Solve $z^4 = 2 - 2\sqrt{3}i$

Problem Solving - What are the terms/strategies I may need? What do I know?

Converting a complex number to polar form:

Plot the complex number on the complex plane

Find the norm of the complex number $|a + bi| = \sqrt{a^2 + b^2}$

Identify the angle from standard position

Write the complex number as $r(\cos\theta + i\sin\theta)$

Find the first root of unity:

If $z^n = r(\cos\theta + i\sin\theta)$ then one solution would be

$$z_1 = r^{\frac{1}{n}}\ (\cos\frac{\theta}{n} + i\sin\frac{\theta}{n})$$

Find the remaining roots of unity by calculating:

$$z_2 = r^{\frac{1}{n}}\ (\cos\frac{\theta+2\pi}{n} + i\sin\frac{\theta+2\pi}{n})$$

$$z_3 = r^{\frac{1}{n}}\ (\cos\frac{\theta+4\pi}{n} + i\sin\frac{\theta+4\pi}{n})$$

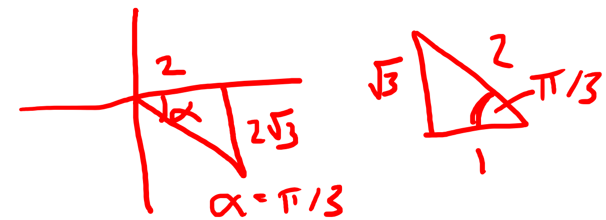
… continue the process until you have found n solutions.

Special triangles

Solve $z^4 = 2 - 2\sqrt{3}i$

Steps & Process

We first convert the complex number into polar form:



$a = 2, \quad b = -2\sqrt{3}$ $\rightarrow |a + bi| = \sqrt{2^2 + (2\sqrt{3})^2} = 4$

$\rightarrow \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

This means $2 - 2\sqrt{3}i = 4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

Thus if $z^4 = 4\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$ we get:

$$z_1 = 4^{1/4}\left(\cos\left(\left(\frac{5\pi}{3}\right) \div 4\right) + i\sin\left(\left(\frac{5\pi}{3}\right) \div 4\right)\right) = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$$

For the remaining roots (3 more of them) we can find them by adding 2π to the original argument before applying the theorem for finding z_1. In this case we get:

$$z_2 = \sqrt{2}\left(\cos\left(\left(\frac{5\pi}{3} + 2\pi\right) \div 4\right) + i\sin\left(\left(\frac{5\pi}{3} + 2\pi\right) \div 4\right)\right) = \sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$
$$z_3 = \sqrt{2}\left(\cos\left(\left(\frac{5\pi}{3} + 4\pi\right) \div 4\right) + i\sin\left(\left(\frac{5\pi}{3} + 4\pi\right) \div 4\right)\right) = \sqrt{2}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$$
$$z_4 = \sqrt{2}\left(\cos\left(\left(\frac{5\pi}{3} + 6\pi\right) \div 4\right) + i\sin\left(\left(\frac{5\pi}{3} + 6\pi\right) \div 4\right)\right) = \sqrt{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$$

Solve $z^4 = 2 - 2\sqrt{3}i$

Solidify Understanding – Explain why the steps makes sense by connecting to math you know.

Why can does converting to polar form help in this question?
Why does finding the first root of unity work?
Why does finding the remaining roots of unity work (tougher question)?

For Video Please click the link below:

Video