Let V be a vector space over a field F. Let F_1 be a field such that $F_1 \subseteq F$. Prove that V is a vector space over F_1.

Problem Solving - What are the terms/strategies I may need? What do I know?

Proof by contradiction:

Assume that the result is false and arrive at a contradiction in the conditions.

Definition of a vector space over a field F:

V is a *vector space* over F if the following axioms are satisfied for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$, $b, c \in F$

(A1) $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$.

(A2) $\mathbf{u}+\mathbf{v} = \mathbf{v}+\mathbf{u}$.

(A3) There exists an element $\mathbf{0} \in V$ such that for all $\mathbf{u} \in V$ we have $\mathbf{u}+\mathbf{0} = \mathbf{u}$.

(A4) For each $\mathbf{u} \in V$, there exists an element $-\mathbf{u} \in V$ such that $\mathbf{u}+(-\mathbf{u}) = \mathbf{0}$.

(S1) $c(\mathbf{u}+\mathbf{v}) = c\mathbf{u}+c\mathbf{v}$

(S2) $(b+c)\mathbf{u} = b\mathbf{u}+c\mathbf{u}$

(S3) $(bc)\mathbf{u} = b(c\mathbf{u})$

(S4) $1\mathbf{u} = \mathbf{u}$

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

Let us assume (for contradiction) that V is not a vector space over F_1:

$\rightarrow$ There is at least one axiom of a vector space that fails over F_1

$\rightarrow$ There is at least one axiom of a vector space that fails over F (as $F_1 \subseteq F$, thus choosing the constants and/or vectors that cause failure in F_1 could be the chosen candidates to show failure in F)

$\rightarrow$ V is not a vector space over a field F

$\rightarrow$ We arrive at a contradiction, thus V is a vector space over F_1

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Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

- Why does a proof by contradiction help here?

For Video Please click the link below:

Video