Determine if $(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) \in span\{(\dot{1},\dot{2},\dot{3},\dot{4},\dot{0}),(\dot{3},\dot{-1},\dot{2},\dot{1},\dot{0})\}$ over $\boldsymbol{Z}_5$

Problem Solving - What are the terms/strategies I may need? What do I know?

Definition of the span of vectors of all linear combinations of vectors $v_1, v_2, \dots, v_n \in \boldsymbol{V}$ (under a field **F**)

$$\mathrm{Span}\{\mathrm{v}_1, \dots, \mathrm{v}_n\} = \{c_1 v_1 + c_2 v_2 + \cdots + c_n v_n | c_1, c_2, \dots, c_n \in \boldsymbol{F}\}$$

Checking if a vector v is inside of a $\mathrm{Span}\{\mathrm{v}_1, \dots, \mathrm{v}_n\}$

Solving the system: $c_1 v_1 + c_2 v_2 + \cdots + c_n v_n = v$ to check if the system is consistent.

Vector space properties:

Vector addition means to add each corresponding entry.

Vector scalar multiplication means multiply all entries by the scalar.

Row operations:

Row replacement

Row swapping

Row scaling

Consistent Systems have no pivot in the last column when the matrix is in RREF.

Determine if $(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) \in span\{(\dot{1},\dot{2},\dot{3},\dot{4},\dot{0}),(\dot{3},\dot{-1},\dot{2},\dot{1},\dot{0})\}$ over $\boldsymbol{Z}_5$

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We set up a system to solve by considering:

$$(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) = c_1(\dot{1},\dot{2},\dot{3},\dot{4},\dot{0}) + c_2(\dot{3},\dot{-1},\dot{2},\dot{1},\dot{0})$$
$$(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) = (\dot{1}c_1,\dot{2}c_1,\dot{3}c_1,\dot{4}c_1,\dot{0}c_1) + (\dot{3}c_2,\dot{-1}c_2,\dot{2}c_2,\dot{1}c_2,\dot{0}c_2)$$
$$(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) = (\dot{1}c_1+\dot{3}c_2,\dot{2}c_1+\dot{-1}c_2,\dot{3}c_1+\dot{2}c_2,\dot{4}c_1+\dot{1}c_2,\dot{0}c_1+\dot{0}c_2)$$

Thus we must solve the system (by equating the entries):

$$\left[\begin{array}{cc|c} \dot{1} & \dot{3} & \dot{4} \\ \dot{2} & \dot{-1} & \dot{4} \\ \dot{3} & \dot{2} & \dot{3} \\ \dot{4} & \dot{1} & \dot{4} \\ \dot{0} & \dot{0} & \dot{0} \end{array}\right] \quad \begin{array}{r} \\ R2 \rightarrow R2-2R1: \\ R3 \rightarrow R3-3R1: \\ R4 \rightarrow R4-4R1: \\ \\ \end{array} \left[\begin{array}{cc|c} \dot{1} & \dot{3} & \dot{4} \\ \dot{0} & \dot{3} & \dot{1} \\ \dot{0} & \dot{3} & \dot{1} \\ \dot{0} & \dot{4} & \dot{3} \\ \dot{0} & \dot{0} & \dot{0} \end{array}\right] \quad \begin{array}{r} \\ R2(\dot{3}^{-1}): \\ R3(\dot{3}^{-1}): \\ R4(\dot{4}^{-1}): \\ \\ \end{array} \left[\begin{array}{cc|c} \dot{1} & \dot{3} & \dot{4} \\ \dot{0} & \dot{1} & \dot{2} \\ \dot{0} & \dot{1} & \dot{2} \\ \dot{0} & \dot{1} & \dot{2} \\ \dot{0} & \dot{0} & \dot{0} \end{array}\right] \quad \begin{array}{r} R1 \rightarrow R1-3R2: \\ \\ R3 \rightarrow R3-R2: \\ R4 \rightarrow R4-R2: \\ \\ \end{array} \left[\begin{array}{cc|c} \dot{1} & \dot{0} & \dot{3} \\ \dot{0} & \dot{1} & \dot{2} \\ \dot{0} & \dot{0} & \dot{0} \\ \dot{0} & \dot{0} & \dot{0} \\ \dot{0} & \dot{0} & \dot{0} \end{array}\right]$$

The matrix has no pivot in the last column and is thus consistent. This means $(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) \in span\{(\dot{1},\dot{2},\dot{3},\dot{4},\dot{0}),(\dot{3},\dot{-1},\dot{2},\dot{1},\dot{0})\}$.

We can verify this by noting that $(\dot{4},\dot{4},\dot{3},\dot{4},\dot{0}) = \dot{3}(\dot{1},\dot{2},\dot{3},\dot{4},\dot{0}) + \dot{2}(\dot{3},\dot{-1},\dot{2},\dot{1},\dot{0})$

Determine if $(\dot{4}, \dot{4}, \dot{3}, \dot{4}, \dot{0}) \in span\{(\dot{1}, \dot{2}, \dot{3}, \dot{4}, \dot{0}), (\dot{3}, -\dot{1}, \dot{2}, \dot{1}, \dot{0})\}$ over $\boldsymbol{Z}_5$

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Consistent Systems have no pivot in the last column when the matrix is in RREF (can you prove this?)

For Video Please click the link below:

Video