QUESTION-32

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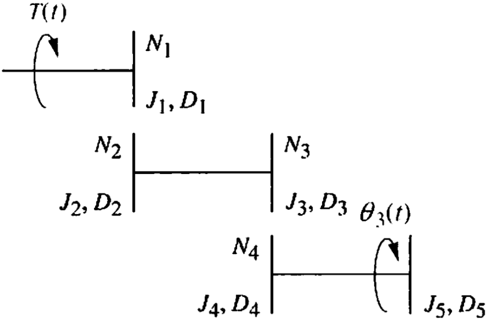
September 9, 2020

1 Problem

2 Solution

Problem

For the rotational mechanical system with gears shown in Figure below ,find the transfer function ,G(s)=$\theta_3(s)$/T(s).The gears have inertia and bearing friction as shown



Solution

So we have to find the Transfer function G(s)

The net equation of motion for the above figure can be written as

$$(J_e s^2 + D_e s)\theta_3(s) = T_{eq}(s)$$

where J_e, D_e are the equivalent impendances of all the inertia and the Viscous dampers involved in the above figure and T_{eq} is the net torque that is reflected onto the output shaft θ_3.
Rotational machanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio

$$\left(\frac{Number of\, teeth\, of\, gear\ on\ destination\ shaft}{Number of\, teeth\, of\, gear on\ source\ shaft}\right)^2 \tag{3.1}$$

Let us see how the Torque T is reflected onto the output shaft θ_3.

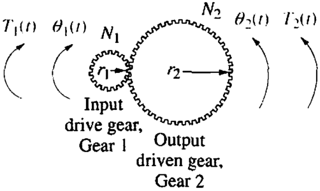


Figure: Gear System

So for the above gear system the rotation in each gear is related as

$$\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$

where N_1 and N_2 is the number of teeth in each gear.

If we assume the gears are lossless,that is they do not absorb or store energy,the energy into Gear1 equals the energy out of Gear2.Since the translational energy of force times displacement becomes the rotational energy of torque times angular displacement,

$$T_1\theta_1 = T_2\theta_2$$

$$\frac{T_1}{T_2} = \frac{\theta_1}{\theta_2} = \frac{N_1}{N_2}$$

So for the above problem since there are two gear systems the torque T is reflected onto the output shaft θ_3 as

$$T_{eq} = T\left(\frac{N_2 N_4}{N_1 N_3}\right)$$

Reflecting impedances onto the output shaft θ_3

To solve the problem we want to reflect all the impedances to the output shaft θ_3.

Let us first reflect the impedances J_1 and D_1 on the input shaft towards the output shaft θ_3.

So here J_1 will be reflected through the gear ratio as $\mathrm{J}_1 \left(\frac{N_2 N_4}{N_1 N_3}\right)^2$ onto the output shaft θ_3 .

Similarliy D_1 will be reflected through the gear ratio as $\mathrm{D}_1 \left(\frac{N_2 N_4}{N_1 N_3}\right)^2$ onto the output shaft θ_3 .

Now we have to reflect the impedances $J_2 + J_3$ and $D_2 + D_3$ on to the output shaft θ_3.

So here $J_2 + J_3$ will be reflected through the gear ratio as

$(\mathrm{J}_2 + J_3)\left(\dfrac{N_4}{N_3}\right)^2$ onto the output shaft θ_3 .

Similarliy $D_2 + D_3$ will be reflected through the gear ratio as

$(\mathrm{D}_2 + D_3)\left(\dfrac{N_4}{N_3}\right)^2$ onto the output shaft θ_3 .

Finally there are impedances $J_4 + J_5$ and $D_4 + D_5$ on the output shaft θ_3.

Equivalent Impedances

So the net equivalent impedances J_e and D_e is given by

$$J_e = (J_4 + J_5) + (J_2 + J_3)\left(\frac{N_4}{N_3}\right)^2 + J_1\left(\frac{N_2 N_4}{N_1 N_3}\right)^2 \tag{3.2}$$

$$D_e = (D_4 + D_5) + (D_2 + D_3)\left(\frac{N_4}{N_3}\right)^2 + D_1\left(\frac{N_2 N_4}{N_1 N_3}\right)^2 \tag{3.3}$$

So the net equation is

$$(J_e s^2 + D_e s)\theta_3(s) = T(s)\left(\frac{N_2 N_4}{N_1 N_3}\right)$$

Result

So the Transfer function is given by $G(s) = \dfrac{\theta_3(s)}{T(s)}$ i.e

$$G(s) = \frac{\theta_3(s)}{T(s)} = \frac{\left(\dfrac{N_2 N_4}{N_1 N_3}\right)}{(J_e s^2 + D_e s)}$$

$$G(s) = \frac{\left(\dfrac{N_2 N_4}{N_1 N_3}\right)}{\left((J_4 + J_5) + (J_2 + J_3)\left(\dfrac{N_4}{N_3}\right)^2 + J_1\left(\dfrac{N_2 N_4}{N_1 N_3}\right)^2\right)s^2 + \left((D_4 + D_5) + (D_2 + D_3)\left(\dfrac{N_4}{N_3}\right)^2 + D_1\left(\dfrac{N_2 N_4}{N_1 N_3}\right)^2\right)s}$$