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```
In [1]: import matplotlib.pyplot as plt
import numpy as np

from pomegranate import State, HiddenMarkovModel, DiscreteDistribution
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In [2]: model = HiddenMarkovModel(name="Climate Model")
```

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In [3]: rainy_emissions = DiscreteDistribution({"Happy":0.9,"Sad":0.1})
rainy_state = State(rainy_emissions,name = "rainy")

cloudy_emissions = DiscreteDistribution({"Happy":0.6,"Sad":0.4})
cloudy_state = State(cloudy_emissions,name = "cloudy")

sunny_emissions = DiscreteDistribution({"Happy":0.2,"Sad":0.8})
sunny_state = State(cloudy_emissions,name = "sunny")
```

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In [4]: model.add_states(rainy_state,cloudy_state,sunny_state)
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In [6]: model.add_transition(model.start, rainy_state, 0.33)
model.add_transition(model.start, cloudy_state ,0.33)
model.add_transition(model.start, sunny_state ,0.34)
#Adding Transitions for Rainy State
model.add_transition(rainy_state, rainy_state , 0.5)
model.add_transition(rainy_state, cloudy_state,0.3)
model.add_transition(rainy_state, rainy_state,0.2)
#Adding Transitions for Cloudy State
model.add_transition(cloudy_state,rainy_state,0.4)
model.add_transition(cloudy_state,cloudy_state, 0.2)
model.add_transition(cloudy_state,sunny_state,0.4)
#Adding Transitions for Sunny State
model.add_transition(sunny_state,rainy_state , 0.0)
model.add_transition(sunny_state,cloudy_state , 0.3)
model.add_transition(sunny_state,sunny_state , 0.7)
```

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In [7]: model.bake()
```

```
In [8]: model.edge_count()
```

```
Out[8]: 11
```

```
In [9]: model.node_count()
```

```
Out[9]: 5
```

```
In [13]: column_order = ["Climate Model-start", "rainy", "cloudy", "sunny", "Climate Model-end"]
column_names = [s.name for s in model.states]
order_index = [column_names.index(c) for c in column_order]
transitions = model.dense_transition_matrix()[ :, order_index][order_index, :]
print("The state transition matrix, P(Xt|Xt-1):\n")
print(transitions)
print("\nThe transition probability from Cloudy to Rainy is {:.0f}%".format(100 * transitions[1, 2]))
```

The state transition matrix, $P(X_t|X_{t-1})$:

```
[[0.  0.33 0.33 0.34 0. ]
 [0.  0.4  0.6  0.   0. ]
 [0.  0.4  0.2  0.4  0. ]
 [0.  0.   0.3  0.7  0. ]
 [0.  0.   0.   0.   0. ]]
```

The transition probability from Cloudy to Rainy is 40%

Calculate Sequence Likelihood

```
In [16]: observations = ['Happy', 'Sad', 'Happy']
assert len(observations) > 0
forward_matrix = np.exp(model.forward(observations))
probability_percentage = np.exp(model.log_probability(observations))
print(" " + " ".join(s.name.center(len(s.name)+6) for s in model.states))
for i in range(len(observations) + 1):
    print(" " if i==0 else observations[i - 1].center(9), end=" ")
    print(" ".join("{:.0f}%".format(100 * forward_matrix[i, j]).center(len(s.name))
                    for j, s in enumerate(model.states)))

print("\nThe likelihood over all possible paths " + \
      "of this model producing the sequence {} is {:.2f}%\n\n"
      .format(observations, 100 * probability_percentage))
```

	cloudy	rainy	sunny	Climate Model-start	Climate Model-end
Climate Model-start	0%	0%	0%	100%	0%
Happy	20%	30%	20%	0%	0%
Sad	11%	2%	9%	0%	0%
Happy	4%	5%	6%	0%	0%

The likelihood over all possible paths of this model producing the sequence ['Happy', 'Sad', 'Happy'] is 14.79%

Decoding the Most Likely Hidden State

Sequence

```
In [17]: observations = ['Happy', 'Sad', 'Happy']
viterbi_likelihood, viterbi_path = model.viterbi(observations)
print("The most likely weather sequence to have generated " + \
      "these observations is {} at {:.2f}%."
      .format([s[1].name for s in viterbi_path[1:]], np.exp(viterbi_likelihood)*100))
```

The most likely weather sequence to have generated these observations is ['rainy', 'cloudy', 'rainy'] at 2.57%.

Forward likelihood vs Viterbi likelihood

```
In [24]: from itertools import product

observations = ['Happy', 'Happy', 'Sad']

p = {'Rainy' : {'Rainy': np.log(.5), 'Cloudy': np.log(.3), 'Sunny': np.log(.2)}, 'Cloudy': {'Rainy': np.log(.3), 'Cloudy': np.log(.5), 'Sunny': np.log(.2)}, 'Sunny': {'Rainy': np.log(.2), 'Cloudy': np.log(.2), 'Sunny': np.log(.5)}}
e = {'Rainy' : {'Sad': np.log(.9), 'Happy': np.log(.1)}, 'Cloudy': {'Sad': np.log(.1), 'Happy': np.log(.9)}, 'Sunny': {'Sad': np.log(.1), 'Happy': np.log(.1)}}
```

```
In [26]: o = observations
k = []
vprob = np.exp(model.viterbi(o)[0])
print("The likelihood of observing {} if the weather sequence is...".format(o))
for s in product(*[['Rainy', 'Cloudy', 'Sunny']]*3):
    k.append(np.exp(np.log(.5)+e[s[0]][o[0]] + p[s[0]][s[1]] + e[s[1]][o[1]] + p[
    print("\t{} is {:.2f}% {}".format(s, 100 * k[-1], " <-- Viterbi path" if k[-1]
print("\nThe total likelihood of observing {} over all possible paths is {:.2f}%"
```

0.025660799999999999

The likelihood of observing ['Happy', 'Happy', 'Sad'] if the weather sequence is...

```
('Rainy', 'Rainy', 'Rainy') is 0.11%
('Rainy', 'Rainy', 'Cloudy') is 0.05%
('Rainy', 'Rainy', 'Sunny') is 0.01%
('Rainy', 'Cloudy', 'Rainy') is 0.22%
('Rainy', 'Cloudy', 'Cloudy') is 0.07%
('Rainy', 'Cloudy', 'Sunny') is 0.05%
('Rainy', 'Sunny', 'Rainy') is 0.00%
('Rainy', 'Sunny', 'Cloudy') is 0.14%
('Rainy', 'Sunny', 'Sunny') is 0.11%
('Cloudy', 'Rainy', 'Rainy') is 0.36%
('Cloudy', 'Rainy', 'Cloudy') is 0.14%
('Cloudy', 'Rainy', 'Sunny') is 0.03%
('Cloudy', 'Cloudy', 'Rainy') is 0.58%
('Cloudy', 'Cloudy', 'Cloudy') is 0.19%
('Cloudy', 'Cloudy', 'Sunny') is 0.13%
('Cloudy', 'Sunny', 'Rainy') is 0.00%
('Cloudy', 'Sunny', 'Cloudy') is 1.15%
('Cloudy', 'Sunny', 'Sunny') is 0.90%
('Sunny', 'Rainy', 'Rainy') is 0.00%
('Sunny', 'Rainy', 'Cloudy') is 0.00%
('Sunny', 'Rainy', 'Sunny') is 0.00%
('Sunny', 'Cloudy', 'Rainy') is 1.73%
('Sunny', 'Cloudy', 'Cloudy') is 0.58%
('Sunny', 'Cloudy', 'Sunny') is 0.38%
('Sunny', 'Sunny', 'Rainy') is 0.00%
('Sunny', 'Sunny', 'Cloudy') is 4.03%
('Sunny', 'Sunny', 'Sunny') is 3.14%
```

The total likelihood of observing ['Happy', 'Happy', 'Sad'] over all possible paths is 14.10%

In []: