Effiziente Ansätze bei Fluid-Struktur-Wechselwirkungen mit großen Deformationen und Topologieänderungen

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Effiziente Ansätze bei Fluid-Struktur-Wechselwirkungen mit großen Deformationen und Topologieänderungen

⇒ Aim: In this project the main concern is to develop efficient and robust discretization techniques for the interaction of flexible thin-walled structures with the surrounding fluid flow, where the structure undergoes large deformations

⇒ Currently we are working on

- Overlapping Domain Decomposition Methods, and
- $\boxed{\text{Coupled Level Set / Extended Finite Element Method}}$

to solve <u>pure fluid</u> problems

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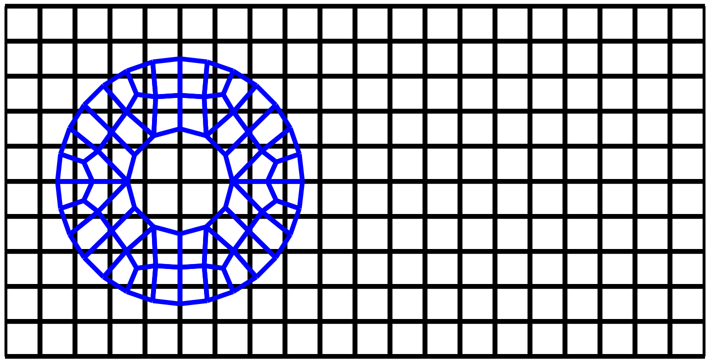
Overlapping Domain Decomposition Methods



Overlapping Domain Decomposition Methods

⇒ Diploma thesis[a], Peter Gamnitzer (Dipl.-Tech.Math.Univ.)

⇒ Pure fluid problems on fixed domain

⇒ Dirichlet-Dirichlet and “Chimera”-Dirichlet coupling

⇒ Mass conserving interpolation

[a] “Overlapping Domain Decomposition Methods in Application to Navier-Stokes Equations”

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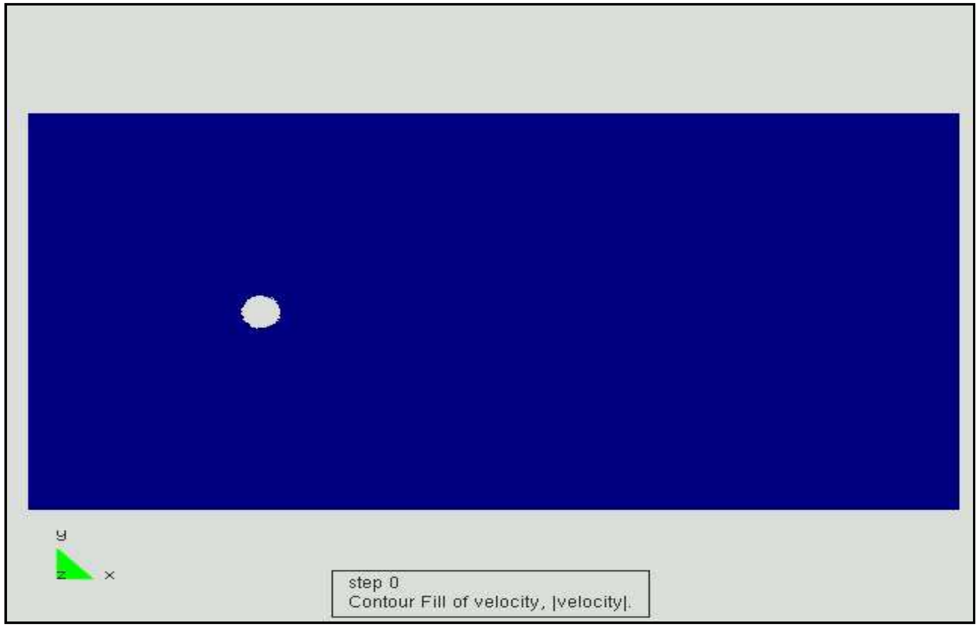
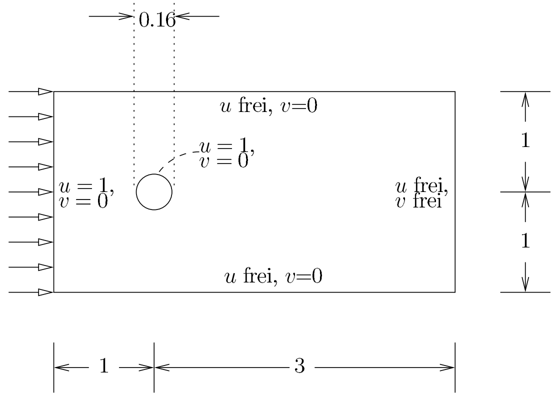
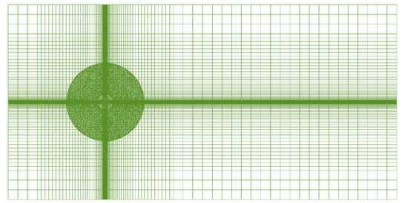
Overlapping Domain Decomposition Methods

⇒ Flow around a cylinder $(Re = 100)$







⇒ Mass conserving interpolation is important especially for pressure field

⇒ Different coupling conditions, e.g., Dirichlet-Neumann, Dirichlet-Robin

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Coupled Level Set / Extended Finite Element Method

Level Set Method (LSM) $\longrightarrow$

To track internal moving boundaries on a fixed mesh

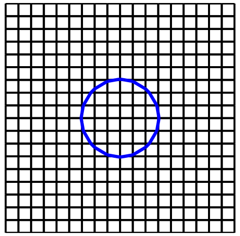
Extended Finite Element Method (XFEM) $\longrightarrow$

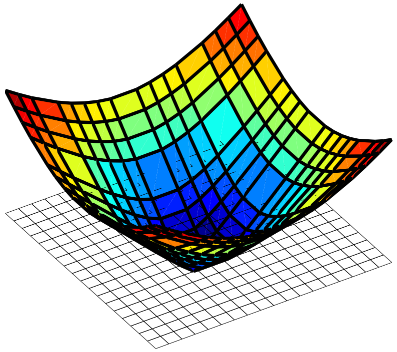
To model internal moving boundaries on a fixed mesh

Level Set Method

⇒ Interface tracking

⇒ Interface is captured on a fixed mesh by locating zero contour of a smooth scalar field ϕ (usually signed distance to the interface)





⇒ Evolution of the interface is governed by first order hyperbolic PDE

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \nabla\phi \cdot \mathbf{u} = 0 \qquad (*)$$

⇒ Streamline Upwind Petrov-Galerkin (SUPG) finite element method has been used to solve $(*)$ numerically

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Level Set Method

$\Rightarrow$ Change in topology and extension to 3D are easy to handle

$\Rightarrow$ Normal vector ($\mathbf{n}$) to the interface and curvature (κ) of the interface can be accurately expressed in terms of smooth level set function, i.e.,

$$\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|},\ \kappa = \nabla \cdot \mathbf{n}$$

$\Rightarrow$ Reinitialization, velocity extension, localization, particle level set method for improved performance

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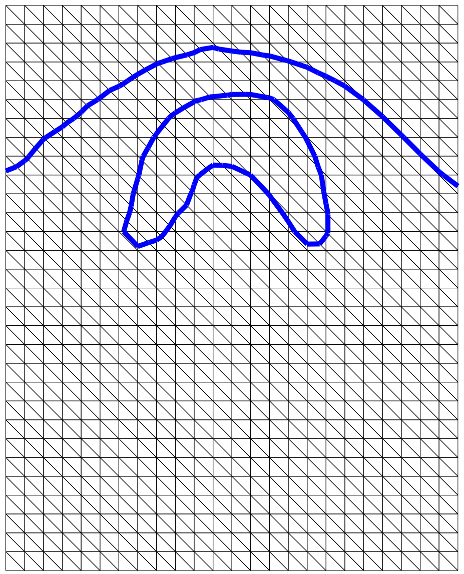
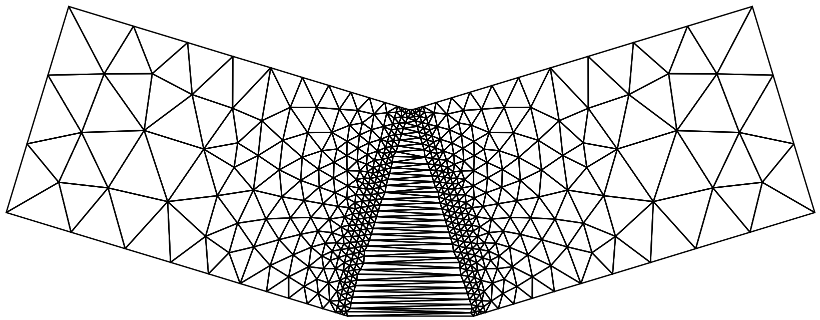
Extended Finite Element Method

$\Rightarrow$ Model moving (or evolving) boundaries on a fixed mesh



Two-phase fluid flow



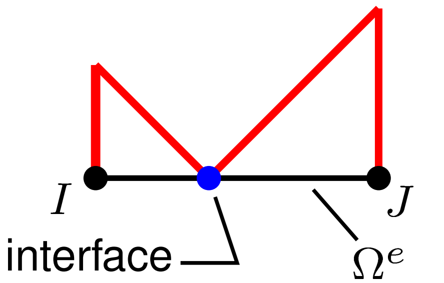
Cohesive crack growth

Extended Finite Element Method

$\Rightarrow$ Idea : Enrich standard finite element approximation around the interface by product of standard nodal finite element shape functions with some suitable enrichment functions (Ψ) such that the kinematical constraints along the interface can be met

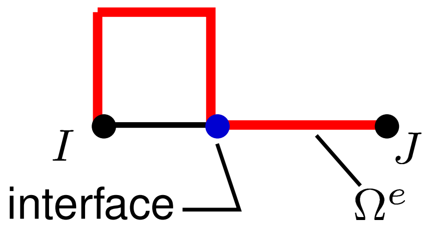
$$\mathbf{u}^h = \underbrace{\sum_{I=1}^{nnode} N_I \mathbf{u}_I}_{\text{standard}} + \underbrace{\sum_{I=1}^{nnode} \chi_I N_I \Psi \bar{\mathbf{u}}_I}_{\text{enriched}}$$

discontinuous 'velocity' gradient



$\Psi = |\phi|$

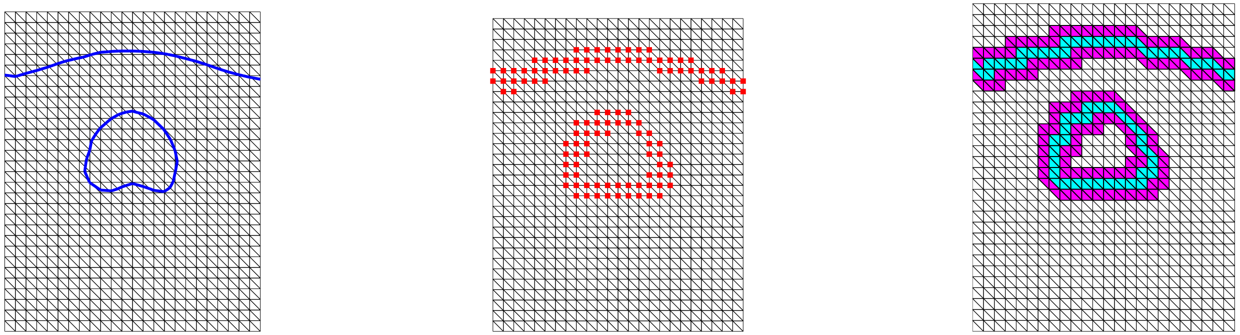
discontinuous 'displacement' field



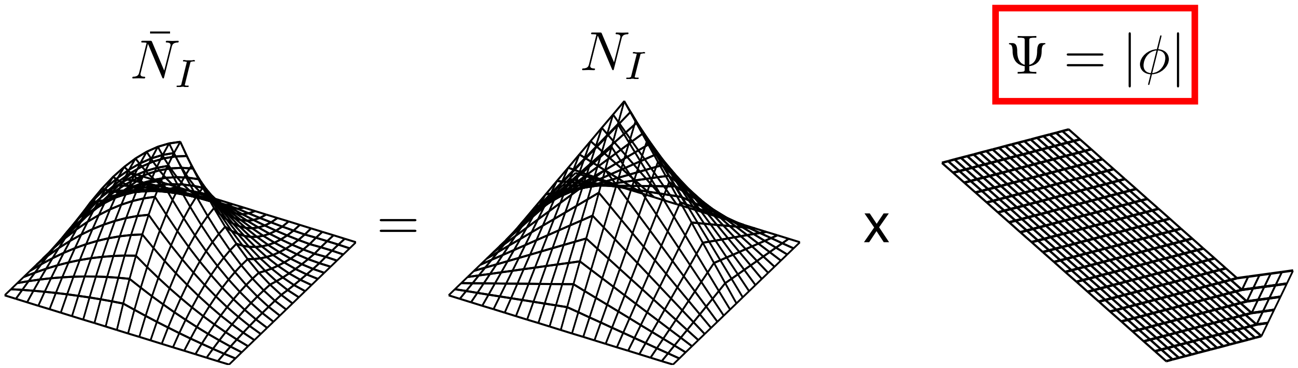
$\Psi = H$ (heaviside function)

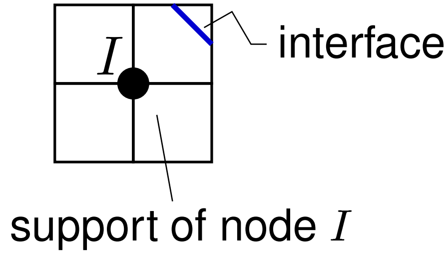
Coupling of XFEM with LSM

⇒ Use the level set function (ϕ) to activate the elements on and around the interface, and the nodes to be enriched

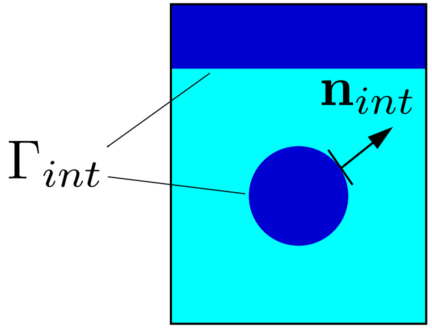


⇒ Use level set function (ϕ) to construct enrichment function (Ψ) if <u>discontinuity</u> in the gradient of a field quantity along the interface is expected





Two-Phase Fluid Flow Simulation



Fluid1 $(\rho_1,\ \mu_1)$
Fluid2 $(\rho_2,\ \mu_2)$

$\Rightarrow$ Navier-Stokes equations

$$\rho_I\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) - 2\mu_I\nabla\cdot\epsilon(\mathbf{u}) + \nabla p = \mathbf{b} \qquad \text{in } \Omega_F \text{ x } (0,T)$$

$$\nabla\cdot\mathbf{u} = 0 \qquad \text{in } \Omega_F \text{ x } (0,T)$$

$\Rightarrow$ Interface conditions assumed

- $\rightarrow$ Velocity is continuous along the interface, i.e., $\boxed{\mathbf{u}_1 = \mathbf{u}_2}$ on Γ_{int}
- $\rightarrow$ Traction is continuous along the interface, i.e., $\boxed{\sigma_1\cdot\mathbf{n}_{int} = \sigma_2\cdot\mathbf{n}_{int}}$ on Γ_{int}

$\Rightarrow$ Solved by coupled level set / extended finite element method

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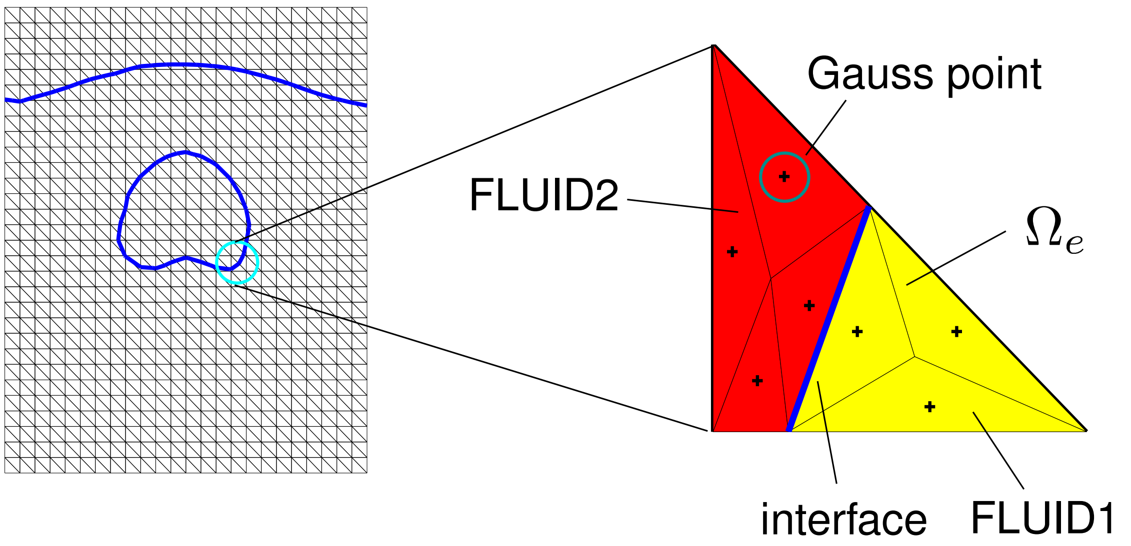
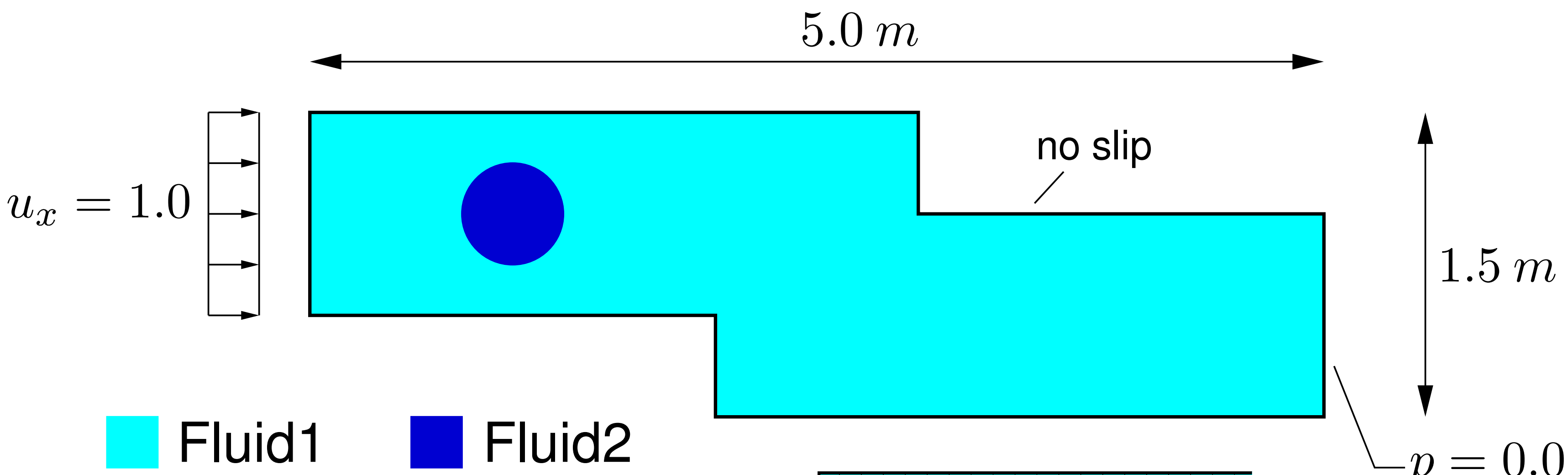
Two-Phase Fluid Flow Simulation

⇒ Level set function (ϕ) is used to construct enrichment function (Ψ) since a jump in the velocity gradient along the interface is expected because of the interface conditions prescribed

⇒ XFEM has been embedded into stabilized Navier-Stokes solver together with the refined integration scheme which is necessary for the elements cut by the interface

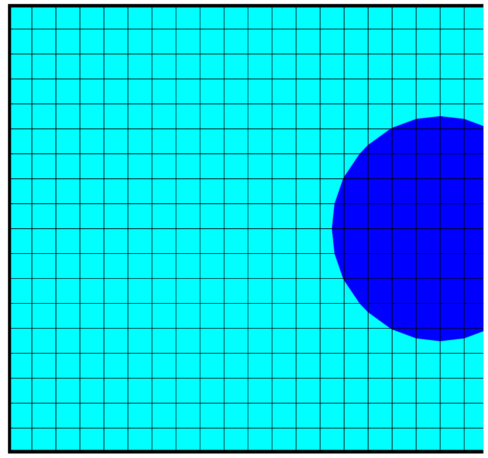


Examples$^{(b)}$ $\rightarrow$ 'Jogged' channel



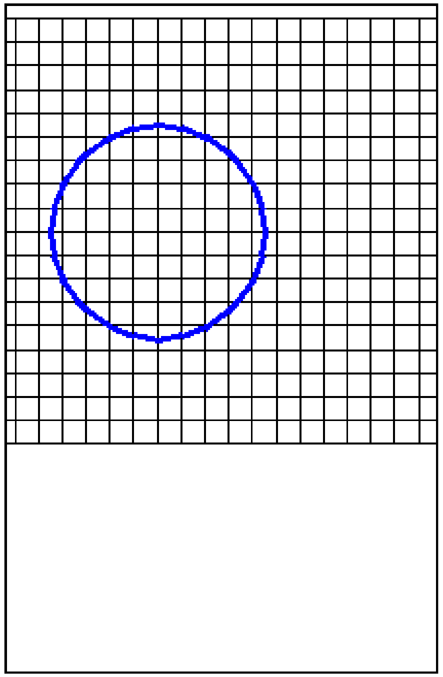
$$\rho_1 = 1\frac{kg}{m^3} \qquad \rho_2 = 4\frac{kg}{m^3}$$

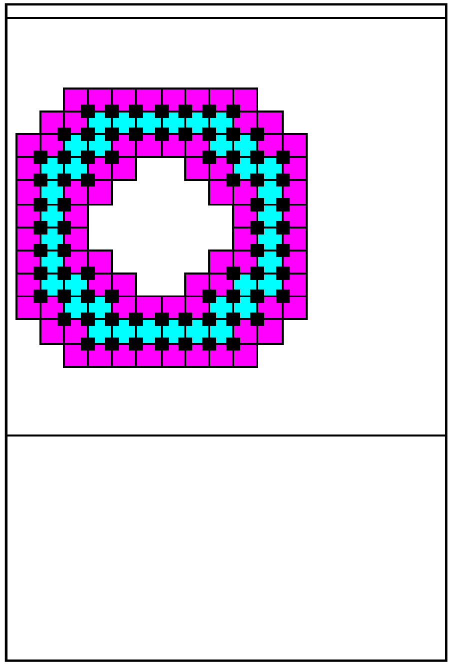
$$\mu_1 = 1\frac{Ns}{m^2} \qquad \mu_2 = 4\frac{Ns}{m^2}$$



$^{(b)}$ (taken from) Chessa J., Belytschko T., "An Extended Finite Element Method for Two-Phase Fluids", ASME, Vol. 70, January 2003

Examples → 'Jogged' channel





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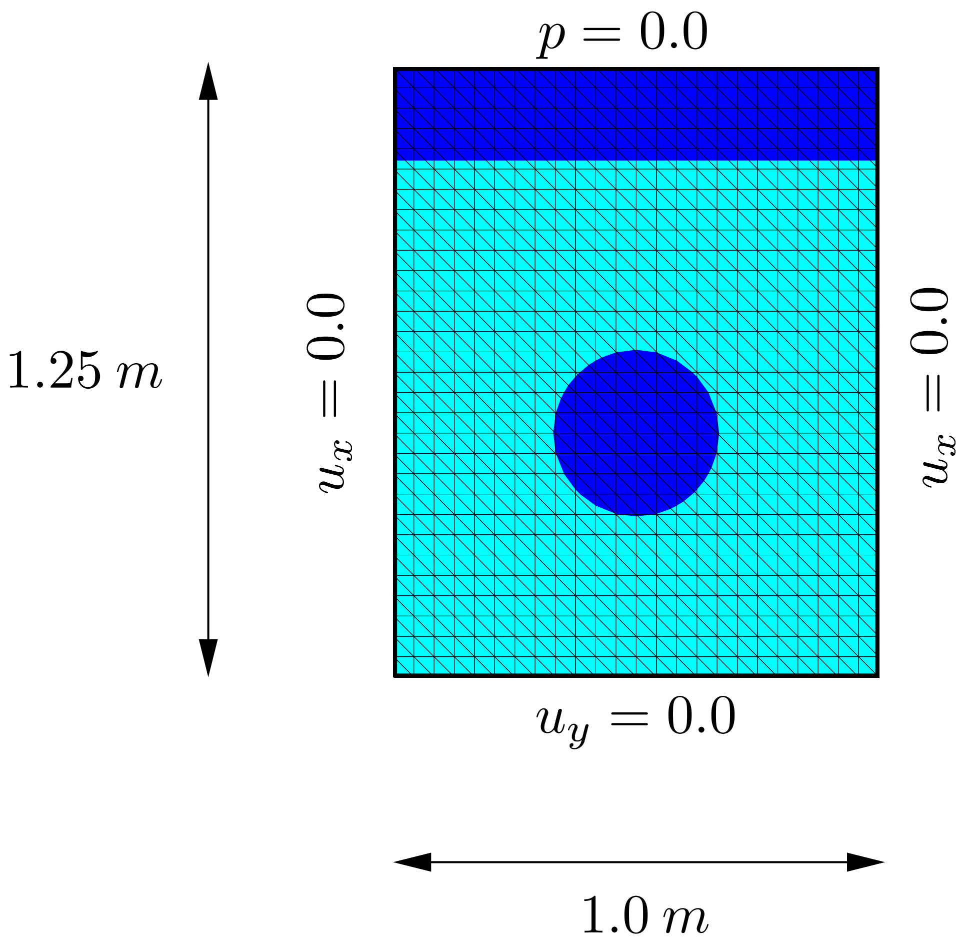
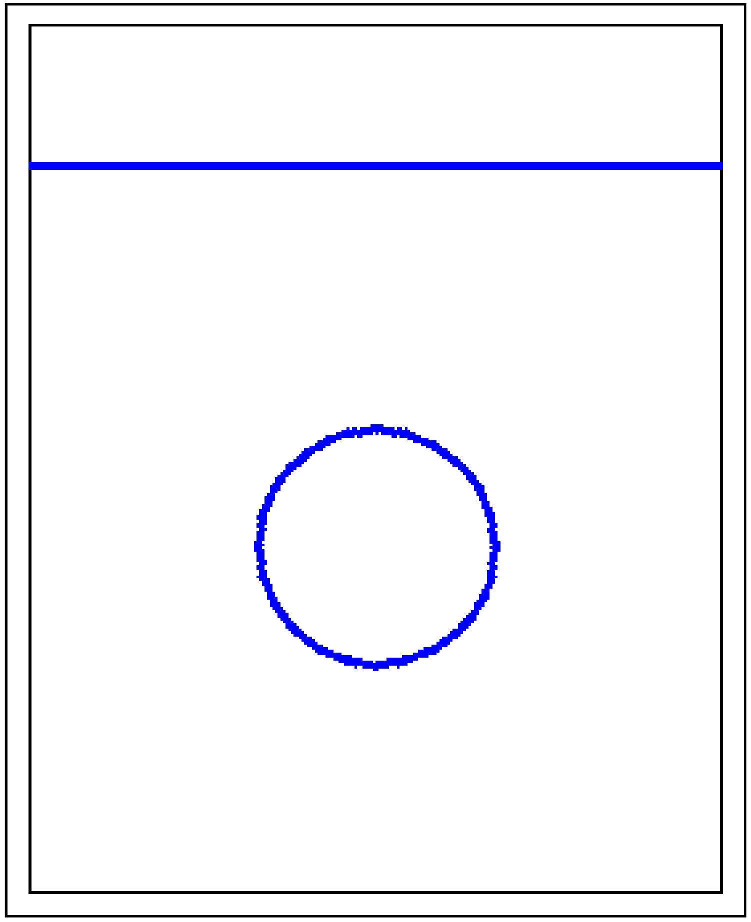
Examples → Rising bubble



Fluid1	Fluid2
$\rho_1 = 10 \frac{kg}{m^3}$	$\rho_2 = 1 \frac{kg}{m^3}$
$\mu_1 = 7 \frac{Ns}{m^2}$	$\mu_2 = 1 \frac{Ns}{m^2}$

Examples → Rising bubble



Outlook

$\Rightarrow$ We are in parallel with the schedule

$\Rightarrow$ Next, we will try to develop a new method for the solution of fluid-structure interaction problems by combining the methods that we have