Is the following linearly independent. If not, explain if it is trivially dependent: $\left\{\begin{bmatrix}-1\\2\\4\\2\end{bmatrix}, \begin{bmatrix}3\\3\\-1\\3\end{bmatrix}, \begin{bmatrix}7\\3\\-6\\4\end{bmatrix}\right\}$

Problem Solving - What are the terms/strategies I may need? What do I know?

Definition of Linearly Independent:

- A set $V = \{v_1, v_2, \ldots, v_n\}$ is linearly independent when the homogenous system:

$c_1v_1 + c_2v_2 + \ldots + c_nv_n = 0$ only has the trivial solution

Solutions to Homogenous Systems:

- A homogenous matrix system only has the trivial solution iff it has a pivot in every column except for the constant column.

Row Operations:

- Swap rows
- Scale a row by a non-zero number
- Row replace using $R_i \rightarrow k_iR_i - k_jR_j$ where k_i is non-zero

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We first place the system into a matrix system:

$$\begin{bmatrix}-1&3&7\\2&3&3\\4&-1&-6\\2&3&4\end{bmatrix} \xrightarrow{(-1)(R1)} \begin{bmatrix}1&-3&-7\\2&3&3\\4&-1&-6\\2&3&4\end{bmatrix} \xrightarrow{R2\to -2R1+R2} \begin{bmatrix}1&-3&-7\\0&9&17\\4&-1&-6\\2&3&4\end{bmatrix} \xrightarrow{R3\to -4R1+R3} \begin{bmatrix}1&-3&-7\\0&9&17\\0&11&22\\2&3&4\end{bmatrix} \xrightarrow{R4\to -2R1+R4}$$

$$\begin{bmatrix}1&-3&-7\\0&9&17\\0&11&22\\0&9&18\end{bmatrix} \xrightarrow{R2\leftrightarrow R4} \begin{bmatrix}1&-3&-7\\0&9&18\\0&11&22\\0&9&17\end{bmatrix} \xrightarrow{(1/9)R2} \begin{bmatrix}1&-3&-7\\0&1&2\\0&11&22\\0&9&17\end{bmatrix} \xrightarrow{R3\to -11R2+R3} \begin{bmatrix}1&-3&-7\\0&1&2\\0&0&0\\0&9&17\end{bmatrix} \xrightarrow{R4\to -9R2+R4}$$

$$\begin{bmatrix}1&-3&-7\\0&1&2\\0&0&0\\0&0&-1\end{bmatrix}$$

Here we see that the system has a pivot in each column (on the left hand side, the constant column which would be all 0 cannot possible have a pivot). Thus the set is linearly independent.

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why to row operations keep the same solutions set?

Why can we set up a matrix system to solve linear independence problems?

Why do we not have to consider the right hand side (the column of zeros) when setting up the matrix?

For Video Please click the link below:

Video