Solve the following system (when consistent) and write the solution in parametric and vector form

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 5 & 9 & 3 \\ -1 & -2 & -3 & 0 \end{bmatrix}$$

Problem Solving - What are the terms/strategies I may need? What do I know?

- Definition for a matrix in RREF:
 - a) All leading entries are 1
 - b) Leading entries go down and to the right
 - c) all zero rows are at the bottom
 - d) All entries above and below a leading 1 are 0.
- Strategy for getting to RREF to solve a system:
 - 1) Choose a leading non-zero row to be your pivot (easiest number possible)
 - 2) Switch this to the first available row without a pivot.
 - 3) Row reduce below the pivot so that all entries are zero below
 - 4) Continue steps 1-3 until you cannot choose any more leading non-zero entries.
 - 5) Start with the last pivot and eliminate all entries above the pivot so they are zero.
 - 6) Repeat step 5 for all pivots
 - 7) Divide all rows so that the leading pivots are 1.
 - 8) Once in RREF, write out the equation in parametric form (solve for each variable) by identifying free variables.
 - 9) Once you have parametric form, you can store the solution in a vector by considering $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

^{*} At any stage, feel free to divide a row to make the numbers smaller, but do so in such a way that you avoid fractions.

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 5 & 9 & 3 \\ -1 & -2 & -3 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

At this stage, we should see that the system is inconsistent as we have [0 0 0 | 2]

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

- How do you get a matrix to RREF?
- Why is having a matrix in RREF so helpful?
- Why are free variables easy to identify when the matrix is in RREF?
- Why does $[0 \ 0 \ 0 \ | \ k]$ mean that there is no solution when $k \neq 0$?

For Video Please click the link below:

<u>Video</u>