Is the following linearly independent. If not, explain if it is trivially dependent:

$$\left\{ \begin{bmatrix} -1\\2\\4\\2 \end{bmatrix}, \begin{bmatrix} 3\\3\\-1\\3 \end{bmatrix}, \begin{bmatrix} 7\\3\\-6\\4 \end{bmatrix} \right\}$$

Problem Solving - What are the terms/strategies I may need? What do I know?

Definition of Linearly Independent:

- A set V = $\{v_1, v_2, ..., v_n\}$ is linearly independent when the homogenous system: $c_1v_1 + c_2v_2 + ... + c_nv_n = 0$ only has the trivial solution

Solutions to Homogenous Systems:

- A homogenous matrix system only has the trivial solution iff it has a pivot in every column except for the constant column.

Row Operations:

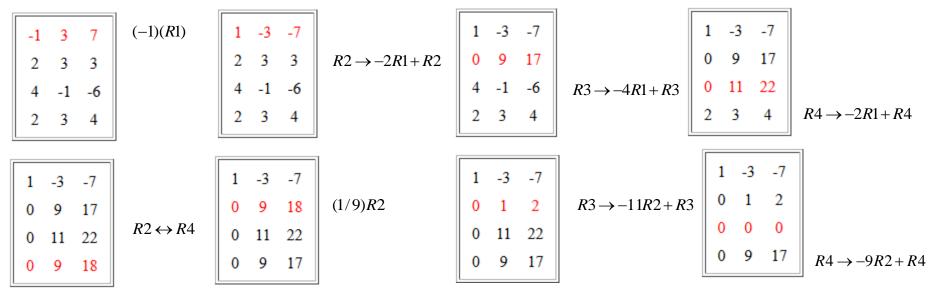
- Swap rows
- Scale a row by a non-zero number
- Row replace using $R_i \rightarrow k_i R_i k_i R_i$ where k_i is non-zero

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We first place the system into a matrix system:



Here we see that the system has a pivot in each column (on the left hand side, the constant column which would be all 0 cannot possible have a pivot). Thus the set is linearly independent.

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

- Why to row operations keep the same solutions set?
- Why can we set up a matrix system to solve linear independence problems?
- Why do we not have to consider the right hand side (the column of zeros) when setting up the matrix?

For Video Please click the link below:

<u>Video</u>