Suppose that A is a nonsingular matrix and A is row equivalent to the matrix B. Prove that B is nonsingular.

Problem Solving - What are the terms/strategies I may need? What do I know?

RREF Properties:

- Pivots are 1
- Pivots go down and to the right
- Everything above and below a pivot is 0
- Zero rows are at the bottom

Singular Matrices:

A non-singular matrix is a matrix that is square and only has the trivial solution to LS(A,0)

A non-singular matrix is a matrix that has RREF as the identity matrix. (Theorem)

Row Operations Are reversible (Previous exercise)

Definition of row equivalent:

There is a sequence of row operations from A to B (or from B to A)

Suppose that A is a nonsingular matrix and A is row equivalent to the matrix B. Prove that B is nonsingular.

Steps & Process – Try to answer the question writing in many steps to avoid small errors.

Let A be a non singular matrix, then:

$\rightarrow$ A has an RREF matrix that is I_n (Theorem)

$\rightarrow$ There is a sequence of row operations $(R_1', R_2', \ldots, R_p')$ that $A \rightarrow R_1' \rightarrow R_2' \rightarrow \cdots \rightarrow R_p'$ gives I_n

$\rightarrow$ Since A is row equivalent to B, There is a sequence of row operations $(R_1, R_2, \ldots, R_k)$ that $B \rightarrow R_1 \rightarrow R_2 \rightarrow \cdots \rightarrow R_k$ gives A

$\rightarrow$ There is a sequence of row operations $(R_1, R_2, \ldots, R_k, R_1', R_2', \ldots, R_p')$ that $B \rightarrow R_1 \rightarrow R_2 \rightarrow \cdots \rightarrow R_k \rightarrow R_1' \rightarrow R_2' \rightarrow \cdots \rightarrow R_p'$ gives I_n

$\rightarrow$ B has RREF matrix I_n

$\rightarrow$ B is non-singular (Theorem)

Suppose that A is a nonsingular matrix and A is row equivalent to the matrix B. Prove that B is nonsingular.

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why is a non-singular matrix (LS(A,0)) equivalent to A having RREF as the identity?

Why are all row operations reversible (can you prove it)?

For Video Please click the link below:

Video