Explain which of the field axioms hold/fail given:

a) $F = Z$ and + and $\cdot$ are the standard addition and multiplication.

Problem Solving - What are the terms/strategies I may need? What do I know?

Field axioms: A *field* is a set F with two operations, denoted $+$ and $\cdot$, that satisfy each of the following properties:

(A1) For all $x, y, z \in F$, $(x + y) + z = x + (y + z)$.

(A2) For all $x, y \in F$, $x + y = y + x$.

(A3) There exists an element $0 \in F$ such that for all $x \in F$ we have $x + 0 = x$.

(A4) For each $x \in F$, there exists an element $-x \in F$ such that $x + (-x) = 0$.

(M1) For all $x, y, z \in F$, $(xy)z = x(yz)$.

(M2) For all $x, y \in F$, $xy = yx$.

(M3) There exists an element $1 \in F$ such that $1 \neq 0$ and for all $x \in F$ we have $x \cdot 1 = x$.

(M4) For each $x \in F$, there exists an element $x^{-1} \in F$ such that $xx^{-1} = 1$.

(DL) For all $x, y, z \in F$, $x(y + z) = xy + xz$.

Explain which of the field axioms hold/fail given:

a) $F = Z$ and + and $\cdot$ are the standard addition and multiplication.

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

Field axioms: A *field* is a set F with two operations, denoted $+$ and $\cdot$, that satisfy each of the following properties:

(A1) For all $x, y, z \in F$, $(x+y)+z = x+(y+z)$.	Passes (Borrows from R)
(A2) For all $x, y \in F$, $x+y = y+x$.	Passes (Borrows from R)
(A3) There exists an element $0 \in F$ such that for all $x \in F$ we have $x+0 = x$.	Passes (Borrows from R)
(A4) For each $x \in F$, there exists an element $-x \in F$ such that $x+(-x) = 0$.	Passes (Borrows from R)
(M1) For all $x, y, z \in F$, $(xy)z = x(yz)$.	Passes (Borrows from R)
(M2) For all $x, y \in F$, $xy = yx$.	Passes (Borrows from R)
(M3) There exists an element $1 \in F$ such that $1 \neq 0$ and for all $x \in F$ we have $x \cdot 1 = x$.	Passes (Borrows from R)
(M4) For each $x \in F$, there exists an element $x^{-1} \in F$ such that $xx^{-1} = 1$.	Fails ($x = 2$ has no integer such that $2q = 1$)
(DL) For all $x, y, z \in F$, $x(y+z) = xy+xz$.	Passes (Borrows from R)

We should also note that the sum and product of integers produces an integer.

Explain which of the field axioms hold/fail given:

a) $F = Z$ and + and $\cdot$ are the standard addition and multiplication.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why is proving that a set over two operations is indeed a field helpful?

For Video Please click the link below:

Video