MCA - 1 MFCS

Linear Algebra:

- Elementary Raw transformation of a Matrix
- Rank of Matrix
- Consistency and Solution of System of linear equation - Gauss - Elimination Method.
- Gauss - Jordan Method and approximate Solution by Gauss - Seidel Method.
- Rayleigh's power Method to find the dominant Eigen Value and the Corresponding Eigen Vector.

* What is Linear Algebra?

- Linear Algebra is a branch of Mathematics that deals with Matrices, Vector, Finite and Infinite Spaces. It is the Study of Vector Spaces, linear equations, linear Functions and Matrices.

* Linear Algebra Equations:

-> The genral linear equation is represen as $u_1x_1 + u_2x_2 + \cdots u_nx_n = V$

- u's = represents the cofficients
- x's = represents the unknowns
- V = represents the constant

There is a Collection of equations Called a System of linear algebric equation.

* What is Matrices ?

→ A Matrix is a way of representing the Numbers in the form of rows and Column and all the Numbers are represented in the cells of this Matrix.

→ We represent the Matrix as $[A]_{m \times n}$ and m represent the Number of rows and n represent the Number of Columns

Eg: 2x2 $\begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}_{2\times2}$ $\begin{bmatrix} 4 & 1 & 4 \\ 3 & 2 & 6 \\ 2 & 3 & 5 \end{bmatrix}_{3\times3}$

* Elementary row transformation of Matrix:

→ As the Name Suggests only the rows of the Matrices are transformed and No changes are Made in the Columns:

① Any two rows are Interchangeable

$R_i \longleftrightarrow R_j$ $\qquad\qquad$ $R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

② All the element of any row can be Multiplied by any non-Zero Number.

$R_i \rightarrow kR_i \; ; k \neq 0$ $\quad [R_3 \rightarrow 2R_3]$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 14 & 16 & 18 \end{bmatrix}$$

(3) All the element of a row Can be added to Corresponding element of another row multiplied by any non-zero Constant.

$R_i \rightarrow R_i + kR_j \quad k \neq 0 \quad R_2 \rightarrow R_2 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 6 & 9 & 12 \\ 7 & 8 & 9 \end{bmatrix}$$

$4+2(1)$	$5+2(2)$
$4+2$	$5+4$
6	9

Example :1

① $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ find A^{-1}

$A = I_A$ (I - Indentity Metrix)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Square matrix
each element
diagonal one 1

$R_2 = -3R_1 + R_2$

$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

$-3(1) + 3$	$-3(2) + 4$
$-3 + 3$	$-6 + 4$
0	-2

$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} A$

$-3(1) + (0)$	$-3(0) + 1$
$-3 + 0$	$0 + 1$
-3	1

$R_1 \rightarrow 1(R_2) + R_1$

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$1(-2)+2$ | $1(0)+1$
$-2+2$ | $0+1$
0 | 1

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -3 & 1 \end{bmatrix}$$

$1(-3)+1$ | $1(1)+0$
$-3+1$ | $1+0$
-2 | 1

$$R_2 = \frac{R_2}{-2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -3/2 & -1/2 \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ -3/2 & -1/2 \end{bmatrix}$$

Rank of Matrix

1st Method

Number of non-zero rows = Rank of matrix.

Rank of Matrix = R(A)

① $A = \begin{array}{c} \left|\begin{matrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & 8 & 0 \end{matrix}\right| \end{array} \begin{array}{l} \to r① \\ \times \\ \to ② \end{array}$ $\qquad R(A) = 2$

② $A = \left|\begin{matrix} 1 & 5 & 0 & 2 \\ 3 & 0 & 2 & 5 \\ 0 & 0 & 0 & 4 \end{matrix}\right|$ $\qquad R(A) = 3$

2^{nd} Method

Find determinde

For 3x3

$$\begin{cases} \text{if it is} \neq 0 & \text{than } R(A) = 3 \\ \text{if it is} = 0 & \text{than } R(A) < 3 \end{cases}$$

① Find Rank of Matrix.

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

$2(4-3) - 2(6-6) + 1(3-4)$

$2(1) - 2(0) + 1(-1)$

$2 + (-1)$

$2 - 1$

1

$|A| \neq 0 \quad R(A) = 3$

② Find Rank of Matrix.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$|A| = 1(45-48) - 2(36-42) + (32-35)$

$= 1(-3) \quad -2(-6) + 3(-3)$

$= -3 + 12 - 9$

$|A| = -12 + 12$

$|A| = 0$

$R(A) < 3$

- Consistency and Solution of System of linear equation.

→ A Solution exists (intersecting or Coinci-dent line)

Conditions:

① Unique Solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

② No Solution = $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

③ Infinite Solution = $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Ex

1) The pair of linear equations

$x + 2y - 5 = 0$

$2x - 4y + 6 = 0$ is lore?

$\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{-4}$ $\frac{c_1}{c_2} = \frac{-5}{6}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Consistent & Unique Solution:

e.g. ① $3x + 2y = 8$
$6x - 4y = 9$

② $5x + 3y + 2z = 0$
$-3x + 6y - 4z = 0$

③ $2x + 4y + 6z = 0$
$4x + 8y + 12z = 0$

④ $5x + 3y + 7z = 0$
$10x + 6y - 6z = 0$

Gauss - Elimination Method

- Gaussian elimination is a row reduction algorithm for solving linear system.

- This method can also help in determing the rank, determinant and inverse of Matrices.

- Gaussian elimination is a method for solving systems of equations in matrix form.

Example: $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

equations:
$$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right]$$

Example:1 Find Solution of linear equation by Gauss- elimination Method.

$$-2b + 3c = 1$$
$$3a + 6b - 3c = -2$$
$$6a + 6b + 3c = 5$$

∴ Lower triangle are allways Zero in Gauss Eliminatio Method.

Solution :-

$$[A|B] = \left[\begin{array}{ccc|c} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$R_2 \longleftrightarrow R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right]$$

$R_3 = -2R_1 + R_3$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{array}\right]$$

$\therefore -2(3) + 6$
$= -6 + 6$
$= 0$

$R_3 = -3R_2 + R_3$

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 6 & -3 & 2 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

Here, $R(A) = 2$ and $R(A|B) = 3$
As $R(A) \neq R(A|B)$

Ans: System of linear equation is not Consistant.

Example: 2 Find Solution of linear equation by Gauss - elimination Method:

$2x + y - z = 1$

$5x + 2y + 2z = -4$

$3x + y + z = 5$

Solution :-

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 5 & 2 & 2 & -4 \\ 3 & 1 & 1 & 5 \end{array}\right]$$

$R_2 = -\frac{5}{2} R_1 + R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -1/2 & 9/2 & -13/2 \\ 3 & 1 & 1 & 5 \end{array}\right]$$

$-\frac{5}{2}(2) + 5$

$-5 + 5$

$= 0$

$R_3 = \frac{-3}{2} R_1 + R_3$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -1/2 & 9/2 & -13/2 \\ 0 & -1/2 & 5/2 & 7/2 \end{array}\right]$$

$R_3 = -1 (R_2) + R_3$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 0 & -1/2 & 9/2 & -13/2 \\ 0 & 0 & -2 & 10 \end{array}\right]$$

$R(A) = 3$ and $R(A|B) = 3$

$\therefore R(A) = R(B)$ so, System is Consistant

We can get its unique Solution

$2x + y - z = 1$ ---- (1)

$-\frac{1}{2} y + \frac{9}{2} z = -\frac{13}{2}$ ---- (2)

$-2z = 10$ ---- (3)

③ $-2z = 10$

$-z = \frac{-10}{2}$

$\boxed{z = -5}$

② $-\frac{1}{2}y + \frac{9}{2}z = -\frac{13}{2}$

$= -\frac{1}{2}y + \frac{9}{2}(-5) = -\frac{13}{2}$

$= -y - 45 = -\frac{13}{2} \times 2$

$-y = -13 + 45$

$-y = 32$

$\boxed{y = -32}$

① $2x - 32 + (-5) = 1$

$2x - 27 = 1$

$2x = 28$

$\boxed{x = 14}$

Gauss - Jordan Method

Gauss Jordan Method is a little Modification of the Gauss Elimination Method. Here, during the Stages of elimination, the Coefficient are elimination in Such a way that the System of equation are reduced to a diagonal Matrix.

Example : 1 Find the Solution of linear equation by Gauss - Jordan Method.

$x + y + z = 3$

$x + 2y - z = 4$ ∴ Converted in

$x + 3y + 2z = 4$ Identity Matrix.

Solution:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & -1 & 4 \\ 1 & 3 & 2 & 4 \end{array}\right]$$

$R_2 = -1(R_1) + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 1 & 3 & 2 & 4 \end{array}\right]$$

$R_3 = -1(R_1) + R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right]$$

$R_1 = -1(R_2) + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 1 & 1 \end{array}\right]$$

$R_3 = -2(R_2) + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 5 & -1 \end{array}\right]$$

$R_3 = \frac{R_3}{5}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1/5 \end{array}\right]$$

$R_1 = -3R_3 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 13/5 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1/5 \end{array}\right]$$

$R_2 = 2R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 13/5 \\ 0 & 1 & 0 & 3/5 \\ 0 & 0 & 1 & -1/5 \end{array}\right]$$

$x = 13/5$

$y = 3/5$

$z = -1/5$

Example 2: Find the Solution of linear equation by Gauss - Jordan Method

$$2x + 8y + 2z = 14$$
$$6x + 6y - z = 13$$
$$2x - y + 2z = 5$$

Gauss- Seidel Method

The Gauss - Seidel Method is defined as an iterative algorithm for solving system of linear equation that utilizes the most recently computed value for each variable during the calculation of subsequent variable. This process continues until the value converge to a solution.

Example: 1 Solve the following equation by using Gauss- Seidel Method:

$20x + y - 2z = 17$

$3x + 20y - z = -18$

$2x - 3y + 20z = 25$

Solution:

$20x + y - 2z = 17 \quad \dots (1)$

$3x + 20y - z = -18 \quad \dots (2)$

$2x - 3y + 20z = 25 \quad \dots (3)$

Step 1

Consider

① $20x + y - 2z = 17$

$20x = 17 - y + 2z$

$x = \frac{1}{20}[17 - y + 2z]$ ----- ④

Consider

② $3x + 20y - z = -18$

$20y = -18 + z - 3x$

$y = \frac{1}{20}[-18 + z - 3x]$ ------ ⑤

Consider

③ $2x - 3y + 20z = 25$

$20z = 25 + 3y - 2x$

$z = \frac{1}{20}[25 - 2x + 3y]$ ----- ⑥

Step :2 First Iteration.

Put $y=0 \quad z=0$ in equation ...(4)

(4)

$$x^{(1)} = \frac{1}{20}[17 - y + 2z]$$

$$x^{(1)} = \frac{1}{20}[17 - 0 + 2(0)]$$

$$x^{(1)} = \frac{1}{20}[17 + 0]$$

$$x^{(1)} = \frac{17}{20}$$

$$\boxed{x^{(1)} = 0.8500}$$

⑤ $y^{(1)} = \frac{1}{20}[-18 - 3(0.8500) + 0]$

$y^{(1)} = \frac{1}{20}[-18 - 2.55 + 0]$

$y^{(1)} = \frac{1}{20}[-20.05]$

$y^{(1)} = \frac{-20.05}{20}$

$\boxed{y^{(1)} = -1.0275}$

⑥ $z^{(1)} = \frac{1}{20}[25 - 2x + 3y]$

$z^{(1)} = \frac{1}{20}[25 - 2(0.8500) + 3(1.0275)]$

$z^{(1)} = \frac{1}{20}[25 - 1.7 - 3.0825]$

$z^{(1)} = \frac{1}{20} [20.2175]$

$z^{(1)} = \frac{20.2175}{20}$

$\boxed{z^{(1)} = 1.0109}$

Second iteration

$x^{(2)} = \frac{1}{20} [17 - (-1.0275) + 2 (1.0109)$

$= \frac{1}{20} [17 + 1.0275 + 2.0218]$

$= \frac{1}{20} [20.0493]$

$= \frac{20.0493}{20}$

$\boxed{x^{(2)} = 1.0025}$

$$y^{(2)} = \frac{1}{20}[-18 + z - 3x]$$

$$y^{(2)} = \frac{1}{20}[-18 + 2(1.0109) - 3(1.0025)]$$

$$y^{(2)} = \frac{1}{20}[-18 + 1.0109 - 3.0075]$$

$$= \frac{1}{20}[-19.9966]$$

$$\boxed{y^{(2)} = -0.9998}$$

$$z^{(2)} = \frac{1}{20}[25 - 2x + 3y]$$

$$z^{(2)} = \frac{1}{20}[25 - 2(1.0025) + 3(-0.9998)]$$

$$= \frac{1}{20}[25 - 2.0025 - 2.9994]$$

$$= \frac{1}{20}[19.9956]$$

$$\boxed{z^{(2)} = 0.9998}$$

3^{rd} iteration

$$x^{(3)} = \frac{1}{20}[17 - y + 2z]$$

$$x^{(3)} = \frac{1}{20}[17 - (-0.9998) + 2(0.9998)]$$

$$= \frac{1}{20}[17 + 0.9998 + 0.9996]$$

$$= \frac{1}{20}[19.9994]$$

$$= 0.99997$$

$$x = \boxed{1.0000}$$

$$y^{(3)} = \frac{1}{20}[-18 - 3x + z]$$

$$= \frac{1}{20}[-18 - 3(1.0000) + 0.9998]$$

$$= \frac{1}{20}[-18 - 3.0000 + 0.9998]$$

$$= \frac{1}{20}[-20.0002]$$

$$\boxed{y^{(3)} = -1.0000}$$

$$z^{(3)} = \frac{1}{20}[25 - 2x + 3y]$$

$$= \frac{1}{20}[25 - 2(1.000) + 3(-1.0000)]$$

$$= \frac{1}{20}[25 - 2 - 3]$$

$$= \frac{1}{20}[20]$$

$$\boxed{z^{(3)} = 1.0000}$$

1st Iteration $x' = 0.8500$

$y' = -1.0275$

$z' = 1.0109$

2nd Iteration $x^2 = 1.0025$

$y^2 = -0.9998$

$z^2 = 0.9998$

3^{rd} iteration $x^3 = 1.0000$

$y^3 = -1.0000$

$z^3 = 1.0000$

eg Solve the following equation by using Gauss - Seidel Method.

$$5x + y - 3z = 15$$

$$3x + 5x + 4z = 17$$

$$2x + 3y + 5z = 15$$

* Rayleigh's power Method to find dominant Eigen Value and Corresponding Eigen Vector.

-> Power Method is particularly used for estimating largest or Smallest Eigen Value and its Corresponding Eigen Vector.

Power Method (Iteration Method)

-> To Find dominant Eigen Value & Eigen Vector
-> To find largest Eigen Value & Eigen Vector

* Dominant -> let $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 2}$ or $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{3\times 3}$ or $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{4\times 4}$

* largest -> let $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 2}$ or $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4\times 4}$

Example : 2 Determine the largest Eigen Value in Magnitude as the Corresponding Eigen Vector of the Matrix

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ using power Method Taking $[1,0,0]$ as the initial approximation its iteration.

Solution :-

$$V^0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$y^1 = AV^0$$

$$= \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 \\ 1+0+0 \\ 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad 1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

x^1 v^1

$$y^2 = AV^1$$

$$= \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+0 \\ 1+2+0 \\ 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 3/7 \\ 0 \end{bmatrix}$$

x^2 v^2

$y^3 = Av^2$

$$= \begin{bmatrix} 1 & 6 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3/7 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 25/7 \\ 13/7 \\ 0 \end{bmatrix}$$

$$= 25/7 \begin{bmatrix} 1 \\ 13/25 \\ 0 \end{bmatrix}$$

largest Eigen Value

Corresponding EigenVector

Example : 3 $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ Find the Dominant Eigen Vector & Eigen Value

Solution :-

$$V^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$AV^0 = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 9 \end{bmatrix} = \underset{x'}{9} \underset{v'}{\begin{bmatrix} 5/9 \\ 1 \end{bmatrix}}$$

$$AV' = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 5/9 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4.12 \\ 6.8 \end{bmatrix}$$

$$= 6.8 \begin{bmatrix} 0.61 \\ 1 \end{bmatrix}$$

$$AV^2 = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 0.61 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4.22 \\ 7.05 \end{bmatrix}$$

$$= 7.05 \begin{bmatrix} 0.6 \\ 1 \end{bmatrix}$$

$$AV^3 = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 0.6 \\ 1 \end{bmatrix}$$

$$= 7 \begin{bmatrix} 0.6 \\ 1 \end{bmatrix}$$