ANOVA:

Q1: To assess the significance of possible variation in performance in a certain test between the convent schools of a city, a common test was given to a number of students taken at a random from the 5th class of the 3 schools concerned. The result is given as follows:

A	B	C
9	13	14
11	12	13
13	10	17
9	15	7
8	5	9

Make the Analysis of Variance of the given data. (Null Hypo: No Significance Variation in the schools).

Solution:
Null Hypothesis = No variation between schools
Alt. Hypothesis = There is variation between schools.

Source of Variation	Sum of Square	Degrees of freedom	Mean Square	F
Between the Sample	SSC	(c-1)	MSC = SSC/df1	F = MSC/MSE
Within the sample	SSE	(n-c)	MSE = SSE/df2	

A	B	C
9	13	14
11	12	13
13	10	17
9	15	7
8	5	9

$\bar{X}_A = \frac{50}{5} = 10$, $\bar{X}_B = \frac{55}{5} = 11$, $\bar{X}_C = \frac{60}{5} = 12$

$\bar{\bar{X}} = \frac{\bar{X}_A + \bar{X}_B + \bar{X}_C}{3} = \frac{10+11+12}{3} = \frac{33}{3} = 11$

SSC Calculation:

$\bar{X}_A - \bar{\bar{X}}$	$(\bar{X}_A - \bar{\bar{X}})^2$	$\bar{X}_B - \bar{\bar{X}}$	$(\bar{X}_B - \bar{\bar{X}})^2$	$(\bar{X}_C - \bar{\bar{X}})$	$(\bar{X}_C - \bar{\bar{X}})^2$
$10-11=-1$	1	$11-11=0$	0	$12-11=1$	1
$10-11=-1$	1	$11-11=0$	0	$12-11=1$	1
$10-11=-1$	1	$11-11=0$	0	$12-11=1$	1
$10-11=-1$	1	$11-11=0$	0	$12-11=1$	1

$10-11=-1$	1	$11-11=0$	0	$12-11=1$	1
Summation	5		0		5

$$SSC = \sum(\bar{X}_A - \bar{\bar{X}})^2 + \sum(\bar{X}_B - \bar{\bar{X}})^2 + \sum(\bar{X}_C - \bar{\bar{X}})^2$$

$$= 5 + 0 + 5$$

$$SSC = 10$$

SSE Calculation:

$A - \bar{X}_A$	$(A - \bar{X}_A)^2$	$B - \bar{X}_B$	$(B - \bar{X}_B)^2$	$C - \bar{X}_C$	$(C - \bar{X}_C)^2$
$9-10=-1$	1	$13-11=2$	4	$14-12=2$	4
$11-10=1$	1	$12-11=1$	1	$13-12=1$	1
$13-10=3$	9	$10-11=-1$	1	$17-12=5$	25
$9-10=-1$	1	$15-11=4$	16	$7-12=-5$	25
$8-10=-2$	4	$5-11=-6$	36	$9-12=-3$	9
$\sum$	16		58		64

$$SSE = \sum(A - \bar{X}_A)^2 + \sum(B - \bar{X}_B)^2 + \sum(C - \bar{X}_C)^2$$

$$= 16 + 58 + 64$$

$$SSE = 138$$

Substituting :-

Source of Variation	Sum of Square	Degrees of freedom	Mean Square	F
Between the Sample	SSC = 10	df = (c-1) = 3-1 = 2	MSC = SSC/df = 10/2 = 5	F = MSC/MSE = 5/11.5 = 0.435
Within the sample	SSE = 138	df = (n-c) = 15 – 3 = 12	MSE = SSE/df = 138/12 = 11.5	

$(F)_{cal.} = 0.435$; $(F)_{tab}$ $\nu_1 = 12$, $\nu_2 = 2$ $= 3.89$

$F_{tab} > F_{cal.}$ (∵ Null Hypothesis is True)

i.e. no variation betⁿ schools A, B & C.

Q2: 2-Way ANOVA

The following data represents the number of Units of Tablet production (in thousands) per day by five different technicians by using 4 different machines.

a. Tell whether the mean productivity of the different machines are same?
b. Test whether the 5 technicians differ w.r.t. the mean productivity?

Machines …… Technicians	A	B	C	D
P	54	48	57	46
Q	56	50	62	53
R	44	46	54	42
S	53	48	56	44
T	48	52	59	48

Solution:

Source of Variance	Sum of Squares	Degree of Freedom	Mean sum of squares	F
Between the columns	SSC	df = c-1	MSC= SSC/(c-1)	MSC/MSE

Between the rows	SSR	df = r-1	MSR = SSR/(r-1)	MSR/MSE
Residual Errors	SSE	df = (c-1)(r-1)	MSE = SSE/(c-1)(r-1)	
Total Sum of Square	SST	df = n-1		

Step1: Calculation of grand total and correction factor.

Mid Value = (42+62)/2 = 52.
However, Let's assume mid value for easy calculation as 50. (You can take 51 or 52 also.)

	A	B	C	D	Row Total
P	54 – 50 = 4	48-50 = -2	57-50 =7	46-50 =-4	5
Q	56-50=6	50-50 =0	62-50 =12	53-50 =3	21
R	44-50=-6	46-50 =-4	54-50 =4	42-50 =-8	-14
S	53-50=3	48-50 =-2	56-50 =6	44-50 =-6	1
T	48-50 = -2	52-50 =2	59-50 =9	48-50 =-2	7
Col Total	5	-6	38	-17	Grand Total = 20

Correction Factor = T^2/N
= (Grand Total)^2/N
=$(20)^2/20$
=20

Note: N = Total observations = 5 * 4 = 20

Step2 : Calculation of SSC

SSC = $\frac{(\Sigma A)^2}{n_A} + \frac{(\Sigma B)^2}{n_B} + \frac{(\Sigma C)^2}{n_C} + \frac{(\Sigma D)^2}{n_D} - \frac{T^2}{N}$ → grand total

$= \frac{5^2}{5} + \frac{(-6)^2}{5} + \frac{(38)^2}{5} + \frac{(-17)^2}{5} - \frac{(20)^2}{20}$

$= 288.8$

Step 3: Calculation of SSR

$$SSR = \frac{(\Sigma P)^2}{n_P} + \frac{(\Sigma Q)^2}{n_Q} + \frac{(\Sigma R)^2}{n_R} + \frac{(\Sigma S)^2}{n_S} + \frac{(\Sigma T)^2}{n_T} - \frac{T^2}{N}$$

$$= \frac{5^2}{4} + \frac{(21)^2}{4} + \frac{(-14)^2}{4} + \frac{1^2}{4} + \frac{7^2}{4} - \frac{(20)^2}{20}$$

Grand total

$$= 158$$

Step 4: Calculation of SST

SST = Sum of square of all observations residuals – correction factor

$$SST = 4^2 + 6^2 + (-6)^2 + 3^2 + (-2)^2 + (-2)^2 + \cdots + (-6)^2 + (-2)^2 - 20$$

$$= 564$$

Source of Variance	Sum of Squares	Degree of Freedom	Mean sum of squares	F
Between the columns	SSC = 288.8	df = c-1 = 4-1 =3	MSC= SSC/(c-1) =288.8/3 =96.27	MSC/MSE = 96.27/9.76 =9.86
Between the rows	SSR = 158	df = r-1 =5-1 =4	MSR = SSR/(r-1) = 158/4 =39.5	MSR/MSE = 39.5/9.76 =4.05
Residual Errors	SSE = SST – (SSC+SSR) = 564 – 288.8 – 158 = 117.2	df = (c-1)(r-1) =(4-1)(5-1) =12	MSE = SSE/(c-1)(r-1) = 117.2/12 = 9.76	
Total Sum of Square	SST = 564	df = n-1 = 20-1 = 19		

Let’s see the Tabulated value of F:

For Between the columns, df1 =12, df2=3. (F)tab = 3.49

(F)tab < (F)calc; Hence, Null is rejected. i.e. there is significant variation between the columns.

For Between the rows, df1 = 12, df2=4, (F)tab = 3.26

(F)tab < (F)calc; Hence, Null is rejected. i.e. there is significant variation between the rows.

Reference:

F-Table for alpha 0.05

Table A.5. F-distribution where α = 0.05

V_1 \ V_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Adapted from E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 1, 1958, pp. 157–63, Table 18, by permission of the Biometrika Trustees.