Determine if the following matrix is singular or non-singular: $\begin{bmatrix} -1 & 2 & 0 & 3 \\ 1 & -3 & -2 & 4 \\ -2 & 0 & 4 & 3 \\ -3 & 1 & -2 & 3 \end{bmatrix}$

Problem Solving - What are the terms/strategies I may need? What do I know?

RREF Properties:

- Pivots are 1
- Pivots go down and to the right
- Everything above and below a pivot is 0
- Zero rows are at the bottom

Row Operations:

- Swap rows
- Scale a row by a non-zero number
- Row replace using $R_i \rightarrow k_i R_i - k_j R_j$ where k_i is non-zero

Square matrix is an n x n matrix.

Singular Matrices:

A non-singular matrix is a matrix that is square and only has the trivial solution to LS(A,0)

A non-singular matrix is a matrix that has RREF as the identity matrix. (Theorem)

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We will use the theorem that determines if the matrix is singular by row reducing the matrix to RREF and seeing if the matrix is an identity matrix:

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 1 & -3 & -2 & 4 \\ -2 & 0 & 4 & 3 \\ -3 & 1 & -2 & 3 \end{bmatrix} \quad R2 \to R2 - R1$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & -1 & -2 & 7 \\ -2 & 0 & 4 & 3 \\ -3 & 1 & -2 & 3 \end{bmatrix} \quad R3 \to R3 + 2R1$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & -1 & -2 & 7 \\ 0 & -4 & 4 & -3 \\ -3 & 1 & -2 & 3 \end{bmatrix} \quad R4 \to R4 + 3R1$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & -1 & -2 & 7 \\ 0 & -4 & 4 & -3 \\ 0 & -5 & -2 & -6 \end{bmatrix} \quad R2 \times -1$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 2 & -7 \\ 0 & -4 & 4 & -3 \\ 0 & -5 & -2 & -6 \end{bmatrix} \quad R3 \to R3 + 4R2$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 2 & -7 \\ 0 & 0 & 12 & -31 \\ 0 & -5 & -2 & -6 \end{bmatrix} \quad R4 \to R4 + 5R2$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 2 & -7 \\ 0 & 0 & 12 & -31 \\ 0 & 0 & 8 & -41 \end{bmatrix} \quad R4 \to -12R4 + 8R3$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 2 & -7 \\ 0 & 0 & 12 & -31 \\ 0 & 0 & 0 & 244 \end{bmatrix} \quad R4 \div 244$$

$$\begin{bmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 2 & -7 \\ 0 & 0 & 12 & -31 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R1 \to R1 + 3R4 \\ R2 \to R2 + 7R4 \\ R3 \to R3 + 31R4 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} R2 \to R2 - 1/6R3 \\ R3 \to R3 \div 12 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R1 \to R1 + 2R2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \therefore \text{The matrix is non-singular as it is the identity matrix}$$

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why is a non-singular matrix (LS(A,0)) equivalent to A having RREF as the identity?

For Video Please click the link below:

Video