Model explanation

The model is conceptually zero-dimensional, and the initial conditions consider the minimal cell concentration recorded by Scalco et al. [2014] as gametogenesis trigger. Cell numbers and available nitrogen are expressed as abundance and concentration, respectively.

The simulation starts with a cell population (P) that enters the sexual reproduction phase after reaching a threshold value of a few thousand cells per ml (see Scalco et al. [2014]). During this phase of variable length, the population stops growing and experiences a prolonged burst of gametogenesis-induced mortality. Within the same span, a P fraction will undergo gametogenesis and generate an offspring population (F_1) that will start to grow the moment ($t_{F_1 I}$) it appears. After t_{AE} days, the growth arrest will end and P will resumes its growth alongside F_1. During the entire course of the simulation (set to ten days) cell growth is modulated by competition over nitrogen and cell size via an allometric relationship (see D'Alelio et al. [2010]). **Fig. 1** shows the relative variation of allometric growth (**plot *a***).

A fundamental assumption, discussed later in the text, is that parental and daughter cells share a common, exclusive space, so that only P and F_1 compete for N made available in such space. This assumption is translated in the model as a decrease in growth rate as the total population ($F_1 + P$) converges toward the carrying capacity, set as the total nitrogen content of initial parental cells.

To understand better how event timing and response amplitudes impact over F_1 recruitment success, we compared simulation outputs across selected parameters range. Our selected parameters are the P fraction that will generate F_1 (α), gametogenesis-induced extra mortality of parental cells (m), duration of the growth arrest (t_{AE}), and growth rates ratio among parental and initial cells ($r_P : r_{F_1}$) We deployed an interactive version of the model at the URL `https://arfalas.shinyapps.io/pns_toy/`, code and figures are available on `https://github.com/bhym/Stec` (currently private)

Model description

Parental (P) and offspring (F_1) dynamics are defined as ODE:

$$\frac{dP}{dt} = \kappa\mu_P P - \delta P \tag{1}$$

$$\frac{dF_1}{dt} = \kappa\mu_{F_1} F_1 \tag{2}$$

where κ is the competition for N, defined as

$$1 - \frac{[N]_t}{1.2[N]_{t=0}}$$

with

$$[N_t] = [P]_t[N]_P + [F_1]_t[N]_{F_1}$$

μ_P and μ_{F_1} are defined as

$$\mu_P = \begin{cases} 0 & \text{if } t < t_{AE} \\ \lambda_P(t) r_P & \text{if } t \geq t_{AE} \end{cases} \quad ; \quad \mu_{F_1} = \begin{cases} 0 & \text{if } t < t_{F_1 I} \\ \lambda_{F_1}(t) r_{F_1} & \text{if } t \geq t_{F_1 I} \end{cases}$$

with $\lambda_i(t)$ representing allometric scaling on length (L), defined as

$$\lambda_i(t) = 0.25 + 0.04L_i(t) - 0.0005L_i(t)^2$$

Length dynamics are based on cell's age (a) and are defined by the rule

$$L_i(t) = L_{0,i} - 0.1a(t) \qquad \text{with } i \text{ either } P \text{ or } F_1$$

Age is counted from the appearance of the population, and does not increase during GA.

Finally, δ is an extra mortality term defined as

$$\begin{cases} m & \text{if } t < t_{AE} \\ 0 & \text{if } t > t_{AE} \end{cases}$$

The parameters values and ranges are reported in **Table 1**.

References

Eleonora Scalco, Krzysztof Stec, Daniele Iudicone, Maria Immacolata Ferrante, and Marina Montresor. The dynamics of sexual phase in the marine diatom p seudo-nitzschia multistriata (b acillariophyceae). *Journal of Phycology*, 50 (5):817–828, 2014. doi: 10.1111/jpy.12225.

Domenico D'Alelio, Maurizio Ribera d'Alcala, Laurent Dubroca, Adriana Zingone, and Marina Montresor. The time for sex: A biennial life cycle in a marine planktonic diatom. *Limnology and Oceanography*, 55(1):106–114, 2010. doi: 10.4319/lo.2010.55.1.0106.

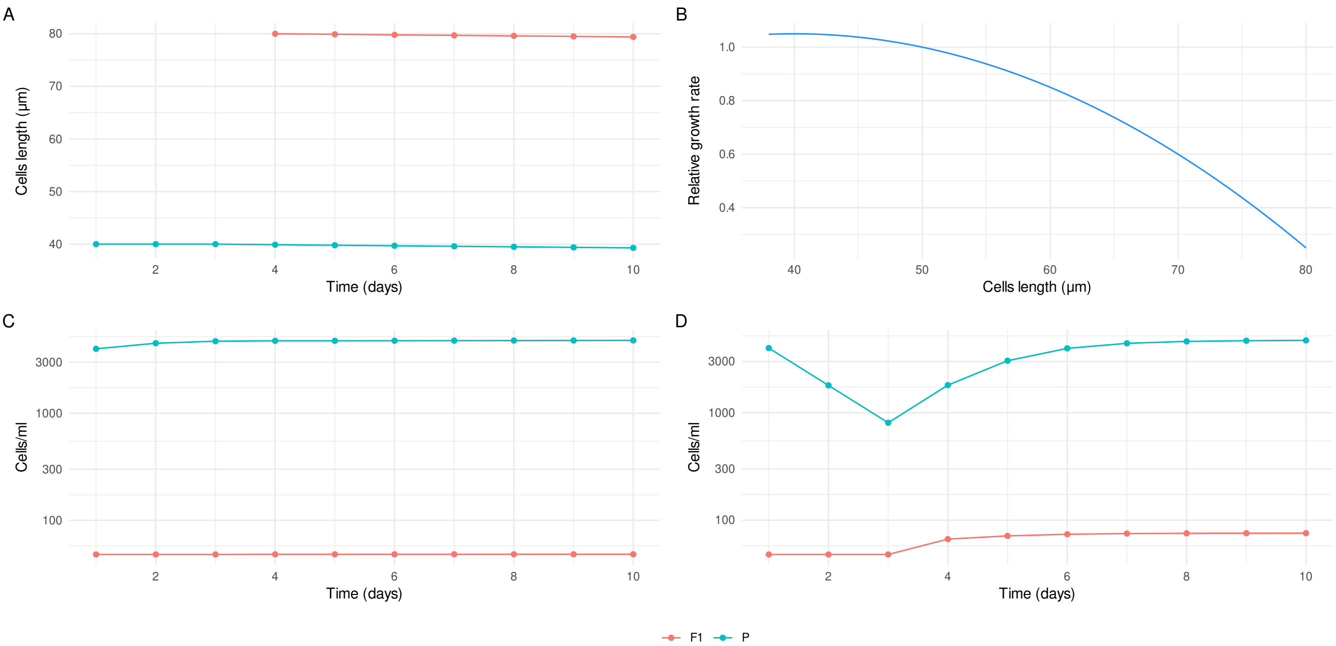

Figure 1: **Simulated size dependent growth and impact on the population dynamics** — *(A)* dependence of specific growth rate on cell length during the time interval considered in the model (10 days); *(B)* Size reduction for the two sub-populations. *(C)* and *(D)*, examples of simulation outputs with the growth rates of the current work ($r_P = 1.06$, $r_{F1} = 0.58$) and *(C)* no growth arrest ($t_{AE} = 0$) or *(D)* duration of growth arrest as in the current work ($t_{AE} = 3$). In C and D:$\alpha = 0.012$, m = 0.4, all the other parameters as per **Tab. 1**.
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Variable	**Meaning**	**Units**	**Value**	**Reference**
P_0	initial concentration of P cells	cells/ml	4E3	Experiment detailed in the paper
α	fraction of P that will generate F_1	—	[0.01, 0.2]	Explored via simulations
r_P	net growth rate of P	day^{-1}	[0.5, 3]	Explored via simulations
r_{F_1}	net growth rate of F_1	day^{-1}	1	—
t_{AE}	end day of the growth arrest	day	[0,6]	Explored via simulations
t_{F_1I}	day of appearence of the offspring	day	3	Experiment detailed in the paper
$L_{0,P}$	starting length for P cells	μm	40	Experiment detailed in the paper
L_{0,F_1}	starting length for F_1 cells	μm	80	Experiment detailed in the paper
m	gametogenesis-induced mortality	day	[0.1, 0.9]	Explored via simulations

Table 1: Values and ranges for model parameters

	0.01	0.03	0.05	0.08	0.1	0.12	0.15	0.17	0.2
3	-1.88	-1.40	-1.16	-0.94	-0.83	-0.74	-0.63	-0.56	-0.47
2.7	-1.88	-1.39	-1.16	-0.94	-0.83	-0.74	-0.62	-0.56	-0.47
2.5	-1.88	-1.39	-1.16	-0.94	-0.83	-0.74	-0.62	-0.56	-0.47
2.2	-1.87	-1.39	-1.15	-0.93	-0.82	-0.73	-0.62	-0.55	-0.46
2	-1.87	-1.38	-1.15	-0.93	-0.82	-0.73	-0.62	-0.55	-0.46
1.7	-1.86	-1.37	-1.14	-0.92	-0.81	-0.72	-0.61	-0.54	-0.46
1.5	-1.86	-1.37	-1.14	-0.92	-0.81	-0.72	-0.61	-0.54	-0.45
1.2	-1.84	-1.35	-1.12	-0.90	-0.80	-0.71	-0.59	-0.53	-0.44
1	-1.83	-1.34	-1.11	-0.89	-0.78	-0.69	-0.58	-0.52	-0.43

(a) $\alpha, r_p : r_{f1}$, with $m = 0.2$

	0.01	0.03	0.05	0.08	0.1	0.12	0.15	0.17	0.2
3	-1.79	-1.30	-1.07	-0.84	-0.73	-0.63	-0.51	-0.44	-0.34
2.7	-1.79	-1.30	-1.06	-0.83	-0.72	-0.63	-0.50	-0.43	-0.33
2.5	-1.78	-1.29	-1.06	-0.83	-0.72	-0.62	-0.50	-0.43	-0.33
2.2	-1.78	-1.28	-1.05	-0.82	-0.71	-0.61	-0.49	-0.42	-0.32
2	-1.77	-1.28	-1.04	-0.81	-0.70	-0.61	-0.48	-0.41	-0.32
1.7	-1.76	-1.26	-1.03	-0.80	-0.69	-0.59	-0.47	-0.40	-0.30
1.5	-1.74	-1.25	-1.02	-0.79	-0.68	-0.58	-0.46	-0.39	-0.29
1.2	-1.72	-1.23	-0.99	-0.76	-0.65	-0.55	-0.43	-0.36	-0.27
1	-1.69	-1.20	-0.96	-0.73	-0.62	-0.53	-0.41	-0.34	-0.24

(b) $\alpha, r_p : r_{f1}$, with $m = 0.4$

	0.01	0.03	0.05	0.08	0.1	0.12	0.15	0.17	0.2
3	-1.73	-1.24	-1.00	-0.76	-0.65	-0.55	-0.42	-0.34	-0.24
2.7	-1.72	-1.23	-0.99	-0.76	-0.64	-0.54	-0.41	-0.34	-0.23
2.5	-1.72	-1.22	-0.98	-0.75	-0.63	-0.53	-0.40	-0.33	-0.22
2.2	-1.70	-1.21	-0.97	-0.74	-0.62	-0.52	-0.39	-0.31	-0.21
2	-1.69	-1.20	-0.96	-0.73	-0.61	-0.51	-0.38	-0.30	-0.20
1.7	-1.68	-1.18	-0.94	-0.70	-0.59	-0.49	-0.36	-0.28	-0.17
1.5	-1.66	-1.16	-0.92	-0.69	-0.57	-0.47	-0.34	-0.26	-0.15
1.2	-1.62	-1.12	-0.88	-0.64	-0.53	-0.42	-0.29	-0.21	-0.11
1	-1.58	-1.08	-0.84	-0.60	-0.48	-0.38	-0.25	-0.17	-0.06

(c) $\alpha, r_p : r_{f1}$, with $m = 0.6$

Table 2: Tabular data of $\log_{10}(F_1/P)$ for different values of parameters showed at bottom and growth arrest duration of three days. All other parameters were assigned as per **Tab. 1**