Let F be a field and let $A, B \in M_{mn}(F)$ such that $\forall v \in F^n$ we have $Av = Bv$. Prove that $A = B$

Problem Solving - What are the terms/strategies I may need? What do I know?

Showing two matrices are equal:

      You must show that each corresponding entry in both matrices are the same.

      Alternatively, you can show that every column of A is the same as every column of B.

Hint:

      Ae_j is the j'th column of A.

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

Our goal will be to show that each column of A is the same as the corresponding column of B:

Let A_i represent the ith column of A, and let B_i represent the ith column of B. Then we have:

$$
\begin{aligned}
A_i \quad &= A e_i \quad &\text{(from hint)} \\
&= B e_i \quad &\text{(as } Bv = Av \text{ for all } v \in F^n \text{ and } e_i \in F^n) \\
&= B_i \quad &\text{(from hint)}
\end{aligned}
$$

Thus we have shown that A = B as all of the columns of A are the same as the columns of B.

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Can you prove the hint?

For Video Please click the link below:

Video