Find integers s and t such that $3451s + 7019t = 1$

Problem Solving - What are the terms/strategies I may need? What do I know?

Euclidean Algorithm for finding $\gcd(a, b)$

Start with finding $a = q_0 b + r_0$

Then continue to iterate $b = q_1 r_0 + r_1$

$r_0 = q_2 r_1 + r_2$

…

Continue until the remainder is 0, then we have that r_{n-1} is our GCD

Note, we found that the $\gcd(3451, 7019) = 1$

Using the extended Euclid algorithm, we can always find $as + bt = \gcd(a, b)$ by working backwards from the Euclid algorithm.

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

If we recall our work for the Euclidean Algorithm, we can use this to help answer this question:

$7019 = 2 \times 3451 + 117$ (We find $\frac{7019}{3451} = 2 + R117$)

$3451 = 29 \times 117 + 58$ (We find $\frac{3451}{117} = 29 + R58$)

$117 = 2 \times 58 + 1$ (We find $\frac{117}{58} = 2 + R1$)

$58 = 58 \times 1 + 0$ (We find $\frac{58}{1} = 58 + 0$)

Ignoring the last line, and working backwards, we can see that

$$1 = 117 - 2 \times 58$$

$$1 = 117 - 2 \times (3451 - 29 \times 117) = (59) \times 117 - 2 \times 3451$$

$$1 = (59) \times (7019 - 2 \times 3451) - 2 \times 3451 = (59) \times (7019) + (-120) \times 3451$$

Therefore we see that $s = -120, t = 59$

Find integers s and t such that $3451s + 7019t = 1$

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why does the Extended Euclid Algorithm work?

For Video Please click the link below:

Video