C45[†] Suppose that $S = \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 1 \end{bmatrix} \right\}$. Let $W = \langle S \rangle$ and let $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$.
Is $\mathbf{w} \in W$? If so, provide an explicit linear combination that demonstrates this.

Problem Solving - What are the terms/strategies I may need? What do I know?

RREF Properties:

- Pivots are 1
- Pivots go down and to the right
- Everything above and below a pivot is 0
- Zero rows are at the bottom

Row Operations:

- Swap rows
- Scale a row by a non-zero number
- Row replace using $R_i \rightarrow k_iR_i - k_jR_j$ where k_i is non-zero

Solving Elements inside Spans using Matrices:

- Set up a matrix from the linear system where the
- Solve the system using row reduction.
- Consistent systems means the vector is in the span, inconsistent systems mean the vector is not in the span

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We first place the system into a matrix system:

$$\begin{bmatrix} -1 & 3 & 1 & -6 & 2 \\ 2 & 1 & 5 & 5 & 1 \\ 1 & 2 & 4 & 1 & 3 \end{bmatrix} \quad (-1)(R1) \quad \begin{bmatrix} 1 & -3 & -1 & 6 & -2 \\ 2 & 1 & 5 & 5 & 1 \\ 1 & 2 & 4 & 1 & 3 \end{bmatrix} \quad R2 \to -2R1 + R2 \quad \begin{bmatrix} 1 & -3 & -1 & 6 & -2 \\ 0 & 7 & 7 & -7 & 5 \\ 1 & 2 & 4 & 1 & 3 \end{bmatrix} \quad R3 \to -R1 + R3$$

$$\begin{bmatrix} 1 & -3 & -1 & 6 & -2 \\ 0 & 7 & 7 & -7 & 5 \\ 0 & 5 & 5 & -5 & 5 \end{bmatrix} \quad R2 \leftrightarrow R3 \quad \begin{bmatrix} 1 & -3 & -1 & 6 & -2 \\ 0 & 5 & 5 & -5 & 5 \\ 0 & 7 & 7 & -7 & 5 \end{bmatrix} \quad (1/5)R2 \quad \begin{bmatrix} 1 & -3 & -1 & 6 & -2 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 7 & 7 & -7 & 5 \end{bmatrix} \quad R3 \to -7R2 + R3$$

$$\begin{bmatrix} 1 & -3 & -1 & 6 & -2 \\ 0 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

Here we see that the system is inconsistent as the last row produces 0 = -2 (which we know is not true). Thus w is not in the span of <S>

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Is $\mathbf{w} \in W$? If so, provide an explicit linear combination that demonstrates this.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why to row operations keep the same solutions set?

Why can we set up a matrix system to solve spans?

For Video Please click the link below:

Video