For each situation, describe when (if possible) the system can have no solution, or a unique solution, infinitely many solutions :

a) A system of 5 equations and 5 unknowns
b) A system of 5 equations and 7 unknowns
c) A system of 7 equations and 5 unknowns

Problem Solving - What are the terms/strategies I may need? What do I know?

- Definition for a matrix in RREF:
  a) All leading entries are 1
  b) Leading entries go down and to the right
  c) all zero rows are at the bottom
  d) All entries above and below a leading 1 are 0.

- Recognizing a solution having from RREF:
  Inconsistent systems will have no solutions. This occurs when we have a row containing: [0 0 0 .. 0 | b] $b \neq 0$
  Consistent systems will have a solution. This occurs when we do not have [0 0 0 .. 0 | b] $b \neq 0$ (that is all zero rows are of the form 0 = 0). There are two types of consistent systems:
    Consistent systems with unique solutions have a pivot in every column.
    Consistent systems with infinitely many solutions do not have a pivot in every column.

For each situation, describe when (if possible) the system can have no solution, or a unique solution, infinitely many solutions :

a) A system of 5 equations and 5 unknowns
b) A system of 5 equations and 7 unknowns
c) A system of 7 equations and 5 unknowns

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

a) A system of 5 equations and 5 unknowns

- This can have no solution (for example, if we have two equations like $x_1 + x_2 + \cdots + x_5 = 1$ and $x_1 + x_2 + \cdots + x_5 = 2$)
- This can have a unique solution (as we can have a system of the form $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5$)
- This can have infinitely many solutions (as we can have a system where all 5 equations are identical, thus giving 4 free variables)

b) A system of 5 equations and 7 unknowns

- This can have no solution (for example, if we have two equations like $x_1 + x_2 + \cdots + x_7 = 1$ and $x_1 + x_2 + \cdots + x_7 = 2$)
This <u>**cannot**</u> have a unique solution (as the most amount of pivots we could have is 5 in RREF, but we have 7 variables)
This can have infinitely many solutions (as we can have a system where all 5 equations are identical, thus giving 6 free variables)

For each situation, describe when (if possible) the system can have no solution, or a unique solution, infinitely many solutions :

a) A system of 5 equations and 5 unknowns
b) A system of 5 equations and 7 unknowns
c) A system of 7 equations and 5 unknowns

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

a) A system of 7 equations and 5 unknowns

- This can have no solution (for example, if we have two equations like $x_1 + x_2 + \cdots + x_5 = 1$ and $x_1 + x_2 + \cdots + x_5 = 2$)
- This can have a unique solution (as we can have a system of the form $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5$, $x_5 = 5$ and $x_5 = 5$)
- This can have infinitely many solutions (as we can have a system where all 7 equations are identical, thus giving 4 free variables)

For each situation, describe when (if possible) the system can have no solution, or a unique solution, infinitely many solutions :

a) A system of 5 equations and 5 unknowns
b) A system of 5 equations and 7 unknowns
c) A system of 7 equations and 5 unknowns

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

- How do you get a matrix to RREF?
- Why is having a matrix in RREF so helpful?
- Why are free variables easy to identify when the matrix is in RREF?

For Video Please click the link below:

Video