Prove the non-zero rows of a matrix in RREF are linearly independent.

Problem Solving - What are the terms/strategies I may need? What do I know?

Definitions: Linearly independent

A set is linearly independent when you can prove

$$c_1v_1 + c_2v_2 + \cdots + c_nv_n = 0 \qquad \rightarrow \qquad c_1 = c_2 = \cdots = c_n = 0$$

Definition of a matrix in RREF

All pivots are 1

Pivots go down and to the right

Zero rows are at the bottom

All entries above and below the pivots are zero.

Prove the non-zero rows of a matrix in RREF are linearly independent.

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

Consider a matrix A in RREF with rows $r_1, r_2, \ldots, r_n$ that are the non-zero rows. Then these rows must have pivots.

Let p_i be the location of the pivot in row $\mathrm{r_i}$.

Since A is in RREF, we know that $(r_i)_{p_i} = 1$ and $(r_i)_{p_j} = 0$ for all $j \neq i$ since all entries above and below pivots are zero, and the pivot values are 1.

Consider the combination $c_1 r_1 + c_2 r_2 + \cdots + c_n r_n = 0$

If we let $1 \leq i \leq n$, then we know that

$[c_1 r_1 + c_2 r_2 + \cdots + c_n r_n]_{p_i} = [0]_{p_i}$

$[c_1 r_1]_{p_i} + [c_2 r_2]_{p_i} + \cdots + [c_n r_n]_{p_i} = [0]_{p_i}$ (Vector addition for row space)

$c_1\, [r_1]_{p_i} + c_1 [r_2]_{p_i} + \cdots + c_n [r_n]_{p_i} = [0]_{p_i}$ (Vector scalar multiplication for row space)

$c_i(1) = 0$ (Since $(r_i)_{p_i} = 1$ and $(r_i)_{p_j} = 0$)

$c_i = 0$

Thus all $c_i = 0$ and our set is linearly independent.

Prove the non-zero rows of a matrix in RREF are linearly independent.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Can we prove this the same way saying that the columns of an RREF matrix are linearly independent?

For Video Please click the link below:

Video