Solve the following system (when consistent) and write the solution in parametric form under Z_{11}

$$\left[\begin{array}{ccc|c} 5 & 4 & 2 & 7 \\ 8 & 1 & 3 & 5 \\ 4 & 9 & 0 & 10 \end{array}\right]$$

Problem Solving - What are the terms/strategies I may need? What do I know?

- Definition for a matrix in RREF:
  - a) All leading entries are 1
  - b) Leading entries go down and to the right
  - c) all zero rows are at the bottom
  - d) All entries above and below a leading 1 are 0.

- Strategy for getting to RREF to solve a system:
  - 1) Choose a leading non-zero row to be your pivot (easiest number possible)
  - 2) Switch this to the first available row without a pivot.
  - 3) Row reduce below the pivot so that all entries are zero below
  - 4) Continue steps 1-3 until you cannot choose any more leading non-zero entries.
  - 5) Start with the last pivot and eliminate all entries above the pivot so they are zero.
  - 6) Repeat step 5 for all pivots
  - 7) Divide all rows so that the leading pivots are 1.
  - 8) Once in RREF, write out the equation in parametric form (solve for each variable) by identifying free variables.
  - 9) Once you have parametric form, you can store the solution in a vector by considering $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Multiplication Inverse Table for Z_{11}:

Element	Inverse
1	1
2	6
3	4
4	3
5	9
6	2
7	8
8	7
9	5
10	10

Z_p operations:

a + b = (a + b) mod p (that is the remainder when dividing (a+b) by p

ab = (ab) mod p (that is the remainder when dividing (ab) by p

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

$$\left[\begin{array}{ccc|c} 5 & 4 & 2 & 7 \\ 8 & 1 & 3 & 5 \\ 4 & 9 & 0 & 10 \end{array}\right] \begin{array}{l} R_2 \to 5R_2 - 8R_1 \\ R_3 \to 5R_3 - 4R_1 \end{array} \quad \left[\begin{array}{ccc|c} 5 & 4 & 2 & 7 \\ 0 & 6 & 10 & 2 \\ 0 & 7 & 3 & 0 \end{array}\right] R_3 \to 6R_3 - 7R_2 \quad \left[\begin{array}{ccc|c} 5 & 4 & 2 & 7 \\ 0 & 6 & 10 & 2 \\ 0 & 0 & 3 & 8 \end{array}\right] \begin{array}{l} R_1 \times 9 \\ R_2 \times 2 \\ R_3 \times 4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 8 \\ 0 & 1 & 9 & 4 \\ 0 & 0 & 1 & 10 \end{array}\right] \begin{array}{l} R_1 \to R_1 - 7R_3 \\ R_2 \to R_2 - 9R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 10 \end{array}\right] R_1 \to R_1 - 3R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 10 \end{array}\right]$$

Since we have a pivot in every column, there is only one solution which is $x_1 = 9, x_2 = 2, x_3 = 10$

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

- How do you get a matrix to RREF?
- Why is having a matrix in RREF so helpful?
- Why are free variables easy to identify when the matrix is in RREF?
- Why do we avoid division when dealing with Mod operations?

For Video Please click the link below:

Video