We note that for Multiplicative Identity, we impose $1 \neq 0$. If we relaxed this to allow $1 = 0$, prove that there is only one element in F. Then explain what the multiplicative inverse of 0 would be.

Problem Solving - What are the terms/strategies I may need? What do I know?

Field axioms: A *field* is a set F with two operations, denoted $+$ and $\cdot$, that satisfy each of the following properties:

(A1) For all $x, y, z \in F$, $(x + y) + z = x + (y + z)$.

(A2) For all $x, y \in F$, $x + y = y + x$.

(A3) There exists an element $0 \in F$ such that for all $x \in F$ we have $x + 0 = x$.

(A4) For each $x \in F$, there exists an element $-x \in F$ such that $x + (-x) = 0$.

(M1) For all $x, y, z \in F$, $(xy)z = x(yz)$.

(M2) For all $x, y \in F$, $xy = yx$.

(M3) There exists an element $1 \in F$ such that $1 \neq 0$ and for all $x \in F$ we have $x \cdot 1 = x$.

(M4) For each $x \in F$, there exists an element $x^{-1} \in F$ such that $xx^{-1} = 1$.

(DL) For all $x, y, z \in F$, $x(y + z) = xy + xz$.

Theorem for Fields:

$$0 \cdot x = 0$$

We note that for Multiplicative Identity, we impose $1 \neq 0$**.** If we relaxed this to allow $1 = 0$**,** prove that there is only one element in F. Then explain what the multiplicative inverse of 0 would be.

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

Let 1 = 0, and let F be a field, consider $x \in F$, we then show that $x = 0$ using our field axioms and/or theorems. This would show that there is indeed only one element in F.

Let $x \in F$:

$$
\begin{aligned}
&x \\
&= (1)(x) && \text{(M3)} \\
&= (0)(x) && \text{(Assuming } 1 = 0 \text{ are the same element)} \\
&= 0 && \text{(Theorem Zero Property)}
\end{aligned}
$$

Thus we conclude that $x = 0$. This means that $\{0\} = F$

$$
\begin{aligned}
\text{In this case we see that } (0)(0) &= 0 && \text{(Theorem Zero Property)} \\
&= 1 && \text{(Assuming } 0 = 1\text{)}
\end{aligned}
$$

Thus 0 is its own multiplicative inverse in this case.

We note that for Multiplicative Identity, we impose $1 \neq 0$. If we relaxed this to allow $1 = 0$, prove that there is only one element in F. Then explain what the multiplicative inverse of 0 would be.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Can you prove that 0x = 0 using axioms of a field?

Why does starting with $x \in F$ then showing $x = 0$ proves that there is only one element in F?

For Video Please click the link below:

Video