Let A and its RREF matrix B below as: A = $\begin{bmatrix} 1 & -3 & 0 & 5 & 0 \\ 2 & -6 & 1 & 9 & 0 \\ 2 & -6 & -1 & 11 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -3 & 0 & 5 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

a) What is Rank(A) and what is Nullity(A)?

b) What is a basis for Col(A)?

c) What is a basis for Row(A)?

d) What is a basis for $N(A)$?

Problem Solving - What are the terms/strategies I may need? What do I know?

Theorems:

1) Rank of A is the number of pivots in the RREF matrix.
2) Nullity of A is the number of non-pivots in A.
3) A basis for the Col Space is the set of pivot columns in A corresponding to the RREF matrix.
4) A basis for the Row Space is the set of pivot rows in the RREF matrix of A.
5) A basis for the Null Space can be found by solving $Ax = 0$
6) A set of two vectors are linearly independent if the vectors are not scalar multiples of one another.

Definition of a Basis

A linearly independent set that spans the space.

Definition of $span\{v_1 v_2, \dots, v_n\} = \{c_1 v_1 + \cdots + c_n v_n | c_i \in F\}$

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

a) The Rank of A is 3 as there are three pivots in the RREF matrix. The nullity of A is 2 as the number of non-pivots in the RREF matrix is 2.

b) A basis for Col(A) would be $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ as these are the corresponding columns in A from the pivots in the B matrix.

c) A basis for Row(A) would be $\{[1\ -3\ 0\ 5\ 0], [0\ 0\ 1\ -1\ 0], [0\ 0\ 0\ 0\ 1]\}$ as these are the corresponding rows in the RREF matrix. (Since row operations preserve the span).

d) To find the basis for the Null(A), we solve $Ax = 0$ which reduces to $Bx = 0$ in this case. Since x_2 and x_4 are free, we get $Null(A) = \left\{ \begin{bmatrix} 3s - 5t \\ s \\ t \\ t \\ 0 \end{bmatrix} \middle| \, s, t \in \boldsymbol{R} \right\} = \left\{ s \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -5 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \middle| s, t \in \boldsymbol{R} \right\} = span \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

Since these two vectors are not scalar multiples of one another, we have that the basis for

$$\text{Null(A)} = \left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why do we select the columns of A for col space but the rows of B for the row space?

Why does Rank + Nullity = m when a matrix is an A_{nm}?

Why could we say that a basis for Col(A) is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ for this question, but not in general?

For Video Please click the link below:

Video