Prove: if a is in **F** and $a^2 = a$, then $a = 1$ or $a = 0$.

Problem Solving - What are the terms/strategies I may need? What do I know?

Field Axioms:

name	addition	multiplication
associativity	$(a+b)+c=a+(b+c)$	$(a\,b)\,c=a\,(b\,c)$
commutativity	$a+b=b+a$	$a\,b=b\,a$
distributivity	$a\,(b+c)=a\,b+a\,c$	$(a+b)\,c=a\,c+b\,c$
identity	$a+0=a=0+a$	$a\cdot 1=a=1\cdot a$
inverses	$a+(-a)=0=(-a)+a$	$a\,a^{-1}=1=a^{-1}\,a$ if $a\neq 0$

0 Property:

If a is in a field **F** with additive inverse 0, then $0a = 0$.

Prove: if a is in **F** and $a^2 = a$, then $a = 1$ or $a = 0$.

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We first note that we have two cases: $a = 0$, or $a \neq 0$.

Case 1: $a = 0$

If $a = 0$ we see that:

a^2	$= (a)(a)$	(Definition of a^2)
	$= (0)(0)$	(Case 1)
	$= 0$	(0 Property)

Thus $a = 0$ will satisfy $a^2 = a$, this means if $a^2 = a$, $a = 0$ is a possible solution.

Case 2: $a \neq 0$

If $a \neq 0$, then there exists an a^{-1} such that $aa^{-1} = a^{-1}a = 1$ (Multiplicative inverses)

Thus, we see that:

$a^2 = a$	
$aa = a$	(Definition of a^2)
$a^{-1}(aa) = a^{-1}a$	(Multiply both sides by a^{-1})
$(a^{-1}a)a = a^{-1}a$	(Multiplication is associative)
$(1)a = 1$	(Multiplicative inverses)
$a = 1$	(Multiplicative identity)

Prove: if a is in **F** and $a^2 = a$, then $a = 1$ or $a = 0$.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why do we need to have the cases $a = 0$ or $a \neq 0$?

Why can we not just substitute $a = 0$, and $a = 1$ and show that $a^2 = a$ in both cases?

For Video Please click the link below:

Video