Cyber Physical Systems

Verification of Train System with Electronically-controlled brake in PRISM

Project Report

Submitted By:

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1.INTRODUCTION

1.1 PRISM Overview:

PRISM is a probabilistic model checker, a tool for formal modelling and analysis of systems that exhibit random or probabilistic behavior. It has been used to analyze systems from many different application domains, including communication and multimedia protocols, randomized distributed algorithms, security protocols, biological systems and many others.

PRISM can build and analyze several types of probabilistic models:

- discrete-time Markov chains (DTMCs)
- continuous-time Markov chains (CTMCs)
- Markov decision processes (MDPs)
- probabilistic automata (PAs)
- probabilistic timed automata (PTAs)

Models are described using the PRISM language, a simple, state-based language. PRISM provides support for automated analysis of a wide range of quantitative properties of these models, e.g. "what is the probability of a failure causing the system to shut down within 4 hours?", "what is the worst-case probability of the protocol terminating in error, over all possible initial configurations?", "what is the expected size of the message queue after 30 minutes?", or "what is the worst-case expected time taken for the algorithm to terminate?". The property specification language incorporates the temporal logics PCTL, CSL, LTL and PCTL*, as well as extensions for quantitative specifications and costs/rewards.

PRISM incorporates state-of-the art symbolic data structures and algorithms, based on BDDs (Binary Decision Diagrams) and MTBDDs (Multi-Terminal Binary Decision Diagrams). It also includes a discrete-event simulation engine, providing support for approximate/statistical model checking, and implementations of various analysis techniques, such as quantitative abstraction refinement and symmetry reduction.

PRISM is free and open source, released under the GNU General Public License (GPL).

1.2 Problem Statement:

Quantize the model of a train with electronically-controlled brakes as described in [1] and to verify the system using the PRISM model checker.

2. METHODOLOGY

2.1 Description of Model:

- Train system having **5 cars.** Each having their own braking system.
- Model Type- **Stochastic.**

2.1.1 Velocity subsystem (first module for PRISM):

- The train starts in the discrete state $q_v = 1$ and remains there until the velocity exceeds a threshold $V_U = 28.5$, switches to the discrete state $q_v = 2$.
- The train remains in the state qv = 2 until one of the brakes engages and it switches to state $q_v = 3$.
- The velocity in states $q_v = 3$ depends on the number of brakes that have been engaged through the braking force term, where Fj is the braking force of car j.
- When all the brakes disengage, the velocity system switches back to the state $q_v = 1$. When in the state $q_v = 1$, the train accelerates to a constant velocity of $V_C = 25$ and oscillates around it with the amplitude 2.5.

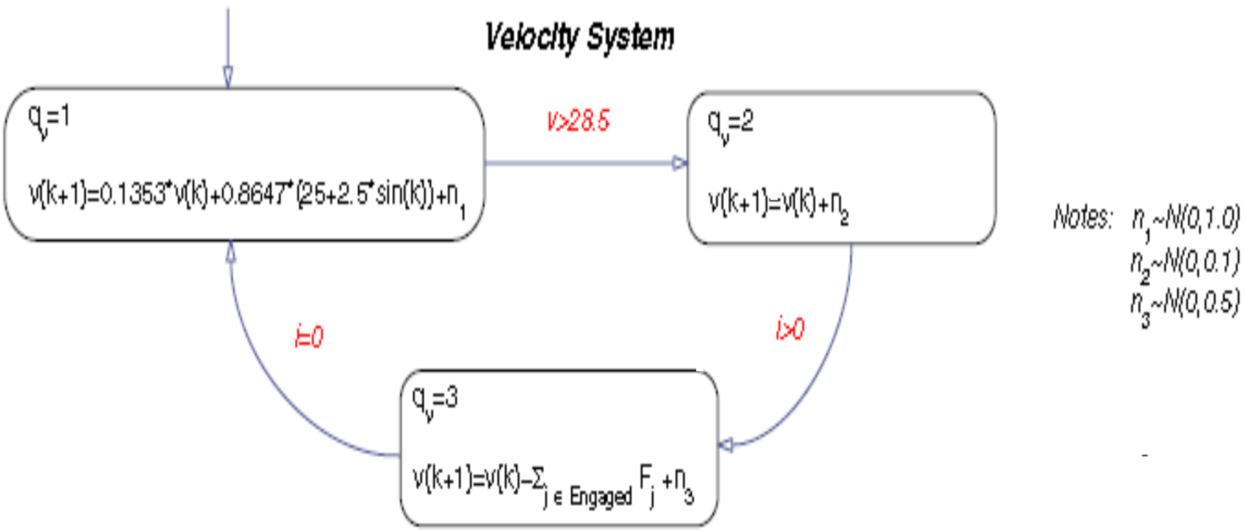


Fig 1. Velocity Subsystem

2.1.2 Braking subsystem for each car (second module for PRISM):

- The train starts in the discrete state $q_b = 1$ and remains there until the velocity exceeds a threshold $V_U = 28.5$, switches to $q_b = 2$.
- Initial value of the timer c_1 in the state $q_b = 2$ is not deterministic, so the duration of time the system remains in $q_b = 2$ is a random variable (until $c_1 > 1$).
- Braking system can fail with a probability $p = 0.1$ and then permanently switch to $q_b = 3$. With the probability $p = 0.9$ it either returns to state $q_b = 1$ if velocity fall below the threshold $V_L = 20$, or switches to $q_b = 4$ and engages in braking sequence otherwise.
- When the brake engages the variable i is increased by 1, thereby affecting the velocity of the train as per velocity module.
- When the velocity falls below $V_L = 20$, the brake disengages after a random amount of time (modeled by the timer c_2 in the state $q_b = 5$), when it switches to the state $q_b = 1$.

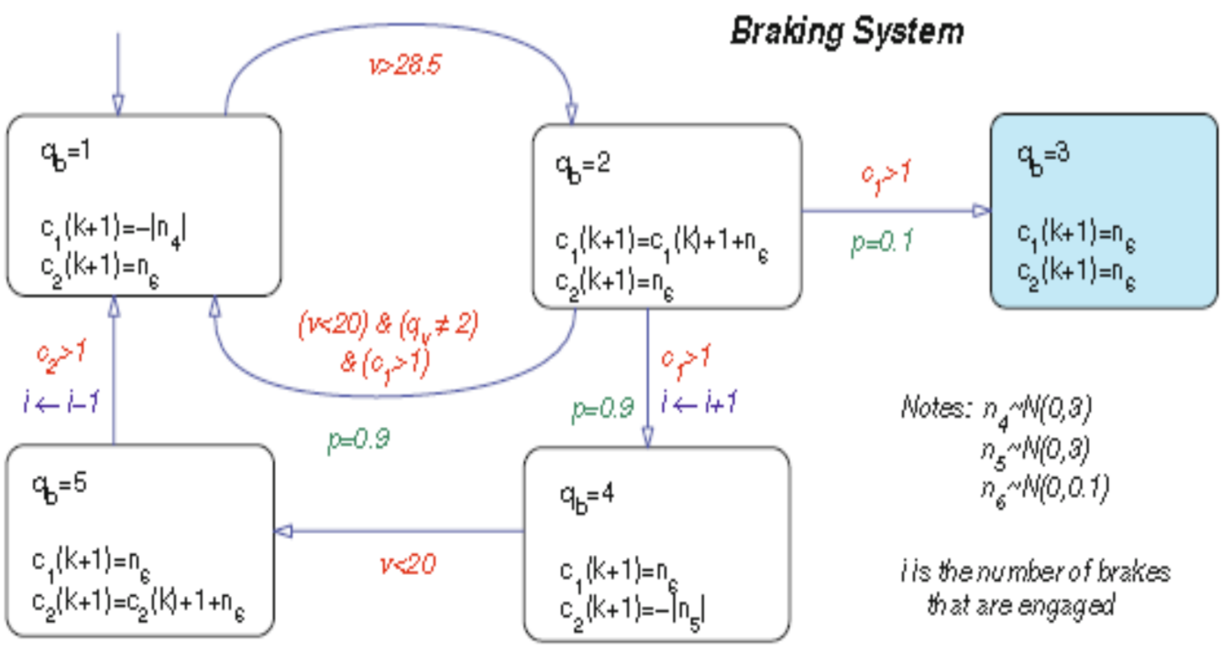


Fig 2. Braking Subsystem

3. IMPLEMENTATION IN PRISM

As can be seen that in the velocity module for the state q_v =1 the velocity equation is given as follows $v(k+1) = 0.1353*v(k) + 0.8647*(25+2.5*sin(k)) + n_1$, where the velocity is discretized at every point k and n_1 is the noise which is the normal gaussian noise represented as **n_1~N (0,1).** For quantization of velocity value kept the quantization constant δ=1 and define a global variable k in the PRISM from 1 to 500 (also act as global timestamp / number of iteration).

global k : [0..500] init 0;

For the function of sin used in the velocity formula which is generated as a lookup function using ? operator as PRISM doesn't allow to define a procedural function and is defined as: *(condition)?(value_returned_if_true):(value_returned_if_false);* so it returns -0.76 whenever k=4 for the line written as (k=4)?(-0.76:4). The system will pick the lowest value and never the value 4 as the actual response 4 is out of bound.

```
formula sin =min [ (k=0?0:4),
                   (k=1?0.84:4),
                   (k=2?0.91:4),
                    :
                    (k=500?-0.47:4)];
```

Now state q_v = 1 is written in PRISM is written as:

[] qv=1-> (v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin)) + n1))& (k'= min(k+1,500);

(guard) (update)

Transition from q_v=1 to q_v=2 for ($V > V_U$ =28.5) written as:

[ ] qv=1 & v_k1>=27 -> 1: (qv'=2);

Similarly, all the states in the velocity and braking subsystem is written in the PRISM as what is represented above using the guard and update as per required by the different states and their transitions.

3.1 Noise Value Quantization:

As the model contains noise at every state which is the Normal Gaussian noise with a zero mean and some variance, this affect the sensor readings for velocity and brake and need to be quantized at every point before adding it in our PRISM.

This can be done by approximating a continuous normal random variable w (noise here) with the density N (0, σ) and the cumulative distribution F(τ) by using a discrete random variable ŵ=kδ (discrete noise that is added finally) so that

$$P[\hat{w} = k\delta] = \begin{cases} F((k+\frac{1}{2})\delta) - F((k-\frac{1}{2})\delta) & -M < k < M \\ F((k+\frac{1}{2})\delta) & k = -M \\ 1 - F((k-\frac{1}{2})\delta) & k = M \\ 0 & \text{otherwise} \end{cases}$$

Where M ∈ N defines the interval of interest for the random variable and calculated as Mδ = 3σ. Here k is an integer ranging from -M to M.

The above equation is implemented in the MTLAB to get the values of k and M for a given σ and assuming δ accordingly. The code to generate that is given in the appendix section of this report **[see appendix for the code].** The probability density function and the cumulative density functions are also obtained for the noise value **n_1~N (0,1).**

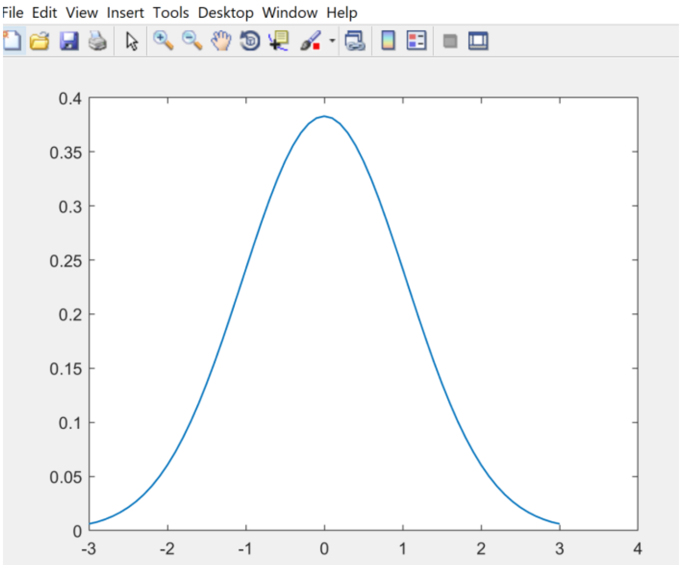


Fig 3. PDF for noise n_1~N (0,1)

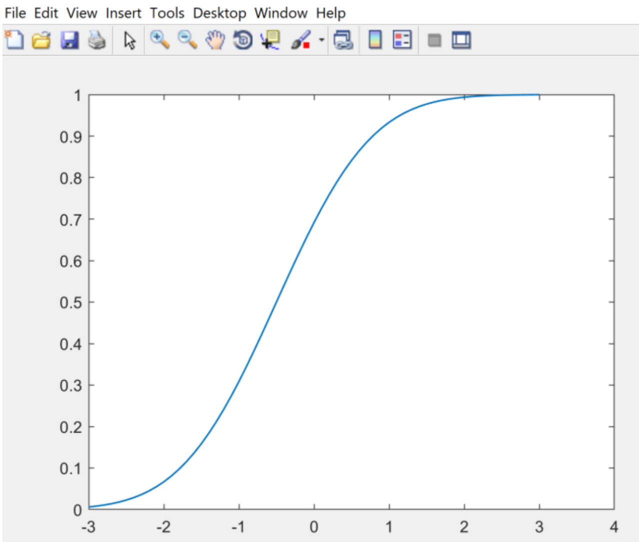


Fig 4. CDF for noise n_1~N (0,1)

For quantization of $n_1 \sim N(0,1)$, having $\mu=0$, $\sigma=1$ and by assuming $\delta=1$.

We get M=3 and -3< k <-3. The probability distribution and corresponding discrete value for this noise are:

Sigma:	**1**	**Delta:**	**1**	**M:**	**3**
k Ranges from -M to M	Probability	W Value Discrete			
-3	0.00621	-3			
-2	0.060598	-2			
-1	0.24173	-1			
0	0.382925	0			
1	0.24173	1			
2	0.060598	2			
3	0.00621	3			
Prob Sum	**1**				

Table 1. Calculation for Quantized value of $n_1 \sim N(0,1)$

Now as noise has the discrete value the velocity equation can be written as:

[] qv=1 -> 0.00621:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-3))& (k'= min(k+1,500))

+ 0.060598:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-2))& (k'= min(k+1,500))

+ 0.24173:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-1))& (k'= min(k+1,500))

+ 0.382925:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-0))& (k'= min(k+1,500))

+ 0.00621:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))+3))& (k'= min(k+1,500))

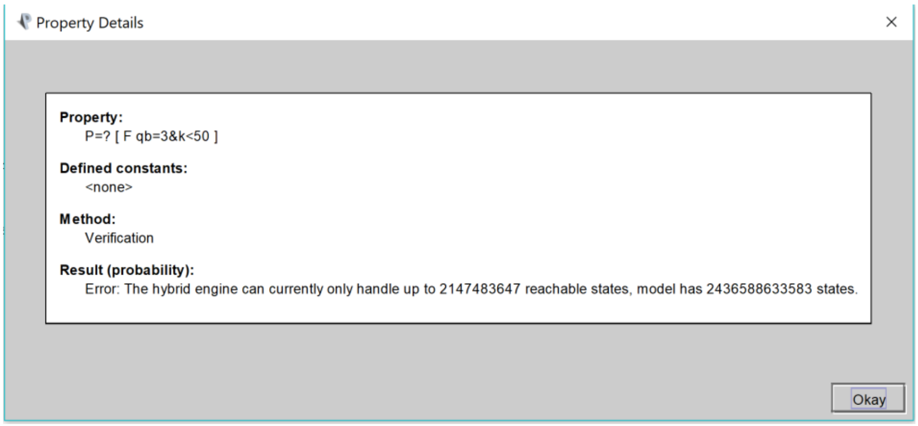
+ 0.060598:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))+2))& (k'= min(k+1,500))

+ 0.24173:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))+1))& (k'= min(k+1,500));

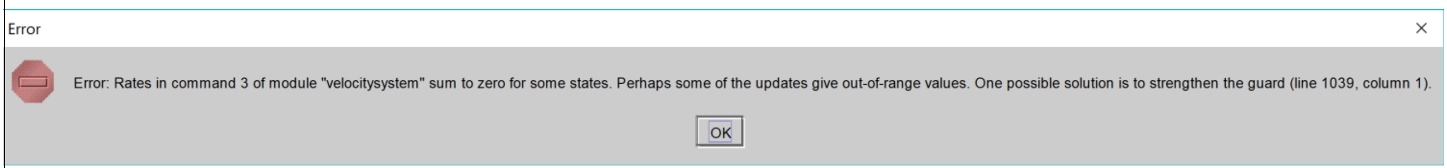
Similarly for all noises **$n_2 \sim N(0,0.1)$, $n_3 \sim N(0,0.5)$, $n_4 \sim N(0,3)$, $n_5 \sim N(0,3)$, $n_6 \sim N(0,0.1)$** quantized values are calculated and then added in the PRISM velocity, braking module.

4. DISCUSSION (Problems Faced):

1) When tried to verify the property **P=?[F (qb=3 & k<50)]** on the 5 car system PRISM has generated total of $>10^{12}$ reachable states for my model and gives an error as its hybrid engine can only handle up to 10^9 states



2) Another problem arise which is shown as follows:



- Both problems are resolved as I have merged my velocity and braking model together such that the number of states and their transitions could be reduced.

- I have strengthened my guard (the left-hand side states in the code) by keeping both the velocity (q_v) and braking (q_b) states together whenever there is a possibility of simultaneous transitions and upgradation for both states. It also implicitly removed the error for "updates give out of range value".

3) PRISM doesn't have inbuild function such as sin(x) so need to implement a look up function for 500 values in a trivial manner as discussed earlier in the Implementation in PRISM part.

4) Also, PRISM during verification of properties takes couple of hours to do just 50 iterations, which was very time consuming.

5. FUTURE WORK:

1) Instead of taking only the brakes into account also take the sensor readings for stress on each car and try to implement that model too.

2) Compute the different value of quantization constant **δ, M** and study what effect it has on the model.

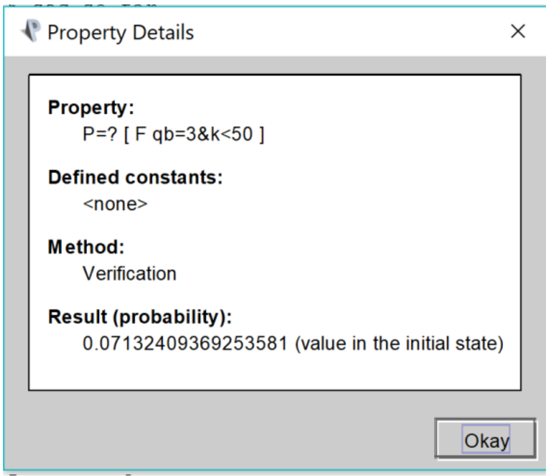
6. RESULTS:

6.1 Model Verification:

1). Initially a simple model with only 3 cars was verified using a simple property:

P=?[F (qb=3 & k<50)] **where F is the eventually (in near future) operator**

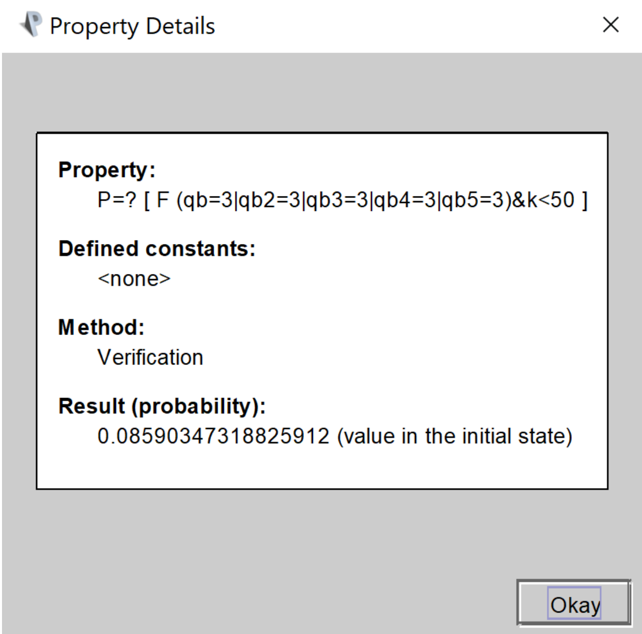
(What is the probability that car 1 brake will fail in less than 50 iterations)



2). **Full implementation** of **5 cars** was verified using property:

P=? [F (qb=3 |qb2=3 |qb3=3 |qb4=3 |qb5=3) & k<50]

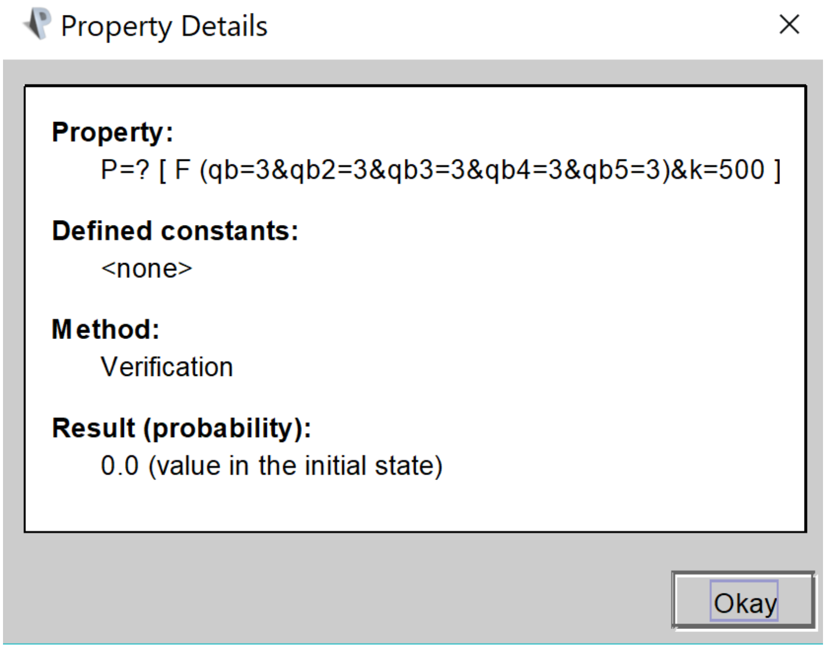
(What is the probability that either car 1 or 2 or 3 or 4 or 5 brake will fail in less than 50 iterations)



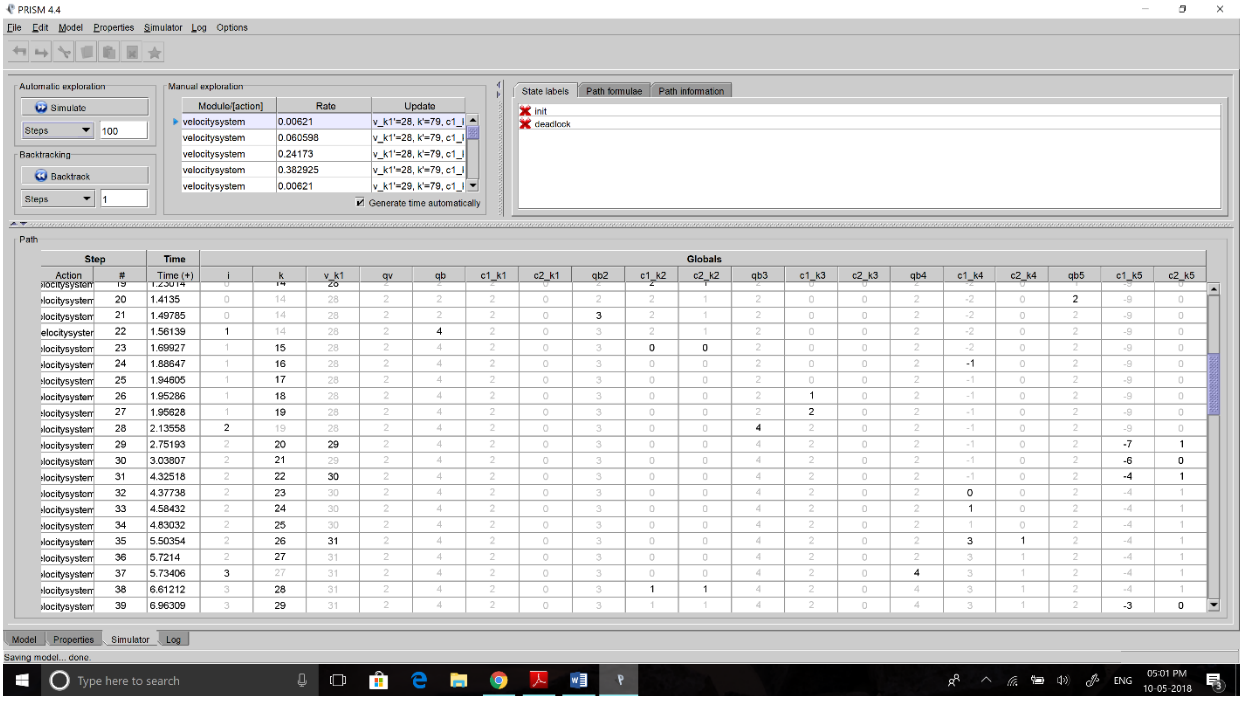
3). **Full implementation** of **5 cars** was verified using property:

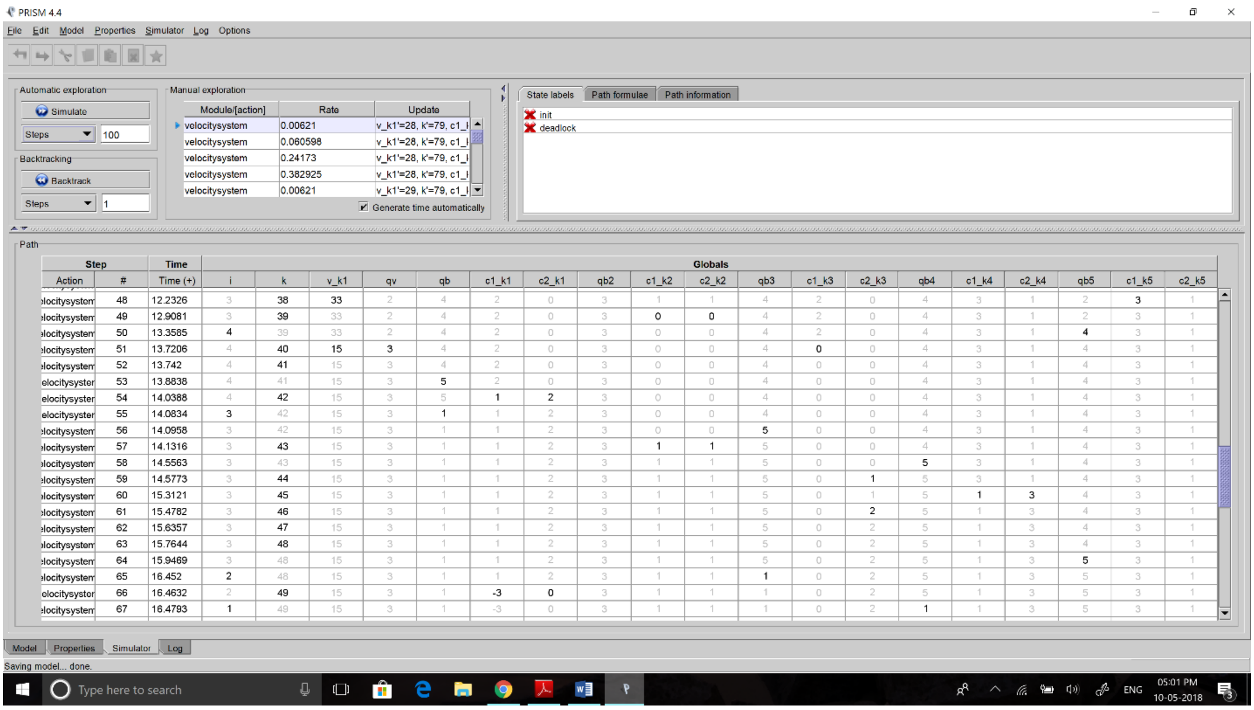
P=? [F (qb=3 &qb2=3 &qb3=3 &qb4=3 &qb5=3) & k=500]

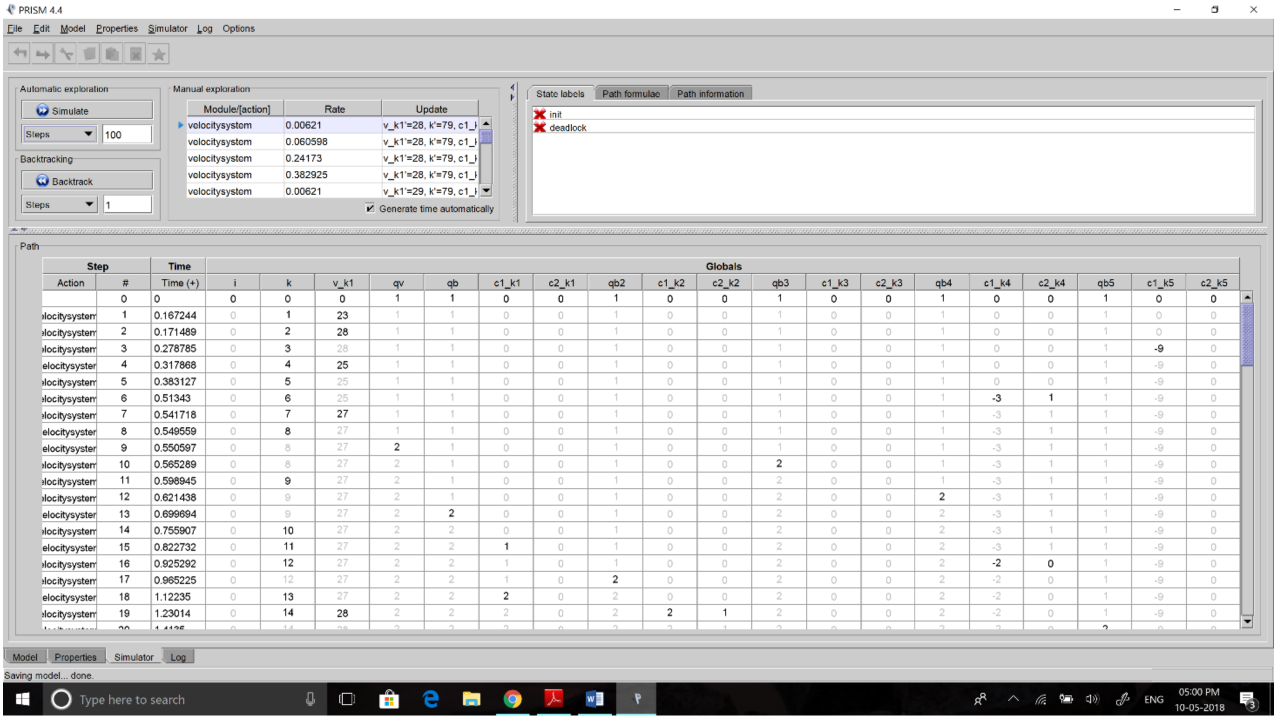
(What is the probability that all 5 cars brake will fail in 500 iterations)

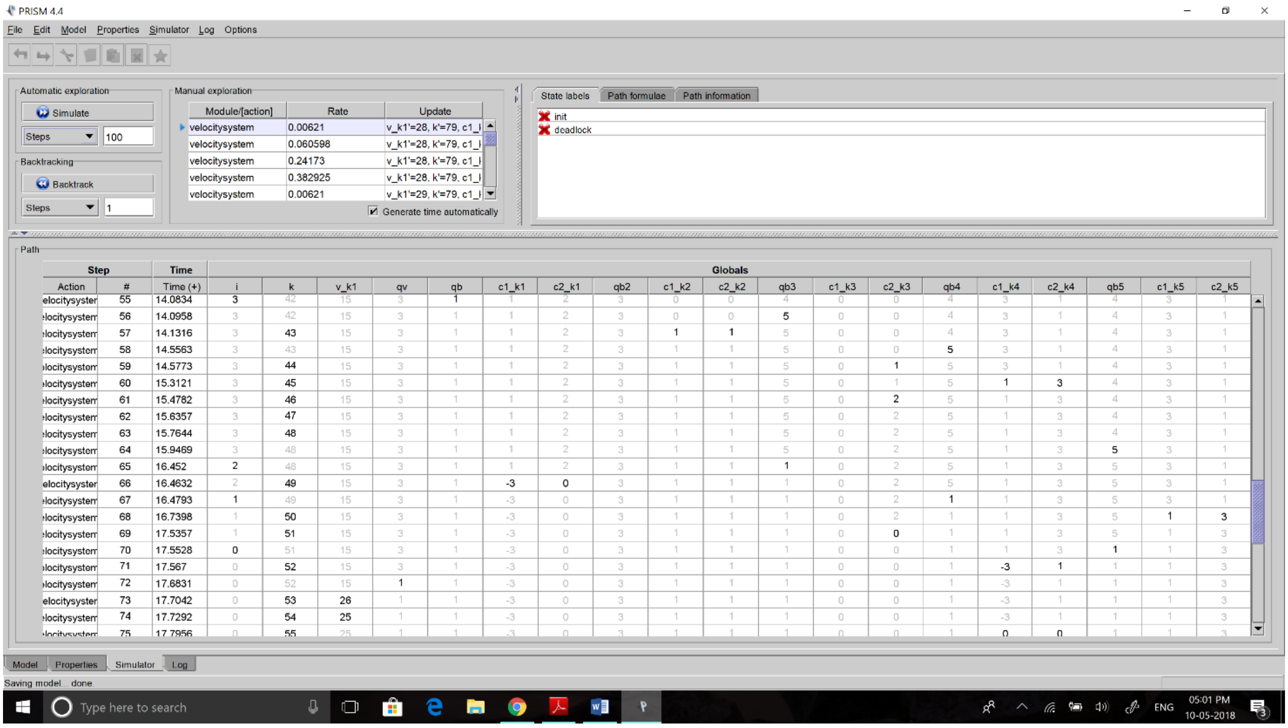


6.2 Simulation:






PRISM 4.4
File Edit Model Properties Simulator Log Options
Automatic exploration
Simulate
Steps
100
Backtracking
Backtrack
Manual exploration
Module/[action]
Rate
Update
Generate time automatically
State labels
Path formulae
Path information
init
deadlock
Path
Step
Time
Globals
Model Properties Simulator Log
Saving model... done.
Type here to search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Generate time automatically
State labels
Path formulae
Path information
init
deadlock
Path
Step
Time
Globals
Model Properties Simulator Log
Type here to search

7. REFRENCES:

[1] P. Sistla, M. Zefran, and Y. Feng, “Runtime Monitoring of Stochastic Cyber-Physical Systems with Hybrid State,” in Runtime Verification, vol. 7186 of Lecture Notes in Computer Science, pp. 276–293, Sept. 2011.
[2] http://www.prismmodelchecker.org/manual/ThePRISMLanguage/Introduction

8. APPENDIX:

MATLAB Code:

For quantization of $n_1 \sim N(0,1)$, having $\mu=0$, $\sigma=1$; Let $\delta=1$

We get M=3. So -3< k <-3

```
mu = 0;
sigma = 0.1;
delta= .1;
M= (3*sigma)/delta;
K = [-M:.1:M];
T1= (K+.5)*delta;
T2= (K-.5)*delta;
cdf1 = normcdf(T1,mu,sigma);
cdf2 = normcdf(T2,mu,sigma);
% -M <K < M
p1=cdf1-cdf2;
% K = -M
p2=normcdf((-M+.5)*delta,mu,sigma);
% K = M
p3=1-(normcdf((M-.5)*delta,mu,sigma));

 x=[p2 p1(2:(end-1)) p3];
 figure,
 plot(K,cdf1,'LineWidth',1)
 figure,
 plot(K,x,'LineWidth',1)
```

PRISM Code:

```
////////// Module 1
module velocitysystem
// qv=1-----qv=1
[] qv=1 ->  0.00621:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-3))& (k'= min(k+1,500))
          + 0.060598:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-2))& (k'= min(k+1,500))
          + 0.24173:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-1))& (k'= min(k+1,500))
          + 0.382925:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))-0))& (k'= min(k+1,500))
          + 0.00621:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))+3))& (k'= min(k+1,500))
          + 0.060598:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))+2))& (k'= min(k+1,500))
          + 0.24173:(v_k1'=ceil (0.1353*v_k1+(0.8647*(25+2.5*sin))+1))& (k'= min(k+1,500));
// qb=1 ----qb=1
[] qb=1 -> 0.00621:(c1_k1'=ceil(0-9)) & (c2_k1'=ceil(0-0.3)) & (k'= min(k+1,500))
          +0.060598:(c1_k1'=ceil(0-6)) & (c2_k1'=ceil(0-0.2)) & (k'= min(k+1,500))
          + 0.24173:(c1_k1'=ceil(0-3)) & (c2_k1'=ceil(0-0.1)) & (k'= min(k+1,500))
          +0.382925:(c1_k1'=ceil(0)) & (c2_k1'=ceil(0)) & (k'= min(k+1,500))
          + 0.00621:(c1_k1'=ceil(0-9)) & (c2_k1'=ceil(0+0.3)) & (k'= min(k+1,500))
          +0.060598:(c1_k1'=ceil(0-6)) & (c2_k1'=ceil(0+0.2)) & (k'= min(k+1,500))
          + 0.24173:(c1_k1'=ceil(0-3)) & (c2_k1'=ceil(0+0.1)) & (k'= min(k+1,500));

// qv=1-----qv=2 When Velocity > Threshold Velocity
// qb=1-----qb=2
[] qv=1 & v_k1>=27&qb=1 -> (qv'=2)&(qb'=2);
// qv=2------qv=2
[] qv=2 &qb=2 &!brake&(c1_k1<=1) -> 0.00621:(v_k1'=ceil(v_k1-0.3)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1-0.3)) & (c2_k1'=ceil(0-0.3))
                  +0.060598:(v_k1'=ceil(v_k1-0.2)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1-0.2)) & (c2_k1'=ceil(0-0.2))
                  +0.24173:(v_k1'=ceil(v_k1-0.1)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1-0.1)) & (c2_k1'=ceil(0-0.1))
                  +0.382925:(v_k1'=ceil(v_k1-0)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1-0)) & (c2_k1'=ceil(0-0))
                  + 0.00621:(v_k1'=ceil(v_k1+0.3)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1+0.3)) & (c2_k1'=ceil(0+0.3))
                  +0.060598:(v_k1'=ceil(v_k1+0.2)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1+0.2)) & (c2_k1'=ceil(0+0.2))
                  +0.24173:(v_k1'=ceil(v_k1+0.1)) & (k'= min(k+1,500))&(c1_k1'=ceil(c1_k1+1+0.1)) & (c2_k1'=ceil(0+0.1));
// qb=2-----qb=3 (Filure), qb=4 (Engage Braking)
[] qb=2 & (c1_k1>1)->  0.1: (qb'=3) +  0.9: (qb'=4)&(i'=i+1);
// qb=3----qb=3 (Error state-System fail)
[] qb=3 ->  0.00621:(c2_k1'=ceil(0-0.3)) & (c1_k1'=ceil(0-0.3)) & (k'= min(k+1,500))
           +0.060598:(c2_k1'=ceil(0-0.2)) & (c1_k1'=ceil(0-0.2)) & (k'= min(k+1,500))
           + 0.24173:(c2_k1'=ceil(0-0.1)) & (c1_k1'=ceil(0-0.1)) & (k'= min(k+1,500))
           +0.382925:(c2_k1'=ceil(0)) & (c1_k1'=ceil(0)) & (k'= min(k+1,500))
           + 0.00621:(c2_k1'=ceil(0+0.3)) & (c1_k1'=ceil(0+0.3)) & (k'= min(k+1,500))
           +0.060598:(c2_k1'=ceil(0+0.2)) & (c1_k1'=ceil(0+0.2)) & (k'= min(k+1,500))
           + 0.24173:(c2_k1'=ceil(0+0.1)) & (c1_k1'=ceil(0+0.1)) & (k'= min(k+1,500));
// qb=2-----qb=1 (if velocity fall below threshold)
[] qb=2&(v_k1<23)&(qv!=2)&(c1_k1>1)-> 0.9: (qb'=1);

// qv=2------qv=3 Brakes Engaged(Braking Force )
// qb=4------qb=4
[] qv=2 &qb=4& brake &(qb>2&qb2>2&qb3>2&qb4>2&qb5>2)&(v_k1>=23)-> 0.00621: (qv'=3) &(v_k1'=ceil(v_k1-force-1.5)) & (k'= min(k+1,500))&(c2_k1'=ceil(0-9)) & (c1_k1'=ceil(0-0.3))
                                          +0.060598: (qv'=3) &(v_k1'=ceil(v_k1-force-1)) & (k'= min(k+1,500))&(c2_k1'=ceil(0-6)) & (c1_k1'=ceil(0-0.2))
                                          +0.24173: (qv'=3) &(v_k1'=ceil(v_k1-force-0.5)) & (k'= min(k+1,500))&(c2_k1'=ceil(0-3)) & (c1_k1'=ceil(0-0.1))
                                          +0.382925: (qv'=3) &(v_k1'=ceil(v_k1-force-0)) & (k'= min(k+1,500))&(c2_k1'=ceil(0)) & (c1_k1'=ceil(0))
                                          +0.00621: (qv'=3) &(v_k1'=ceil(v_k1-force+1.5)) & (k'= min(k+1,500))&(c2_k1'=ceil(0-9)) & (c1_k1'=ceil(0+0.3))
                                          +0.060598: (qv'=3) &(v_k1'=ceil(v_k1-force+1)) & (k'= min(k+1,500))&(c2_k1'=ceil(0-6)) & (c1_k1'=ceil(0+0.2))
                                          +0.24173: (qv'=3) &(v_k1'=ceil(v_k1-force+0.5)) & (k'= min(k+1,500))&(c2_k1'=ceil(0-3)) & (c1_k1'=ceil(0+0.1));

// qb=4---qb=5 (velocity reaches below threshold)
[] qb=4 & (v_k1<23) -> (qb'=5);
// qb=5----qb=5 (waiting time to disengage the brakes- until counter c2>1)
[] qb=5 &(c2_k1<=1)-> 0.00621:(c2_k1'=ceil(c2_k1+1-0.3)) & (c1_k1'=ceil(0-0.3)) & (k'= min(k+1,500))
                    +0.060598:(c2_k1'=ceil(c2_k1+1-0.2)) & (c1_k1'=ceil(0-0.2)) & (k'= min(k+1,500))
                    + 0.24173:(c2_k1'=ceil(c2_k1+1-0.1)) & (c1_k1'=ceil(0-0.1)) & (k'= min(k+1,500))
                    +0.382925:(c2_k1'=ceil(c2_k1+1-0)) & (c1_k1'=ceil(0)) & (k'= min(k+1,500))
                    +0.00621:(c2_k1'=ceil(c2_k1+1+0.3)) & (c1_k1'=ceil(0+0.3)) & (k'= min(k+1,500))
                    +0.060598:(c2_k1'=ceil(c2_k1+1+0.2)) & (c1_k1'=ceil(0+0.2)) & (k'= min(k+1,500))
                    + 0.24173:(c2_k1'=ceil(c2_k1+1+0.1)) & (c1_k1'=ceil(0+0.1)) & (k'= min(k+1,500));
// qb=5----qb=1 (breakes are disengaged)
[] qb=5 & (c2_k1>1)&(i>0) -> (qb'=1) & (i'=i-1);
// qv=3------qv=1 Brakes Disengaged
[] qv =3 & disengage ->(qv'=1);
endmodule


// Module for car 2
module velocitysystem2=velocitysystem
[qb=qb2,c1_k1=c1_k2,c2_k1=c2_k2]
endmodule
// Module for car 3
module velocitysystem3=velocitysystem
[qb=qb3,c1_k1=c1_k3,c2_k1=c2_k3]
endmodule
// Module for car 4
module velocitysystem4=velocitysystem
[qb=qb4,c1_k1=c1_k4,c2_k1=c2_k4]
endmodule
// Module for car 5
module velocitysystem5=velocitysystem
[qb=qb5, c1_k1=c1_k5,c2_k1=c2_k5]
endmodule
```