Make it Pop: The Physics of Popping Popcorn

Bhuvan Manojh (261048662), Mariah Zeroug (261054280), Emilia Vlahos (260983702)

December 6, 2023

Abstract

We seek to model a distribution of radii and popping times of popcorn through variation of the kernel's parameters. This will be done by modelling thermal conduction for a popcorn kernel using the spherical heat equation and finite differencing to determine the popping temperature, as well as the radius expansion from the pressure differential during the popping time. It was found that on average, with a sample size of 1000 kernels, the kernel has a mean final radius of 8.07mm and a mean popping time of 87.3s. This popping time is slightly lower than values found in literature, however many assumptions were made in our model to make it ideal which contributed to the low popping time. Overall, our ideal model with its underlying physics showcases the effects of the parameters of the kernel on the popping process.

1 Introduction

Have you ever prepared popcorn and wondered why there is such a long wait between the initial and final 'pop'? Maybe not, but you did not grow up with social media, so your attention span is likely much longer than ours.

In this project, we will examine how variations in kernel composition affect popping time and radial expansion when cooking popcorn. We begin by modelling the thermal conduction of heat into the kernel to determine when the kernel becomes sufficiently hot to pop. We calculate the kernel's expansion through its pop from its change in pressure during the pop. We then generate a sample of kernels with a variety of compositions/thermal properties based on experimental data to generate a theoretical distribution of pop times and popcorn sizes. We utilize many numerical methods including finite differencing, the Backwards Euler Method, non-uniform random generators, and curve-fitting.

2 Problem Tackled and Theory

2.1 Critical Temperature

The popcorn kernel comprises a hard outer hull or pericarp encasing a starchy endosperm. The endosperm is made up of around 15% moisture content. This moisture content present in the kernel will affect various parameters, primarily the dimensions and shape of the kernel. We can ignore the changes to the sphericity of the kernel as we are assuming it to be a perfect sphere. As such, the density of the kernel will decrease with an increasing moisture level according to a linear relation:

$$\rho_c = 1398 - 10M_c$$

where density is in units of kgm^{-3} and M_c is the moisture level in percentage.

When popcorn cooks, the temperature increase within the endosperm causes this moisture to partially evaporate. Water vapour within the kernel creates an internal pressure which increases with temperature. Most literature on this topic models the internal pressure at a given temperature to be water's temperature-dependent vapour pressure [1]. Once this pressure exceeds the critical pressure set by the the pericarp's hull strength, the pericarp will burst. Estimates of the critical pressure can be found through the equation:

$$P_c = \frac{2t_h}{R} \cdot 10\text{MPa}, \tag{1}$$

where t_h is the hull thickness and R is the kernel radius. We solve for the critical temperature at which the kernel pops from the Clausius–Clapeyron relation:

$$T_c = \frac{T_0}{1 - R_c T_0 / L_v M \ln(P_c/P_0)}, \tag{2}$$

where $P_0 = 1$ atm, $T_0 = 373$K are the standard boiling conditions of water, $R_c = 8.3$ J/K$\cdot$ mol is the ideal gas constant, $M = 18$ g/mol is the molar mass of water, and $L_v = 2.3 \times 10^6$ J/kg is the molar mass of water.

Once critical temperature is reached within the kernel, the pericarp bursts and all the heated moisture turns into steam. This cools down the kernel, which is why it doesn't burn. As the steam ruptures the pericarp, the starch inside the kernel expands into bubble-like structures and forms the popcorn structure we all know and love.

2.2 Conductive Heat Transfer

Our primary objective becomes modelling the heat transfer into a kernel to determine when the critical temperature is reached. This is a complicated endeavour since there are many contributing factors to the heat in/out of a kernel. For simplicity, we model popcorn cooked in a pot with oil. We assume that the kernel is fully coated in a layer of hot oil at any given time and that the oil conducts heat into the kernel evenly. Intuitively, the oil at the bottom of the kernel that is in contact with the pot is at a higher temperature than the oil coating the top of the kernel that may radiate away heat into the pot. Furthermore, we do not expect oil to stay coated to the kernel, but rather to drip down to the bottom of the pot. These observations suggest that heat is not transferred evenly into the kernel. However, we assert that regular mixing of the cooking popcorn will continuously change the orientation of the kernel resulting in heating that is on average symmetrically distributed across the pericarp. We, therefore, model the heating of the kernel by modelling the conductive heat transfer into the kernel from the oil.

Note that within the pot there are likely many forms of heat transfer (advective, convective, etc.), However, we consider only the internal heat transfer within the kernel, which is primarily conductive.

Fourier's law of conduction equation is defined as follows:

$$\mathbf{q} = -k\nabla T$$

where k is the thermal conductivity and q describes the heat flux at a given position and time. The total thermal energy density within a material is

$$E = C_p \rho T,$$

where C_p is the material's specific heat capacity, and ρ is its mass density. By conservation of energy, the change in thermal energy/volume can be expressed as both:

$$\frac{\partial E}{\partial t} = C_p \rho \frac{\partial T}{\partial t}$$
$$-\nabla q = k\nabla^2 T.$$

This stems from the fact that the rate of change in the energy contained in some volume of material is opposite the rate at which energy leaves said volume. Equating these rates of change, we recover the heat equation:

$$C_p \rho \frac{\partial T}{\partial t} = k\nabla^2 T$$
$$\frac{\partial T}{\partial t} = \frac{k}{C_p \rho}\nabla^2 T$$
$$\therefore \frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

where $\alpha = k/C_p\rho$ is the thermal diffusivity constant.

For simplicity, we will model the kernel as a perfect sphere. The heat equation in spherical coordinates is:

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial T}{\partial r}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2 \sin\phi}\frac{\partial}{\partial \phi}(\sin\phi \frac{\partial T}{\partial \phi}))$$

Since we assume that the kernel heats evenly from all side, we expect no the thermal differential along the angles θ and ϕ. The second and third terms of the Laplace operator are therefore zero

and the heat equation reduces to the 1D equation

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{\alpha}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) \\ &=\alpha(\frac{2}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2})\end{aligned} \tag{3}$$

We will solve this form of the heat equation numerically to model the movement of heat through a spherical popcorn kernel. When the interior of the kernel reaches the critical temperature, the kernel pops.

2.3 Initial Parameters

Factors such as the kernel size, hull thickness and thermal conductivity all affect the time and temperature at which the kernel pops. We randomly generate a sample of kernels by varying these parameters according to their experimentally determined ranges. We then study the resulting distribution of stop times. The numerical distributions of the initial parameters are given in Section 3.2.

2.4 Radius Expansion

We finally model the expansion of the radius during the pop. We follow [2], and treat the expansion as an adiabatic expansion with and initial pressure corresponding to the kernel's critical temperature and the final pressure corresponding to atmospheric pressure. We can therefore model the kernel's change in volume (and radius) according to

$$P_i V_i^\gamma = P_f V_f^\gamma, \tag{4}$$

where $\gamma = 1.3$ is the adiabatic index of water vapour.

3 Coding Methods

3.1 Temperature Distribution of Kernel

The first section of our code implements the backwards Euler method to evaluate the thermal evolution of the layers of the kernel, as described in Eq. 3. We begin by finite differencing the RHS of Eq. 3 to transform the PDE into a system of coupled ODEs.

We first substitute $\frac{\partial T}{\partial r} = \frac{T_{i+1}-T_{i-1}}{2\Delta r}$ and $\frac{\partial^2 T}{\partial r^2} = \frac{T_{i+1}-2T_i+T_{i-1}}{(\Delta r)^2}$ into Eq. 3:

$$\begin{aligned}\frac{dT_i}{dt} &= \alpha\left(\frac{2}{r_i}\left(\frac{T_{i+1}-T_{i-1}}{2\Delta r}\right) + \left(\frac{T_{i+1}-2T_i+T_{i-1}}{(\Delta r)^2}\right)\right) \\ &= \alpha\left(\left(\frac{1}{\Delta r^2} - \frac{1}{r_i \Delta r}\right)T_{i-1} - \frac{2}{\Delta r^2}T_i + \left(\frac{1}{\Delta r^2} + \frac{1}{r_i \Delta r}\right)T_{i+1}\right)\end{aligned} \tag{5}$$

The above provides a system of coupled ODE's describing the rate of change in temperature, T_i at a given layer of the kernel radius r_i. To solve this system, we must consider the temperature evolution at the boundaries $r = R$ and $r = 0$. We assume that the exterior of the kernel is held in thermal equilibrium with the hot oil which remains at a constant temperature near its smoke point (460K). The change in temperature at the external boundary is therefore zero. The interior boundary is slightly more complicated since the temperature at the center of the kernel is neither known nor held fix; it is the parameter we wish to solve for. We circumvent this by extending the array of radii past the origin and applying the transformation from $r \longrightarrow -r$, as suggested in [3]. We must be careful, since the Laplace operator in spherical coordinates is derived for $r > 0$. Through symmetric reasoning, we expect the T_{i+1} and T_{i-1} coefficients in Eq. 5 to become inverted in this negative region (we are iterating towards r=0 rather than away from it). Fortunately, this arises naturally for negative r, and our equation need not be modified. We apply the same boundary condition to $r = -R$ as to $r = R$ and we obtain the following system of equations.

$$\begin{bmatrix}\dot{T}_0\\ \dot{T}_1\\ \dot{T}_2\\ \dot{T}_3\\ \vdots\\ \dot{T}_n\end{bmatrix} = -\alpha \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & 0 \\ -\left(\frac{1}{\Delta r^2} - \frac{1}{r_1 \Delta r}\right) & \frac{2}{\Delta r^2} & -\left(\frac{1}{\Delta r^2} + \frac{1}{r_1 \Delta r}\right) & 0 & \dots & 0 \\ 0 & -\left(\frac{1}{\Delta r^2} - \frac{1}{r_2 \Delta r}\right) & \frac{2}{\Delta r^2} & -\left(\frac{1}{\Delta r^2} + \frac{1}{r_2 \Delta r}\right) & \dots & 0 \\ 0 & 0 & -\left(\frac{1}{\Delta r^2} - \frac{1}{r_3 \Delta r}\right) & \frac{2}{\Delta r^2} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix}T_0\\ T_1\\ T_2\\ T_2\\ \vdots\\ T_n\end{bmatrix}$$

The above is a system of the form $\dot{\mathbf{T}} = -C\mathbf{T}$ which we solve using the Backwards Euler Method. We assume that at time 0, the room temperature kernel is brought into contact with the oil, and the boundary conditions are applied. We then update temperature at each timestep according to

$$\mathbf{T}_{j+1} = (1 + C\Delta t)^{-1}\mathbf{T}_j. \tag{6}$$

3.2 Distribution of Parameters

We model the distribution of parameters based on ranges found in literature. The parameters varied are the hull thickness h_t in μm [1], thermal conductivity constant in W/mK k [4] and the initial radius $r_{initial}$ in mm. The density ρ in kgm^3 is also varied using the linear relation found to the moisture content discussed in Section 2.1 [5]. With a value of 1390 for the density and a moisture level of 15%, we get a density of 1240. We give an uncertainty of 10% to the moisture to get a value of $(15 \pm 1.5)\%$ and this gives us a range of density values. The distribution for these parameters is given below in Table 1.

Table 1: Distribution of Parameters

ρ (kg/m^3)	h_t (μm)	k (W/mK)	$r_{initial}$ (mm)
1240 $\pm$ 15	160 $\pm$ 40	0.1745 $\pm$ 1.25	5 $\pm$ 1

Each parameter is then organized in a Gaussian distribution. We then randomly select a value from each parameter to model a varied range of 1000 popcorn kernels. Then, using Eq. 1 and the Clausius-Clapeyron relation (Eq. 2), we can calculate the critical pressure and temperature, i.e. the pressure and temperature at which a will pop.

We then made a histogram of the final radii of popcorn and of popping times and fit a Gaussian to them. We will discuss those more in Sec. 4.1

4 Results/Analysis

4.1 Final Radius and Popping Time Histograms

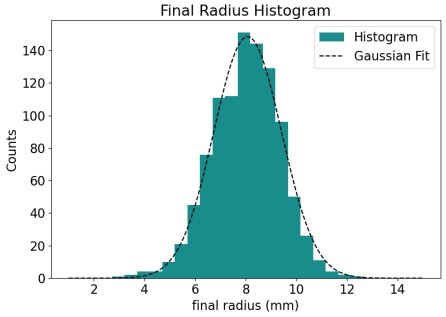

Figure 1: The final radius histogram of 1000 popcorn kernels with a Gaussian fit of the data.

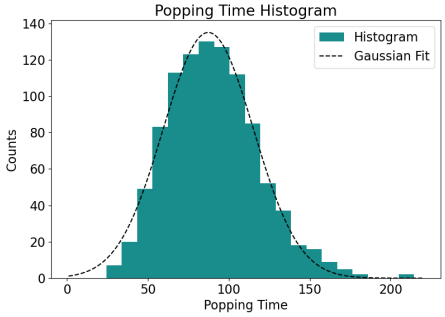

Figure 2: The popping time histogram of 1000 popcorn kernels with a Gaussian fit of the data.

After implementing the backwards Euler method using a timestep of 0.2, 1000 kernels and 500 layers within one kernel, we first plotted a histogram of the final radius of popcorn in Fig. 1. We can

see that the radii seem normally distributed. From the Gaussian fit, we obtain a mean of 8.07mm with a standard deviation of 1.34mm. We also plotted a histogram of the popping time along with a Gaussian fit shown in Fig 2. From this Gaussian fit, we extract a mean of 87.3s and a standard deviation of 27.9s. We also found that 1% of kernels do not end up popping. Note that these values will slightly change from run to run since we are selecting random parameters each time.

5 Code Tests

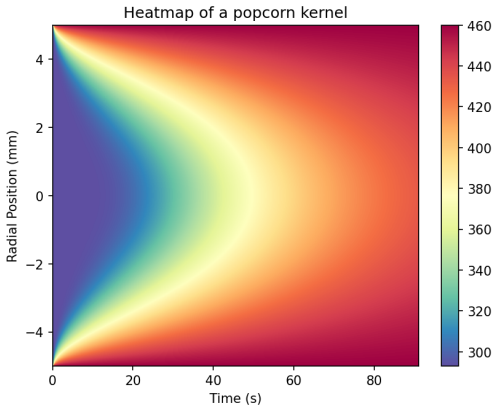

Figure 3: Time evolution of thermal gradient across the diameter of a cooking popcorn kernel

To verify the time-dependent temperature solution, we first tested the code on a single kernel with all inputs at their average value. We plotted the temperature gradient in the kernel at each timestep on a color map (Fig. 3). We see that initially the exterior of the kernel is hot and the interior is cool. As time progresses, the outer layers get hot first, followed by the inner layers, as the kernel approaches thermal equilibrium with the boundary. This behaviour is exactly what we expect from a sphere heated at its surface. Note also that the heating is symmetric between the 'positive' and 'negative' radii. This is necessary since 'positive' and 'negative' can be defined for any orientation.

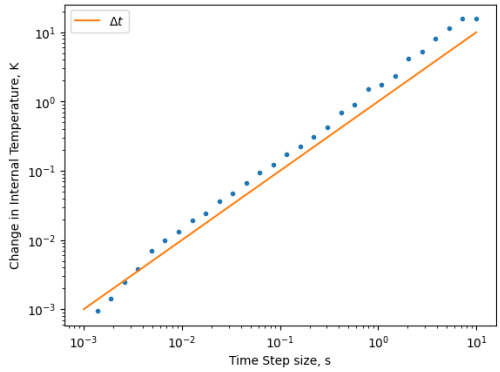

Figure 4: Difference in interior (temperature after 100s) for increased timesteps

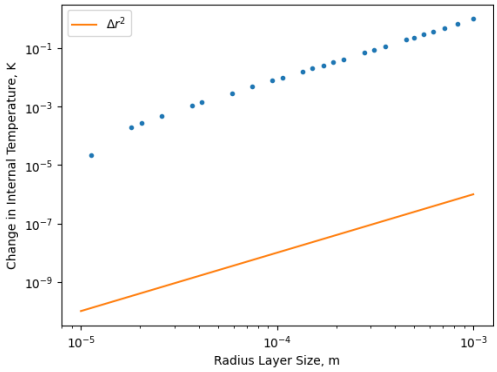

Figure 5: Difference in interior (temperature after 100s) for increased layer thickness

We further tested the solver by examining how error scales with step size in both time and radius. Since we have no known true value to test error from, we calculated the temperature at the center of the kernel after 100s for a range of step sizes. We assumed that the smallest step size corresponds to the least error, and plotted the difference from this temperature value for at each

step size. Note that while this is not to scale, it should give an approximate of the error scaling. We find that when varying timestep, the error scales with Δt and when varying layer thickness, the error scales with Δr^2. This follows from the known error scalings of the Backwards Euler Method ($\sim h$) and the Central Finite Differencing Method ($\sim h^2$).

6 Discussion

As seen in Figure 1 and 2, both parameters reasonably follow a normal distribution. The number of kernels we used for this model is quite high at $N = 1000$. In reality, one serving of popcorn contains around 100-200 kernels. If we were to conduct our model with this number of kernels, we would get a very similar mean and standard deviation. This measurement of the mean might be slightly different since we generate some of our parameters with a Gaussian spread, causing slight fluctuations in either values.

If we look at the distribution for the popping time, we notice that the descent after the mean is more spread out than the spread before the mean, which follows a slightly steeper ascent. We can look at the effect of critical temperature on the popping time for this slight quirk. As the critical temperature increases and approaches the oil temperature, the popping time is increased. Thus, for deviations in the critical temperature that are closer to the oil temperature, we can observe a clear one-sided bias for radii greater than the mean of the popping temperature, since the popping times will be a lot higher for this case. In addition, this histogram does not include popping times of 0 seconds, which represent the unpopped kernels. Thus, from these 2 effects, we notice that our data follows the Gaussian distribution well when it's around one standard deviation less than the mean, but for 1 and more standard deviations greater than the mean, the popping times are more spread out.

Experimental data from [6] suggests that the average popcorn kernel takes between 90s and 130s to pop. Our mean popping time of 87.3s falls on the low end of this range. While the popping times that we observe are not impossible, we realistically expect higher pop times (based in part on our lived experience making popcorn). We note that the boundary conditions applied in our thermal evolution model are highly simplified. In reality, the kernel loses and gains thermal energy in unpredictable ways, and cannot be treated as a closed system with fixed boundary conditions. In fact, not even the pot can be treated as a closed system, since small amounts of steam and heat are typically allowed to escape. This is likely the reason that we calculate relatively short pop times.

Similarly, we expect the typical radius of the popped kernel to be larger than the mean value of 8.07 mm. Treating the kernel's pop as an adiabatic expansion fails to account for the sudden evaporation of remaining water contents, nor the complex diffusion of moisture and heat through the endosperm, nor gelling and cooling of the starch. These mechanisms that are responsible for propelling the kernel's expansion are unfortunately poorly understood. We note that while the values for pop time and popped radius obtained in this project are likely underestimations due to simplified models, the underlying physics in these models successfully displays how various properties of the kernel affect its popping. While the adiabatic expansion model is incomplete, it successfully encapsulates how the kernel's change in the kernel's critical pressure affects its expansion.

Finally we note that while we found just 1% of the kernels to go unpopped, this percentage may, in practice, be greater. This is because the external heat source is not the only thing that determines whether a kernel will pop. In practice, a kernel may fail to pop if it has insufficient moisture content to reach its critical pressure, or if the hull is somehow weakened such that vapour leaks out slowly, rather than all at once. This is why, no matter how hot the kernels are heated to, there will always be some that do not pop.

7 Conclusion

In this paper, we model the transformation of a popcorn kernel into popcorn by mainly using the Backward Euler method. We made a few assumptions for the problem to be feasibly solvable. We assumed that a kernel is a perfect sphere, that it is uniformly coated with oil that is at the same temperature at the kernel's surface. We also assumed that the expansion of the kernel as it's popping is adiabatic. We found that on average, using a time step of 0.2, 1000 kernel and with 500 kernel layers, we get a mean final radius of 8.07mm and a mean popping time of 87.3s. These results are on the lower end of what experimental data shows, which is expected if we consider our approximations. Therefore, we conclude that given the limit of our assumptions, our model accurately represents the physics of popcorn. For future simulations, further research should be conducted into limit assumptions that we made in this paper(to a feasible limit): for example, not assuming the kernel is a perfect sphere, or finding a way to model heat loss and using this to modify the boundary conditions for the system. These methods would increase the accuracy of our pop time compared to literature values. There are also experimental procedures that could be conducted to study relations we weren't able to quantify analytically in our simulation, such as the relation

between the parameters of the kernel to other variables such as the oil temperature.

Author Contribution Statement

Every member of this group contributed equally to this report. We all worked on parts of the coding. Bhuvan wrote the abstract. Emilia and Mariah wrote the Introduction. Everyone has written parts of the Problem Tackled and Theory section and Coding Methods. Mariah wrote the results section. Emilia wrote the Code Test section. Emilia and Bhuvan wrote the discussion. Mariah and Bhuvan wrote the conclusion.

References

[1] E. Virot and A. Ponomarenko, "Popcorn: Critical temperature, jump and sound," Mar 2015. 1, 6

[2] D. C. Hong and J. A. Both, "Controlling the size of popcorn," *Physica A: Statistical Mechanics and its Applications*, vol. 289, p. 557–560, Jan. 2001. 4

[3] K. Mohseni and T. Colonius, "Numerical treatment of polar coordinate singularities," *Journal of Computational Physics*, vol. 157, no. 2, p. 787–795, 2000. 5

[4] C. S. Chang, "Thermal conductivity of wheat, corn, and grain sorghum as affected by bulk density and moisture content," Jan 1986. 6

[5] E. Karababa, M. Amin, N. Aviara, C. Aydin, D. Balasubramanian, E. Baryeh, K. Çarman, V. Chandrasekar, S. Deshpande, S. Dutta, and et al., "Physical properties of popcorn kernels," Jan 2005. 6

[6] E. Shimoni, E. Dirks, and T. Labuza, "The relation between final popped volume of popcorn and thermal–physical parameters," *LWT - Food Science and Technology*, vol. 35, no. 1, p. 93–98, 2002. 8