Solve the following system (when consistent) and write the solution in parametric form under Z_3

$$\left[\begin{array}{ccccc|c} 2 & 2 & 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 & 2 & 2 \end{array}\right]$$

Problem Solving - What are the terms/strategies I may need? What do I know?

- Definition for a matrix in RREF:
  - a) All leading entries are 1
  - b) Leading entries go down and to the right
  - c) all zero rows are at the bottom
  - d) All entries above and below a leading 1 are 0.

- Strategy for getting to RREF to solve a system:
  1) Choose a leading non-zero row to be your pivot (easiest number possible)
  2) Switch this to the first available row without a pivot.
  3) Row reduce below the pivot so that all entries are zero below
  4) Continue steps 1-3 until you cannot choose any more leading non-zero entries.
  5) Start with the last pivot and eliminate all entries above the pivot so they are zero.
  6) Repeat step 5 for all pivots
  7) Divide all rows so that the leading pivots are 1.
  8) Once in RREF, write out the equation in parametric form (solve for each variable) by identifying free variables.
  9) Once you have parametric form, you can store the solution in a vector by considering $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Z_p operations:

a + b = (a + b) mod p (that is the remainder when dividing (a+b) by p

ab = (ab) mod p (that is the remainder when dividing (ab) by p

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

$$\left[\begin{array}{ccccc|c} 2 & 2 & 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 & 2 & 2 \end{array}\right] R_2 \leftrightarrow R_1 \quad \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 2 & 2 & 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 & 2 & 2 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_1 \end{array} \quad \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 & 0 & 2 \end{array}\right] R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] R_2 \times 2 \quad \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] R_1 \to R_1 - R_2 \quad \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right]$$

Here we should see that x_4 and x_5 do not have pivots, thus they are free variables. Let $x_4 = s$ and $x_5 = t$ and we get:

$x_1 = 2 - 2s - t$

$x_2 = 1$

$x_3 = 1 - s - t$

$x_4 = s$

$x_5 = t$

Where $s, t \in Z_3$

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

- How do you get a matrix to RREF?
- Why is having a matrix in RREF so helpful?
- Why are free variables easy to identify when the matrix is in RREF?
- Why do we avoid division when dealing with Mod operations?
- How many different solutions are there for this question?

For Video Please click the link below:

Video