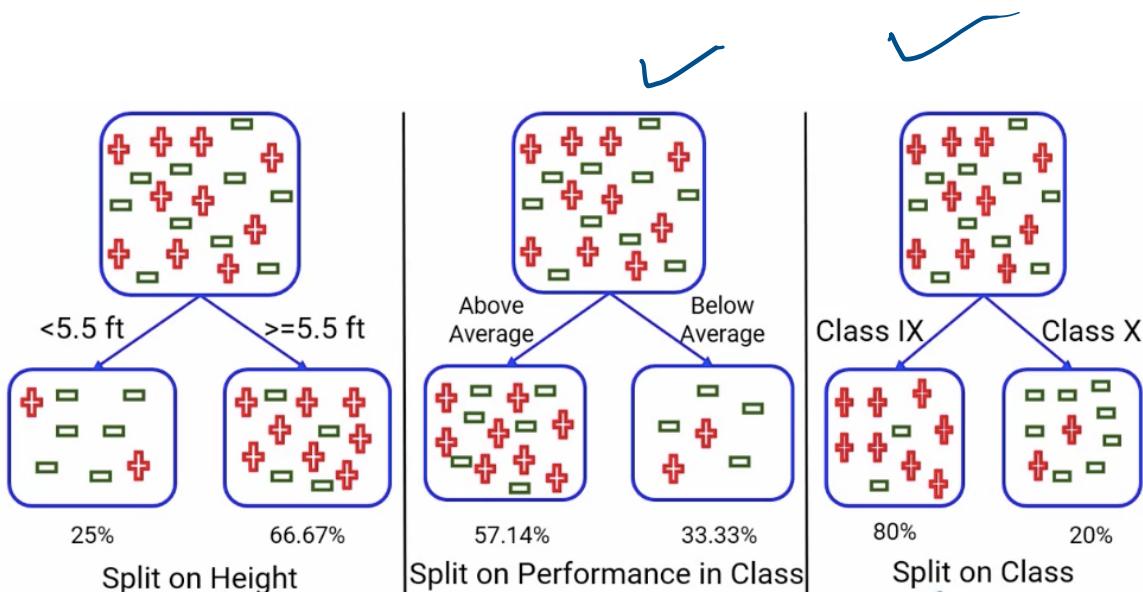
## ② Chi - Square

→ statistical significance of any node

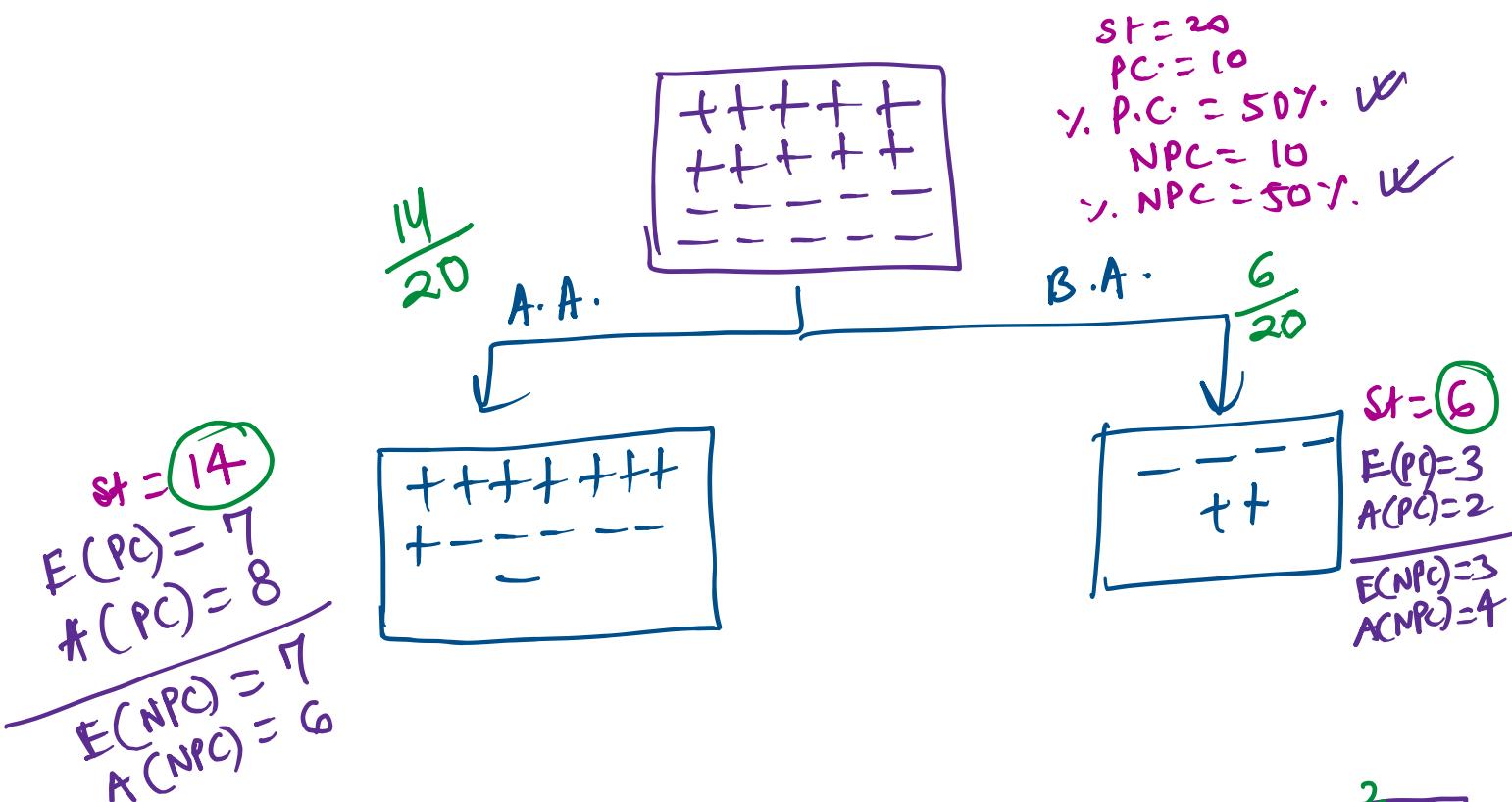
→ Mostly used for Categorical Var.

$$\rightarrow \text{chi-Sq} (\chi^2) = \sqrt{\frac{\text{Actual} - \text{Expected}}{\text{Expected}}}^2$$

→ Higher the value of  $\chi^2$ , higher is the homogeneity of node.



Split!!: Split on perf -

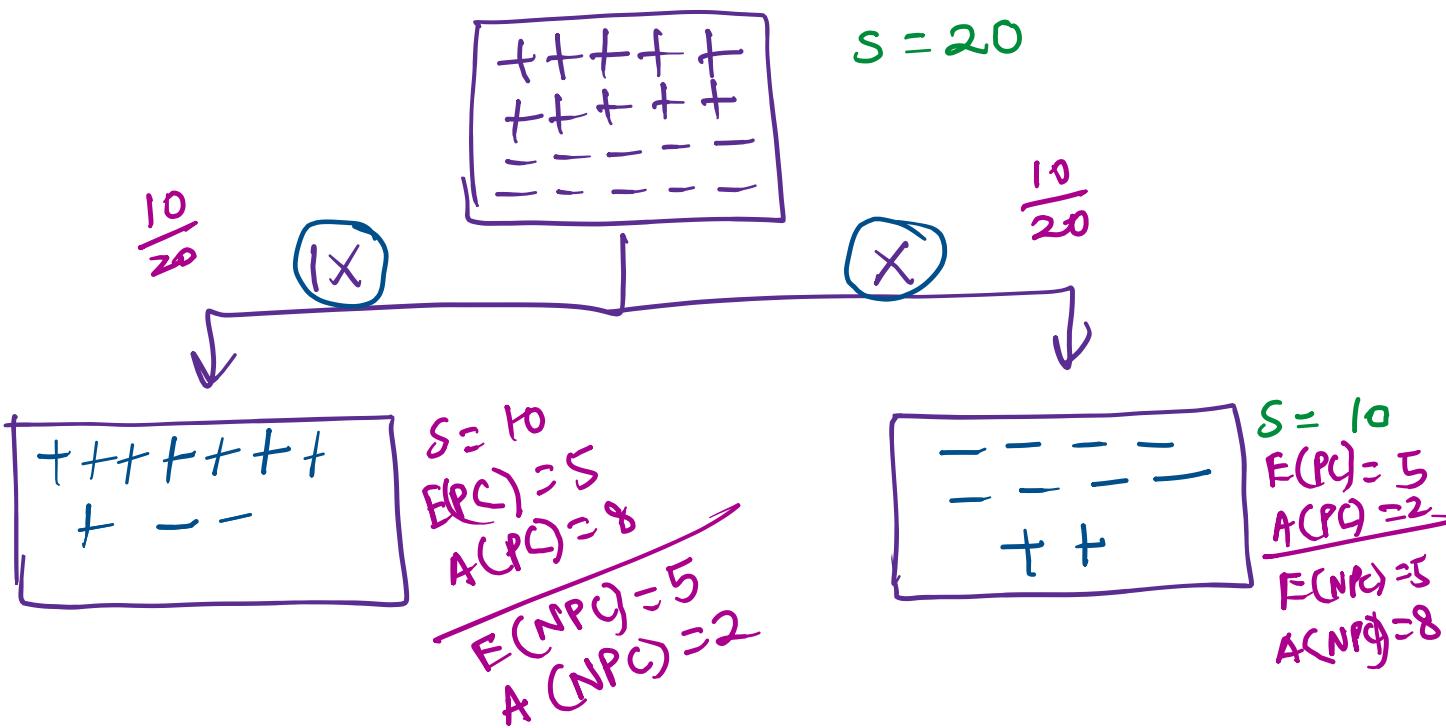


Node	A(P)	A(NPC)	E(P)	E(NPC)	Dev(PC)	Dev(NPC)	$\chi^2(P)$	$\chi^2(NPC)$
AA	8	6	7	7	$\frac{8-7}{=1} = 1$	-1	0.37	0.37
BA	2	4	3	3	$\frac{2-3}{= -1} = -1$	1	0.57	0.57

Total Chi-sq. for this split =  $0.37 + 0.37 + 0.57 + 0.57$

$$= 1.88$$

## split 2: split on class



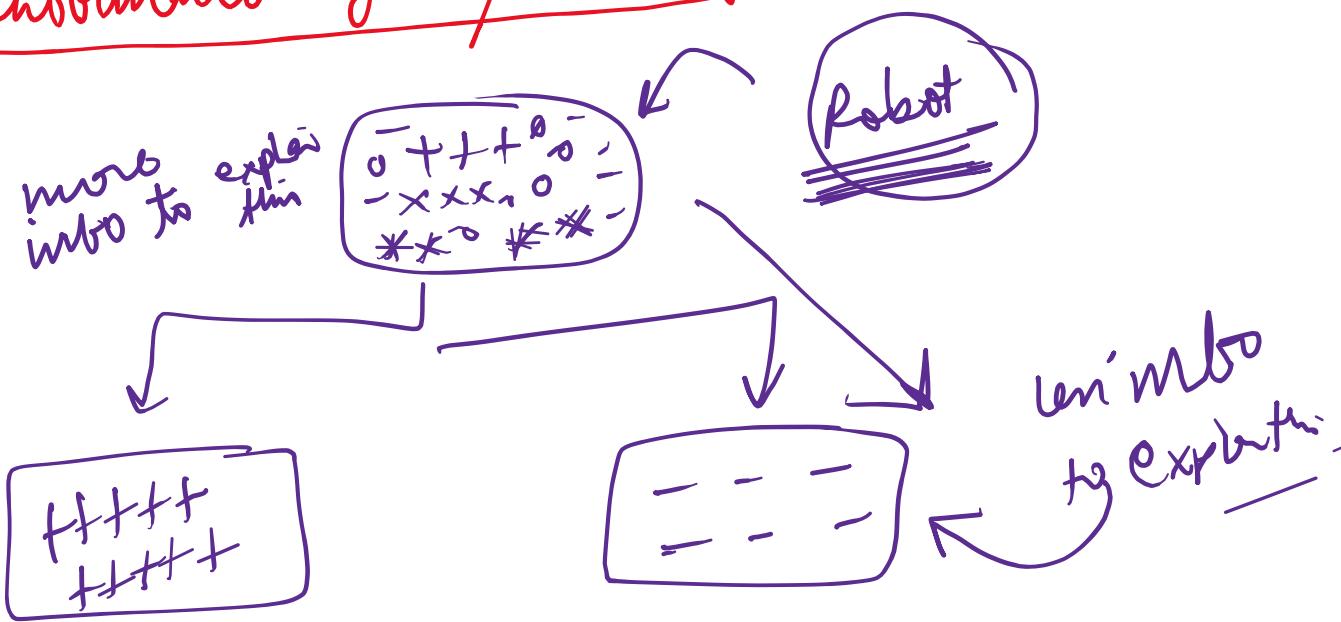
Node	$A(PC)$	$A(NPC)$	$E(PC)$	$E(NPC)$	$D(PC)$	$D(NPC)$	$\chi^2(PC)$	$\chi^2(NPC)$
1X	8	2	5	5	3	-3	1.34	1.34
X	2	8	5	5	-3	3	1.34	1.34

$$\begin{aligned} \text{Total chisq.} &= 1.34 + 1.34 + 1.34 + 1.34 \\ &= 5.36 \end{aligned}$$

decision

split	chi- <del>sq</del> -
part	1.88
clsn	5.36

### ③ Information gain / Entropy :-



→ Higher information gain leads to more pure nodes.

$$\rightarrow I.G. = 1 - \text{Entropy}$$

$$\text{Entropy} : -P_1 \log_2 P_1 - P_2 \log_2 P_2 - \dots - P_n \log_2 P_n$$

$$\approx -P \log_2 P - Q \log_2 Q$$

let:

+++ + +
---- - -

A

$$P(+)=0.5 \\ P(-)=0.5$$

--- --- -
--- --- -

B

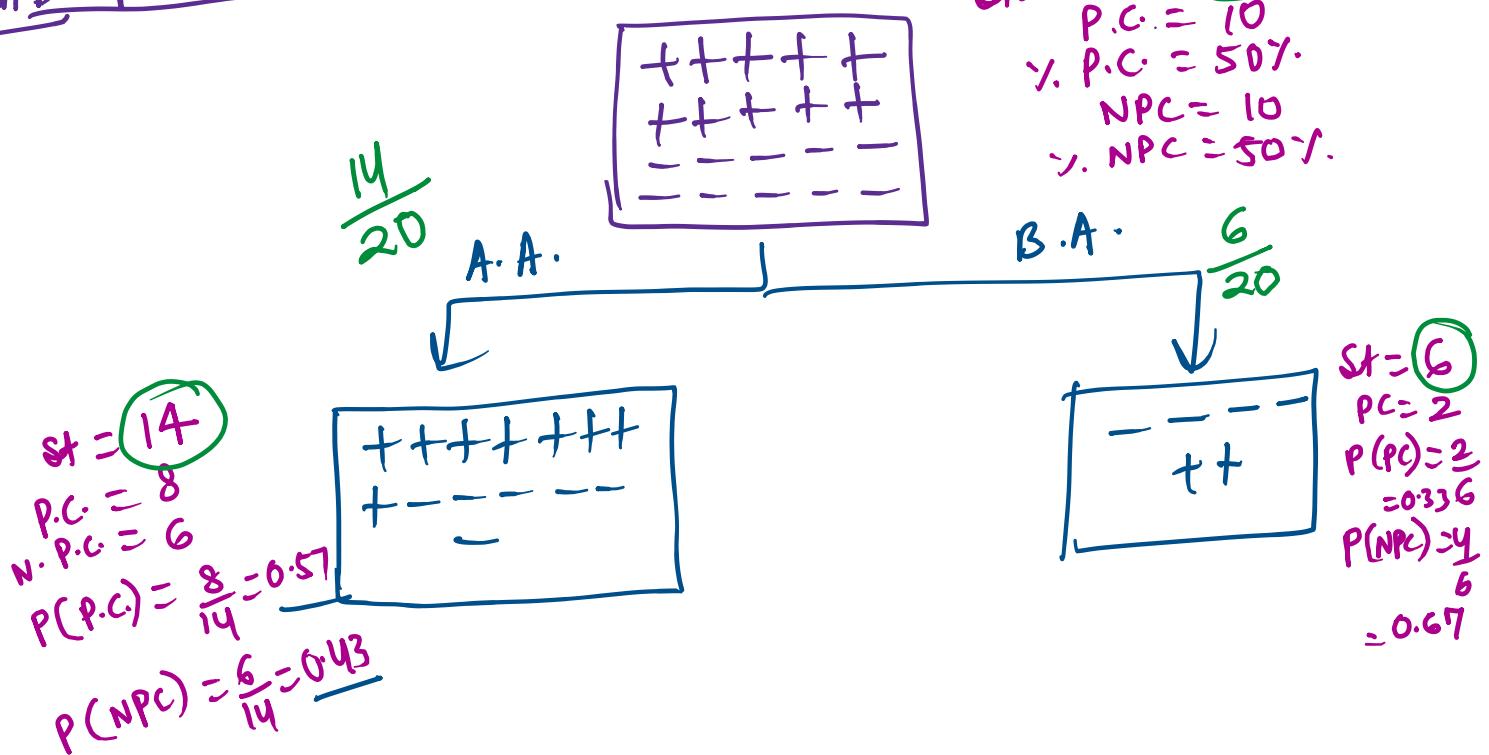
$$P(-)=1 \\ P(+)=0$$

$$\begin{aligned}\text{Entropy } (A) &= -P \log_2 P - Q \log_2 Q \\ &= -0.5 \log 0.5 - 0.5 \log 0.5 \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Entropy } (\beta) &= -1 \log 1 - 0 \log 0 \\ &= 0\end{aligned}$$

Higher Entropy  $\rightarrow$  less pure node

### split1: Split on Performance



$$\textcircled{1} \quad E(\text{Parent}) = -0.5 \log 0.5 - 0.5 \log 0.5 = 1$$

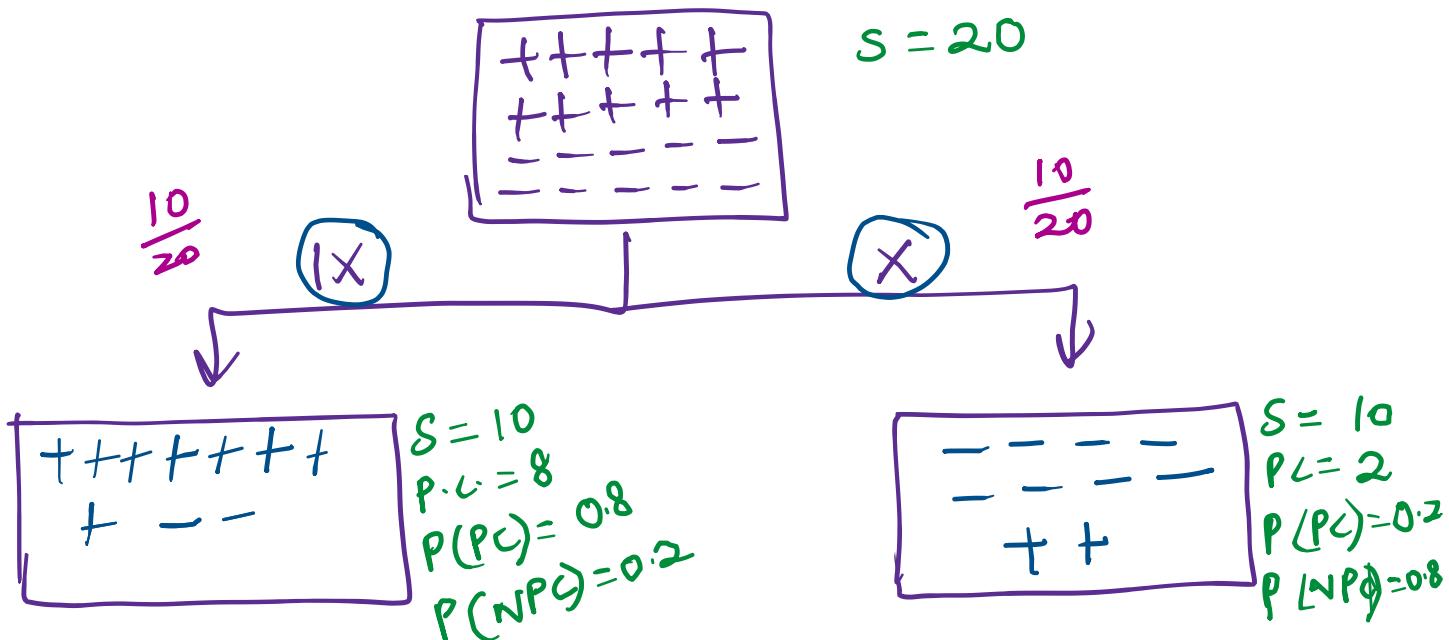
$$\textcircled{2} \quad E(\text{Subnode } AA) = -0.57 \log 0.57 - 0.43 \log 0.43 \\ = 0.98$$

$$\textcircled{3} \quad E(\text{Subnode } BA) = -0.33 \log 0.33 - 0.67 \log 0.67 \\ = 0.91$$

$$\textcircled{4} \quad \text{weighted entropy} = \frac{14}{20} \times (0.98) + \frac{6}{20} \times (0.91) \\ = 0.95$$

-

### split 2: split on class



$$\textcircled{1} \quad E(\text{Parent}) = 1$$

$$\textcircled{2} \quad E(Ix) = -0.8 \log 0.8 - 0.2 \log 0.2 \\ = 0.72$$

$$\textcircled{3} \quad E(x) = -0.2 \log 0.2 - 0.8 \log 0.8 \\ = 0.72$$

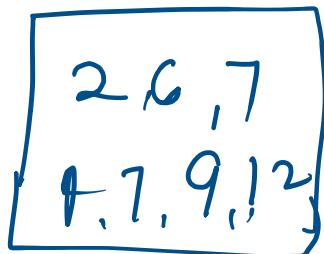
$$\textcircled{4} \quad \begin{aligned} & \text{Weighted Entropy for this Split (by class)} \\ &= \frac{10}{20} \times 0.72 + \frac{10}{20} \times 0.72 \\ &= 0.72 \end{aligned}$$

decision

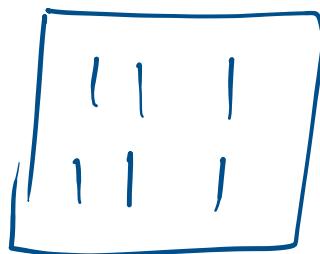
SPLIT	Entropy	I.G.
Parent	0.95	0.05
Class	0.72	0.28

## ④ Reduction in variance-

A



B



$$\text{Mean} = \frac{2+6+7+4+7+9+12}{7}$$

$$= \frac{47}{7} = 6.7$$

$$(\text{Var})_A = \frac{\sum (x - \mu)^2}{n}$$

$$(2-6)^2 + (6-6)^2$$

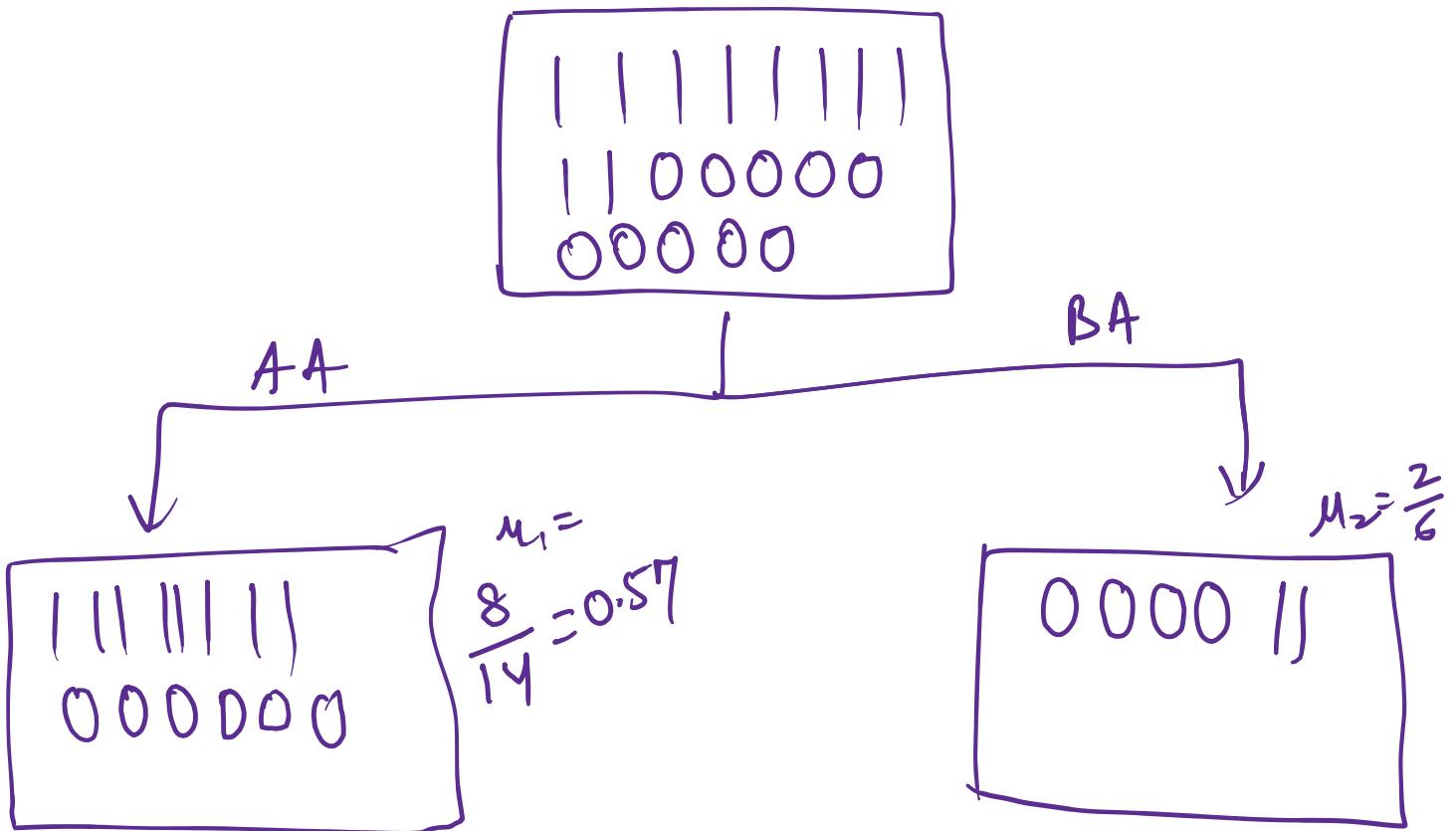
$$\frac{67}{7} = 9.$$

$$\text{Mean} = \frac{6}{6} = 1$$

$$(\text{Var})_B = 0$$

Lesser the variance  $\rightarrow$  more pure node  
 $\rightarrow$  Homogeneous

split 1 : Performance



$$\textcircled{1} \quad V(AA) = \frac{\sum (x - \mu)^2}{n} = \frac{8(1 - 0.57)^2}{14} + \frac{6(0 - 0.57)^2}{14}$$

$$= 0.24$$

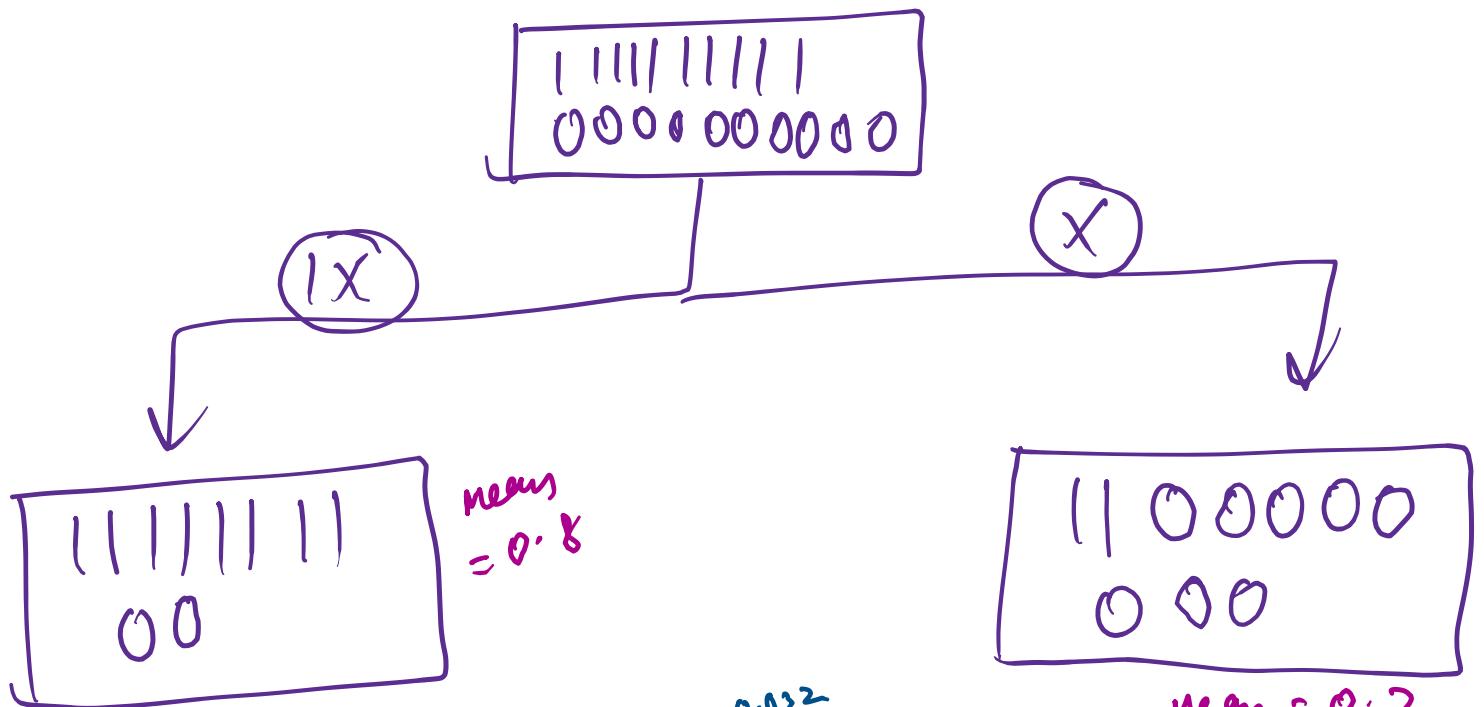
$$\textcircled{2} \quad V(BA) = \frac{\sum (x - \mu)^2}{n} = \frac{2(1 - 0.33)^2}{6} + \frac{4(0 - 0.33)^2}{6}$$

$$= 0.22$$

$$\textcircled{3} \quad \text{Weighted Var.} = \frac{14}{20} \times (0.24) + \frac{6}{20} \times (0.22)$$

$$= 0.23$$

split 2: Split by class



$$\textcircled{1} \quad \text{Var}(1x) = \frac{8(1-0.8)^2}{10} + \frac{2(0-0.8)^2}{10} = 0.16$$

$$\textcircled{2} \quad \text{Var}(x) = \frac{2(1-0.2)^2}{10} + \frac{8(0-0.2)^2}{10} = 0.16$$

$$\textcircled{3} \quad \text{weighted var.} = \frac{10}{20} \times (0.16) + \frac{10}{20} \times (0.16)$$

$$= 0.16$$

decision

split	w. var.
perf	0.24
clan	0.16

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