Suppose A is a square matrix, and B is a matrix in reduced row echelon form that is row equivalent to A (that is, B is the RREF of A). Prove that if A is singular, then the last row of B is a zero row.

Problem Solving - What are the terms/strategies I may need? What do I know?

RREF Properties:

- Pivots are 1
- Pivots go down and to the right
- Everything above and below a pivot is 0
- Zero rows are at the bottom

Singular Matrices:

A non-singular matrix is a matrix that is square and only has the trivial solution to LS(A,0)

A non-singular matrix is a matrix that has RREF as the identity matrix. (Theorem)

Row Operations Are reversible (Previous exercise)

Proof by Contraposition

Instead of proving $A \to B$ prove $\sim B \to \sim A$

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Steps & Process – Try to answer the question writing in many steps to avoid small errors.

We will prove this by considering the contrapositive:

If B does not have the last row as a zero row, then A is nonsingular.

If B does not have the last row as a zero row

→ B does not have any zero rows (definition of RREF, all zero rows must be at the bottom. So if B does not have a zero row at the bottom, it does not have any zero rows)

→ B has a pivot in every row (as every row must have a leading one or be a zero row)

→ B has a pivot in every column (Since A is square, B is also square as the RREF matrix is the same size as the original matrix)

→ $B = I_n$ (as by definition of RREF, pivots go down and to the right. If it was not I_n then there would be two adjacent columns where pivots go up to the right (can you prove this?))

→ A is nonsingular by theorem (as the RREF matrix is I_n)

Thus we have proven the claim using the contrapositive.

Suppose A is a square matrix, and B is a matrix in reduced row echelon form that is row equivalent to A (that is, B is the RREF of A). Prove that if A is singular, then the last row of B is a zero row.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why is a non-singular matrix (LS(A,0)) equivalent to A having RREF as the identity?

Why does the contrapositive help?

Why is it that a matrix in RREF with a pivot in every row and column must be I_n?

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Video