

Let  $O_{mn} \in M_{mn}(F)$  such that  $[O_{mn}]_{ij} = 0$  prove that if  $A \in M_{mn}$ , then  $O_{rm}A = O_{rn}$  and  $AO_{nr} = O_{mr}$

**Problem Solving** - What are the terms/strategies I may need? What do I know?

Matrix product definition/theorem:

$$[AB]_{ij} = \sum_{k=1}^n [A]_{ik} [B]_{kj}$$

Let  $O_{mn} \in M_{mn}(F)$  such that  $[O_{mn}]_{ij} = 0$  prove that if  $A \in M_{mn}$ , then  $O_{rm}A = O_{rn}$  and  $AO_{nr} = O_{mr}$

**Steps & Process** – Try to answer the question writing in many steps to avoid small errors.

Our goal is to show that the product is  $O_{rn}$ , thus we need to show that  $[O_{rm}A]_{ij} = 0$

$$\begin{aligned}[O_{rm}A]_{ij} &= \sum_{k=1}^m [O_{rm}]_{ik} [A]_{kj} && \text{(Matrix product formula)} \\ &= \sum_{k=1}^m (0) [A]_{kj} && \text{(Definition of } O_{mn}) \\ &= \sum_{k=1}^m 0 && \text{(Zero property of a field)} \\ &= 0 && \text{(Sum of zero is zero)}\end{aligned}$$

Thus the product is  $O_{rn}$

Similarly to show that the product is  $O_{mr}$ , thus we need to show that  $[AO_{nr}]_{ij} = 0$

$$\begin{aligned}[AO_{nr}]_{ij} &= \sum_{k=1}^n [A]_{ik} [O_{nr}]_{kj} && \text{(Matrix product formula)} \\ &= \sum_{k=1}^n [A]_{ik} (0) && \text{(Definition of } O_{mn}) \\ &= \sum_{k=1}^n 0 && \text{(Zero property of a field)} \\ &= 0 && \text{(Sum of zero is zero)}\end{aligned}$$

Thus the product is  $O_{mr}$

Let  $O_{mn} \in M_{mn}(F)$  such that  $[O_{mn}]_{ij} = 0$  prove that if  $A \in M_{mn}$ , then  $O_{rm}A = O_{rn}$  and  $AO_{nr} = O_{mr}$

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**Solidify Understanding** – Explain why the steps makes sense by connecting to math you know.

Why does our matrix product formula work?

For Video Please click the link below:

[Video](#)