Program : **B.Tech**

Subject Name: **Analog and Digital Communication**

Subject Code: **IT-404**

Semester: **4th**

Unit-I: Signals and Systems: Block diagram of a communication system, signal-definition, types of signals continuous, discrete, deterministic, non-deterministic, periodic, non-periodic, energy, power, analog and digital signals. Electromagnetic Spectra, Standard signals- DC, sinusoidal, unit step, ramp, signum, rectangular pulse, impulse(delta) signal. System definition, classification of systems, linear, nonlinear, time variant, time invariant, causal, non causal, stable and unstable systems. Fourier transforms: Time domain and frequency domain representation of signal, Fourier Transform and its properties, conditions for existence, Transform of Gate, unit step, constant, impulse, sine and cosine wave. Shifting property of delta function, convolution, time and frequency convolution theorems.

1.1. Introduction to Communication System

Communication is a process whereby information is enclosed in a package and is channelled and imparted by a sender to a receiver via some medium. The receiver then decodes the message and gives the sender a feedback. So the basic elements of communication systems are:

- Transmitter: originates the signal
- Receiver: receives transmitted signal after it travels over the medium
- Medium: guides the signal from the transmitter to the receiver.

In a data transmission system, the transmission medium is the physical path between transmitter and receiver. For guided media, electromagnetic waves are guided along a solid medium, such as copper twisted pair, copper coaxial cable, and optical fibre. For unguided media, wireless transmission occurs through the atmosphere, outer space, or water.
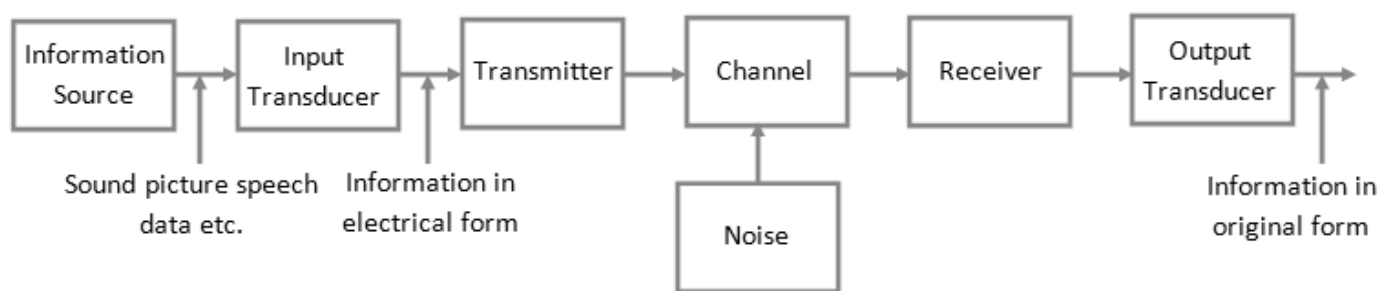

Figure1.1. Basic communication system

Elements of Communication system:

1. Information source: The message or information originates in the information source which has to be transmitted.

2. Input transducer: A transducer is the device which converts one form of energy to another form. In communication system transducer is usually required to convert the output of a source into an electrical signal that is suitable for transmission.

3. The Transmitter: The transmitter process the electrical signal into a form that is suitable for transmission through the transmission medium. The transmitter performs the signal processing of the message signal such as restriction of range of audio frequencies, amplification and modulation. All these processing are done to ease the transmission of the signal through the channel.

4. The Channel and the Noise: The communications channel is the physical medium that is used to send the signal from the transmitter to the receiver. In wireless transmission, the channel is usually the free space. On the other hand, telephone channels usually employ a variety of physical media, including wire lines, optical fibre cables, and wireless microwave radio. Whatever the physical medium for signal transmission, transmitted signal is corrupted in a random manner by noise.

5. The Receiver: The function of the receiver is to recover or reproduce the message signal contained in the received signal. If the message signal is transmitted by carrier modulation, the receiver performs *carrier demodulation* in order to extract the message from the sinusoidal carrier.

6. Output transducer: It is the final stage use to convert an electrical message signal into its original form.

1.2. Signal and its types:

A signal is a way of conveying information. Any time varying physical phenomenon that can convey information is called signal. Some examples of signals are human voice, electrocardiogram, sign language, videos etc. Technically - A signal is a function of one or more independent variables. a function of time, space, or another observation variable that conveys information

Signals are classified into the following categories:

- Continuous Time, Discrete Time Signals and Digital signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Real and Imaginary Signals

1. **Continuous Time Signal:** If the independent variable (t) is continuous, then the corresponding signal is continuous time signal. A finite, real-valued, smooth function x(t) of a variable *t* which usually represents time. Both *s* and *t* in X (t) are continuous. A continuous-time signal is a signal that can be defined at every instant of time. It is denoted by *x(t)*. Figure 1.2 shows continuous-time signal.
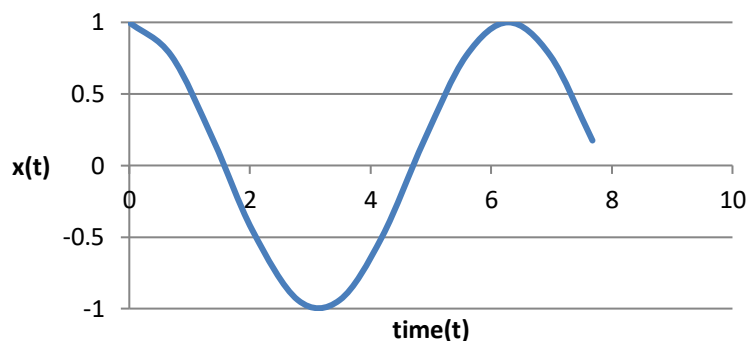

Figure 1.2.Continuous-time signal.

• Discrete Time Signal If the independent variable (t) takes on only discrete values, for example t = ±1, ±2, ±3, ... A *discrete-time* signal is a bounded, continuous-valued sequence x[n]. Alternately, it may be viewed as a continuous-valued function of a discrete index n. Discrete time signals can be obtained by sampling a continuous-time signal. It is denoted as *x(n)*.Figure 1.3 shows discrete-time signal.
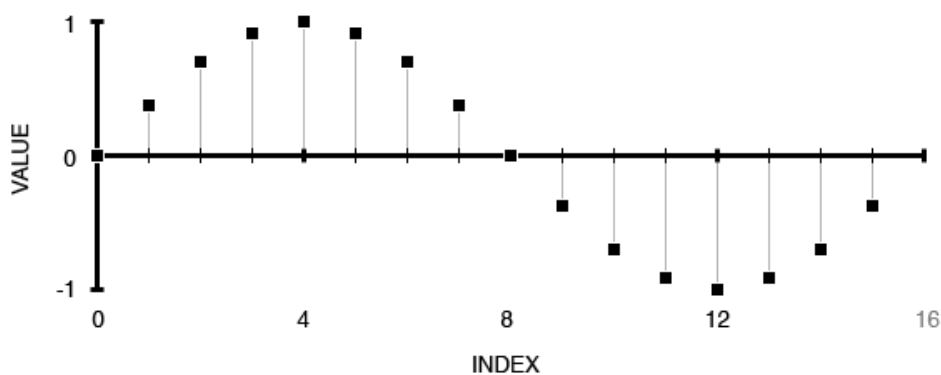

Figure 1.3.Discrete-time signal

Digital Signal: The signals that are discrete in time and quantized in amplitude are called digital signal. The term "digital signal" applies to the transmission of a sequence of values of a discrete-time signal in the form of some digits in the encoded form.

    2. **Random (Non-deterministic) and Deterministic Signal:**

A random signal cannot be described by any mathematical function, where as a deterministic signal is one that can be described mathematically. A common example of random signal is noise. Random signal and deterministic signal are shown in the Figure 1.4 and 1.5 respectively. A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modeled in probabilistic terms. A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time.
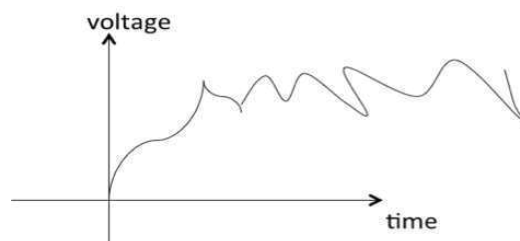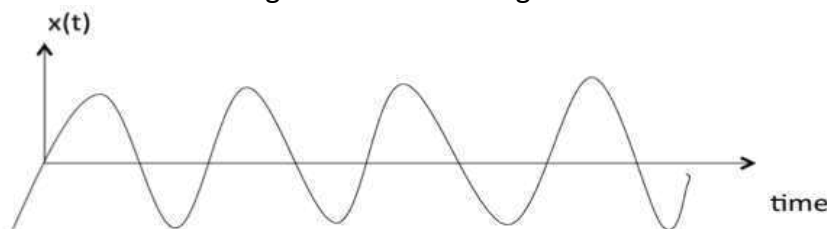

Figure 1.4.Random signal

Figure 1.5.Deterministic signal

Even and Odd Signals

A signal is said to be even when it satisfies the condition x(t) = x(-t)

Example 1: t^2, t^4… cos t etc.
    Let x(t) = t^2
    x(-t) = $(-t)^2$ = t^2 = x(t)
    t^2 is even function

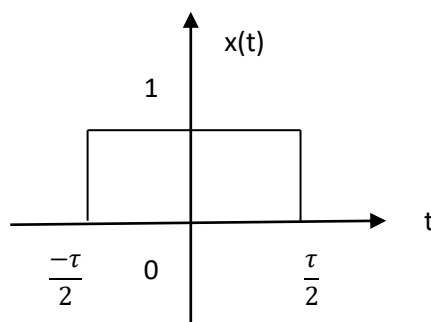**Example 2:** As shown in the following diagram, rectangle function x(t) = x(-t) so it is also even function.

Figure 1.6.Even function

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A signal is said to be odd when it satisfies the condition $x(t) = -x(-t)$

Example: t, t^3 ... And sin t

Let $x(t) = \sin t$

$x(-t) = \sin(-t) = -\sin t = -x(t)$

∴, sin t is odd function.

Any function $x(t)$ can be expressed as the sum of its even function $x_e(t)$ and odd function $x_o(t)$.

$x(t) = x_e(t) + x_0(t)$

where

$x_e(t) = ½[x(t) + x(-t)]$

$x_0(t) = ½[x(t) - x(-t)]$

A simple way of visualizing even and odd signal is to imagine that the ordinate *[x(t)]* axis is a mirror. For even signals, the part of *x(t)* for *t > 0* and the part of *x(t)* for *t < 0* are mirror images of each other. In case of an odd signal, the same two parts of the signals are negative mirror images of each other.

Periodic and Aperiodic Signals

A signal is said to be periodic if it satisfies the condition $x(t) = x(t + T)$ or $x(n) = x(n + N)$.

Where T = fundamental time period, $1/T = f$ = fundamental frequency.

Figure 1.7.Periodic signal

The above signal will repeat for every time interval T_0 hence it is periodic with period T_0.

3. Energy and Power Signals

Energy and power for Continuous time signal

A signal is said to be energy signal when it has finite energy.

Energy $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

A signal is said to be power signal when it has finite power.

Power $P = \lim_{t \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$

NOTE: A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal. If 0< E< ∞, then the signal x(t) is called an energy signal. However, there are signals where this condition is not satisfied. For such signals we consider the power. If 0< P< ∞, then the signal is called a power signal. Note that the power for an energy signal is zero (P = 0) and that the energy for a power signal is infinite (E = ∞). Some signals are neither energy nor power signals.

Power of energy signal = 0

Energy of power signal = ∞

Energy and power for discrete-time signal: The definition of signal energy and power for discrete signals is similar definitions for continuous signals.

Definition: The signal energy in the discrete-time signal x(n) is:

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

The signal power in the signal x(n) is:

$P = \lim_{n \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

4. **Real and Imaginary Signals**

A signal is said to be real when it satisfies the condition x(t) = x*(t)
A signal is said to be imaginary when it satisfies the condition x(t) = -x*(t)
Example:
If x(t)= 3 then x*(t)=3*=3 here x(t) is a real signal.
If x(t)= 3j then x*(t)=3j* = -3j = -x(t) hence x(t) is a imaginary signal.
Note: For a real signal, imaginary part should be zero. Similarly for an imaginary signal, real part should be zero.

5. **Causal, Non-causal and Anti-causal Signal:**

 Signal that are zero for all negative time, that type of signals are called causal signals, while the signals that are zero for all positive value of time are called anti-causal signal.

A non-causal signal is one that has non zero values in both positive and negative time. Causal, non-causal and anti-causal signals are shown below in the Figure 1.8(a), 1.8(b) and 1.8(c) respectively.
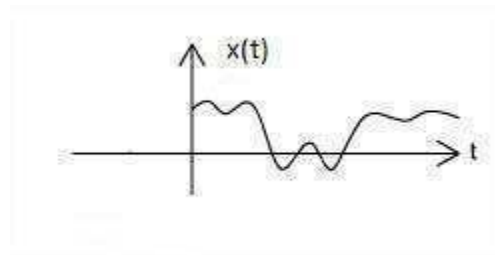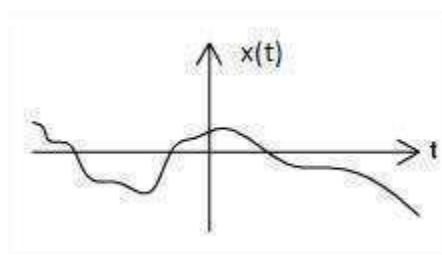

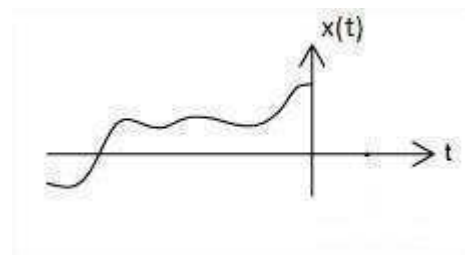Fig.1.8(a) Causal signal   Fig.1.8(b) Non-causal signal   Fig.1.8(c) Anti-causal signal

Standard Signal:

1. **The DC Signal:** The "DC" or constant signal simply takes a constant value. In continuous time it would be represented as: s(t)=1. In discrete time, it would be s[n] =1. The number "1" may be replaced by any constant. The DC signal typically represents any constant offset from 0 in real-world signals. The analog DC signal has bounded amplitude and power and is smooth.

2. **Unit Step Function**

   The "unit step", also often referred to as a *Heaviside* function, is literally a step. It has 0 value until time 0, at which point, it abruptly switches to 1 from 0. The unit step represents events that change state, e.g. the switching on of a system, or of another signal. It is usually represented as u(t) in continuous time and u[n] in discrete time.

Unit step function is denoted by u(t). It is defined as

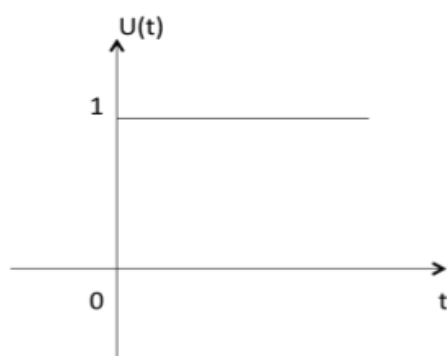$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Discrete time: $u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$

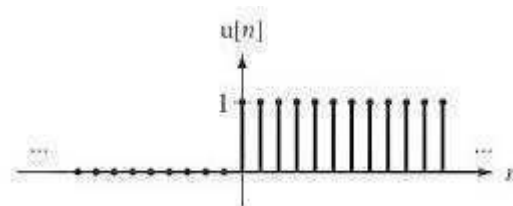Figure 1.9(a) continuous time Unit step signal   Figure 1.9(b) Discrete time Unit step signal

3. **Unit Impulse Function**

The unit impulse function also known as the Dirac Delta Function is denoted by δ(t) and it is defined as

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad \text{at } t = 0$$

$$\delta(t) = 0 \quad t \neq 0$$

Consider a function

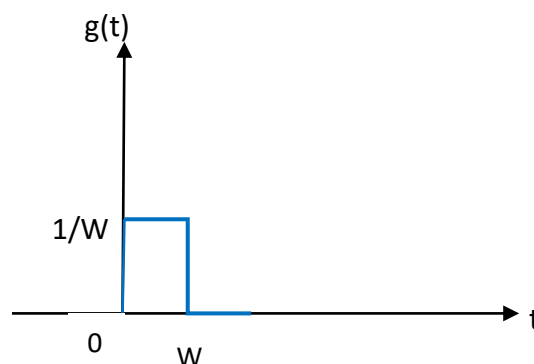$$g(t) = \begin{cases} \frac{1}{W} & 0 < t < W \\ 0 & otherwise \end{cases}$$

Figure 1.10.Gate function

One thing of note about g(t) is that

$$\int_{0^-}^{W} g(t)\, dt = 1:$$

The lower limit 0^- is a infinitesimally small amount less than zero. Now, suppose that the width w gets very small, indeed as small at 0^+, a number an infinitesimal amount bigger than zero. At that point, g(t) has become like the δ function, a very thin, very high spike at zero, such that

$\int_{-\infty}^{\infty} \delta(t)\ dt = \int_{0^-}^{0^+} \delta(t)\ dt = 1$

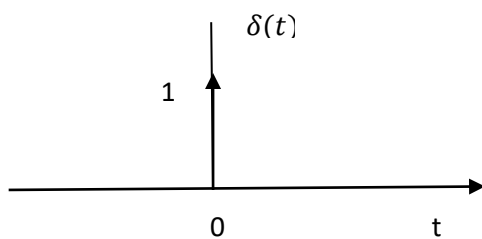As w becomes very small the function g(t) turns into a function δ (t) indicated by the arrowed spike.

Figure 1.11.Unit Impulse signal

$\int_{-\infty}^{t} \delta(t)\ dt = u(t)$

$\delta(t) = \frac{du(t)}{dt}$

4. Ramp Signal

Ramp signal is denoted by r(t), and it is defined as r(t) = $\begin{cases} t & t \geqslant 0 \\ 0 & t < 0 \end{cases}$
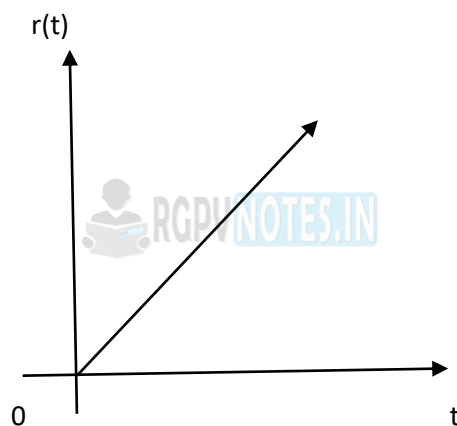

Figure 1.12.Ramp signal

$\int u(t) = (t)$
$u(t) = dr(t)/dt$
Area under unit ramp is unity.

5. Parabolic Signal

Parabolic signal can be defined as x(t) = $\begin{cases} \frac{t^2}{2} & t \geqslant 0 \\ 0 & t < 0 \end{cases}$

$\iint u(t)dt = \int r(t)dt = \int t\,dt = t^2 =$ parabolic signal
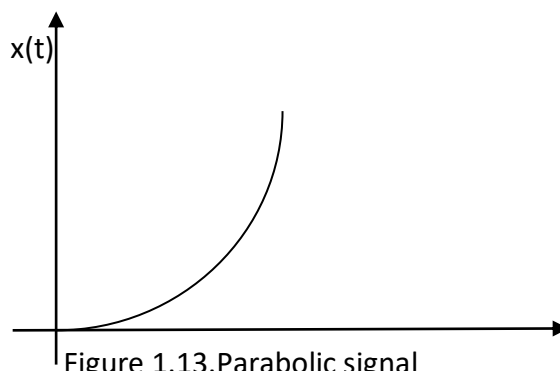$\Rightarrow u(t) = d^2x(t)/dt^2$
$\Rightarrow r(t) = dx(t)/dt$

Figure 1.13.Parabolic signal

6. Signum Function

Signum function is denoted as sgn (t). It is defined as sgn (t) = $\begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$
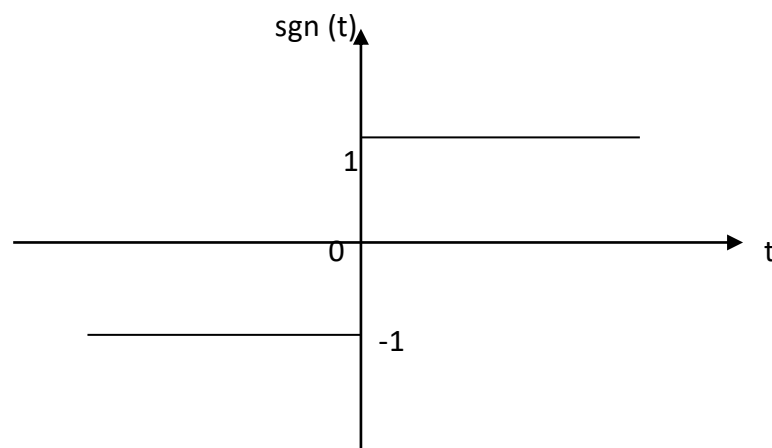
sgn(t) = 2u(t) − 1

Figure 1.14.Signum signal

7. Exponential Signal

Exponential signal is in the form of x(t) = $e^{\alpha t}$.

The shape of exponential can be defined by α.

Case i: if α = 0 → x(t) = e^0 = 1

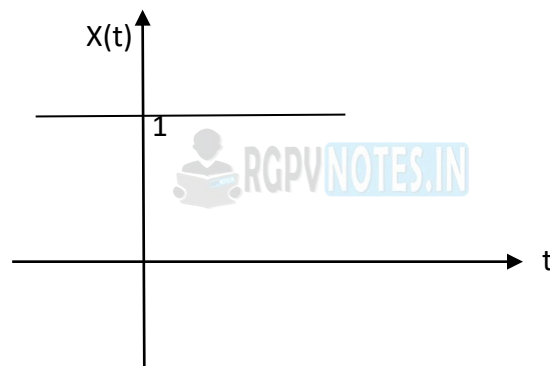
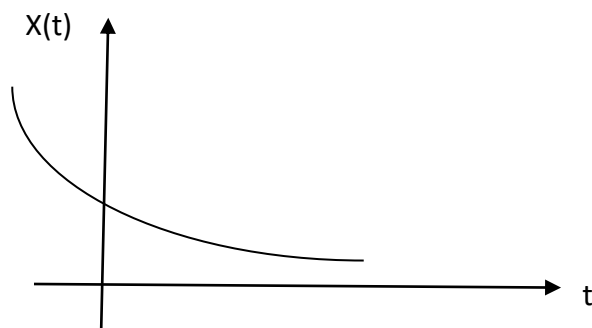
Figure 1.15(a) Exponential function at α=0

Case ii: if α < 0 i.e. -ve then x(t) = $e^{-\alpha t}$. The shape is called decaying exponential.

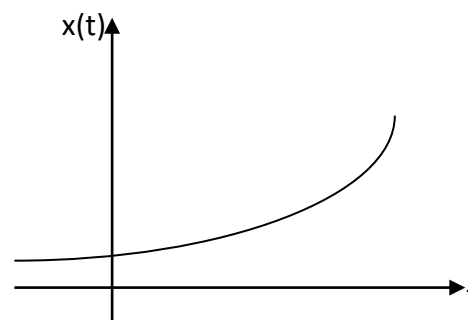**Case iii:** if α > 0 i.e. +ve then x(t) = $e^{\alpha t}$. The shape is called raising exponential.

Figure 1.15(b).Exponential decaying and raising signals

8. Rectangular Signal

Let it be denoted as x(t) and it is defined as

$$x(t) = A\ rect\left[\frac{r}{\tau}\right]$$

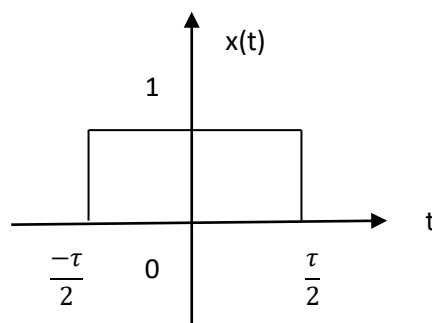Figure 1.16.Rectangular function

9. Triangular Signal

Let it be denoted as $x(t)= A\left[1-\frac{|t|}{T}\right]$
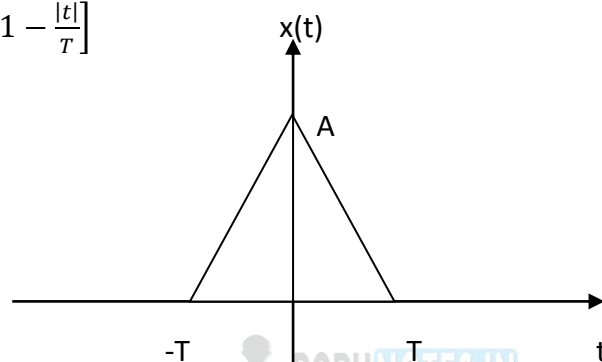
Figure 1.17.Triangular function

10. Sinusoidal Signal

Sinusoidal signal is in the form of x(t) = A cos (w$_0$±φ) or A sin(w$_0$±φ)

Where $T_0 = 2\pi w_0$
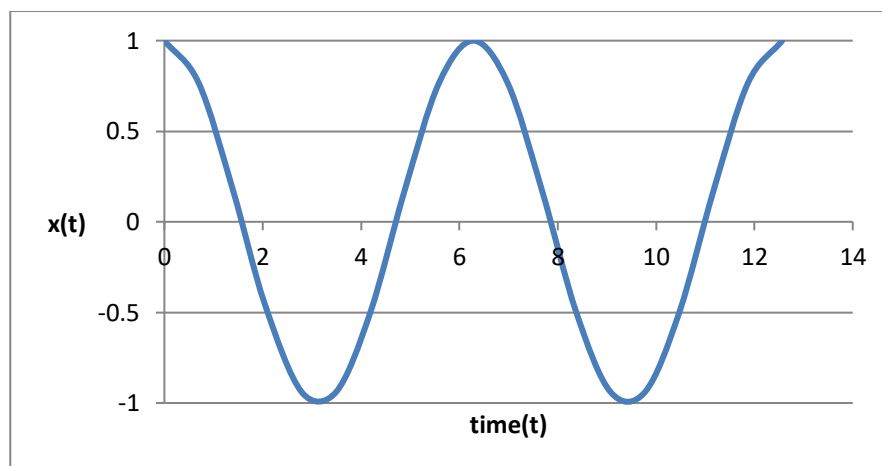
Figure 1.18.Cosinusoidal function

11. Sinc function:

It is denoted as sinc (t) and it is defined as

sinc(t)=sin (πt)/πt

=0   for t=±1, ±2 ,±3...
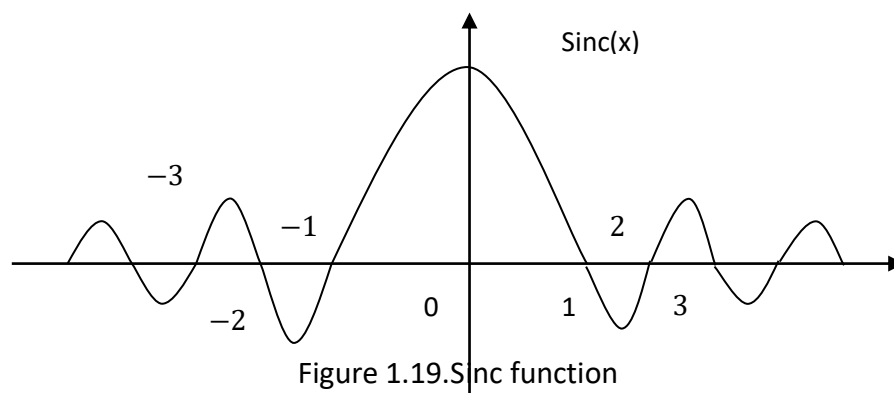
Figure 1.19.Sinc function

12. Sampling Function

It is denoted as sa(t) and it is defined as

Sa (t)=sin t/t

=0   for   t=±π,±2π,±3π.....
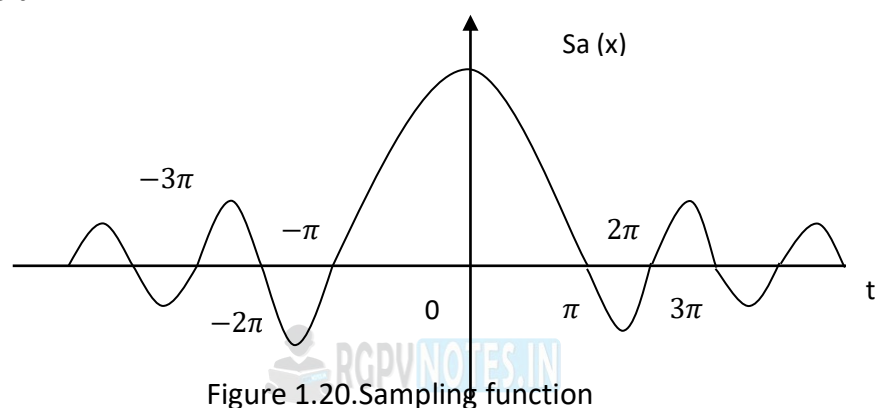
Figure 1.20.Sampling function

1.3. Electromagnetic Spectra: Electromagnetic spectrum ranges from dc to light. The lower radio frequencies are designated mainly by frequency. The optical ranges are referred by wavelength.

Signal parameters: Amplitude is the height of a wave. It is measured from a wave's midpoint to its peak. It is normally expressed in Volts (V). Frequency refers to the number of times a wave cycles past a given point each second. It is normally expressed in Hertz (Hz).Wave Length is the distance from the start to the end of a single wave cycle. It is typically expressed in meters.
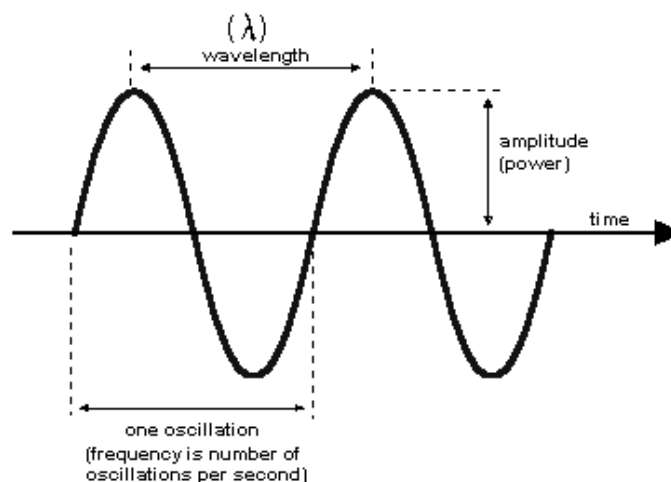
Figure 1.21.Signal Parameters
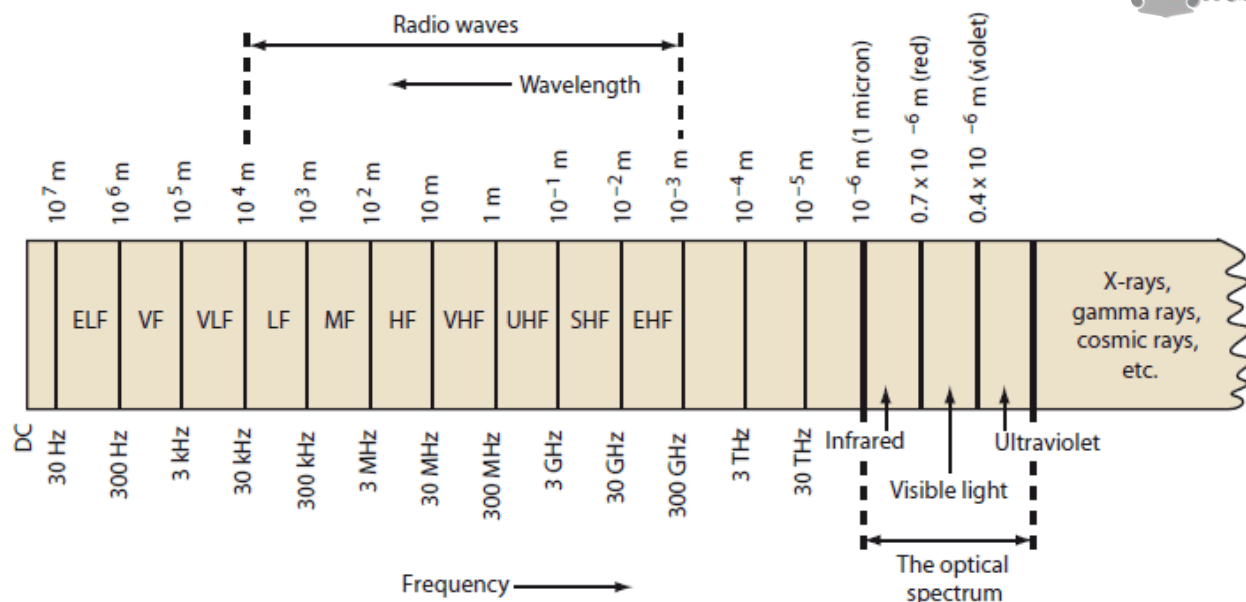
Figure 1.22.Electromagnetic spectra
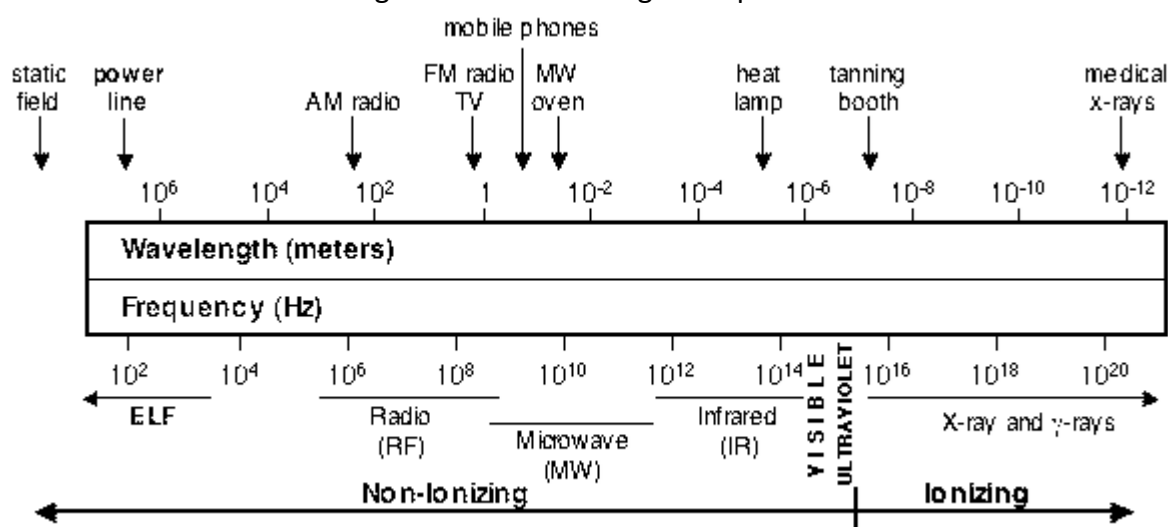
Figure 1.23.Electromagnetic Spectrum for communication systems

1.4. System definition and classification of systems:

System can be considered as a physical entity which manipulates one or more input signals applied to it. The system description specifies the transformation of the input signal to the output signal.
 Systems are classified into the following categories:

- Linear and Non-linear Systems
- Time Variant and Time Invariant Systems
- Linear Time variant and Linear Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Stable and Unstable Systems

1. **Liner and Non-liner Systems**

A system is said to be linear when it satisfies superposition and homogeneity principles. Consider two systems with inputs as $x_1(t)$, $x_2(t)$, and outputs as $y_1(t)$, $y_2(t)$ respectively. Then, according to the superposition and homogenate principles,

Principle of homogeneity: $T[a_1*x_1(t)] = a_1*y_1$, $T[a_2*x_2(t)] = a_2*y_2$

Principle of superposition: $T[x_1(t)] + T[x_2(t)] = a_1*y_1 + a_2*y_2$

Linearity: $T[a_1 x_1(t)] + T[a_2 x_2(t)] = a_1 y_1(t) + a_2 y_2(t)$

From the above expression, is clear that response of overall system is equal to response of individual system.

Example:

$y(t) = x^2(t)$

$y_1(t) = T[x_1(t)] = x_1^2(t)$

$y_2(t) = T[x_2(t)] = x_2^2(t)$

$T[a_1 x_1(t) + a_2 x_2(t)] = [a_1 x_1(t) + a_2 x_2(t)]^2$

Which is not equal to $a_1 y_1(t) + a_2 y_2(t)$. Hence the system is said to be non linear.

2. **Time Variant and Time Invariant Systems**

   A system is said to be time variant if its input and output characteristics vary with time. If the system response to an input signal does not change with time such system is termed as time invariant system. The behavior and characteristics of time variant system are fixed over time. The condition for time invariant system is:

   In time invariant systems if input is delayed by time t_0 the output will also gets delayed by t_0.

   Mathematically it is specified as follows

   $$y(t-t_0) = T[x(t-t_0)]$$

   For a discrete time invariant system the condition for time invariance can be formulated mathematically by replacing t as $n*T_s$ is given as

   $$y(n-n_0) = T[x(n-n_0)]$$

3. **Linear Time variant (LTV) and Linear Time Invariant (LTI) Systems**

   If a system is both linear and time variant, then it is called linear time variant (LTV) system.

   If a system is both linear and time Invariant then that system is called liner time invariant (LTI) system.

4. **Static and Dynamic Systems**

   Static system is memory-less whereas dynamic system is a memory system.

   Example 1: $y(t) = 2 x(t)$

   For present value t=0, the system output is $y(0) = 2x(0)$. Here, the output is only dependent upon present input. Hence the system is memory less or static.

   Example 2: $y(t) = 2 x(t) + 3 x(t-3)$

   For present value t=0, the system output is $y(0) = 2x(0) + 3x(-3)$.

   Here $x(-3)$ is past value for the present input for which the system requires memory to get this output. Hence, the system is a dynamic system.

5. **Causal and Non-Causal Systems**

A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input. For non causal system, the output depends upon future inputs also.

Example 1: y(n) = 2 x(t) + 3 x(t-3)

For present value t=1, the system output is y(1) = 2x(1) + 3x(-2).

Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Example 2: y(n) = 2 x(t) + 3 x(t-3) + 6x(t + 3)

For present value t=1, the system output is y(1) = 2x(1) + 3x(-2) + 6x(4) Here, the system output depends upon future input. Hence the system is non-causal system.

6. **Stable and Unstable Systems**

   The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system then it is said to be unstable.

   Note: For a bounded signal, amplitude is finite.

- **Example 1:** y (t) = $x^2(t)$

  Let the input is u(t) (unit step bounded input) then the output y(t) = u2(t) = u(t) = bounded output.

  Hence, the system is stable.

- **Example 2:** y (t) = $\int x(t)dt$

  Let the input is u (t) (unit step bounded input) then the output y(t) = $\int u(t)dt$ = ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when t → infinite).

  Hence, the system is unstable.

1.5. Time domain and frequency domain representation of signal

An electrical signal either, a voltage signal or a current signal can be represented in two forms: These two types of representations are as under:

i) Time Domain representation-: In time domain representation a signal is a time varying quantity as shown in Fig.1.24
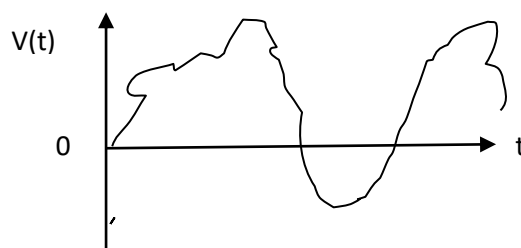

Fig 1.24 An arbitrary time domain signal

ii) Frequency Domain Representation: In frequency domain, a signal is represented by its frequency spectrum as shown in Fig 1.25
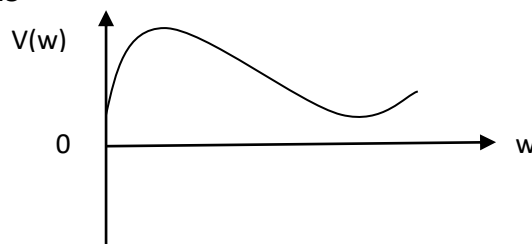

Fig 1.25 Frequency domain representation of time domain signal

1.6. Fourier Transform and its properties

Fourier Transform pair

Fourier transform may be expressed as

X(w)=F[x(t)]=$\int_{-\infty}^{\infty} x(t)\, e^{-jwt}$dt

In the above equation X(w) is called the Fourier transform of x(t). In other words X(w) is the frequency domain representation of time domain function x(t). This means that we are converting a time domain signal into its frequency domain representation with the help of fourier transform. Conversely if we want to convert frequency domain signal into corresponding time domain signal, we will have to take inverse fourier transform of frequency domain signal. Mathematically, Inverse fourier transform.

$$F^{-1}[X(w)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)e^{jwt}\,dw$$

Example

Q.1 Find the fourier transform of a single-sided exponential function $e^{-at}u(t)$.

Solution: $e^{-at}u(t)$ is single sided function because her the main function e^{-at} is multiplied by unit step function u(t), then resulting signal will exist only for t>0.

u(t)=   {1   for t>0

={0 for elsewhere

Now, given that x(t)= $e^{-at}u(t)$

X(w)=F[x(t)]=$\int_{-\infty}^{\infty} x(t)\, e^{-jwt}$dt

Or   X(w)=$\int_{-\infty}^{\infty} e^{-at}\, u(t)\, e^{-jwt}$dt

=$\int_{0}^{\infty} e^{-at}\, e^{-jwt}$dt

=$\int_{0}^{\infty} e^{-t(a+jw)}$dt

$\frac{-1}{(a+jw)}[e^{-\infty} - e^{0}]=$ $\frac{-1}{(a+jw)}[0\text{-}1]= \frac{1}{(a+jw)}$

To obtain the above expression in the proper form we write

X(w)= $\frac{-1}{(a+jw)} * \frac{(a-jw)}{(a-jw)}$

X(w)= $\frac{(a-jw)}{(a^2+w^2)} = \frac{a}{(a^2+w^2)}$ - $\frac{jw}{(a^2+w^2)}$

Obtaining the above expression in polar form

X(w)=$\frac{1}{\sqrt{a^2+w^2}}\, e^{-jtan^{-1}(\frac{w}{a})}$

As we know that

X(w)=$|X(w)|e^{j\varphi(jw)}$

On comparision amplitude spectrum

$|X(w)|=\frac{1}{\sqrt{a^2+w^2}}$

$$\varphi(w) = -tan^{-1}(\frac{w}{a})$$

Properties of Continuous Time Fourier Transform (CTFT)

1. **Time Scaling Function**

Time scaling property states that the time compression of a signal results in its spectrum expansion and time expansion of the signal results in its spectral compression. Mathematically,

If x(t) $\longleftrightarrow$ X(w)

Then, for any real constant a,

x(at) $\longleftrightarrow$ $\frac{1}{|a|}X(\frac{w}{a})$

proof: The general expression for fourier transform is

X(w)=F[x(t)]=$\int_{-\infty}^{\infty} x(t)\,e^{-jwt}$ dt

Now F[x(at)]=$\int_{-\infty}^{\infty} x(at)\,e^{-jwt}$ dt

Putting

At=y

We have dt=$\frac{dy}{a}$

Case (i): When a is positive real constant

F[x(at)]= $\int_{-\infty}^{\infty} x(y)\,e^{-j\left(\frac{w}{a}\right)y}\frac{dy}{a}$ = $\frac{1}{a}\int_{-\infty}^{\infty} x(y)\,e^{-j\left(\frac{w}{a}\right)y}$ = $\frac{1}{a}$X($\frac{w}{a}$)

Case (ii): When a is negative real constant

F[x(at)]= $\frac{-1}{a}$ X($\frac{w}{a}$)

Combining two cases, we have

F[x(at)]= $\frac{1}{|a|}$X($\frac{w}{a}$) Or x(at) $\longleftrightarrow$ $\frac{1}{|a|}$X($\frac{w}{a}$)

The function x(at) represents the function x(t) compressed in time domain by a factor a. Similarly, a function X($\frac{w}{a}$) represents the function X(w) expanded in frequency domain by the same factor a.

2. **Linearity Property**

Linearity property states that fourier transform is linear. This means that

If $x_1(t)$ $\longleftrightarrow$ $X_1(w)$

And $x_2(t)$ $\longleftrightarrow$ $X_2(w)$

Then $a_1\,x_1(t)$ + $a_2\,x_2(t)$ $\longleftrightarrow$ $a_1X_1(w)$ + $a_2X_2(w)$

3. **Duality or Symmetry Property**

If x(t) $\longleftrightarrow$ X(w)

Then   X(t) $\longleftrightarrow$ 2πx(-w)

Proof

The general expression for fourier transform is

$$F^{-1}[X(w)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)e^{jwt}\,dw$$

Therefore,

x(-t)= $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)e^{-jwt}\,dw$

2πx(-t)= $\int_{-\infty}^{\infty} X(w)e^{-jwt}\,dw$

Since w is a dummy variable, interchanging the variable t and w we have

2πx(-w)= $\int_{-\infty}^{\infty} X(t)e^{-jwt}\,dw$=F[X(t)]

Or   F[X(t)]= 2πx(-w)

Or   X(t)   ⟵ 2πx(-w)
For an even function x(-w)=x(w)

Therefore , X(t)   ⟷   2πx(w)

Example (1)
The fourier transform $F[e^{-t}u(t)]$ is equal to $\frac{1}{1+j2\pi f}$. Therefore $F[\frac{1}{1+j2\pi f}]$ is equal to

Solution:
Using Duality property of Fourier Transform, we have

If   x(t)   ⟵ X(f)⟶
Then   X(t)   ⟵ x(-f)⟶
Therefore,
$e^{-t}u(t)$ ⟵ $\frac{1}{1+j2\pi f}$ ⟶
Then $\frac{1}{1+j2\pi t}$ ⟵ $e^{-f}u(f)$

4. Time Shifting property

Time Shifting property states that a shift in the time domain by an amount b is equivalent to multiplication by e^{-jwb} in the frequency domain. This means that magnitude spectrum $|X(w)|$

Remains inchanged but phase spectrum θ(w) is changed by -wb.

If   x(t)   ⟵ X(w)⟶
Then   X(t-b)   ⟵ X(w) e^{-jwb}
Proof: $X(w)=F[x(t)]=\int_{-\infty}^{\infty} x(t)\, e^{-jwt}$dt
And $F[x(t-b)]=\int_{-\infty}^{\infty} x(t-b)\, e^{-jwt}$dt
Putting t-b=y, so that dt=dy

$F[x(t-b)]=\int_{-\infty}^{\infty} x(y)\, e^{-jw(b+y)}$dy $=\int_{-\infty}^{\infty} x(y)\, e^{-jwb}e^{-jwy}$ dy

Or $F[x(t-b)]= e^{-jwb} \int_{-\infty}^{\infty} x(y)\, e^{-jwy}$ dy
Since y is a dummy variable, we have
$F[x(t-b)]= e^{-jwb}$X(w)=X(w) e^{-jwb}
Or x(t-b)   ⟷   X(w) e^{-jwb}

5. Frequency Shifting Property

Frequency shifting property states that the multiplication of function x(t) by $e^{jw_0 t}$ is equivalent to shifting its fourier transform X(w) in the positive direction by an amount w_0. This means that the spectrum X(w) is translated by an amount c. hence this property is often called frequency translated theorem. Mathematically .

If   x(t)   ⟵ X(w)⟶
Then $e^{jw_0 t}$ x(t)   ⟵ X(w-w_0)
Proof: General expression for fourier transform is

$X(w) = F[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$

Now, $\quad F[e^{jw_0 t} x(t)] = \int_{-\infty}^{\infty} x(t) e^{jw_0 t} e^{-jwt} dt$

Or $\qquad F[e^{jw_0 t} x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j(w-w_0)t} dt$

Or $\qquad F[e^{jw_0 t} x(t)] = X(w - w_0)$

Or $\quad e^{jw_0 t} x(t) \longleftarrow X(w-w_0)$

6. Time Differentiation Property

The time differentiation property states that the differentiation of a function x(t) in the time domain is equivalent to multiplication of its fourier transform by a factor jw. Mathematically

If x(t) $\longleftarrow X(w)$

Then $\quad \frac{dx(t)}{dt} x(t) \longleftarrow jw\, X(w)$

Proof: The general expression for fourier transform is

$$F^{-1}[X(w)] = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w) e^{jwt} dw$$

Taking differentiation, we have

$\frac{dx(t)}{dt} = \frac{1}{2\pi} \frac{d}{dt} [\int_{-\infty}^{\infty} X(w) e^{jwt} dw]$

Interchanging the order of differentiation and integration, we have

$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt}[X(w) e^{jwt}] dw$

$\qquad = \frac{1}{2\pi} \int_{-\infty}^{\infty} jw\, X(w) e^{jwt} dw$

Or $\frac{dx(t)}{dt} = F^{-1}[jwX(w)]$

Or $F[\frac{dx(t)}{dt}] = jwX(w)$

Or $\frac{dx(t)}{dt} \longleftrightarrow jwX(w)$ Hence proved

1.7 Transform of Gate

A gate function is rectangular pulse. Figure 1.3 shows gate function. The function or rectangular pulse shown in figure 1.3 is written as rect $(\frac{t}{\tau})$.
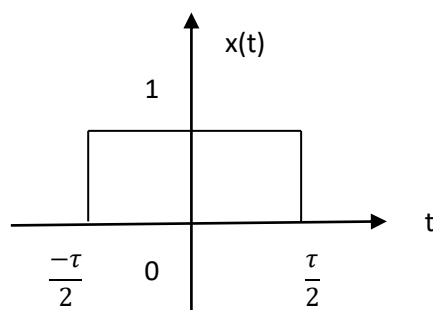

Fig2.3 A Gate Function

From the above figure it is clear that rect $(\frac{t}{\tau})$ represents a gate pulse of height or amplitude unity and width τ.

x(t)= rect $(\frac{t}{\tau}) = \{1 \quad for\ \frac{-\tau}{2} < t < \frac{\tau}{2}\}$

$\{0 \quad otherwise\}$

Sampling Function Or Interpolation Function Or Sinc function

The functions $\frac{sinx}{x}$ is the "sine over argument" and denoted by sinc(x). This function plays an important role in signal processing. It is also known as the filtering or interpolating function. Mathematically,

Sinc(x)=$\frac{sinx}{x}$

Or

Sa(x)= $\frac{sinx}{x}$
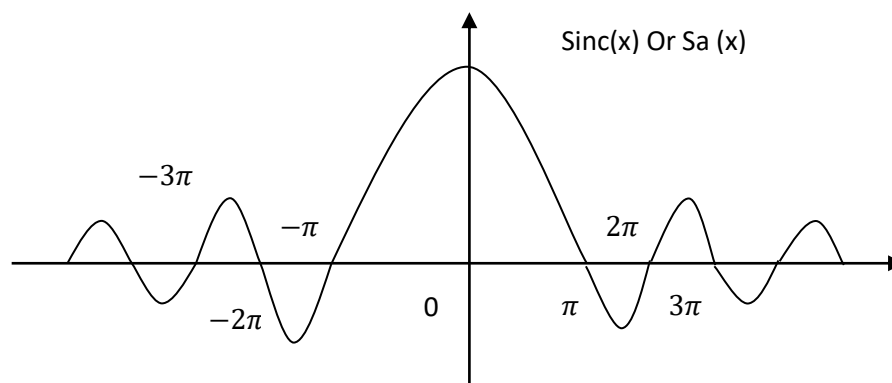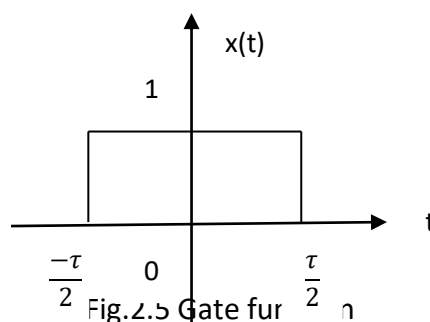

Fig.2.4 Sample function

From the figure, following points may be observed about the sampling function :

(i)   Sa(x) or sinc(x) is an even function of x.

(ii)   Sinc(x) =0 when sinx=0 except at x=0, where it is indeterminate. This means that sinc(x)=0 for x=$\pm$nπ , here n=$\pm1, \pm2$ ….

(iii)   Sinc(x) is the product of oscillating signal sinx of period 2π and a decreasing function $\frac{1}{x}$. Therefore, sinc(x) exhibits sinusoidal oscillations of period 2π with amplitude decreasing continuously as 1/x.

Example 2: Find the fourier transform of the gate function shown in figure 1.5.

Fig.2.5 Gate function

Sol.   x(t)= rect ($\frac{t}{\tau}$) =$\left\{1 \quad for \frac{-\tau}{2} < t < \frac{\tau}{2}\right\}$
$\{0 \quad otherwise\}$

X(w)=F[x(t)]=$\int_{-\infty}^{\infty} x(t)\, e^{-jwt}$dt

X(w)=F[x(t)]=$\int_{-\infty}^{\infty}$ rect ($\frac{t}{\tau}$) e^{-jwt}dt

=$\int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} 1.\, e^{-jwt}$dt= $\left[\frac{-e^{-jwt}}{jw}\right]_{\frac{-\tau}{2}}^{\frac{\tau}{2}}$

$=\frac{-1}{jw}[e^{-j\frac{w\tau}{2}}-e^{j\frac{w\tau}{2}}]$

$=\frac{1}{jw}[e^{j\frac{w\tau}{2}}-e^{-j\frac{w\tau}{2}}]$ --------(1)

We know that $e^{j\theta}$=cosθ +jsinθ

And $e^{-j\theta}$=cosθ –jsinθ

Hence   2cosθ=$e^{j\theta}+e^{-j\theta}$

2jsinθ=$e^{j\theta}-e^{-j\theta}$

Putting $\theta=\frac{w\tau}{2}$, we get

2jsin$\frac{w\tau}{2}$=$e^{j\frac{w\tau}{2}}-e^{-j\frac{w\tau}{2}}$ --------(2)

From (1) and (2)

X(w)= $\frac{1}{jw}$[2jsin$\frac{w\tau}{2}$]

By multiplying and dividing the equation by τ

$=\frac{2\tau}{jw\tau}$[jsin$\frac{w\tau}{2}$]

$=\frac{\tau}{\frac{w\tau}{2}}$[sin$\frac{w\tau}{2}$]

$= \tau\ [\frac{\sin\frac{w\tau}{2}}{\frac{w\tau}{2}}]$

$= \tau$sinc($\frac{w\tau}{2}$)

Now, since sinc(x)=0, when x=$\pm$nπ

Therefore, sinc($\frac{w\tau}{2}$) =0, when $\frac{w\tau}{2}$=$\pm$nπ

Or w=$\frac{\pm2n\pi}{\tau}$

Figure 2.6 shows the plot of X(w) $\frac{2\pi}{\tau}$
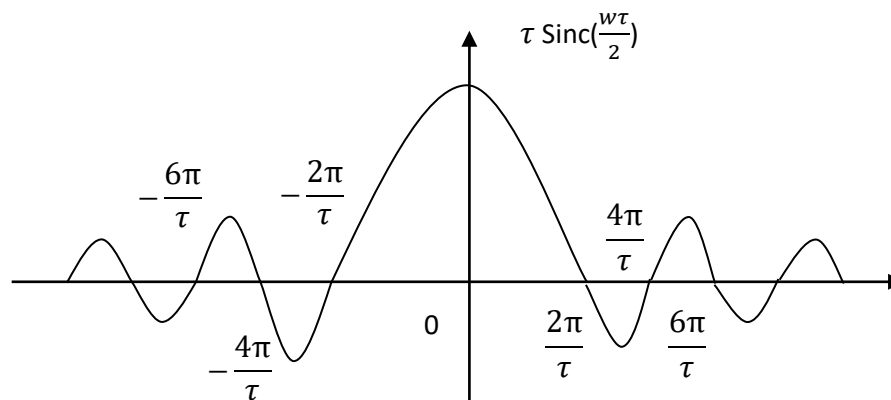

Fig. 2.6

1.8 Impulse Functions

Unit Impulse functions:

A unit impulse function was invented by P.A.M. Diarc and so it is also called as Delta function. It is denoted by (t).

Mathematically,

$\delta(t) = 0$, $t \neq 0$

And, $\int_{-\infty}^{\infty} \delta(t)dt = 1$

Figure 1.6 shows the graphical representation of an unit impulse function. The following points may be observed about an unit-impulse function:
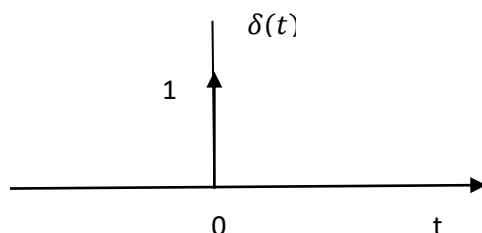
Fig.2.7 The Unit Impulse function

i)   The width of pulse is zero. This means that pulse exist only at t=0.
ii)   The height of the pulse goes to infinity
iii) The area under the pulse-curve is always is always unity.

Shifting Property of the Impulse function:

If we take the product of unit impulse function $\delta(t)$ and any given function x(t) which is continuous at t=0,then this product will provide the function x(t) existing only at t=0 since $\delta(t)$ exist only at t=0. Mathematically,

$\int_{-\infty}^{\infty} x(t)\delta(t)dt = x(0) \int_{-\infty}^{\infty} \delta(t)dt = x(0).1 = x(0)$

The equation is also known as shifting or sampling property of the impulse function because the impulse shifts the value of x(t0 at t=0. This means that the value of x(t) has been sampled at t=0. to The shifting or sampling may be also done at any, instant t=t_0, if we define the impulse function at the instant. Mathematically,

$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)dt = x(t_0)$

The above equation states that the product of a continuous function x(t) with an impulse function $\delta(t - t_0)$ provides the sampled value of x(t) at t=t_0.

Q.1 Find the fourier transform of an impulse function x(t) = $\delta(t)$ Also draw the spectrum

Sol. Expression of the fourier transform is given by

X(w)=F[x(t)]=$\int_{-\infty}^{\infty} x(t) e^{-jwt}dt = \int_{-\infty}^{\infty} \delta(t) e^{-jwt}$dt

Using shifting property of impulse function

X(w)=$[e^{-jwt}]_{at\ t=0}$

X(w)=1

$\delta(t)$ ←1→

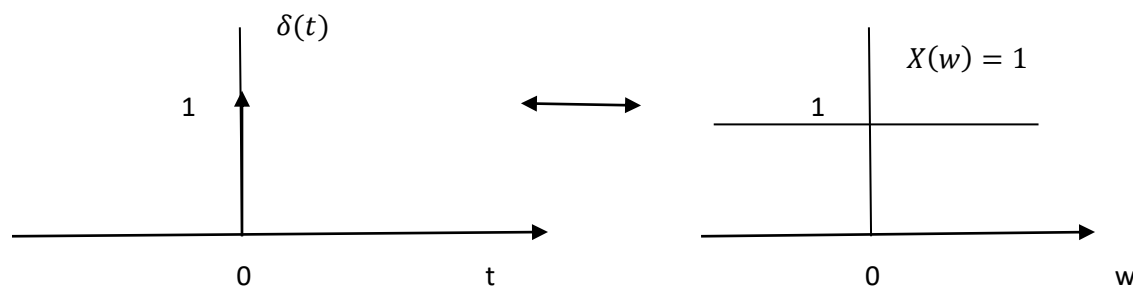Hence the fourier transform of an impulse function is unity.

Fig 2.8 Time and frequency domain of impulse function

Figure 2.8 shows an unit impulse function and its fourier transform or spectrum. From the figure1.7 it is clear that an unit impulse contains the entire frequency components having identical magnitude. This means that the bandwidth of the unit impulse function is infinite. Also, since spectrum is real, only magnitude spectrum is required. The phase spectrum $\theta(w)=0$, which means that all the frequency components are in the same phase.

Q.(2) Find the inverse fourier transform of $\delta(w)$
Solution. Inverse fourier transform is expressed as

$$F^{-1}[X(w)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)e^{jwt}dw$$

$$F^{-1}[\delta(w)] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(w)e^{jwt}dw$$

$F^{-1}[\delta(w)]=\frac{1}{2\pi}[\ e^{jwt}]_{\text{at w=0}}$

$F^{-1}[\delta(w)]=\frac{1}{2\pi}[\ e^{0}]= \frac{1}{2\pi}.1\ =\frac{1}{2\pi}$

$F[\frac{1}{2\pi}]= \delta(w)$

$\frac{1}{2\pi} \qquad \delta(w) \longrightarrow$

$1 \longleftrightarrow 2\pi\delta(w)$
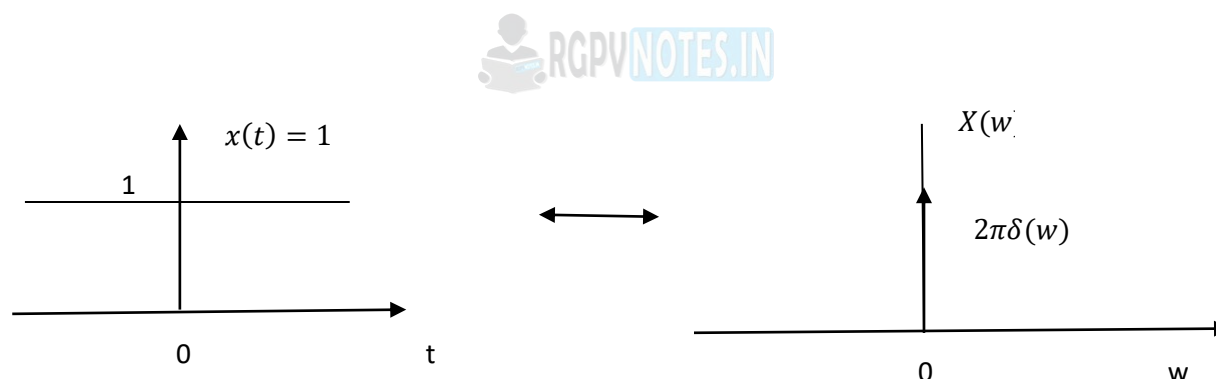
Fig.2.9 representation of Inverse fourier transform

This shows that the spectrum of a constant signal x(t)=1 an impulse function $2\pi\delta(w)$. This can also be interpreted as that x(t) =1 is a d.c. signal which has single frequency. W=0(dc).

Q.(3) Find the inverse fourier transform of $\delta(w-w_0)$
Solution. Inverse fourier transform is expressed as

$$F^{-1}[X(w)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)e^{jwt}dw$$

Or

$$F^{-1}[\delta(w - w_0)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(w - w_0)e^{jwt}dw$$

Using shifting or sampling property of impulse function, we get

$F^{-1}[\delta(w - w_0)]=\frac{1}{2\pi}[\ e^{jwt}]_{\text{at}}\ w = w_0$

$F^{-1}[\delta(w - w_0)] = \frac{1}{2\pi}[e^{jw_0 t}]$

$F[\frac{1}{2\pi}e^{jw_0 t}] = \delta(w - w_0)$

$\frac{1}{2\pi}e^{jw_0 t} \qquad \delta(w - w_0)$

Or

$e^{jw_0 t} \qquad 2\pi\,\delta(w - w_0)$

The above expression shows that the spectrum of an overlasting exponential $e^{jw_0 t}$ is a single impulse at w=0.

Similarly,

$e^{-jw_0 t} \qquad 2\pi\,\delta(w + w_0)$

1.9.Fourier Transform of Cosine wave

Q.4 Find the fourier transform of overlasting sinusoid $\cos w_0 t$.

Solution: We know that Euler's identity is given by

$e^{j\theta} = \cos\theta + j\sin\theta$

And $\qquad e^{-j\theta} = \cos\theta - j\sin\theta$

Hence $\qquad 2\cos\theta = e^{j\theta} + e^{-j\theta}$ Or

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

And

$2j\sin\theta = e^{j\theta} - e^{-j\theta}$ Or

$\sin\theta = \frac{e^{j\theta} + e^{-j\theta}}{2j}$

Hence, $\cos w_0 t = \frac{e^{jw_0 t} + e^{-jw_0 t}}{2}$

We know that

$e^{jw_0 t} \qquad 2\pi\,\delta(w - w_0)$

And $\qquad e^{-jw_0 t} \qquad\qquad)\,2\pi\,\delta(w + w_0$

So that $\cos w_0 t \qquad \frac{1}{2}[2\pi\,\delta(w - w_0) + 2\pi\,\delta(w + w_0]$

Or

$\cos w_0 t \qquad [\pi\,\delta(w - w_0) + \pi\,\delta(w + w_0]$

1.10. Fourier Transform of Periodic Function

Fourier transform of periodic function could also be found out. This means that Fourier transform may be used as a universal mathematical tool to analyze both periodic and non-periodic waveform over the entire interval.

Let us find the fourier transform of periodic function x(t). x(t) may be expressed in terms of complex fourier series as

x(t) = $\sum_{n=-\infty}^{\infty} C_n e^{jnw_0 t}$

Taking fourier transform of both the side

F[x(t)] = F[$\sum_{n=-\infty}^{\infty} C_n e^{jnw_0 t}$] $\qquad\qquad = \sum_{n=-\infty}^{\infty} C_n . F[1. e^{jnw_0 t}]$

Using frequency shifting shifting theorem, we can write

$F[1. e^{jnw_0 t}] = 2\pi\,\delta(w - nw_0)$

Hence, F[x(t)] = $\sum_{n=-\infty}^{\infty} C_n 2\pi\,\delta(w - nw_0) = 2\pi \sum_{n=-\infty}^{\infty} C_n\,\delta(w - nw_0)$

Hence, the fourier transform of a periodic function consist of a train of equally spaced impulses. These impulses are located at the harmonic frequencies of the signal and the strength or area of each impulse is given by $2\pi C_n$
.

1.11. Convolution

Convolution is a mathematical operation used to express the relation between input and output of an LTI system. It relates input, output and impulse response of an LTI system as

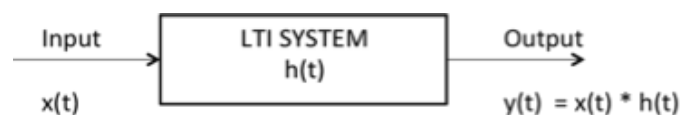y(t)=x(t)∗h(t)

Where y (t) = output of LTI

x (t) = input of LTI

h (t) = impulse response of LTI

There are two types of convolutions:

- Continuous convolution

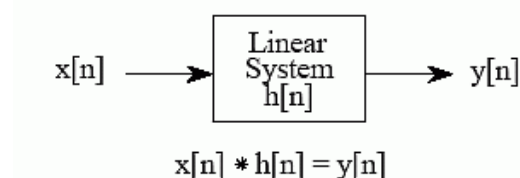- Discrete convolution

Continuous Convolution

y(t)=x(t)∗h(t)

$$= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$
$$= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

A convolution is a mathematical operation that represents a signal passing through a LTI (Linear and Time-Invariant) system or filter.

Discrete Convolution

y(n)=x(n)∗h(n)

$$= \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$
$$= \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

Delta function, symbolized by the Greek letter delta, δ[n] is a normalized impulse, that is, sample number zero has a value of one, while all other samples have a value of zero. For this reason, the delta function is called the unit impulse.

Impulse response is the signal that exits a system when a delta function (unit impulse) is the input. If two systems are different in any way, they will have different impulse responses. The input and output signals are often called x[n] and y[n], the impulse response is usually given the symbol, h[n]. Any impulse can be represented as a shifted and scaled delta function.

Properties of Convolution

1. Commutative Property: $x_1(t)*x_2(t)=x_2(t)*x_1(t)$
2. Distributive Property: $x_1(t)*[x_2(t)+x_3(t)]=[x_1(t)*x_2(t)]+[x_1(t)*x_3(t)]$
3. Associative Property: $x_1(t)*[x_2(t)*x_3(t)]=[x_1(t)*x_2(t)]*x_3(t)$
4. Shifting Property: $x_1(t)*x_2(t)=y(t)$
   $x_1(t)*x_2(t-t_0)=y(t-t_0)$
   $x_1(t-t_0)*x_2(t)=y(t-t_0)$
   $x_1(t-t_0)*x_2(t-t_1)=y(t-t_0-t_1)$
5. Convolution with Impulse: $x_1(t)*\delta(t)=x(t)$
   $x_1(t)*\delta(t-t_0)=x(t-t_0)$
6. Convolution of Unit Steps: $u(t)*u(t)=r(t)$
   $u(t-T_1)*u(t-T_2)=r(t-T_1-T_2)$
   $u(n)*u(n)=[n+1]u(n)$
7. Scaling Property: If $x(t)*h(t)=y(t)$
   then $x(at)*h(at)=\frac{1}{|a|}y(at)$
8. Differentiation of Output: If $y(t)=x(t)*h(t)$
   then $\frac{dy(t)}{dt} = \frac{dx(t)}{dt}*h(t)$

or

$$\frac{dy(t)}{dt} = x(t) * \frac{dh(t)}{dt}$$

Note:
- Convolution of two causal sequences is causal.
- Convolution of two anti causal sequences is anti causal.
- Convolution of two unequal length rectangles results a trapezium.
- Convolution of two equal length rectangles results a triangle.
- A function convoluted itself is equal to integration of that function.

Limits of Convoluted Signal
If two signals are convoluted then the resulting convoluted signal has following range:
Sum of lower limits < t < sum of upper limits
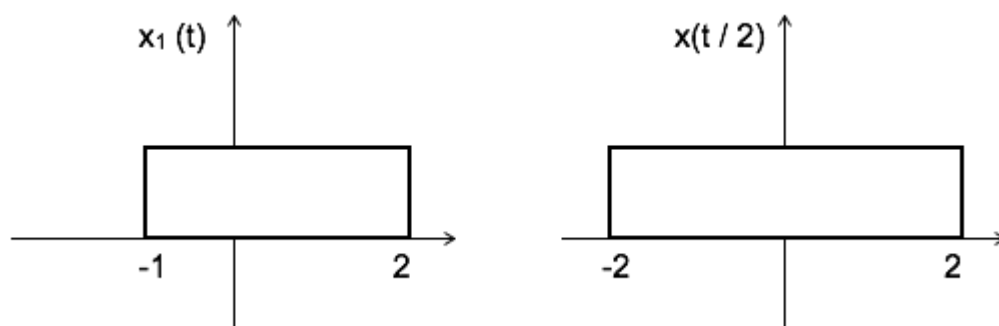
Here, we have two rectangles of unequal length to convolute, which results a trapezium.
The range of convoluted signal is:
Sum of lower limits < t < sum of upper limits
$-1+-2<t<2+2$
$-3<t<4$
Hence the result is trapezium with period 7.

Area of Convoluted Signal
The area under convolved signal is given by $A_y=A_xA_h$

Where A_x = area under input signal

A_h = area under impulse response

A_y = area under output signal

Proof: y(t)= $= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Take integration on both sides

$\int y(t)dt = \int \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \; dt$

$= \int x(\tau)d\tau \int_{-\infty}^{\infty} h(t-\tau)dt$

We know that area of any signal is the integration of that signal itself.

$\therefore A_y = A_x A_h$

DC Component

DC component of any signal is given by

DC component=area of the signal/period of the signal

NOTE:

1. The temporal output is the temporal input CONVOLVED with the Impulse Response Function.
2. The frequency domain output is the frequency domain input MULTIPLIED by the Transfer Function.
3. The frequency domain signal is the Fourier Transform of the temporal signal

Mathematically, it must be that the FT of a convolution is a product. y (t) = x(t) * h(t) $\xrightarrow{FT}$ Y (ω) = X(ω)H(ω)

1.12. Convolution Theorems:

Convolution of signals may be done either in time domain or frequency domain. So there are following two theorems of convolution associated with Fourier transforms:

1. Time convolution theorem
2. Frequency convolution theorem

Time convolution theorem: The time convolution theorem states that convolution in time domain is equivalent to multiplication of their spectra in frequency domain.

 Mathematically, if

           $x_1(t) \leftrightarrow X_1(\omega)$

And $x_2(t) \leftrightarrow X_2(\omega)$

Then $x_1(t) * x_2(t) \leftrightarrow X_1(\omega)X_2(\omega)$

Proof:

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty}[x_1(t) * x_2(t)] \, e^{-jwt} \, dt$

We have $x_1(t) * x_2(t) = \int_{-\infty}^{\infty}[x_1(\tau)x_2(t-\tau)]d\tau$

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty}\{\int_{-\infty}^{\infty}[[x_1(\tau)x_2(t-\tau)] \, d\tau\}e^{-jwt} \, dt$

Interchanging the order of integration, we have

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty}[x_2(t-\tau)]e^{-jwt} \, dt] \, d\tau$

Letting t-τ = p, in the second integration, we have t=p+τ and dt = dp

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty}[x_2(p)e^{-jw(p+\tau)}dp] \, d\tau$

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) \int_{-\infty}^{\infty}[x^2(p)e^{-jwp}dp]e^{-jw\tau} \, d\tau$

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) X_2(w) \, e^{-jw\tau} \, d\tau$

$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau) \, e^{-jw\tau} \, d\tau \, X_2(w)$

$x_1(t) * x_2(t) \leftrightarrow X_1(\omega)X_2(\omega)$

This is time convolution theorem

Frequency convolution theorem:

The frequency convolution theorem states that the multiplication of two functions in time domain is equivalent to convolution of their spectra in frequency domain.

Mathematically, if

$x_1(t) \leftrightarrow X_1(\omega)$

And $x_2(t) \leftrightarrow X_2(\omega)$

Then $x_1(t) x_2(t) \leftrightarrow 1/2\pi [X_1(\omega) * X_2(\omega)]$

Proof:

$F[x_1(t) x_2(t)] = \int_{-\infty}^{\infty} [x_1(t) x_2(t)]\ e^{-j\omega t}dt$

By definition of inverse Fourier transform

$= \int_{-\infty}^{\infty} [\frac{1}{2\pi} \int_{-\infty}^{\infty} x_1(\lambda)e^{j\lambda t}d\lambda]\ x_2(t)]\ e^{-j\omega t}dt$

Interchanging the order of integration, we get

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} x_1(\lambda)[\int_{-\infty}^{\infty} x_2(t)\ e^{-j\omega t}e^{j\lambda t}d\lambda]\ dt$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} x_1(\lambda)[\int_{-\infty}^{\infty} x_2(t)\ e^{-(\omega-\lambda)t}dt]\ d\lambda$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} x_1(\lambda) X_2(\omega-\lambda)\ d\lambda$

$= \frac{1}{2\pi} [X_1(\omega) * X_2(\omega)]$

This is frequency convolution theorem in radian frequency in terms of frequency, we get

$F[x_1(t) x_2(t)] = X_1(f) * X_2(f)$

Parseval's Energy theorem:

For continuous time signals x(t) energy of the signal is expressed as:

$E = \int_{-\infty}^{\infty} x^2(t)dt$

According to Parseval's theorem

$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dw$

Let $x^*(t)$ is conjugate of x(t).

Therefore, $x(t) . x^*(t) = |x(t)|^2$ ....(1)

On integrating the above equation with respect to t, we get

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} x(t) . x^*(t)dt$............(2)

From the definition of inverse Fourier transform

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(jw)e^{jwt} dw$ ....(3)

Taking conjugate to both sides

$x^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(jw)e^{-jwt} dw$...............(4)

Putting the above value in equation (2)

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} x(t) . \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(jw)e^{-jwt} dw\ dt$

Changing the order of integration we get

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(jw) \int_{-\infty}^{\infty} x(t)\ e^{-jwt}\ dt\ dw$

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(jw)X(jw)\ dw$

$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(jw)|^2 dt$ Hence proved.

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