For each of the transformations $T: \mathbb{R}^2 \to \mathbb{R}^2$ below, show that T is not linear:

a)
$$T(x,y) = (xy,y)$$

b)
$$T(x,y) = (e^x, e^y)$$
 c) $T(x,y) = (1,0)$

c)
$$T(x,y) = (1,0)$$

Problem Solving - What are the terms/strategies I may need? What do I know?

Definition of a linear map:

$$T(u + v) = T(u) + T(v)$$
$$T(cu) = c(T(u))$$

Proving T is not linear:

Finding a counter example that shows one of the two properties fail.

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

a) Often when coming up with a counter example, we avoid using 0 or 1. In this case, let us try x = 2, y = 3 We try scalar multiplication first (although you could show addition as well):

$$T(3,2) = (6,2)$$

 $T(5(3,2)) = T(15,10) = (150,10) \neq 5(6,2) = 5(T(3,2))$

Thus, this is not a linear transformation.

b)
$$T(3,2) = (e^3, e^2)$$

 $T(5(3,2)) = T(15,10) = (e^{15}, e^{10}) \neq 5(e^3, e^2) = 5(T(3,2))$

Thus, this is not a linear transformation.

c)
$$T(3,2) = (1,0)$$

 $T(5(3,2)) = T(15,10) = (1,0) \neq 5(1,0) = 5(T(3,2))$

Thus, this is not a linear transformation.

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Can you find disproofs by contradicting the sum property instead?

For Video Please click the link below:

<u>Video</u>