Prove the following is not a subspace $S = \{f(x) \in P_4(R) \text{ with } f(1) = 2\}$

Problem Solving - What are the terms/strategies I may need? What do I know?

Subspace Test:

The space is a subset of a known vector space.

The zero vector is in the space.

The space is closed under addition.

The space is closed under scalar multiplication

Disproving a space:

Showing that one of the properties fails.

Prove the following is not a subspace $S = \{\mathrm{f}(x) \in P_4(R) \text{ with } \mathrm{f}(1) = 2\}$

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We note that all subspace test properties will fail (except for being a subset), so we will disprove one of them (you could easily disprove it using any of the 3)

Zero vector Disproof:

The zero vector in $P_4(R)$ is $\mathrm{z(x)} = 0 + 0x + 0x^2 + 0x^3 + 0x^4$

However, $z(1) = 0$ and so $z(x)$ is not in the set S.

Thus we have that S is not a subspace of $P_4(R)$.

(Based off of what we know about adding polynomials if f and g are in S, then (f+g)(1) = f(1) + g(1) = 4 which disproves the addition property)

(Based off of what we know about coefficients for polynomials if f is in S, and we have k be 0 which is a real number, then kf is the zero vector which is not in S which disproves the scalar multiplication property)

Prove the following is not a subspace $S = \{\mathrm{f}(x) \in P_4(R) \text{ with } \mathrm{f}(1) = 2\}$

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why does the subspace test work?

For Video Please click the link below:

Video