(a) Find invertible 2×2 matrices A and B for which A + B is not invertible.
(b) Find invertible 2×2 matrices A and B for which A+B is also invertible, but $(A+B)^{-1} \neq A^{-1} + B^{-1}$.

Problem Solving - What are the terms/strategies I may need? What do I know?

Showing two matrices are equal:

You must show that each corresponding entry in both matrices are the same.

Theorem:

A matrix $A \in M_2$ is invertible if and only if A row reduces to the identity matrix I_2.

Definition of inverse matrices for square matrices:

$AB = I = BA$ if and only if $B = A^{-1}$

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

(a) Let $A = I_2$ and $B = -I_2$.

Observe that $A^2 = I_2$ and $B^2 = I_2$, which implies that both A and B are invertible with $A = A^{-1}$ and $B = B^{-1}$.

However $A + B = I_2 + (-I_2) = 0_2$ implies that $A + B$ is not invertible since 0 doesn't row reduce to I_2.

(b) Let $A = B = I_2$.

Observe that

$$A + B = I_2 + I_2 = 2I_2$$

From this it follows that $A + B$ is invertible since

$$\frac{1}{2}I_2 \times (A + B) = (A + B) \times \frac{1}{2}I_2 = I_2$$

Next observe that

$$(A + B)^{-1} = \frac{1}{2}I_2 \qquad \text{and} \qquad A^{-1} + B^{-1} = I_2^{-1} + I_2^{-1} = I_2 + I_2 = 2I_2$$

This shows that $(A + B)^{-1} \neq A^{-1} + B^{-1}$ since $\frac{1}{2}I_2 \neq 2I_2$.

(a) Find invertible 2×2 matrices A and B for which A + B is not invertible.
(b) Find invertible 2×2 matrices A and B for which A+B is also invertible, but $(A+B)^{-1} \neq A^{-1} + B^{-1}$.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Was there anything special about the identity matrix or could we have found other matrices to solve questions (a) and (b)?

For Video Please click the link below:

Video