Tutorial Sheet - 4

Solution

Q-1.

a) p : I run

q : I will get there quicker.

The premises of argument are

$P_1 : p \Rightarrow q$

$P_2 : q$

& Conclusion is $Q : p$

The Argument form is

$$\begin{array}{l} P_1 \\ P_2 \\ \hline \therefore Q \end{array} \quad \text{or} \quad (P_1 \wedge P_2) \Rightarrow Q$$

which means

$$\begin{array}{l} p \Rightarrow q \\ q \\ \hline \therefore p \end{array}$$

The Argument is invalid due to converse type of fallacy.

b) The atomic statements are same as in a).

Premises are $P_1 : p \Rightarrow q$

$P_2 : \neg p$

& Conclusion is $Q : \neg q$

The Argument given is

$$\begin{array}{c} p \Rightarrow q \\ \neg p \\ \hline \therefore \neg q \end{array}$$

The given argument is invalid due to inverse type fallacy.

c) p ; Bennett University is in Greater Noida.
q ; Bennett University is in UP.

The premises are

$P_1 : p \Rightarrow q$

$P_2 : q$

& The conclusion $Q : p$

The given argument is

$$\begin{array}{c} p \Rightarrow q \\ q \\ \hline \therefore p \end{array}$$

$\therefore$ Given argument is invalid.

d) p : Jatin is a policeman.
q : Jatin is a footballer.
r : Jatin has big feet.

The premises are $P_1 : p \vee q$

$P_2 : p \Rightarrow r$

$P_3 : \neg r$

and conclusion is $Q : q$

The given argument is

$$\begin{array}{l} p \vee q \\ p \Rightarrow r \\ \neg r \\ \hline \therefore \ p \vee q \\ \quad \neg p \quad (\because \text{By Modus Tollens with } P_2 \ \& \ P_3) \\ \hline \therefore \ q \quad (\because \text{By Disjunctive syllogism}) \end{array}$$

$\therefore$ The given argument is valid.

(But not sound as the 2^{nd} premise is not true in real life)

e) p: The company invests in renewable energy.

q: The company will reduce its carbon footprint.

r: The company faces public backlash.

Here, premises are $P_1 : p \Rightarrow q$

$P_2 : p$

$P_3 : q \vee r$

& conclusion is $Q : \neg r$

The given argument is

$$\begin{array}{l} p \Rightarrow q \\ p \\ q \vee r \\ \hline \therefore \ \neg r \end{array}$$

By Modus Ponens for P_1 & P_2, we have

$$\begin{array}{l} q \\ q \vee r \\ \hline \therefore \neg r \end{array}$$

$\therefore$ The argument is not valid due to the fallacy of the type disjunctive syllogism.

f) p : A student attend lectures of DMS.

q : Student's understanding of CS will improve.

r : Student is not present on campus.

s : DMS examination is not tough.

Here, Premises are $P_1 : p \Rightarrow q$

$P_2 : \neg q$

$P_3 : \neg p \Rightarrow r \vee s$

$P_4 : \neg r$

and the conclusion is $Q : s$

The argument is given as

$$\begin{array}{l} p \Rightarrow q \\ \neg q \\ \neg p \Rightarrow r \vee s \\ \neg r \\ \hline \therefore \neg p \end{array}$$

(By Modus Tollens in P_1 & P_2)

$$\begin{array}{l} \neg p \Rightarrow r \vee s \\ \neg r \end{array}$$

$\therefore r \vee s$ (By Modus ponens with $\neg P$ & P_3)

$\neg r$

$\therefore s$ (By Disjunctive syllogism)

As The conclusion is infered from the given arguments, therefore argument is valid.

Q-2

a) $P(x)$: x is a lawyer.
$Q(x)$: x is a liar.

$\therefore \forall x : (P(x) \Rightarrow Q(x))$

b) $\exists x : (P(x) \wedge \neg Q(x))$

c) $P(x)$: x is a person.
$Q(x)$: x becomes politician.
$R(x)$: x will get corrupt.

$\forall x : ((P(x) \wedge Q(x)) \Rightarrow R(x).)$

d) $P(x)$: x is a doctor.
$Q(x)$: x know calculus.

$\forall x : (P(x) \Rightarrow \neg Q(x))$

e) $P(x,y)$: y is greater than x

& $Q(x)$: x is a positive integer

$$\forall x\,(Q(x) \rightarrow \exists y\,(Q(y) \wedge P(x,y)))$$

[redacted]

f) $P(x)$: x is an integer.

$Q(x)$: x is positive.

$R(x)$: x is negative.

$$\forall x\,\{P(x) \Rightarrow (Q(x) \vee R(x))\}$$

Q-3

a) Let UoD = Universe of Indian festival

& $P(x)$: x is celebrated based on solar calendar.

$Q(x)$: x is celebrated based on lunar calendar.

$$\therefore \quad \forall x : P(x) \vee Q(x).$$

b) Let UoD = Universe of citizens.

$P(x)$: x is eligible to vote.

$Q(x)$: x is above 18 years of age.

$$\forall x : P(x) \Leftrightarrow Q(x).$$

c) Let UoD = Universe of family member.

P(x) : x enjoys spicy food.

Q(x) : x dislikes sweet

[redacted]

$$\forall x : (P(x) \land Q(x)) \lor (\neg Q(x) \land P(x))$$

d) $\forall x : (Even(x) \land x > 2) \Rightarrow (\exists p \exists q : Prime(p) \land Prime(q) \land (x = p + q))$

Q-4

~~(x) No program invokes a function~~

~~or~~

~~Program do not invoke any function.~~

a) There is only one book on the table.

b) All prime numbers greater than 2 are odd.

c) There is atleast one real number x for all real numbers such that addition of x in any real number result the same real number.
(Defining zero element).

Q-5

a) True → For $x = 9$, $R(9)$ is true

b) False → For $y = 10$

c) True → For any real number, there always exist one real number that is less than it.

d) False → There does not exist any real number which is greater or equal to all other real numbers.

e) True → For any two real numbers x & y, either $x > y$ or $x \leq y$.

f) True → As there exist at least one real number greater than 9 and all real numbers are not equal to 9.

g) False → There does not exist any real number greater than 9 which is greater that or equal to all real numbers.

h) True → As $\forall x: R(x)$ is false

$\Rightarrow \forall x \forall y: R(x) \wedge S(y)$ is false

$\Rightarrow \forall x \forall y: (R(x) \wedge S(y)) \Rightarrow Q(x, y)$ is true.

(6) (a) dom = all foods

$H(x)$: x is healthy to eat

$G(x)$: x tastes Good.

$Y(x)$: you eat x.

$\forall x(H(x) \rightarrow \neg G(x))$ — (1)

$H(Tofu)$ — (2)

$\forall x(Y(x) \rightarrow G(x))$ — (3)

$\neg Y(Tofu)$

To check its validity =>

apply universal instantiation in (1) and get

$H(Tofu) \rightarrow \neg G(Tofu)$ — (4)

apply modus ponens in (4) & (2) → and get

$\neg G(Tofu)$ — (5)

apply universal instantiation in (3) and get

$Y(Tofu) \rightarrow G(Tofu)$ — (6)

apply Modus Tollens in (5) & (6) and get

$\neg Y(Tofu)$

which is our conclusion. So, it is a valid argument.

(b) domain = all people

$\forall x(B(x) \rightarrow I(x))$

$\exists x(B(x) \wedge \neg S(x))$

$\forall x(I(x) \rightarrow \neg S(x))$

It is not valid because the conclusion can never be true for all elements if one of the premise is existentially true. In above example, premise 2 is existentially true.

(c) domain = all particles

$\forall x(P(x) \rightarrow S(x))$

$\forall x(N(x) \rightarrow S(x))$

$\forall x(P(x) \rightarrow N(x))$

→ It is not valid. It is a fallacy of Hypothetical syllogism.