Let A $= \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$, $B_1 = \left\{ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. Find $[A]_{B1}$ and $[A]_{B2}$

Problem Solving - What are the terms/strategies I may need? What do I know?

Definition of a standard $[v]_B$ when $B = \{v_1, v_2, \dots, v_n\}$

When we determine $v = c_1 v_1 + c_2 v_2 + \cdots + c_n v_n$ we define $[v]_B = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{bmatrix}$.

Setting up the linear combinations and solving linear systems.

Let A $= \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$, $B_1 = \left\{ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. Find $[A]_{B_1}$ and $[A]_{B_2}$

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

To find $[A]_{B_1}$ we first want to write:

$$A = c_1 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} + c_4 \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} c_1 & -c_1 + c_2 \\ -c_1 + c_2 + c_3 & c_1 + c_2 - c_3 - c_4 \end{bmatrix}$$

Here we see that we require $c_1 = 1, c_2 = -2 + 1 = -1, c_3 = 3 + 1 + 1 = 5$, and $c_4 = 5 - 1 + 1 + 5 = -10. \therefore [A]_{B_1} = \begin{bmatrix} 1 \\ -1 \\ 5 \\ -10 \end{bmatrix}$

To find $[A]_{B_2}$ we first want to write:

$$A = c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + c_4 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$$

Here we see that we require $c_1 = 1, c_2 = -2, c_3 = 3$, and $c_4 = 5. \therefore [A]_{B_2} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 5 \end{bmatrix}$

Let $\mathrm{A} = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}$, $B_1 = \left\{ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. Find $[A]_{B_1}$ and $[A]_{B_2}$

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why are coordinate vectors helpful?

What can we do if we cannot spot the linear combination by inspection?

For Video Please click the link below:

[Video]