

Solve the following system:

$$x_1 + 2x_2 - 4x_3 - x_4 = 32$$

$$x_1 + 3x_2 - 7x_3 - x_5 = 33$$

$$x_1 + 2x_3 - 2x_4 + 3x_5 = 22$$

Problem Solving - What are the terms/strategies I may need? What do I know?

RREF Properties:

- Pivots are 1
- Pivots go down and to the right
- Everything above and below a pivot is 0
- Zero rows are at the bottom

Row Operations:

- Swap rows
- Scale a row by a non-zero number
- Row replace using $R_i \rightarrow R_i - k_j R_j$

Solving Systems using Matrices:

- Set up a matrix from the linear system
- Put the matrix into RREF using row operations
- Identify pivots and free variables.
- Write all pivot variables in terms of free variables.

Solve the following system:

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$$x_1 + 3x_2 - 7x_3 - x_5 = 33$$

$$x_1 + 2x_3 - 2x_4 + 3x_5 = 22$$

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We first place the system into a matrix system:

$$x_1 + 2x_2 - 4x_3 - x_4 = 32$$

$$x_1 + 3x_2 - 7x_3 - x_5 = 33$$

$$x_1 + 2x_3 - 2x_4 + 3x_5 = 22$$

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$$\begin{bmatrix} 1 & 2 & -4 & -1 & 0 & 32 \\ 1 & 3 & -7 & 0 & -1 & 33 \\ 1 & 0 & 2 & -2 & 3 & 22 \end{bmatrix}$$

$R2 \rightarrow R2 - R1$

$$\begin{bmatrix} 1 & 2 & -4 & -1 & 0 & 32 \\ 0 & 1 & -3 & 1 & -1 & 1 \\ 1 & 0 & 2 & -2 & 3 & 22 \end{bmatrix}$$

$R3 \rightarrow R3 - R1$

$$\begin{bmatrix} 1 & 2 & -4 & -1 & 0 & 32 \\ 0 & 1 & -3 & 1 & -1 & 1 \\ 0 & -2 & 6 & -1 & 3 & -10 \end{bmatrix}$$

$R3 \rightarrow R3 + 2R2$

$$\begin{bmatrix} 1 & 2 & -4 & -1 & 0 & 32 \\ 0 & 1 & -3 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 1 & -8 \end{bmatrix}$$

$R2 \rightarrow R2 - R3$

$$\begin{bmatrix} 1 & 2 & -4 & -1 & 0 & 32 \\ 0 & 1 & -3 & 0 & -2 & 9 \\ 0 & 0 & 0 & 1 & 1 & -8 \end{bmatrix}$$

$R1 \rightarrow R1 + R3$

$$\begin{bmatrix} 1 & 2 & -4 & 0 & 1 & 24 \\ 0 & 1 & -3 & 0 & -2 & 9 \\ 0 & 0 & 0 & 1 & 1 & -8 \end{bmatrix}$$

$R1 \rightarrow R1 - 2R2$

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 5 & 6 \\ 0 & 1 & -3 & 0 & -2 & 9 \\ 0 & 0 & 0 & 1 & 1 & -8 \end{bmatrix}$$

$$\therefore S = \left\{ \begin{bmatrix} 6 - 5x_5 - 2x_3 \\ 9 + 2x_5 + 3x_3 \\ x_3 \\ -8 - x_5 \\ x_5 \end{bmatrix} \mid x_3, x_5 \in C \right\}$$

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$$x_1 + 2x_3 - 2x_4 + 3x_5 = 22$$

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why to row operations keep the same solutions set?

For Video Please click the link below:

[Video](#)